

① Factorizar LU

$$\begin{pmatrix} 3 & 7 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ -11 \end{pmatrix} \quad \frac{6}{3} = 2$$

$$A = LU \quad L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 3 & 7 \\ 0 & -13 \end{pmatrix}$$

$$Ax = b$$

$$Ly = b$$

$$Ux = y$$

$$A = LU \quad \therefore \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 7 \\ 0 & -13 \end{pmatrix} = \begin{pmatrix} 3 & 7 \\ 6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 \\ -11 \end{pmatrix} \quad y_1 = -1$$

$$2y_1 + y_2 = -11 \quad -11 + 2 = -9$$

$$\begin{pmatrix} 3 & 7 \\ 0 & -13 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ -9 \end{pmatrix} \quad x_2 = \frac{-9}{-13} = \frac{9}{13}$$

$$3x_1 + 7x_2 = -1 \rightarrow 3x_1 = -1 - 7\left(\frac{9}{13}\right) = \frac{-74}{13}$$

$$x = \begin{pmatrix} \frac{-74}{39} \\ \frac{9}{13} \end{pmatrix}$$

$$x_1 = \frac{-74}{39}$$

③ LU:  $\frac{2}{3}n^3$ 

$$Ly = b, \quad Ux = y = 2n^2$$

$$\text{FLOPs} = \frac{2}{3}n^3 + (500)2n^2 = \frac{2}{3}(10^9) + 1000(10^6) = \frac{2}{3} \times 10^9 + 1 \times 10^9$$

$$\frac{0.6}{1.6} = \frac{2}{5} = 0.4 \quad \therefore 24 \text{ segundos} \quad \therefore \text{operación completa} = 60s$$



②

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 3 & 4 \\ 3 & 1 & 8 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

$$R_2 = R_2 - 2R_1 \rightarrow \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$R_3 = R_3 - R_1$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = LUV$$

$$Ly = b \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

$$y_1 = 0$$

$$2y_1 + y_2 = 1$$

$$y_1 + y_3 = 3$$

$$Ux = y \quad \begin{pmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

$$3x_3 = 3 \rightarrow x_3 = 1$$

$$x_2 = 1$$

$$3x_1 + 1x_2 + 2x_3 = 0$$

$$x_1 = -1$$

$$x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$