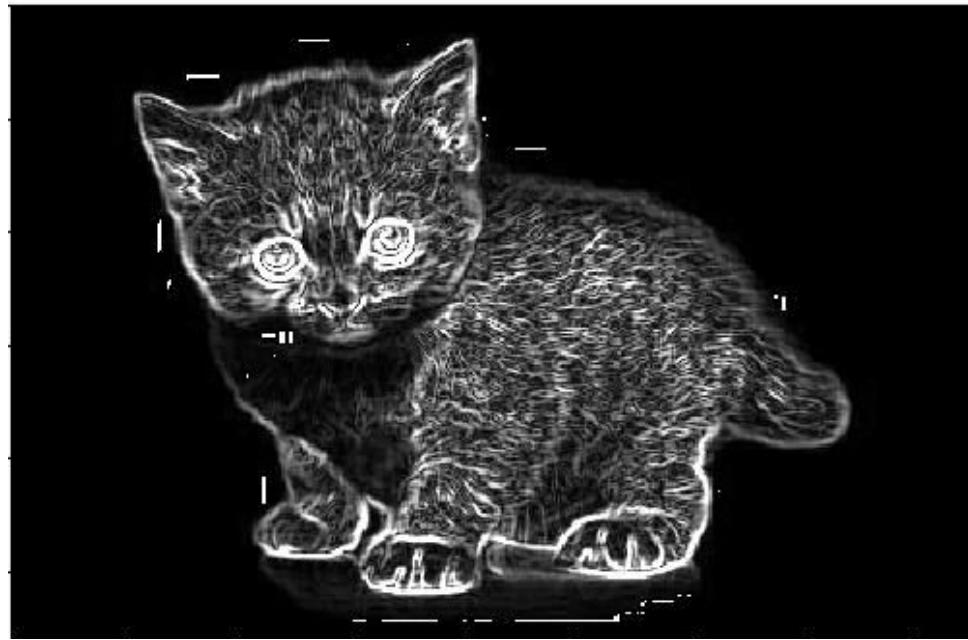


CS-GY 6643

Computer Vision

**Lecture 3: Image filtering +
convolutions + edge detection**

Prof. Erdem Varol



Today's menu

Announcements

Image filtering continued (Szeliski 3.3)

Mid-level imaging features (Szeliski 7.2)

Coding lab



Announcements



Homework 1 was due at 10:59am

Thank you for completing!

1 point will be deducted for every hour the assignment is late (rounded down).

Make sure your assignment is submitted on brightspace asap.

We plan to release HW grades and solutions by 9/26.

CS GY 6643 - Computer Vision, Fall 2024

Erdem Varol

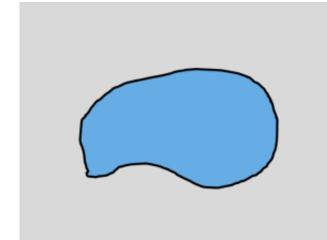
Homework 1

Out: 09/12/2024

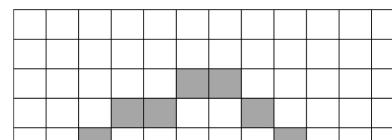
Due: 09/18/2024 11:59 PM

1. Connectivity: Connected component labeling

The famous theorem of Jordan states that every simple closed curve in the plane separates the complement into two connected nonempty sets: an interior region and an exterior. Although this looks simple and intuitive, the proof can get very complex (for those who want to look it up: [Link](#)). Elements are a closed curve and the complement, which is the plane, which is divided into 2 regions. Now let us explore how the continuous case transfers to our discrete domain.



Given the following pixel image with white areas and a gray closed pixel curve. How many connected components can you detect?



Project 1 out

Due October 4, 10:59am (~2 weeks)

15 % of total course grade.

A python notebook with executed outputs + a pdf write up of each step explained to be submitted on brightspace.

You are encouraged to discuss with other students and look online for examples.

Project must be submitted individually (**We will check for blatant code copying**).

1 point will be deducted for every hour the project is late (rounded down).

CS GY 6643 - Computer Vision, Fall 2024

Project 1

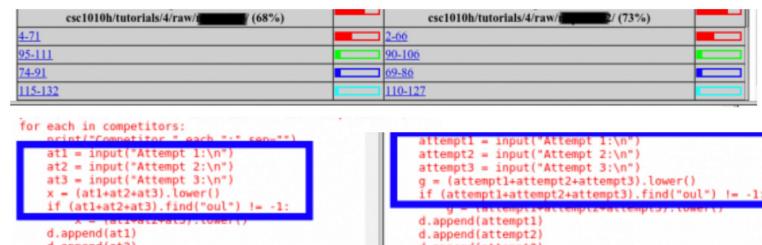
Due: 2024/10/04 11:59 PM

Note: For the project you are not allowed to use pre-built libraries to implement the core algorithm of the problem. You can use pre-built libraries to load/process the images. You have to submit a final jupyter notebook as a submission. Use markdown cells to write text explanation whenever needed. Upload the jupyter notebook to google drive and add the sharable link to brightspace.

1 Histogram Equalisation

Imagine you're a detective analyzing a mysterious photograph below 1 where a person is sitting on something, but you can't quite tell what it is. The image is washed out, with a bright laptop screen stealing all the attention, leaving the rest of the scene in low contrast and hidden details. Your mission, should you choose to accept it, is to enhance the image and reveal the secret!

Using the concept of histogram equalization, your task is to write a function histogram.equalisation that takes this grayscale image as input and enhances the contrast, making the scene clearer. Once you process the image, uncover what the person is sitting on. Can you bring the hidden details to light and solve the mystery?



```
csc1010h/tutorials/4/raw/1/ (68%) csc1010h/tutorials/4/raw/1/ (73%)
4-71 2-66
95-111 90-106
74-91 69-86
115-132 110-127
```

```
for each in competitors:
    attempt1 = input("Attempt 1:\n")
    attempt2 = input("Attempt 2:\n")
    attempt3 = input("Attempt 3:\n")
    g = attempt1+attempt2+attempt3.lower()
    if (attempt1+attempt2+attempt3).find("out") != -1:
        g = attempt1+attempt2+attempt3.lower()
    d.append(attempt1)
    d.append(attempt2)
    d.append(g)
```

```
attempt1 = input("Attempt 1:\n")
attempt2 = input("Attempt 2:\n")
attempt3 = input("Attempt 3:\n")
g = attempt1+attempt2+attempt3.lower()
if (attempt1+attempt2+attempt3).find("out") != -1:
    g = attempt1+attempt2+attempt3.lower()
d.append(attempt1)
d.append(attempt2)
d.append(g)
```



Please complete final project team selection!

Only 37 out of 65 students have chosen teams or asked to be assigned on google spreadsheet.

Only 5 submitted on brightspace...

Due September 22, 10:59am (3 days!)

Worth 2% of total course grade

	Assignment	New Submissions	Completed
<input type="checkbox"/>	Project area + Teammate selection Due on Sep 22, 2024 10:59 AM	5	5/65

Each student must submit the names and NYU ID's of teammates and project area on brightspace.

Use the match-making spreadsheet to declare project teams + ask to join other teams.

<https://docs.google.com/spreadsheets/d/1RkjD30T0BsH8VO79zaxYJUE6GJFWTgVNu7nE4SJ1UYw/edit?usp=sharing>

A	B	C	D	E	F	G	H	I	J
Project area	Project tentative title	Team member 1 name	Team member 1 email	Team member 2 name	Team member 2 email	Team member 3 name	Team member 3 email	Team member 4 name	Team member 4 email
Object tracking	Cell tracking in C. elegans microsc	Maren Eberle	mie8014@nyu.edu	Nalini Ramanathan	nar8991@nyu.edu	Ryan Fleishman	rmf9265@nyu.edu	Neel Gandhi	njg9191@nyu.edu
Biometrics	TBA	Allison Lee	al6897@nyu.edu	ASSIGN ME TEAMMATES		Varad Naik	vvn7114@nyu.edu		
Object tracking	Ball Tracking in Sports	Pratham Shah	prathamshah@nyu.edu	Ohm Patel		Sidhved Warik	sw6071@nyu.edu	Vedangi Kirtane	vrk3359@nyu.edu
Image Reconstruction	TBA	Jaydeep Mulani	jgm9413@nyu.edu	Aditya Azad	aa10878@nyu.edu	Jialin Li	jl10897@nyu.edu	Remo Shen	ys6345@nyu.edu
Image Alignment	TBA	Tianxiao He	th3129@nyu.edu	Yurong Liu	y16624@nyu.edu	Roman Vakhrushev	rv1057@nyu.edu		
Superresolution	TBA	Shambhavi Seth	ss17936@nyu.edu	Aryan Prasad	ap7949@nyu.edu	Bowen Gong	bg1941@nyu.edu	Lucy Shi	ls4900@nyu.edu
Image Alignment	TBA	Jizheng Dong	jd4587@nyu.edu	Yifei Zhuang	y29865@nyu.edu				
Image Colorization	TBA	Nika Emami	ne2213@nyu.edu	ASSIGN ME TEAMMATES					
		Wei-Lun Hung	wh2528@nyu.edu	Yu-Yuan Chang	yc6549@nyu.edu	Guan-Cherng Lin	gl2547@nyu.edu	Da Hye Kim	dk3343@nyu.edu
		Gordon Lu	gl1589@nyu.edu	Suemy Inagaki	si2324@nyu.edu	Felipe Oliveira	fd2264@nyu.edu	Chhatrapathi Sivaji Lakk	cl7203@nyu.edu
Image Generation	TBA	Daoyu Li	dl5312@nyu.edu	Ziming Song	zs2815@nyu.edu	ASSIGN ME TEAMMATES		Charan K Raja	ck3470@nyu.edu
		Aravindsrinivas Krishnar	ak11115@nyu.edu	Priyangshu Pal	pp2833@nyu.edu			Bharat S	



Project ideas

Check slack channel #project-ideas for more suggestions.

What is COCO?



COCO is a large-scale object detection, segmentation, and captioning dataset. COCO has several features:

- ✓ Object segmentation
- ✓ Recognition in context
- ✓ Superpixel stuff segmentation
- ✓ 330K images (>200K labeled)
- ✓ 1.5 million object instances
- ✓ 80 object categories
- ✓ 91 stuff categories
- ✓ 5 captions per image
- ✓ 250,000 people with keypoints

<https://cocodataset.org>

WormID.org

Home Datasets Code Apps How to use Contact

ID your worm neurons

WormID.org is a resource for the *C. elegans* neuroscience research community to enable data standardization, storage, and sharing, and to provide data conversion and pipelines for analysis of whole-brain imaging recordings, including cell identification.



Step-by-step guide

<https://www.wormid.org/>

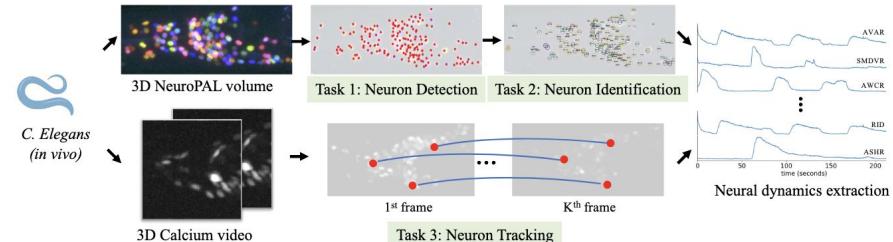


Figure 1: Task overview. To extract neural dynamics from microscopy images of *in vivo* *C. elegans*, WormID benchmarks each step of the pipeline: neuron detection, identification, and tracking.



Take advantage of office hours to discuss the next project + final project ideas



Instructor: Prof. Erdem Varol

Email: ev2240@nyu.edu

Office Hours: On Zoom, 15 min appointments.



TA: Rishabh Raj

Email: rr4574@nyu.edu

Office Hours: Tuesdays 1-2pm on Zoom.



TA: Malhar Patel

Email: mkp6112@nyu.edu

Office Hours: Mondays 1-2pm on Zoom.

Also in person in 370 Jay Street, room 1166



Recap of previous lecture



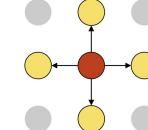
Recap of previous lecture

We learned

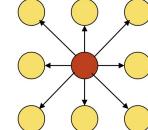
- How images are represented digitally and topologically
- How we can manipulate images through pixel and patchwise operations, and filters.

