CS GY 6643 - Computer Vision, Fall 2024

Homework 1

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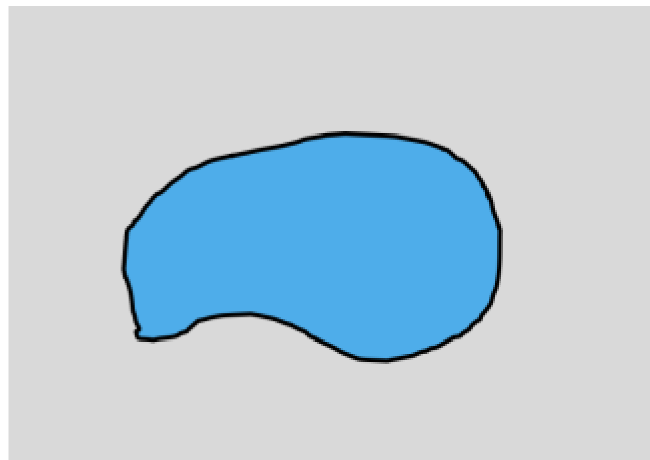
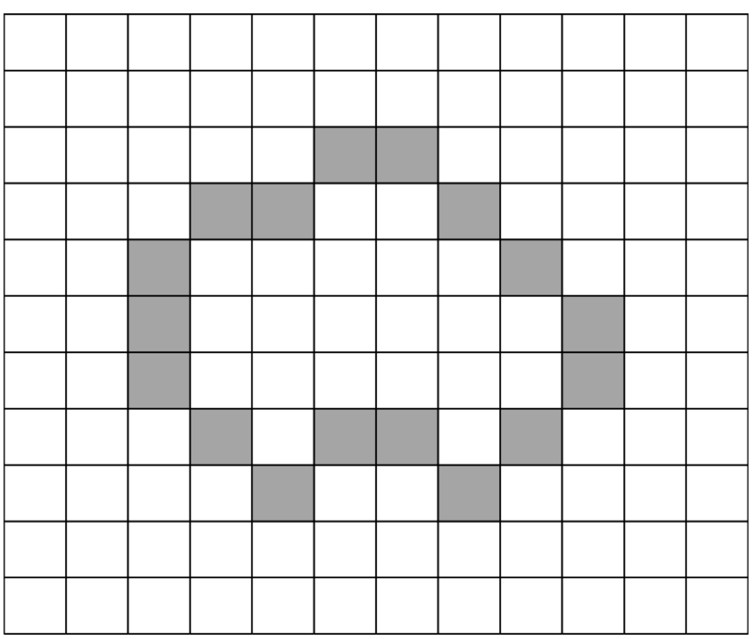
1. Connectivity: Connected component labeling **(25)**

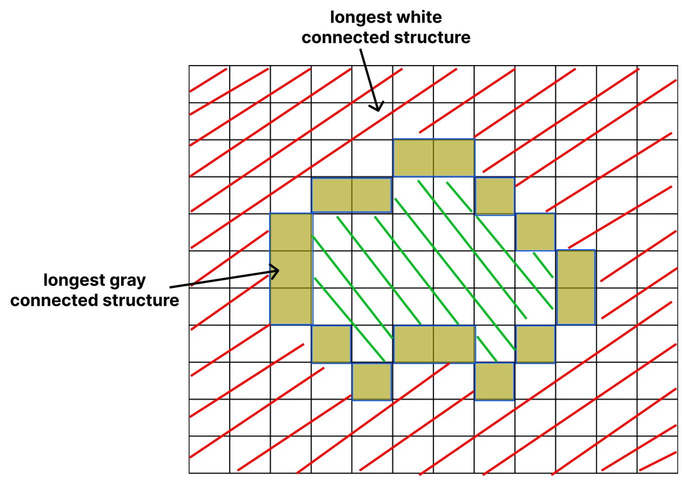
The famous theorem of Jordan states that every simple closed curve in the plane separates the complement into two connected nonempty sets: an interior region and an exterior. Although this looks simple and intuitive, the proof can get very complex (for those who want to look it up: Link). Elements are a closed curve and the complement, which is the plane, which is divided into 2 regions. Now let us explore how the continuous case transfers to our discrete domain.



