Tarea 3-02

Resuelva el sistema de ecuaciones no lineales

$$3x_1 - \cos(x_2 x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 81(x_2 + 0.1)^2 + \operatorname{sen} x_3 + 1.06 = 0$$

$$e^{-(x_1 x_2)} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

en la forma $x = G(x)$ despejando x_1 en la primera ecuación, x_2 en la segunda y x_3 en la tercera.

Pruebe que el sistema debe tener solución única en el intervalo

$$D = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : -1 \leq x_i \leq 1, \forall i \right\}$$

y que puede resolverse usando el método de punto fijo

$$x_1 = \frac{\cos(x_2 x_3) + 1/2}{3}$$

$$x_2 = -0.1 + \frac{1}{9}\sqrt{x_1^2 + \operatorname{sen}(x_3) + 1.06}$$

$$x_3 = -\frac{1}{20} e^{-x_1 x_2} - \frac{10\pi - 3}{60}$$

$$G(x) = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$g_1 = \frac{1}{3}\left(\cos(x_2 x_3) + \frac{1}{2}\right) \rightarrow \left(-\frac{1}{3}\right)\left(\frac{1}{2}\right) \leq g_1 \leq \left(\frac{1}{3}\right)\left(\frac{3}{2}\right) \rightarrow -\frac{1}{6} \leq g_1 \leq \frac{1}{2}$$

$$g_2 = -0.1 + \frac{1}{9}\sqrt{x_1^2 + \operatorname{sen} x_3 + 1.06} \rightarrow \sqrt{0.06} \leq \sqrt{x_1^2 + \operatorname{sen} x_3 + 1.06} \leq \sqrt{3.06} < 3$$

$$\rightarrow -0.1 + \frac{\sqrt{0.06}}{9} \leq g_2 \leq -0.1 + \frac{1}{3}$$

$$g_3 = -\frac{1}{20} e^{-x_1 x_2} - \frac{10\pi - 3}{60} \rightarrow -\frac{e}{20} - \frac{10\pi - 3}{60} \leq g_3 \leq -\frac{e^{-1}}{20} - \frac{10\pi - 3}{60}$$

$g_1(x_1, x_2, x_3) \rightarrow \left[-\frac{1}{6}, \frac{1}{2}\right]$

$g_2(x_1, x_2, x_3) \rightarrow [-0.072783, 0.2\overline{3}]$

$g_3(x_1, x_2, x_3) \rightarrow [-0.609512, -0.491992]$

$\therefore G(D) \subset D$

$$\|JG(x)\|_\infty = \max_{x_j} \sum_{ij} \left|\frac{\partial g_i}{\partial x_j}\right|$$

$g_1 = \frac{1}{3}\left(\cos(x_2 x_3) + \frac{1}{2}\right)$

$g_2 = -0.1 + \frac{1}{9}(\sqrt{\Phi})$

$g_3 = -\frac{1}{20} e^{-x_1 x_2} - \frac{10\pi - 3}{60}$

$\Phi = x_1^2 + \operatorname{sen} x_3 + 1.06$

$\frac{\partial g_1}{\partial x_1} = 0 \qquad \leq 1/3$

$\frac{\partial g_1}{\partial x_2} = -1/3 \operatorname{sen}(x_2 x_3) x_3 \qquad |-.280490| \leq 1/3$

$\frac{\partial g_1}{\partial x_3} = -1/3 \operatorname{sen}(x_2 x_3) x_2 \qquad |-.280490| \leq 1/3$

$\frac{\partial g_2}{\partial x_1} = \frac{x_1}{9\sqrt{\Phi}} \qquad |0.065230| \leq 1/3$

$\frac{\partial g_2}{\partial x_2} = 0 \qquad \leq 1/3$

$\frac{\partial g_2}{\partial x_3} = \frac{\cos x_3}{18\sqrt{\Phi}} \qquad |0.017622| \leq 1/3$

$\frac{\partial g_3}{\partial x_1} = \frac{x_2 e^{-x_1 x_2}}{20} \qquad |0.018393| \leq 1/3$

$\frac{\partial g_3}{\partial x_2} = \frac{x_1 e^{-x_1 x_2}}{20} \qquad |0.018393| \leq 1/3$

$\frac{\partial g_3}{\partial x_3} = 0 \qquad \leq 1/3$