243	239	240	225	206	185	188	218	211	206	216	225
242	239	218	110	67	3	34	152	213	206	208	221
243	242	123	58	94	82	132	77	108	208	208	215
235	217	115	212	243	236	247	139	91	209	208	211
233	208	131	222	219	226	196	114	74	208	213	214
232	217	131	116	77	150	69	56	52	201	228	223
232	232	182	186	184	179	159	123	93	232	235	235
232	236	201	154	216	133	129	81	175	252	241	240
235	238	230	128	172	138	65	63	234	249	241	245
237	236	247	143	59	78	3	94	255	248	247	251
234	237	245	193	55	33	115	144	213	255	253	251
248	245	161	128	149	109	138	65	47	156	239	255
190	107	39	102	94	73	114	58	3	51	137	
3	32	33	148	168	203	179	43	27	3	35	24
26	160	255	255	109	26	19	35	24			

4 connected



8 connected

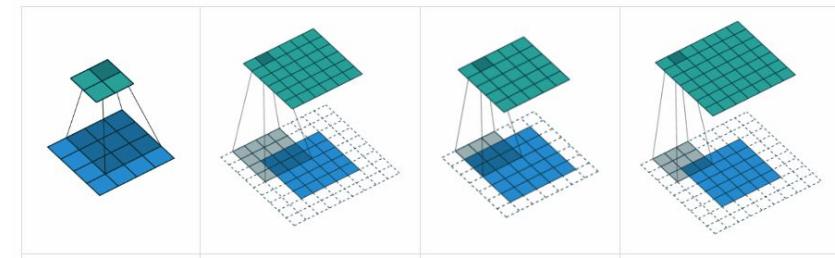


Today we will learn

More filtering, convolutions

Non-linear filtering

Going from pixel-wise to
feature-wise understanding of
images



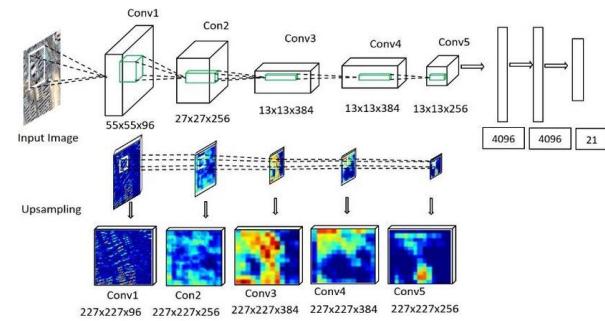
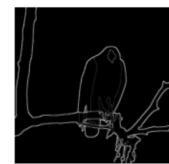
pixels

243

243	239	240
242	239	218
243	242	123

patches

features



Low level → High level



Recall the box filter

243	222	171
203	145	137
209	132	193



121	111	85
101	183	68
104	66	96

$$f: pa \rightarrow p'$$

Input **patch**

Output pixel

$I[0, 0]$	$I[0, 1]$	$I[0, 2]$
$I[1, 0]$	$I[1, 1]$	$I[1, 2]$
$I[2, 0]$	$I[2, 1]$	$I[2, 2]$

Input

Filter g

$\frac{1}{9}$	1	1	1
1	1	1	1
1	1	1	1

(Dot product in 1D)

	$J[1, 1]$	

Output

$$J[1, 1] = \frac{1}{N} \sum_{x,y} I[x, y]$$



Input Image



Output: Filtered image
(mimic focus blur)

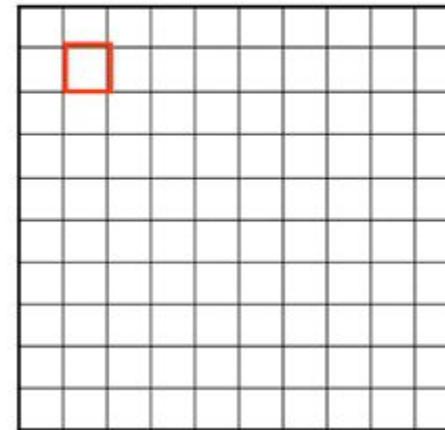
This filter computes dot products then slides

$$g(i, j) = \sum_{k,l} f(i + k, j + l)h(k, l).$$

$F[x, y]$

0	0	0		0	0	0	0	0	0	0	0
0	0	0		0	0	0	0	0	0	0	0
0	0	0		90	90	90	90	90	0	0	0
0	0	0		90	90	90	90	90	0	0	0
0	0	0		90	90	90	90	90	0	0	0
0	0	0		90	0	90	90	90	0	0	0
0	0	0		90	90	90	90	90	0	0	0
0	0	0		0	0	0	0	0	0	0	0
0	0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$



$$\frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Sliding + computing dot products is known as cross-correlation

- 1D

$$g(i) = \sum_k f(i+k)h(k)$$

- 2D

$$g(i, j) = \sum_{k, l} f(i+k, j+l)h(k, l)$$

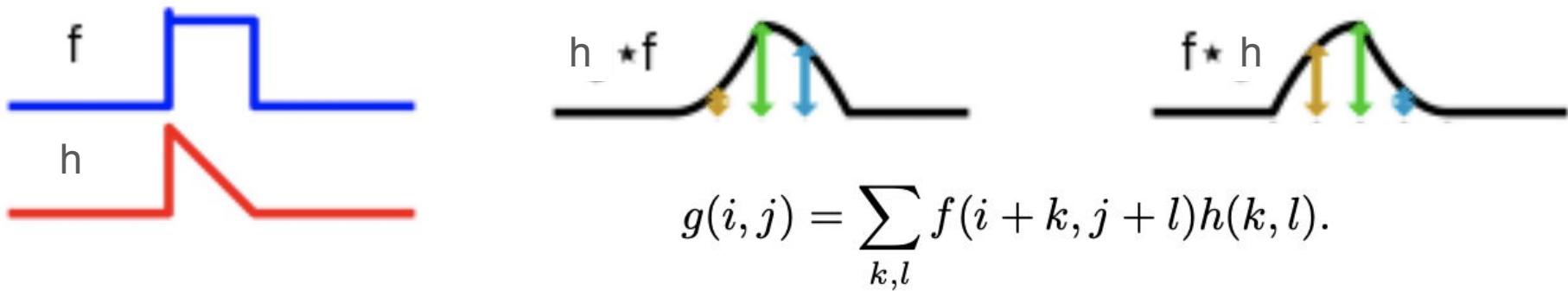
$$g = f \otimes h$$

\otimes : correlation operator



Why is cross-correlation not generally used as a filter?

Non-commutative



Why do we care? We want to be able to potentially apply many filters without caring about their order. Flexibility.

Solution: Convolutions!

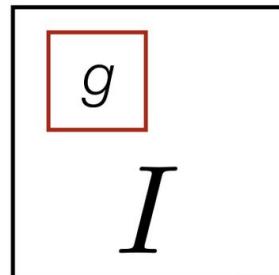
Convolution

Let I be the input image and g be the filter (conv. **kernel**).

$$J[x, y] = I * g = \sum_{k, l} I[x-k, y-l]g[k, l]$$

Indexes go backward!

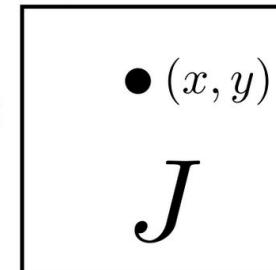
Patch centered at (x, y)



Input image



Image intensity at (x, y)



Convention:
“flipped” dot product



53

Output filtered image

Source: F. Durand

Convolution example

90	3	241	65
242	143	122	71
203	103	45	167
29	239	2	93

Input image

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline -1 & 0 & 1 \\ \hline -1 & -2 & -3 \\ \hline \end{array}$$

Convolution kernel

$$\begin{array}{|c|c|c|} \hline -3 & -2 & -1 \\ \hline 1 & 0 & -1 \\ \hline 1 & 2 & 3 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline -270 & -6 & -241 \\ \hline 242 & 0 & -122 \\ \hline 203 & 206 & 135 \\ \hline \end{array}$$

Element-wise product

$$\begin{array}{|c|c|} \hline 147 \\ \hline \end{array}$$

Sum

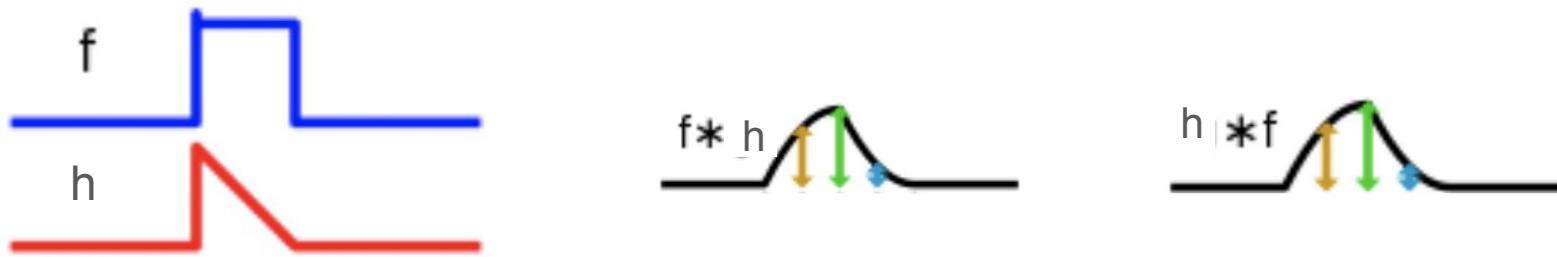
Step 1. Flip the kernel
(top-bottom, left-right)

Step 2. Dot product

(Same for the rest 3 patches)



Convolution is commutative



$$g(i, j) = \sum_{k,l} f(i - k, j - l)h(k, l) = \sum_{k,l} f(k, l)h(i - k, j - l),$$

Other convolution properties

Commutative:

$$h[n] \circ g[n] = g[n] \circ h[n]$$

Associative:

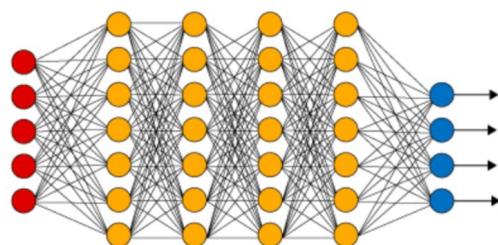
$$h[n] \circ g[n] \circ f[n] = h[n] \circ (g[n] \circ f[n]) = (h[n] \circ g[n]) \circ f[n]$$

Distributive with respect to the sum:

$$h[n] \circ (f[n] + g[n]) = h[n] \circ f[n] + h[n] \circ g[n]$$



Flexibility of convolutions makes them ideal filtering operators



What you see in the media

The revolutionary AlexNet:
5 layers of convolution



Q: Is box filter a convolution?

$I[x, y]$: Image intensity at pixel (x,y)

$I[0, 0]$	$I[0, 1]$	$I[0, 2]$
$I[1, 0]$	$I[1, 1]$	$I[1, 2]$
$I[2, 0]$	$I[2, 1]$	$I[2, 2]$

Input

$$\xrightarrow{\text{Filter } g}$$
$$\frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

(Dot product in 1D)

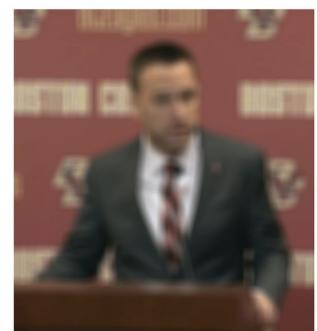
	$J[1, 1]$	

Output

$$J[1, 1] = \frac{1}{N} \sum_{x,y} I[x, y]$$



Input Image



Output: Filtered image
(mimic focus blur)



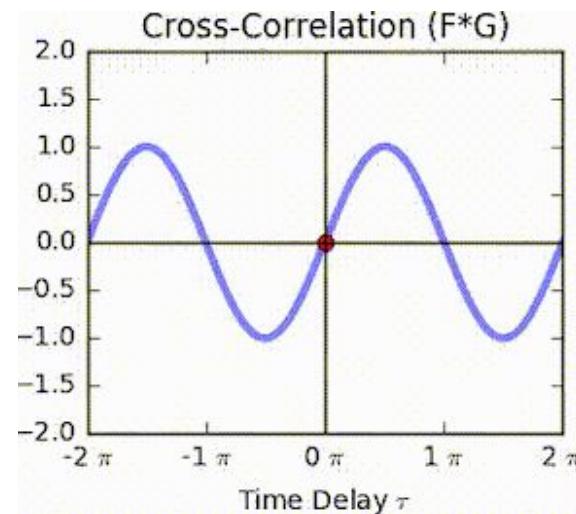
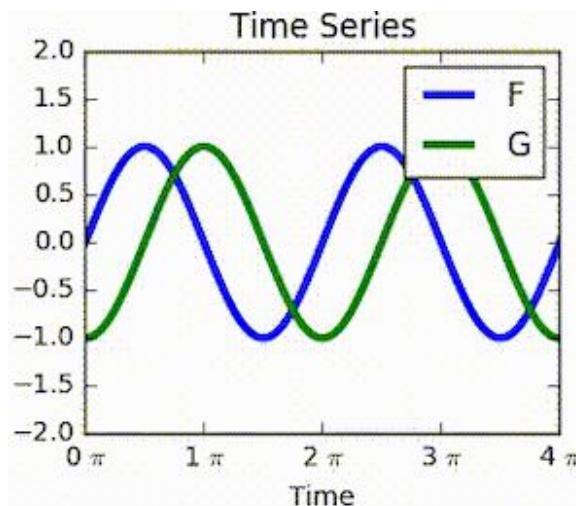
Yes - it's both a convolution and a cross-correlation.