Given the following pixel image with white areas and a gray closed pixel curve. How many connected components can you detect?



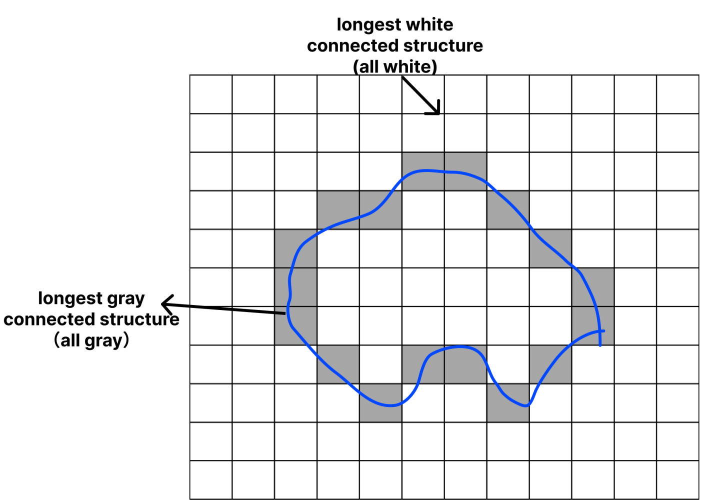
a) Apply **d**$_4$ connectivity to all gray and white structures. Number of 4-connected structures? Label one or all connected structures and report the longest structure.**(7)**



Answer:

1. number of 4-connected structures
   gray: 11   white: 2
2. Label one or all connected structures
   For gray structures, I label all 11 connected structures with yellow blocks.
   For white structures, there are only two connected structures. One is the area outside the gray (red slash), the other is the area inside the gray (green slash).
3. **I labeled the longest connected structures of white and gray with arrows**

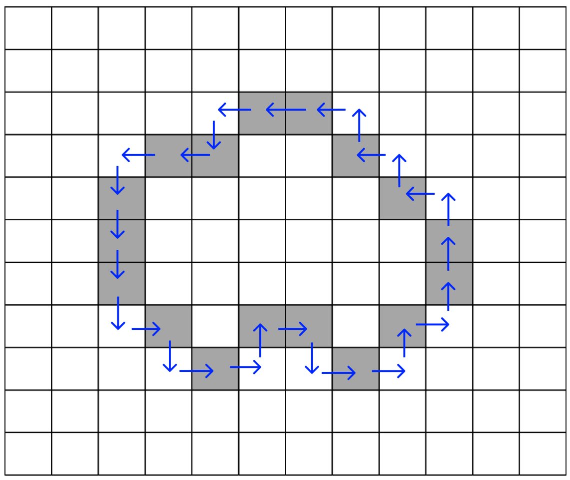
b) Apply **d**$_8$ connectivity to all gray and white structures. Number of 8-connected structures and label one or all connected structure and report the longest path.**(7)**



Answer:

1. number of 8-connected structures
   gray: 1   white: 1
2. Label one or all connected structures
   All gray is a whole connected structure.
   All white is a whole connected structure.
3. **I labeled the longest connected structures of white and gray with arrows**

c) Now think about the continuous domain where we have 3 connected elements, one curve and two regions (exterior and interior). How could you get the same result given **d_4** and **d_8** metrics? Explain briefly and also calculate the length and mark the trajectory on the curve.**(11)**



In the d4 metrics, it allows to move in the horizontal and vertical directions, each move has distance of one. In the d8 metric, it allows to move in not only horizontal and vertical but also in diagonal directions, also each move has distance of one.

To get the same result given d4 and d8 metrics, the key point is to convert diagonal moves into horizontal and vertical steps in d8 metrics. See the blue arrows I marked in the left picture.

The length is 28.

2. Filtering: **(25)**

Given the following 1-D filter and 1-D pixel image:

a) Apply a 3 pixel average box filter and add the resulting values into the empty boxes below.**(9)**

(1/3)	1	1	1

Box Filter

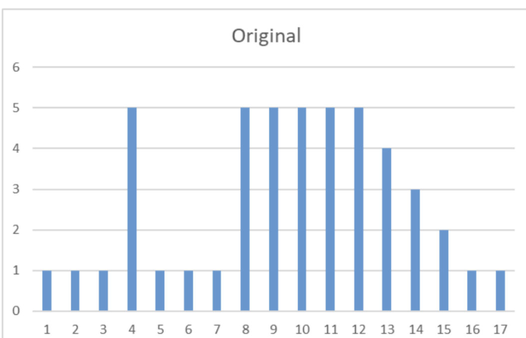
Image Signal

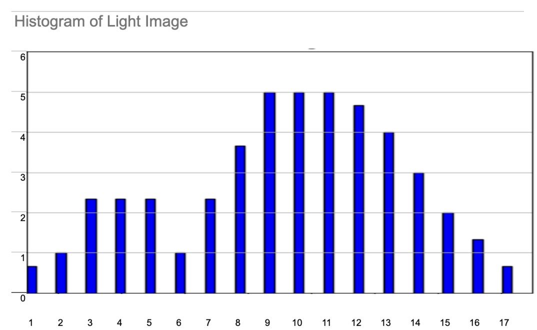
0	1	1	1	5	1	1	1	5	5	5	5	5	4	3	2	1	1	0

Filtered Signal

	2/3	**1**	**7/3**	**7/3**	**7/3**	**1**	**7/3**	**11/3**	**5**	**5**	**5**	**14/3**	**4**	**3**	**2**	**4/3**	**2/3**	

b) Plot the original image signal into the left graph, and the filtered signal into the right one. The horizontal axis is the pixel location, and the vertical axis the intensity value**(9)**



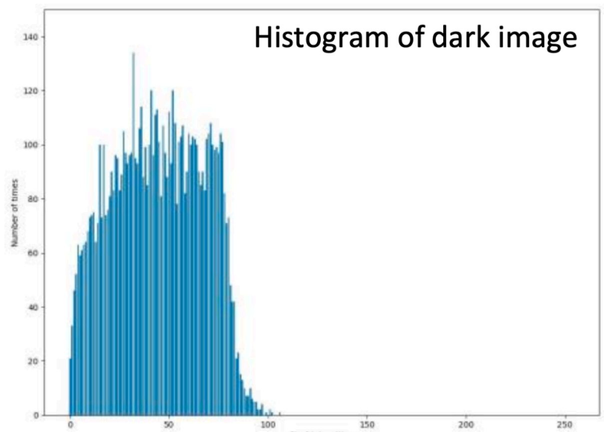


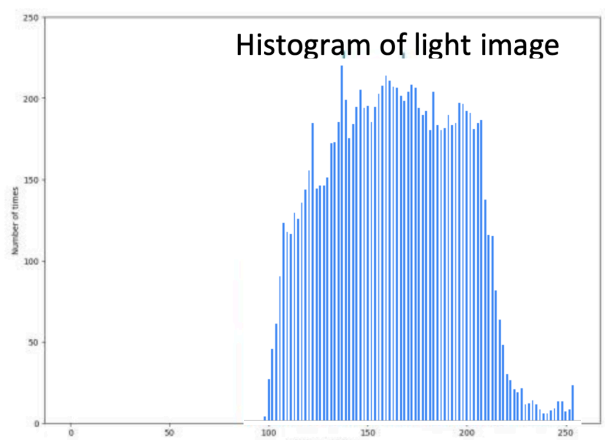
c) Briefly discuss the result after linear filtering when comparing the two plots.**(7)**

1. **The original histogram seems to highlight the specific intensity values and the according distributions, but the filtered signal emphasis more on the overall distribution of intensity values. It pays more attention on the trend.**
2. **Compared with the original signal, the filtered signal reveals how the box filter has sharp intensity peaks and balance the gaps between pixels. The result of filtered signal owns smoothing effect result.**
3. **Due to the averaging effect of the linear filter, the filtered signal's histogram typically shows reduced variability and fewer extreme values.**