- Convolution same as cross correlation with kernel transposed around each axis.
- The two operations (correlation and convolution) are **the same** if the kernel is symmetric about the axes.

$$g = f \otimes h = f * h^{\wedge} \quad h^{\wedge}: \text{reflection of } h$$



- General consensus:
 - Cross-correlation: measure the similarity between two signals
 - Convolution: filtering operation on a signal

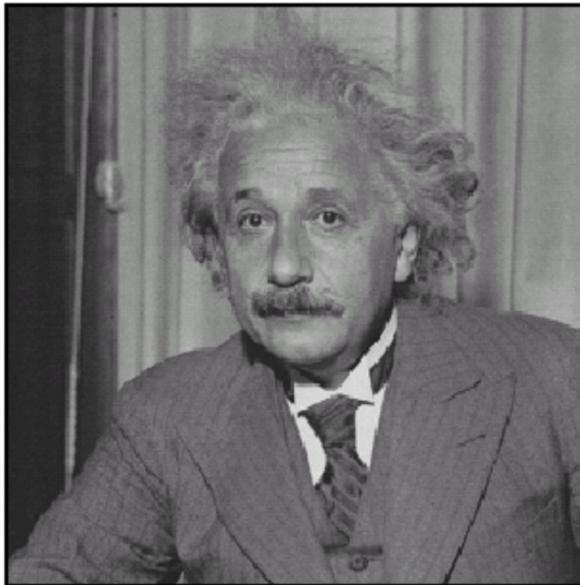


Filters are stackable

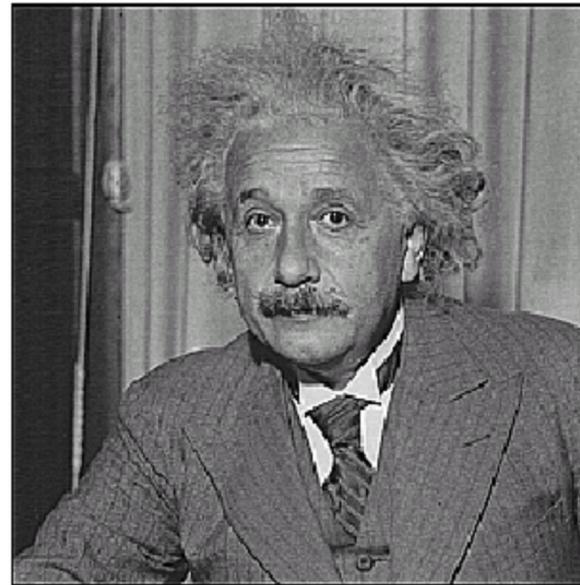


How can we design a sharpening filter?

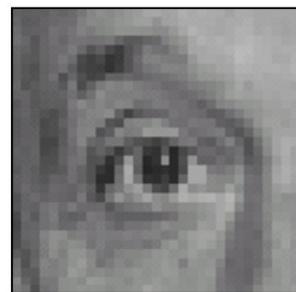
Input



Sharpened result

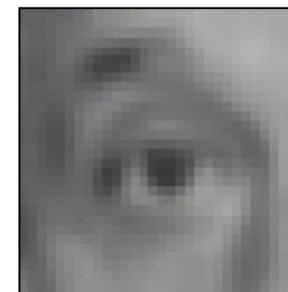


Box filter: blurring effect



Original

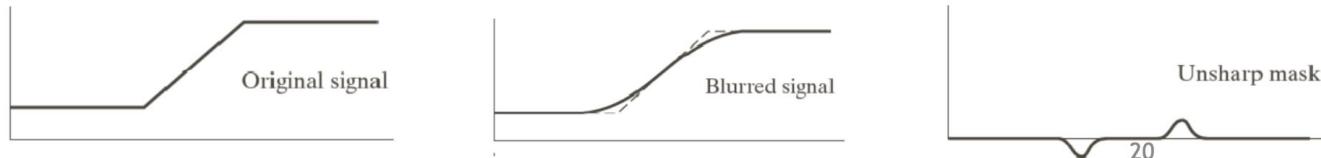
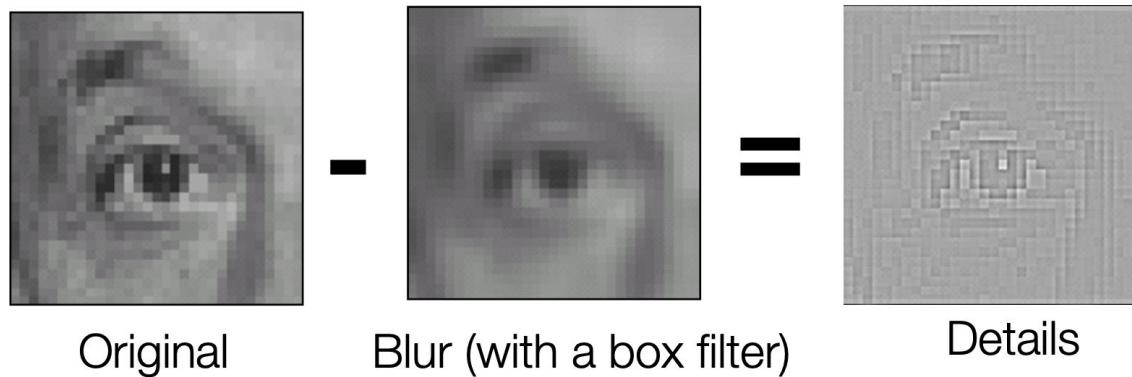
$$* \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$



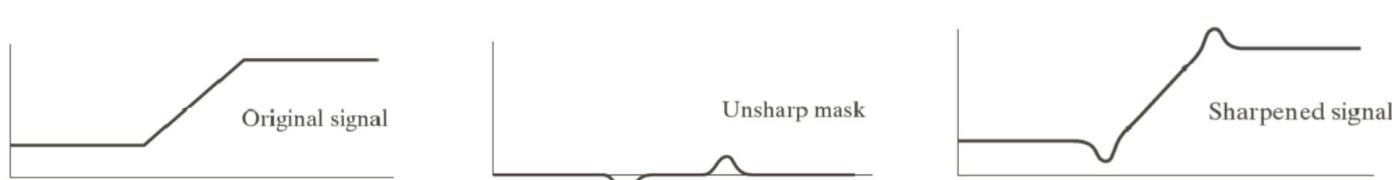
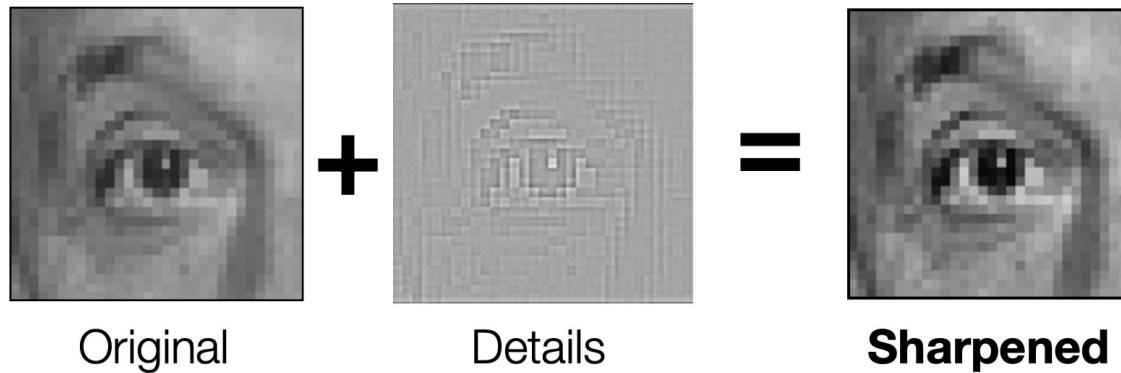
Blur (with a box filter)



Sharpen filter: get details



Sharpen filter: add details



Impulse filter: Identity

Box size = [1,1]

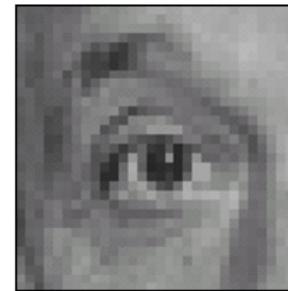


Original

*

0	0	0
0	1	0
0	0	0

=



Filtered
(no change)

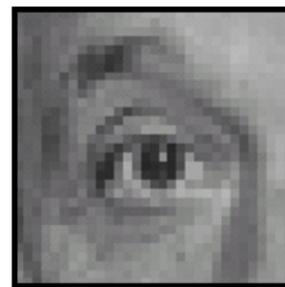
Sharpen filter: put them together

Remember: convolution filters are linear!



$$* \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$

+

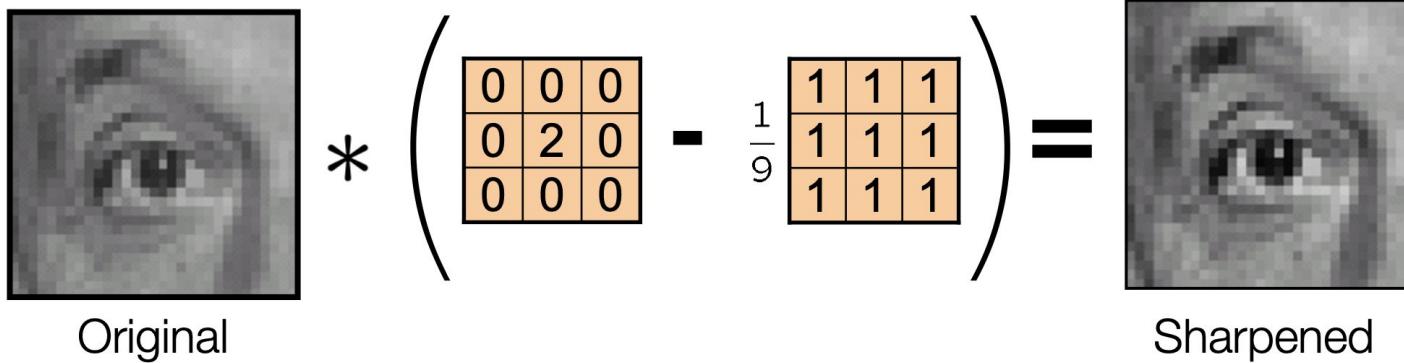


$$* \left(\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} - \frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \right)$$

Original

Details

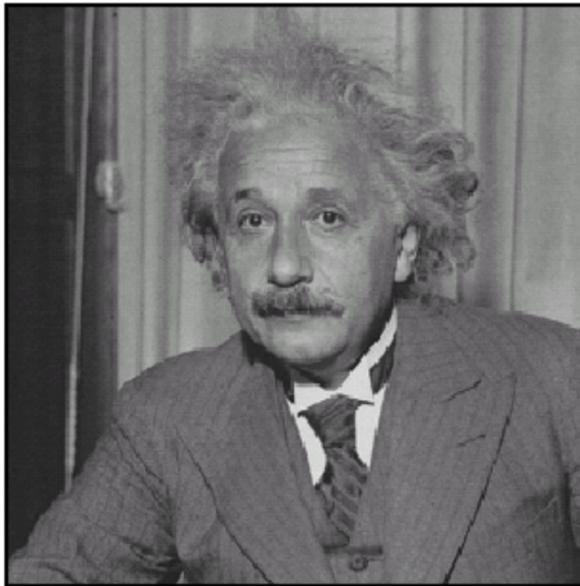
Sharpen filter: one filter

$$\text{Original} \quad * \left(\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 2 & 0 & \\ 0 & 0 & 0 & \end{array} - \frac{1}{9} \begin{array}{ccc|c} 1 & 1 & 1 & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & \end{array} \right) = \text{Sharpened}$$


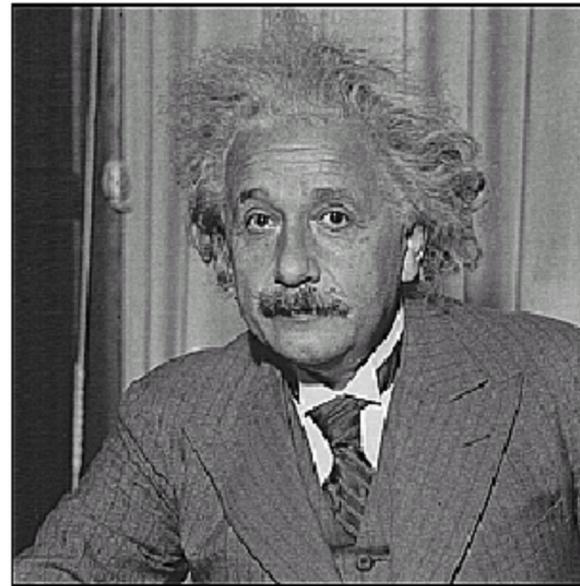
$$\frac{1}{9} \begin{array}{ccc|c} -1 & -1 & -1 & \\ -1 & 17 & -1 & \\ -1 & -1 & -1 & \end{array}$$

Result

Input



Sharpened result



Box filters are ... blocky.



$$\frac{1}{9} \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} * \begin{matrix} & & \\ & & \\ & & \end{matrix}$$

Box filter

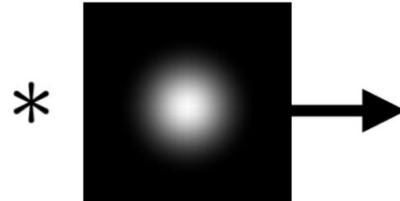


“Ghosting grid artifact”



Can we make this smoother?