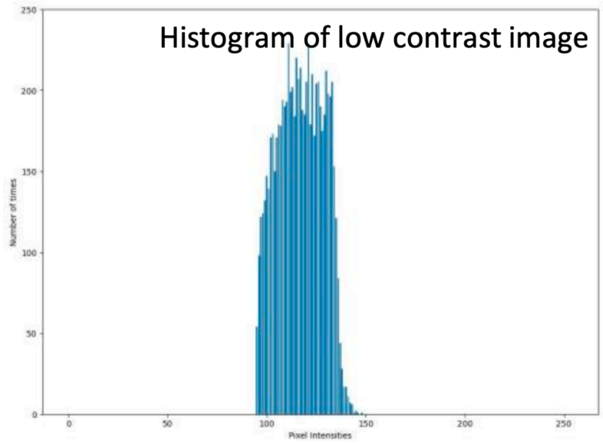
3. Histograms **(25)**

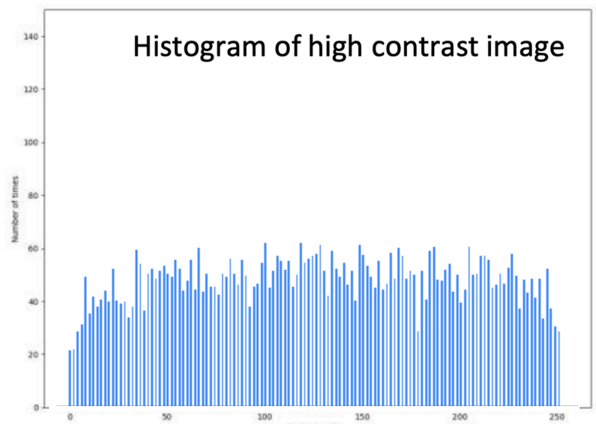
Below are a few examples of the histograms of the images. Roughly sketch the histogram for the **a)** light image and **b)** high contrast image and **c)**briefly explain your reasoning.**(10)(10)(5)**





Briefly explain the reason: **The histogram for a light image will be skewed towards the higher end of the intensity scale because the image contains mostly bright pixels. This results in a peak near the value of 255 and low frequency of lower intensities.**





Briefly explain the reason: **In high contrast images, we can see that the histogram shows a uniform distribution due to the wide range of pixel values spanning from black to white. Even though the picture has distinct bright and dark, the inclusion of intermediate gray levels cause the histogram to be more than concentrated at the extremes. Instead, it shows a more even distribution.**

4. Non-Linear Filtering and Shift Invariance (25)

The median, z, of a set of numbers is such that half the values in the set are below z and the other half are above it. With filtering such as cross-correlation and convolution, we discussed properties such as:

Shift invariance: Recall the definition, if g(x) is the response of the system to f(x), then g(x-x0) is the response of the system to f(x-x0):

$$g = w \circ f \qquad g(x) = \sum_{s=-a}^{s=a} w(s) f(x+s) = w \circ f(x) \qquad g(x - x_0) = \sum_{s=-a}^{s=a} w(s) f(x - x_0 + s) = w \circ f(x - x_0)$$

a) Discuss in a few words why or why not a median filter fulfills the shift invariance properties:**(5)**

First of all, I think a median filter doesn't fulfills the shift invariance properties. Essentially, it is because the process of finding the median is discontinuous. You can't write this function in a simple mathematical expression f(x) -> g(x). From the perspective of the image itself. For a median filter, shifting the input does not lead to a simple shift in the output, because the sorting of the values within each window changes after shifting, which leads to a different median calculation.

b) Show by numerical example (e.g. create two 1-D pixel images with 3 elements each) if a median filter is a linear or nonlinear operator. Remember that the additivity property for linearity requires that OP(A+B) = OP(A)+OP(B), with OP representing an operator, here the median, and A and B are two small 1-D pixel images, for example.**(5)**

Let's define:
A = [1, 2, 3]; B = [4, 5, 6]

Median of A: Median(A) = 2
Median of B: Median(B) = 5

Median of A+B: Median(A+B) = 7

In this case: Median(A+B) = Median(A) + Median(B)

Consider another case:
Let's define:
A=[2, 1, 3] B = [6, 2, 5]