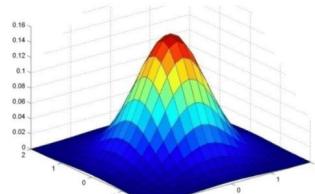
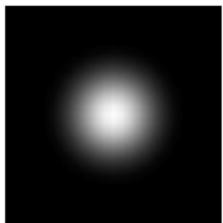
Idea: weight of neighborhood pixels according to their closeness to the center



Gaussian filter

Box vs. gaussian

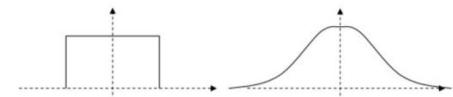
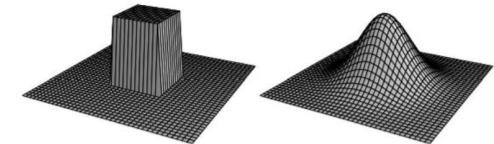
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



Convolution kernel \mathbf{g}

0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

($5 \times 5, \sigma = 1$)



0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

(a)

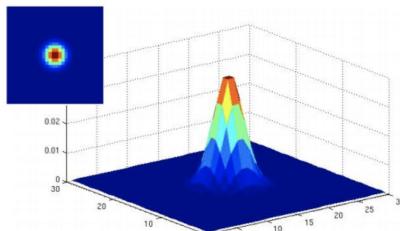
0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0

(b)

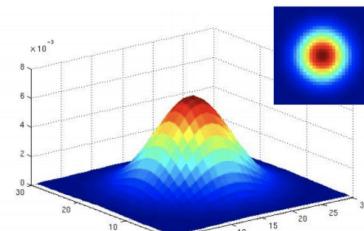
- Divide a constant factor to make the kernel sum to 1

Property 1: Bigger sigma means more blurring (and also higher computational cost. Why?)

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



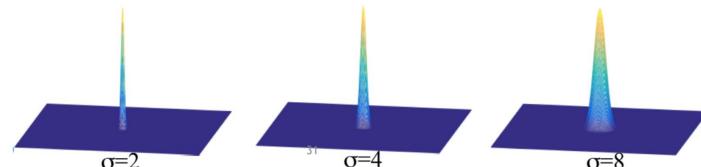
kernel: $\sigma = 2$ with 30×30



kernel: $\sigma = 5$ with 30×30



- Bigger σ
- More blurry
 - Bigger effective kernel size



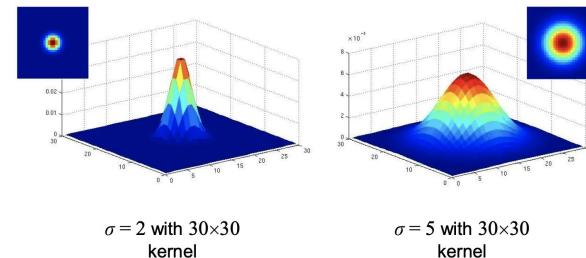
- Standard deviation σ : determines extent of smoothing

Complexity detour: Each filter can be written as a linear matrix operation

$$\begin{bmatrix} 72 & 88 & 62 & 52 & 37 \end{bmatrix} * \begin{bmatrix} 1/4 & 1/2 & 1/4 \end{bmatrix} \Leftrightarrow \frac{1}{4} \begin{bmatrix} 2 & 1 & . & . & . \\ 1 & 2 & 1 & . & . \\ . & 1 & 2 & 1 & . \\ . & . & 1 & 2 & 1 \\ . & . & . & 1 & 2 \end{bmatrix} \begin{bmatrix} 72 \\ 88 \\ 62 \\ 52 \\ 37 \end{bmatrix}$$

Figure 3.12 One-dimensional signal convolution as a sparse matrix-vector multiplication, $\mathbf{g} = \mathbf{H}\mathbf{f}$.

- Convert 1D or 2D images into raster-ordered vectors \mathbf{g} and \mathbf{f}
- \mathbf{H} is sparse matrix of convolution (respectively correlation)



More zeros in filter, less multiplication operations.

$\sim O(n^2)$ operations instead of $O(n^3)$ for small kernels.

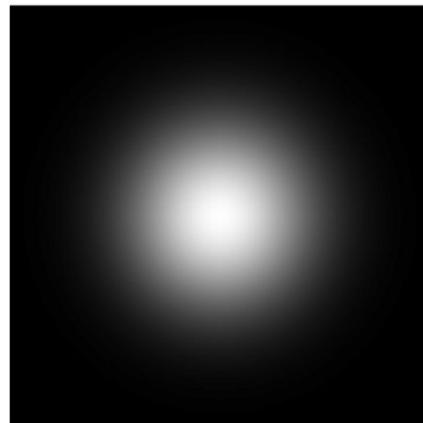


Property 2: Recursive

Convolve a Gaussian filter with itself? Get another Gaussian.

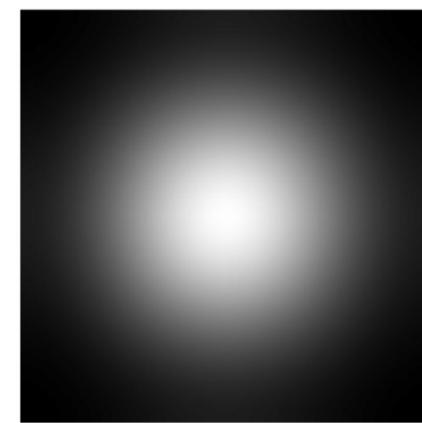


*



σ

=



$\sigma\sqrt{2}$



Property 3: separability

Thanks to the commutativity of convolution!

It's a **separable kernel**.

- Blur with 1D Gaussian in one direction, then the other.
- Fast! For an $n \times n$ kernel, $\mathcal{O}(n)$ instead of $\mathcal{O}(n^2)$



$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot [1 \quad 2 \quad 1]$$

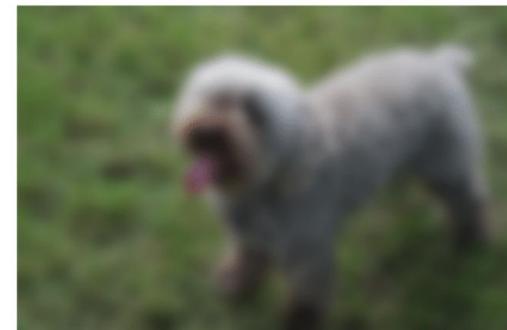
Separable kernel



I



blur_x(I)



blur_y(blur_x(I))

Complexity detour: separability improves efficiency

- Compute time

- MxM mask
- NxN image

}

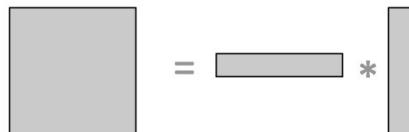
$O(M^2N^2)$

"for" loops are nested 4 deep

- Special case: *separable*

Two 1D kernels

$$w = \overbrace{w_x * w_y}^{}$$



$$w * f = (\underbrace{w_x * w_y}_{O(M^2N^2)}) * f = w_x * (\underbrace{w_y * f}_{O(MN^2)})$$

Due to associativity

Examples of separable filters

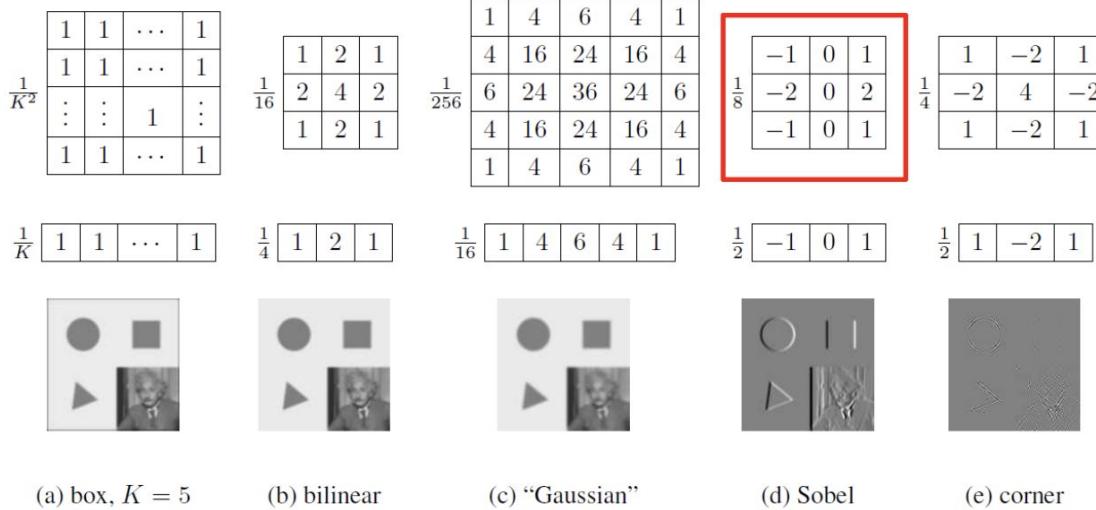
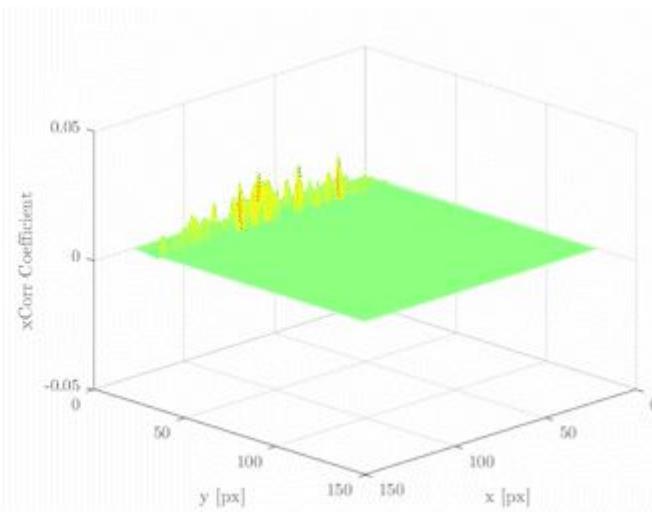
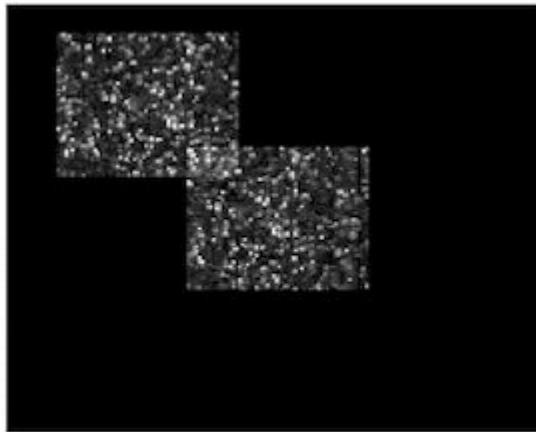


Figure 3.14 Separable linear filters: For each image (a)–(e), we show the 2D filter kernel (top), the corresponding horizontal 1D kernel (middle), and the filtered image (bottom). The filtered Sobel and corner images are signed, scaled up by 2× and 4×, respectively, and added to a gray offset before display.

So far, we have treated filters as things separate from images that are to be filtered.



Images can be their own self filters:



One key application of filtering - pattern matching

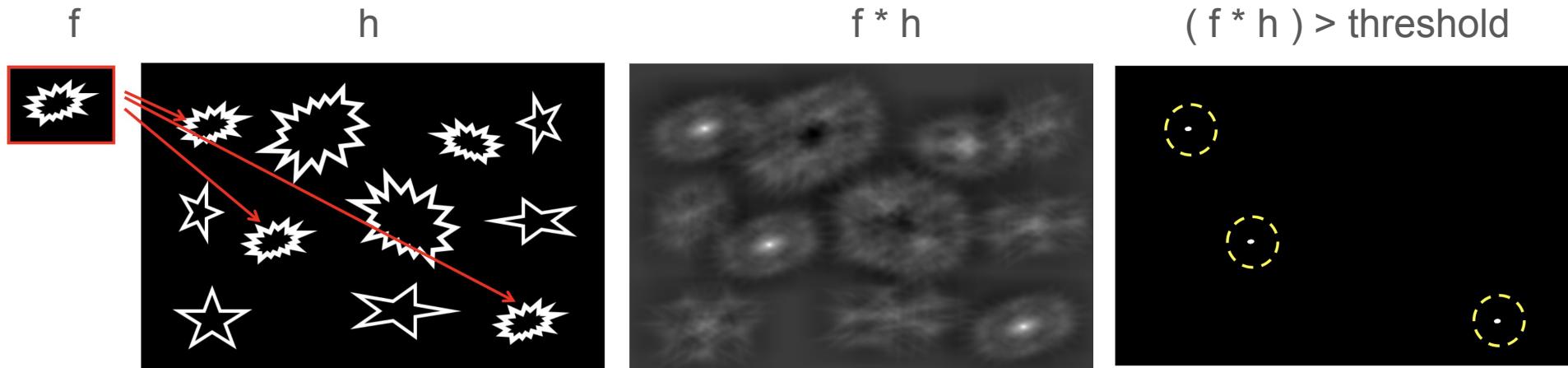
- The optimal (highest) response from a filter is the autocorrelation evaluated at position zero

$$\max_{\bar{x}} C_{ff}(\bar{x}) = C_{ff}(0) = \int f(\bar{s})f(\bar{s})d\bar{s}$$

- A filter responds best when it matches a pattern that looks itself
- Strategy
 - Detect objects in images by correlation with “matched” filter