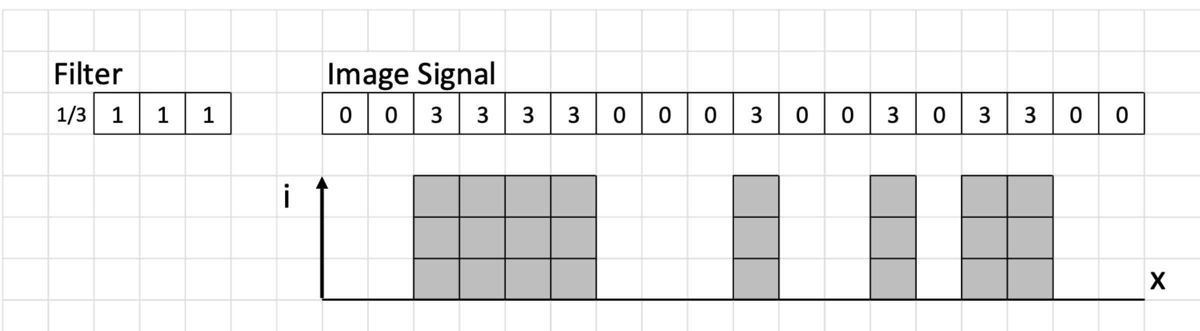
Median of A: Median(A) = 2
Median of B: Median(B) = 5

Median of A+B: Median(A+B) = 8

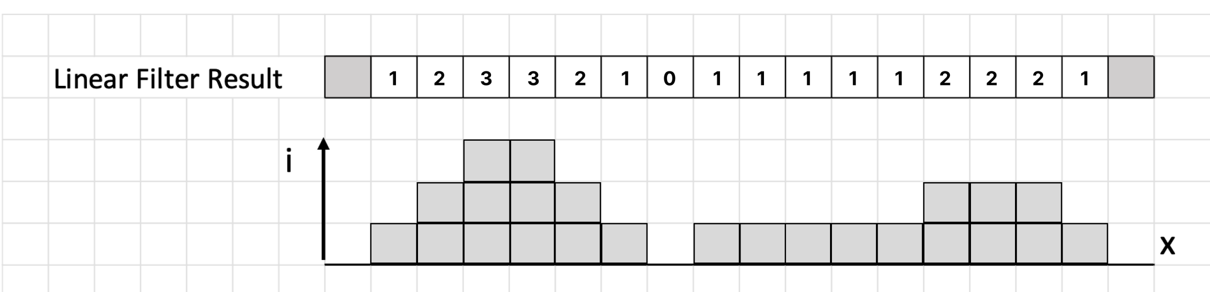
In this case: Median(A+B) != Median(A) + Median(B)

So, median filter is nonlinear operator, because it can't always satisfy OP(A+B) = OP(A) + OP(B)

c) Consider the following 1-D pixel images, shown with numbers and also a plot of x versus i(x).

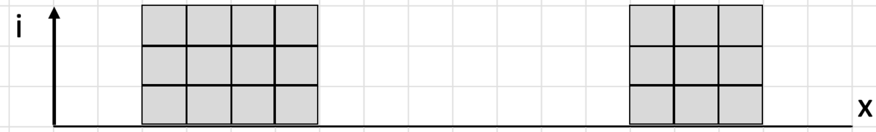


Apply a linear filter as shown above to the left to the pixel image. Provide the result as pixel numbers and also a plot of x versus i(x).**(5)**



d) Apply a median filter of width 3 to the image signal. Provide the result as pixel numbers and also a plot of x versus i(x).**(5)**

Median Filter Result		0	3	3	3	3	0	0	0	0	0	0	0	3	3	3	0	



e) Briefly compare and discuss the two results.**(5)**

According to the result histogram, it seems that linear filters pays more attention on the whole pixels area. Linear filters often smooth or blur the pictures. They can effectively reduce noise but may also blur important edges and details, so they may not handle outliers well.

However, Median filters may work well on remove some noise while keeping the edges. It seems that they can maintain the sharpness of the image better than linear filters because they do not smooth and blur the edges.