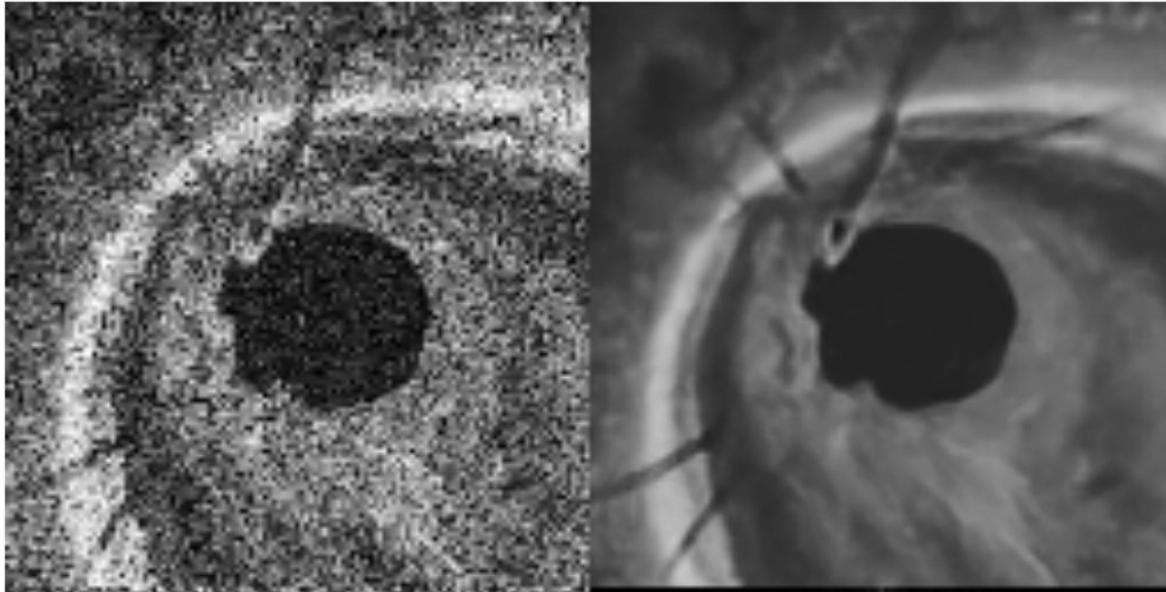


Convolving template against larger image - pattern matching



*project 1 problem 3

Convolving linear filters ultimately have its limits



Example OCT
(optical coherence
Tomography for studying
Eye diseases)

Result from Shijie Li, new
deep learning-based
methodology

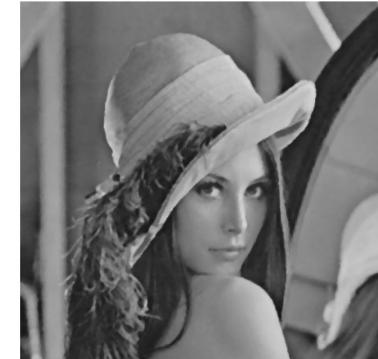
Noise patterns might be non-linear, anisotropic

High amount of outliers = median filter

- Also: “shot noise”, “salt&pepper”
- Replace certain % of pixels with samples from pdf
- Best filtering strategy: filter to avoid outliers



gaussian

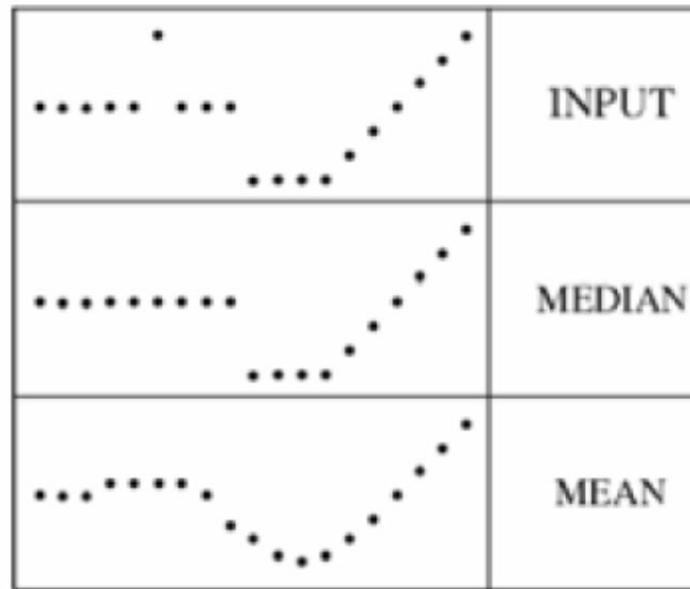


median



- What advantage does median filtering have over Gaussian filtering?
 - Robustness to outliers

filters have width 5 :

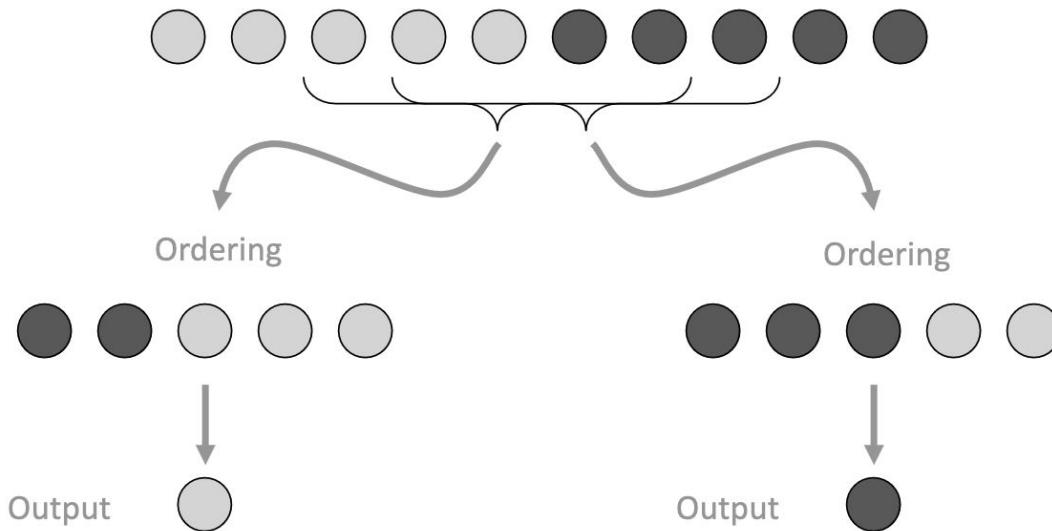


Insight: Can we use our assumptions about images to help filter them?

Source: K. Grauman



Inductive bias in median filtering: images are piecewise “flat”

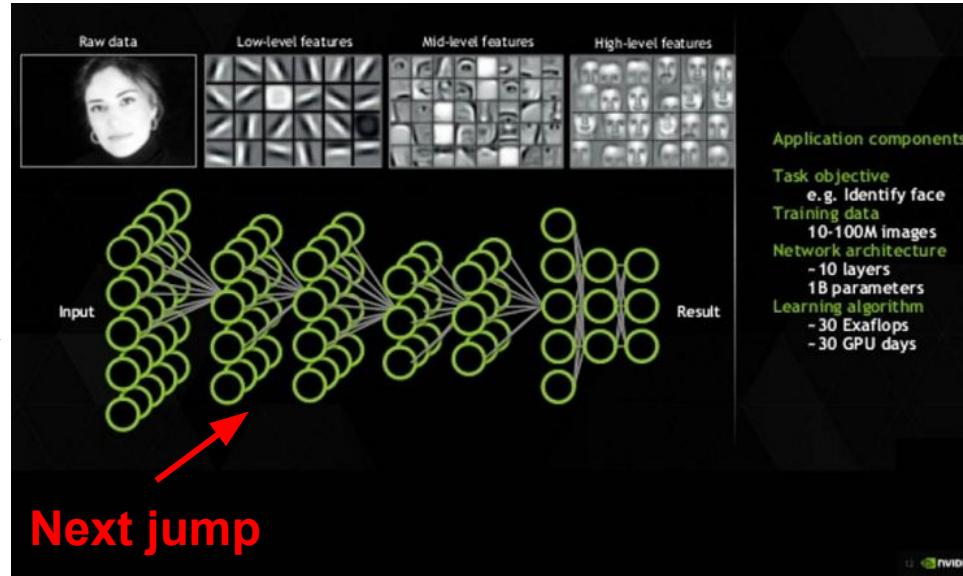


Piecewise Flat Image Models

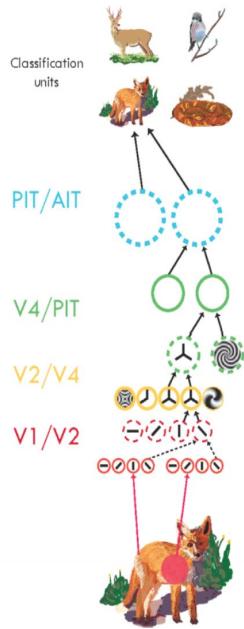
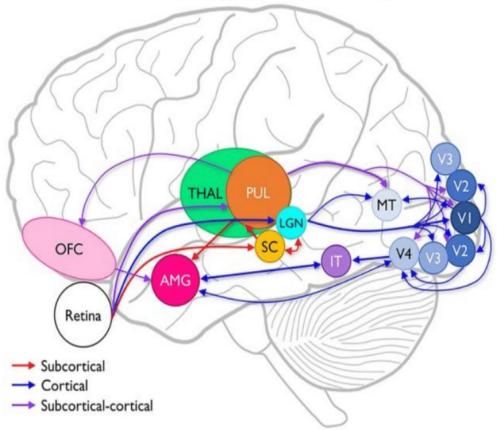
- Image piecewise flat -> average only within similar regions
- Problem: don't know region boundaries



Linear filters
are here



We need to make a new jump in our evolution to the next layer of the visual cortex to understand region boundaries



High-level vision
(object, scene)

After midterm

Mid-level vision
(segmentation)

Up to midterm

Low-level vision
(Edge, corner)



features

Lecture 3

243	239	240
242	239	218
243	242	123

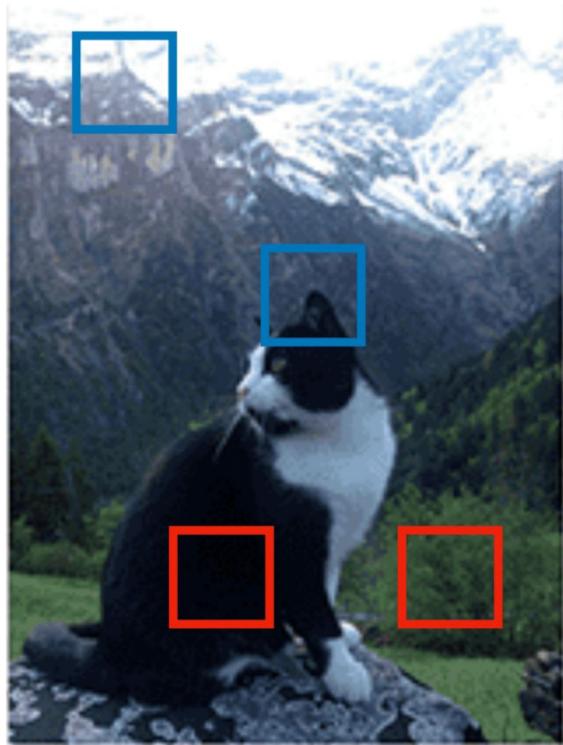
patches

Lecture 2

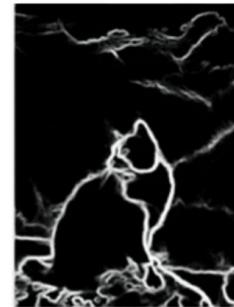
pixels

start

Intermediate computation in the brain



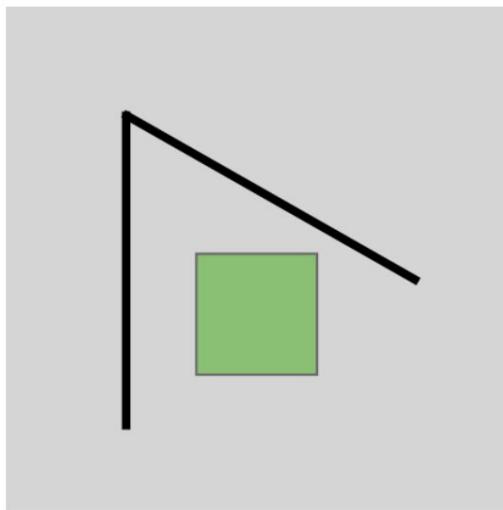
Edge



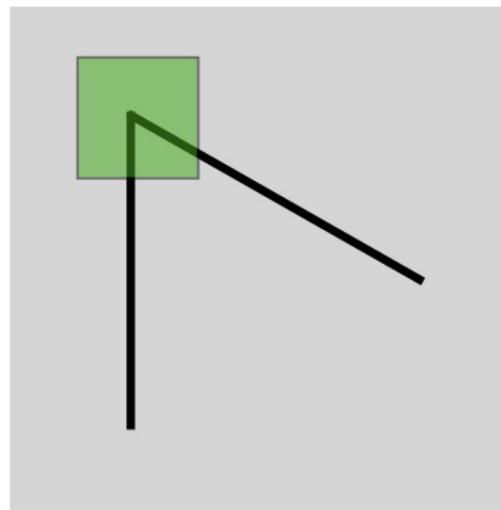
Corner

Texture
(fur, leaves)

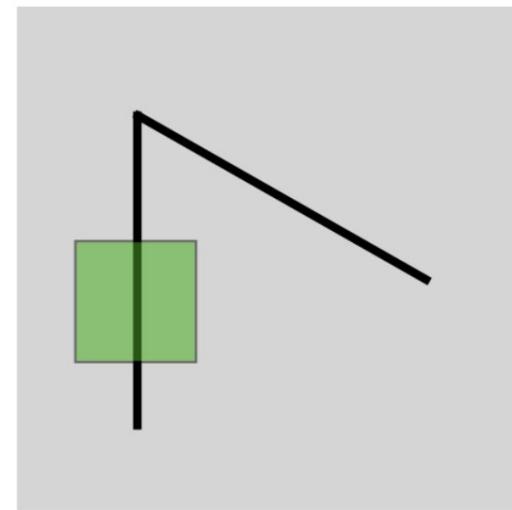
Three types of regions



Textured region
(appearance)



Corner
(anchor for matching)



Edge
(object contour)

Quick detour on textures

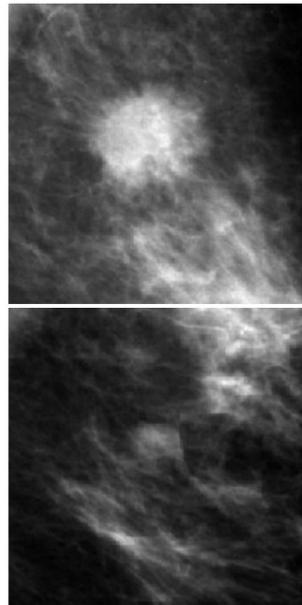
- Structural approach: a texture is composed of fundamental texture elements, called “textons”.



Bela Julesz,
"Textons, the
Elements of Texture
Perception, and their
Interactions". Nature.
1981.

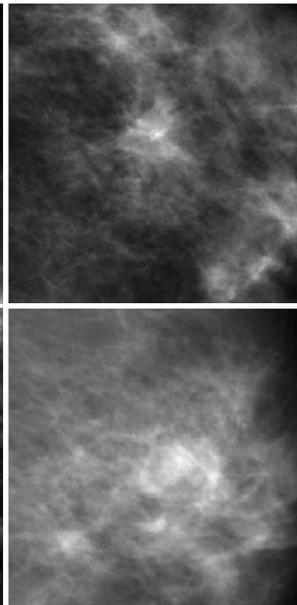
Non-deterministic textures

Malignant



42

Benign

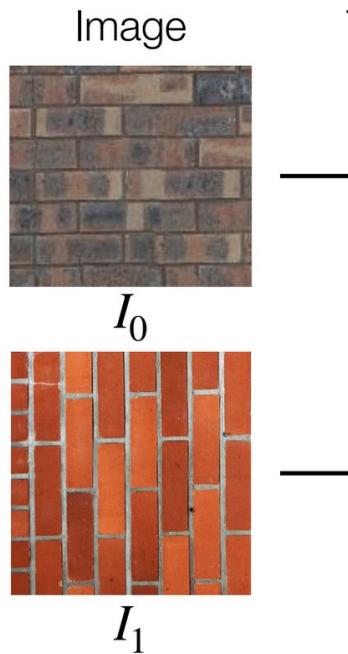


medical images (mammography)

- Stochastic approach: a texture is “a stochastic, possibly periodic, image field”

G. R. Cross and A. K. Jain. Markov Random Field Texture Models. 1983

We want series of filters that can yield textural invariances



Desired property: **Invariant (intra-class)**

- The features have similar values for the same texture despite the pixel-level difference
- e.g. robust to color, orientation changes

We can also use non-linear filters to retexture input images (generative models)



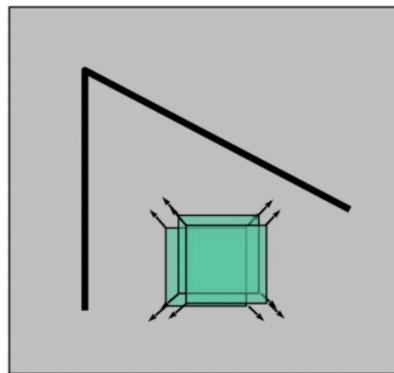
input images



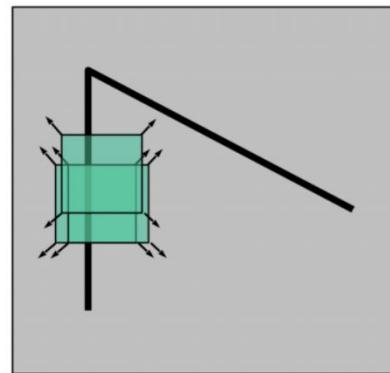
quilting results

Quick detour on corner detection

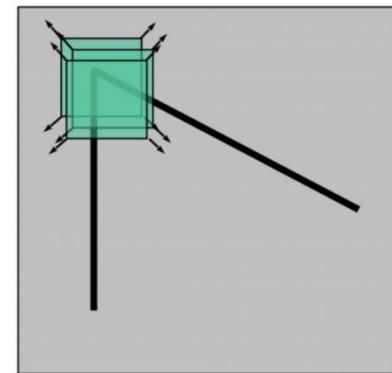
Intuition: Find the patches where a window shift in *any direction* to give a *large change* in intensity.



“flat” region:
no change in any
direction

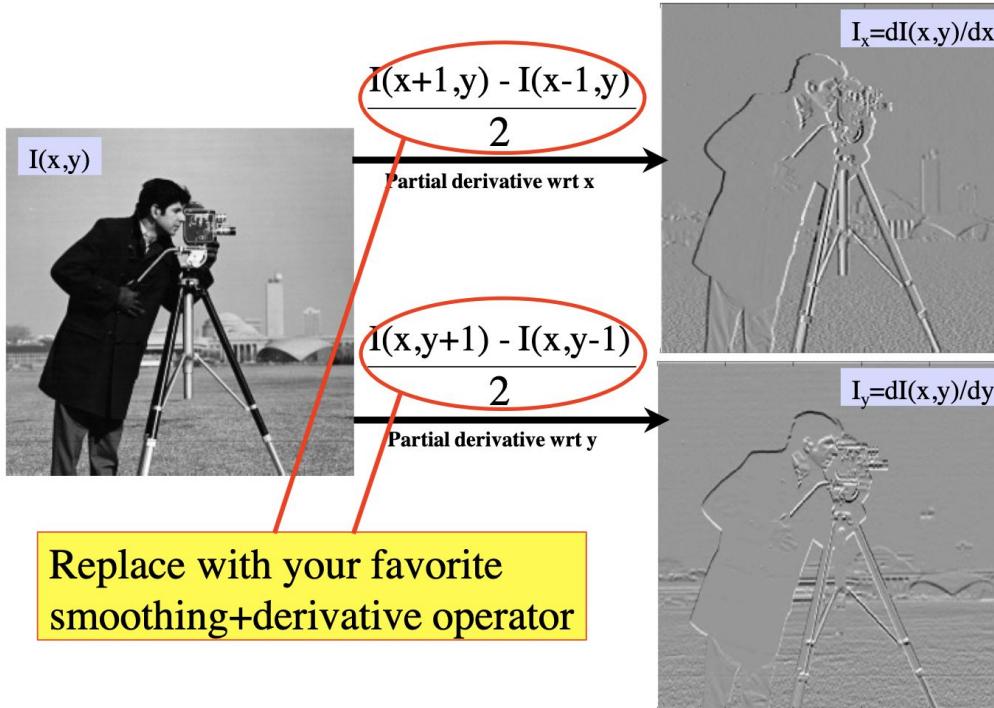


“edge”:
no change along the
edge direction



“corner”:
significant change in
all directions⁴⁶

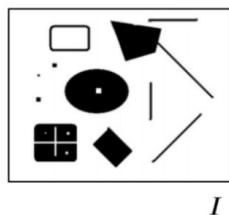
Another quick detour for computing “change”



Harris Corner Detector [Harris88]

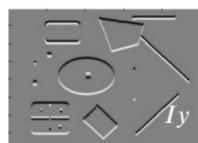
James Hays

Input image



Step 1. Edges

-1(black)~1(white)



Step 2. Compute \mathbf{M}

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$



Step 3. Compute cornerness

$$C = \det(\mathbf{M}) - \alpha \operatorname{trace}(\mathbf{M})^2$$

Step 4. Threshold on C

+ Non-maximal suppression

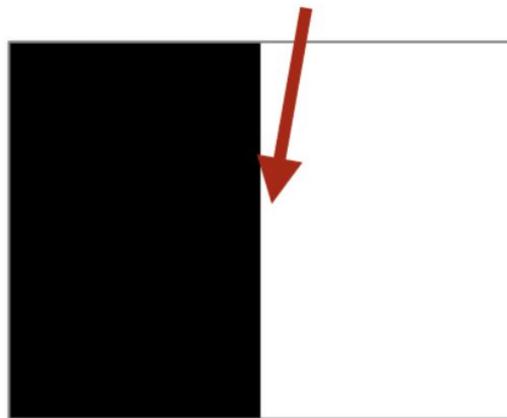


Corner detection result

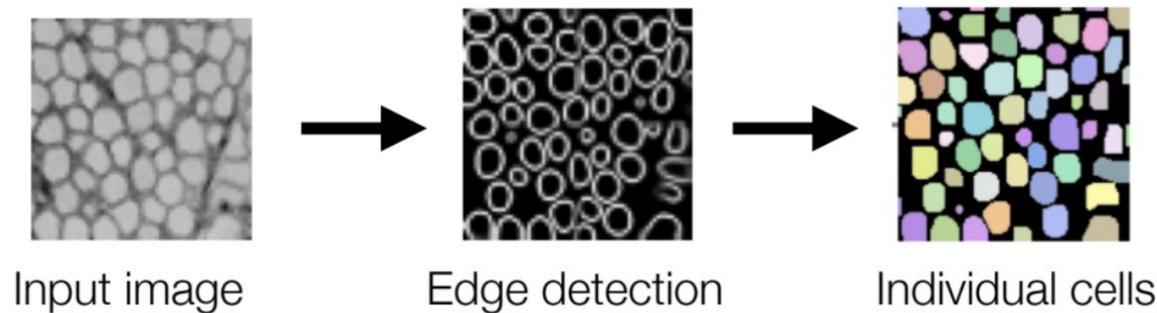
47

Edge detection

Edge: discontinuity between
image appearance



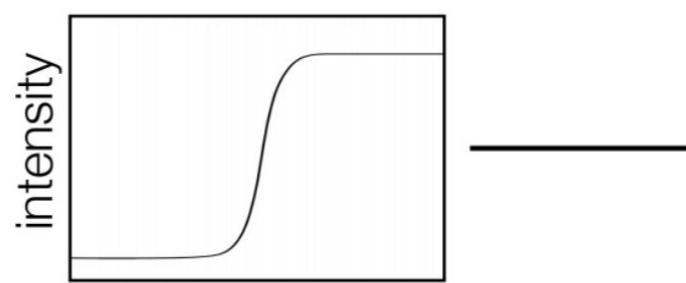
Application: cell segmentation (grouping)



Edge = an area of maximum change

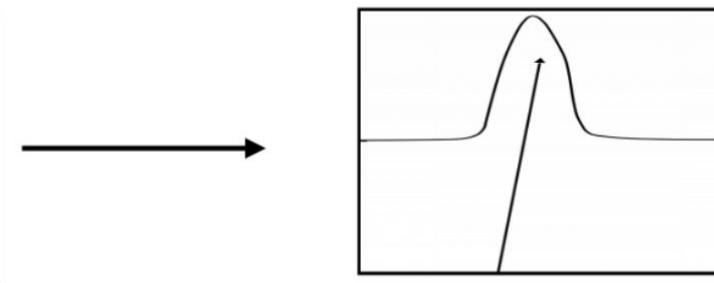


Image



1D profile

$f(x)$



Edge filter

$$f(x+1) - f(x) = f(x) * [1, -1]$$

$$\begin{array}{|c|c|c|} \hline f[0] & f[1] & f[2] \\ \hline \end{array} * [1, -1] = \begin{array}{|c|c|} \hline f[1]-f[0] & f[2]-f[1] \\ \hline \end{array}$$

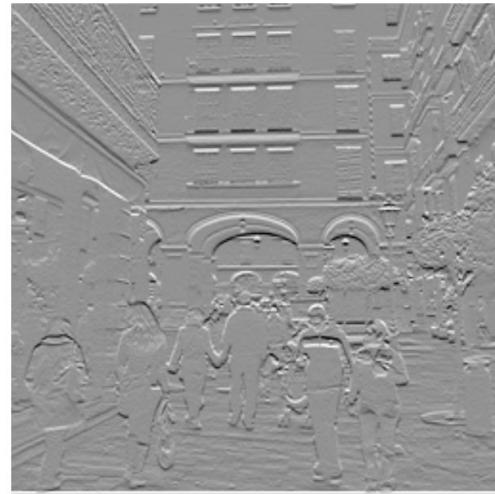
(flip kernel+dot product) 50

Horizontal edge: $[-1 \ 1]^T$



Input image

$$* [-1, 1] =$$



Horizontal edge

Vertical edge: [-1 1]



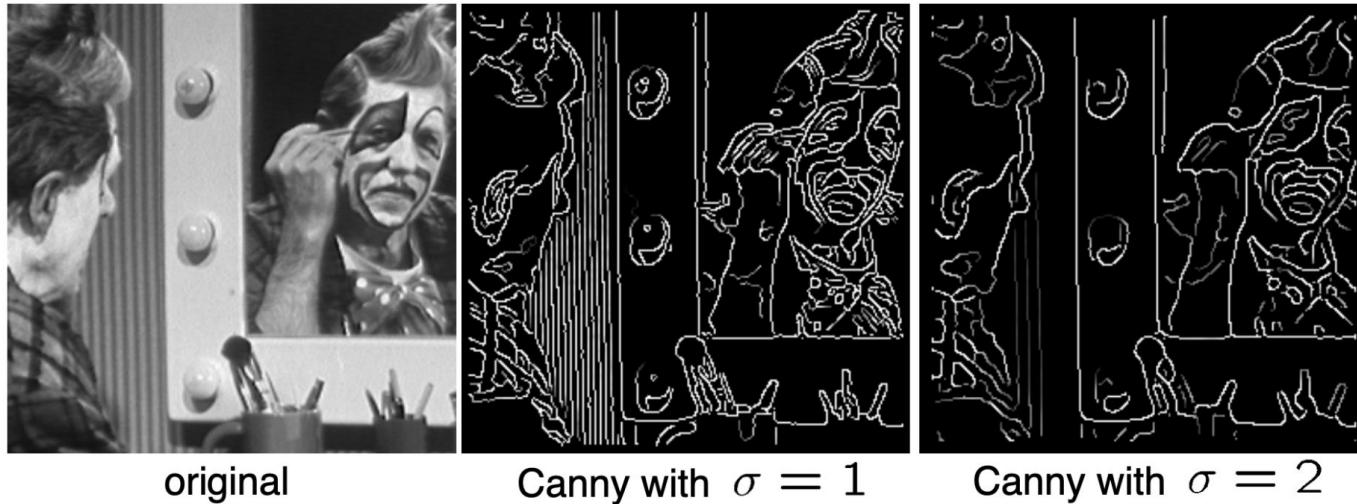
Input image

* [-1, 1] =



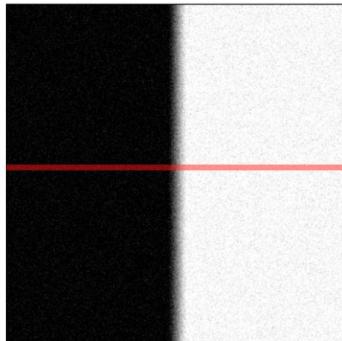
Vertical edge

Canny edge detector

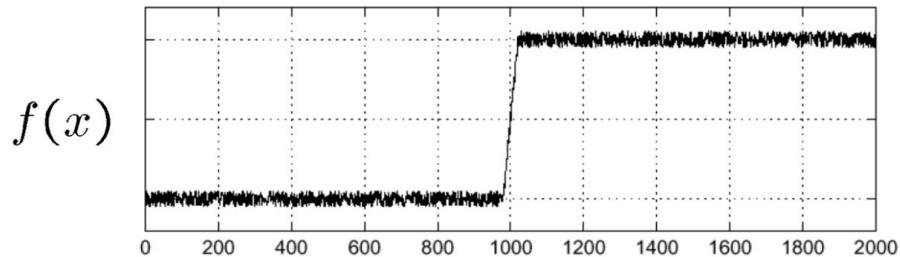


- The choice of σ depends on desired behavior
 - large σ detects “large-scale” edges
 - small σ detects fine edges

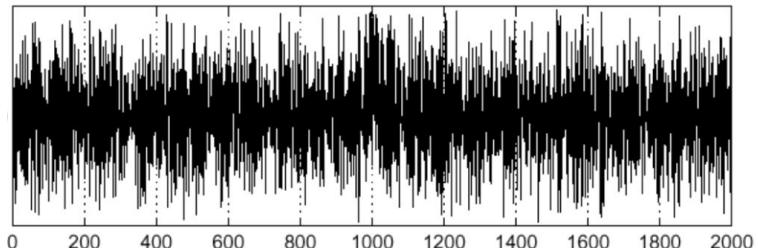
Idea 1: Robust to noise



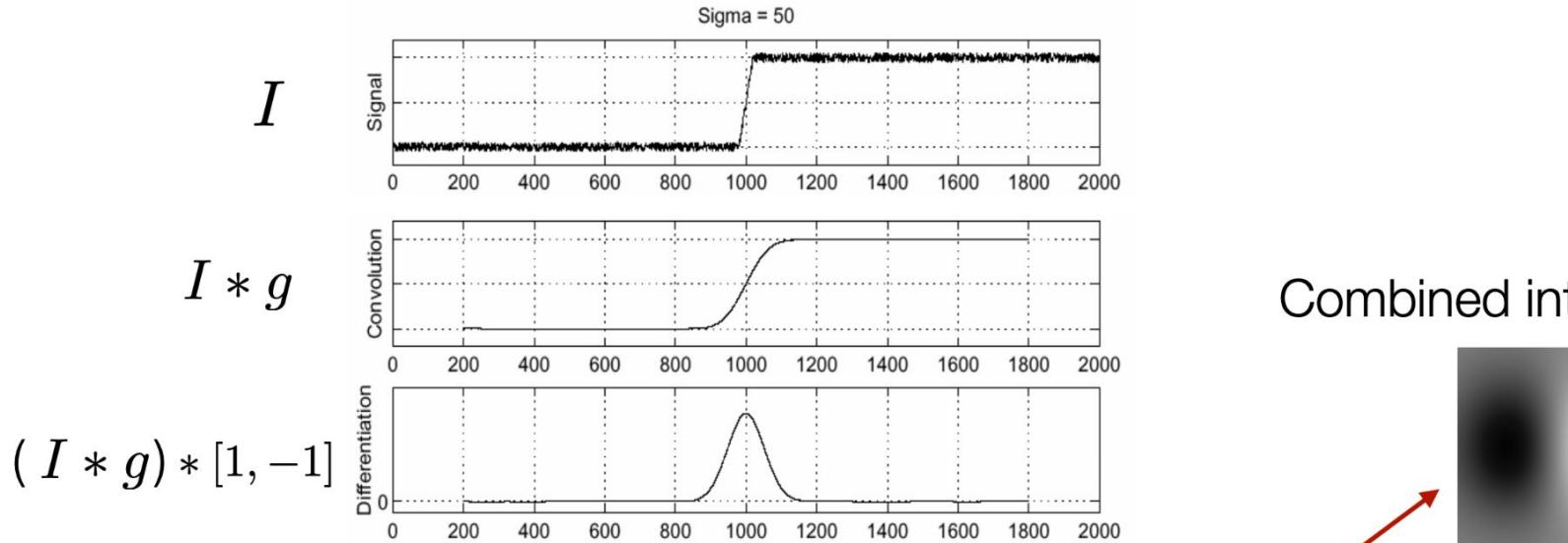
Noisy input image



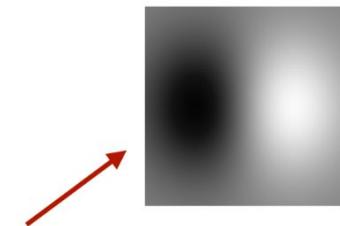
$$\ast [1, -1]$$



Solution: smooth image first



Combined into 1 filter



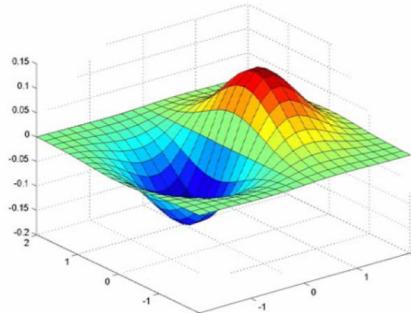
Associativity
of convolution

$$(I * \text{[blurred image]}) * \text{[vertical bar]} = I * (\text{[blurred image]} * \text{[vertical bar]})$$

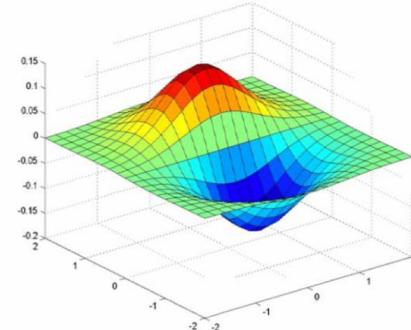
$$(a+b)+c = a+(b+c)$$

55

Derivative of Gaussian filter

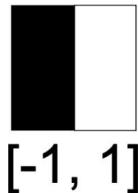


x-direction

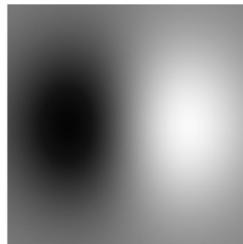


y-direction

Naive edge filter



Robust edge filter

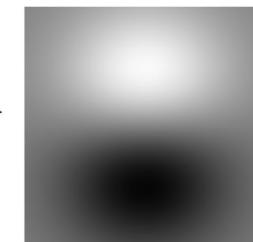


DoG-x

$$[-1, 1]$$



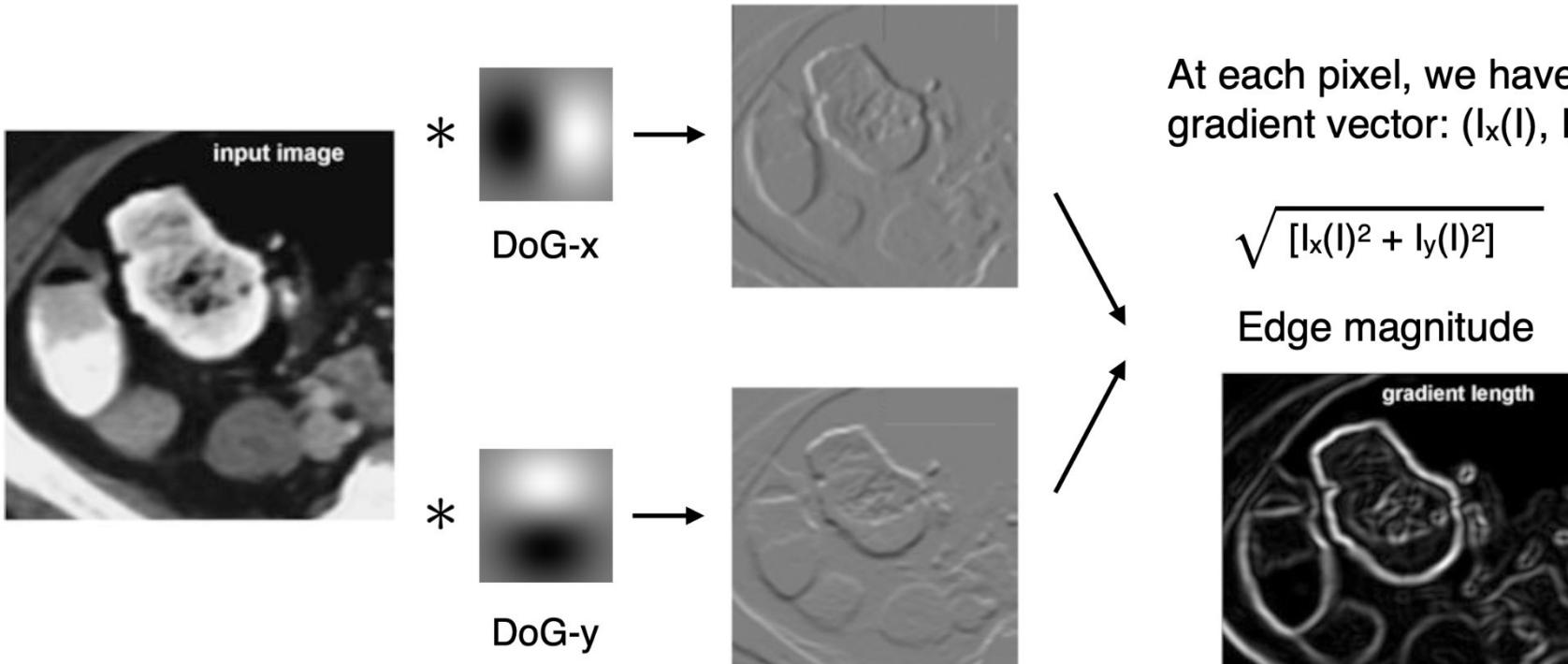
$$[-1, 1]^T$$



DoG-y

56

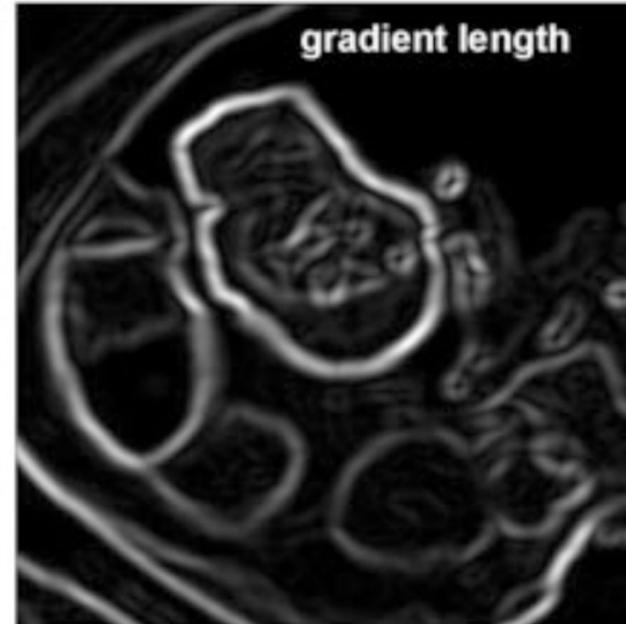
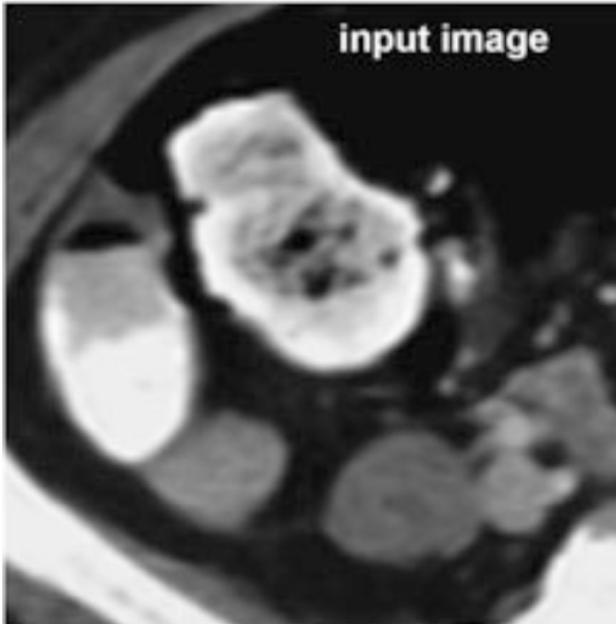
Edge: Norm of the gradient vector



*We lose linearity here

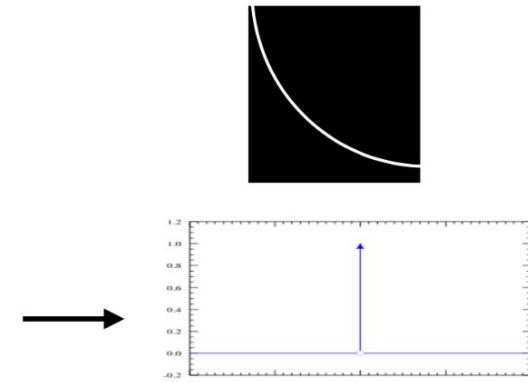
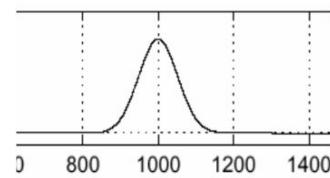
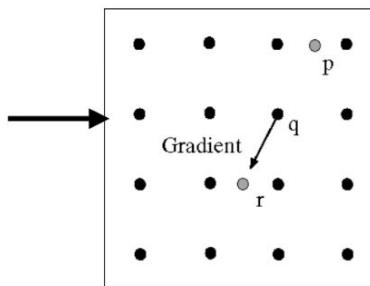
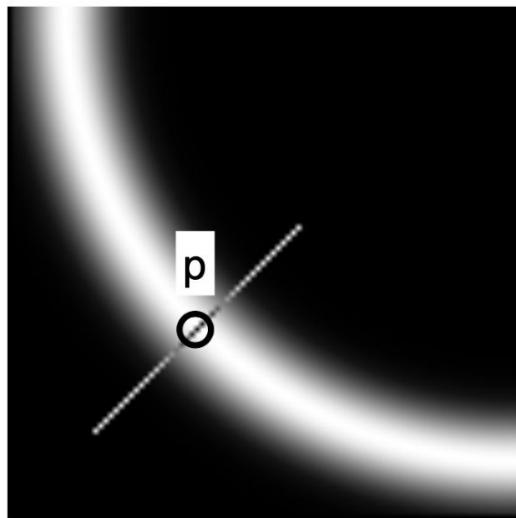
57

Idea 2: Need thin/sharp edge



Edge is too thick!

Non-maximum suppression (NMS)

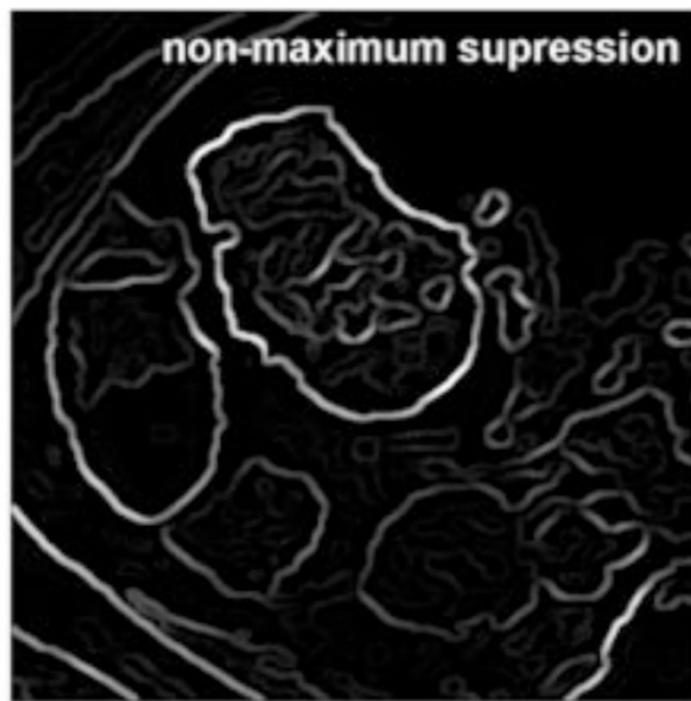
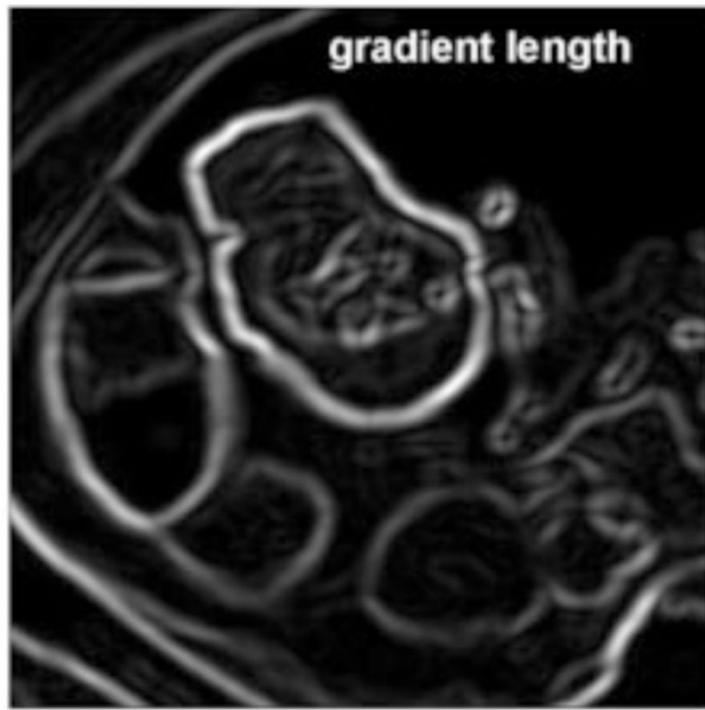


Thick edge

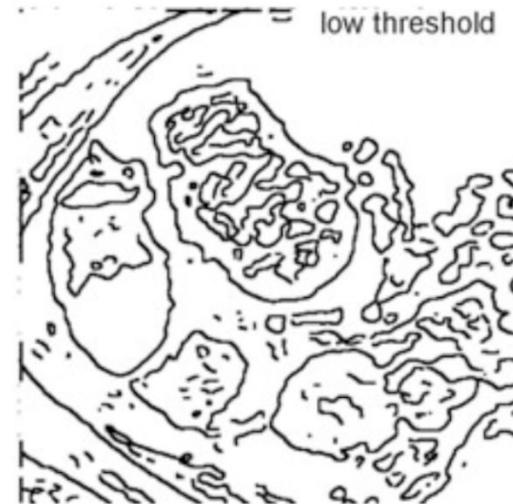
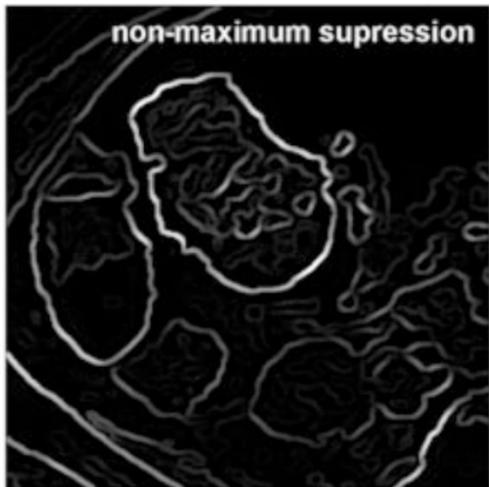
Profile along the gradient direction

suppress non-max to 0

After Non-max Suppression (NMS)



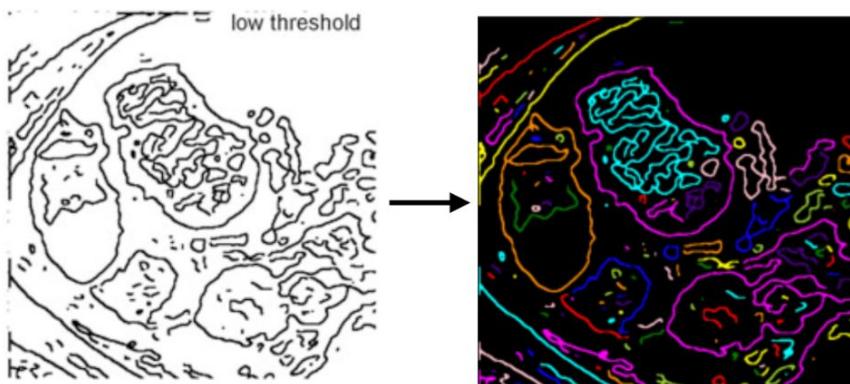
Idea 3: Two types of edges



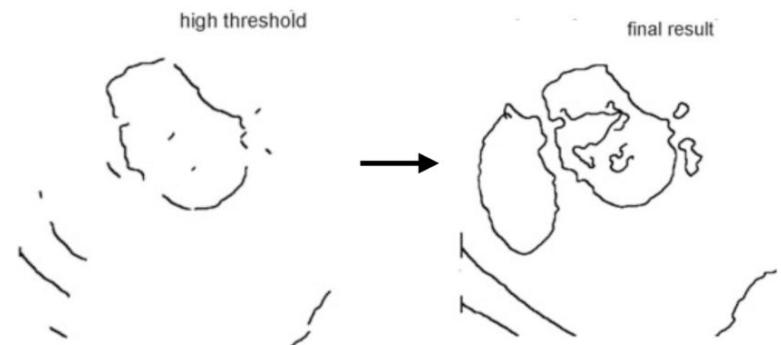
- Good: strong edges
- Bad: broken/short edges
- Good: long edges
- Bad: noisy weak edges

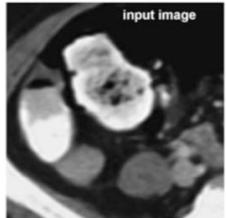
Connecting edges (hysteresis thresholding)

1. Find connected component

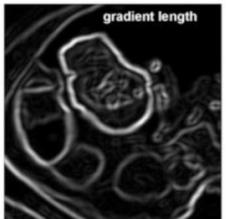


2. Use strong edge as **seeds** and select weak components



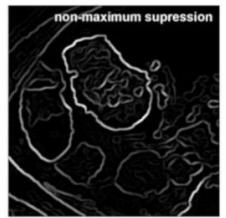


Put it together: Canny edge detector



1. Filter image with derivative of Gaussian and compute magnitude

(P1. Robust to noise)



2. Non-maximum suppression

(P2. Thin edges)



3. Linking and thresholding (hysteresis):

(P3. Long/Strong edges)

- Define two thresholds: low and high
- Use the high threshold to start edge curves and the low threshold to continue them

63

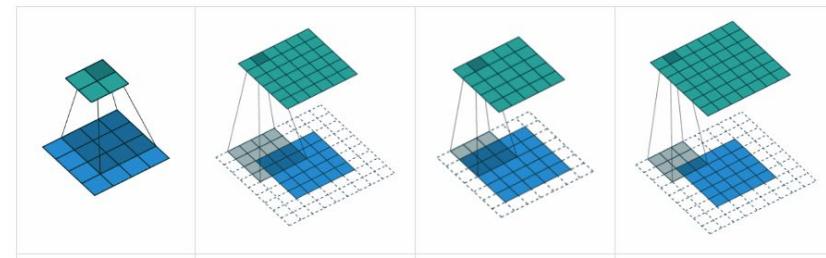


Recap of today

Showed convolutions as the basic building block of most filtering operations

Used combinations of simple filtering to yield more complex filters from images

Each filter has its own implicit target information within an image (which makes it ripe for learning!)



pixels

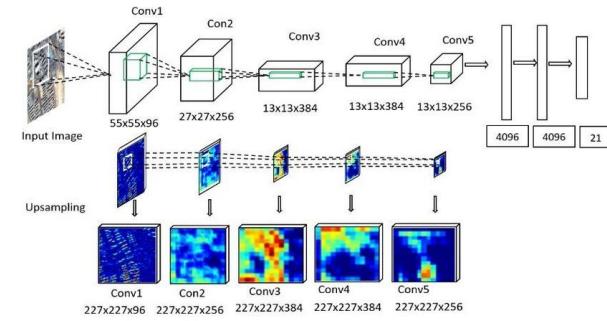
243

243	239	240
242	239	218
243	242	123

patches

227x227x96	227x227x256	227x227x384	227x227x384	227x227x256
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features



Low level → High level

Next: code lab



Next week: Feature detection and matching

