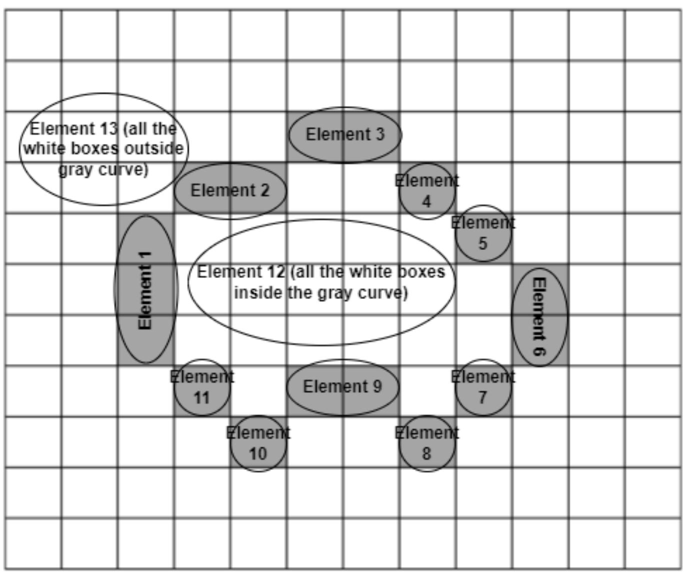
ASSIGNMENT 1

Q1 Connectivity: Connected component labeling (25)
The famous theorem of Jordan states that every simple closed curve in the plane separates the complement into two connected nonempty sets: an interior region and an exterior. Although this looks simple and intuitive, the proof can get very complex (for those who want to look it up: Link). Elements are a closed curve and the complement, which is the plane, which is divided into 2 regions. Now let us explore how the continuous case transfers to our discrete domain.

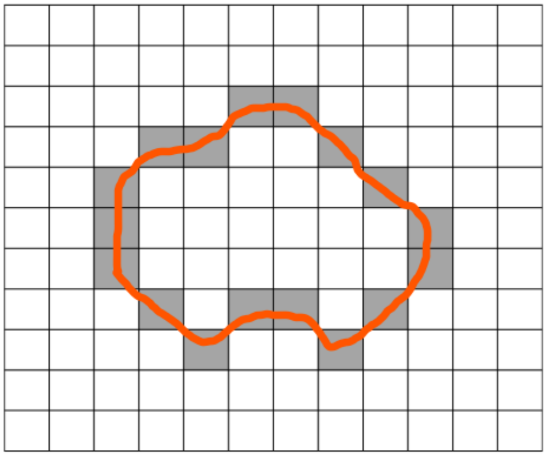
Given the following pixel image with white areas and a gray closed pixel curve. How many connected components can you detect?
By d4 adjacency, we get 13 connected components and by d8 adjacency, we get 2 connected components.

a) Apply d4 connectivity to all gray and white structures. Number of 4-connected structures? Label one or all connected structures and report the longest structure.(7)



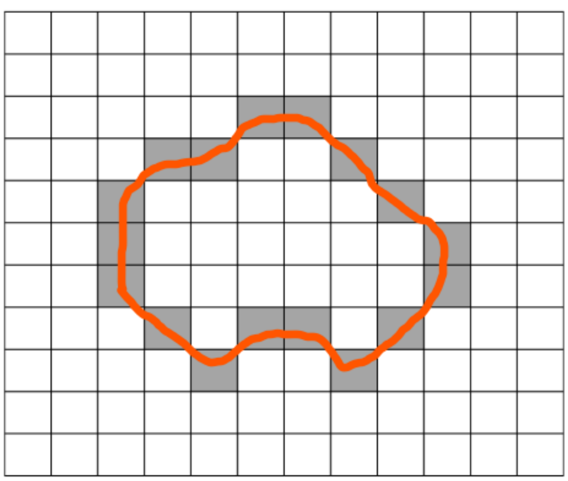
There are a total of 13 4-connected structures labeled as element 1 to element 13 in the above image. There are a total of 11 gray elements and 2 white elements. As the components are 4-connected, the structures within the components are connected with neighboring structures that are either horizontally or vertically next to the structure. The largest gray element is Element 1 with length 3 where as the largest connected structure in the image is Element 13 which includes all the white structures outside the gray enclosure. The length of the largest gray connected structure, i.e. length of Element 1 = 3.

b) Apply d8 connectivity to all gray and white structures. Number of 8-connected structures and label one or all connected structure and report the longest path.(7)



There are a total of 2 8-connected structures, 1 gray and 1 white. The gray structure is traced via the red line creating an enclosure. As the components are 8-connected, they can be connected diagonally as well, hence forming 1 continuous structure. Similarly all the white grid boxes are connected with eachother forming another singular continuous structure. The longest path is marked by the element connecting all the white structures. The longest gray element has length = 17

c) Now think about the continuous domain where we have 3 connected elements, one curve and two regions (exterior and interior). How could you get the same result given d4 and d8 metrics? Explain briefly and also calculate the length and mark the trajectory on the curve.(11)



In the above image, if we assume the gray structures to be connected via d8, it forms 1 single enclosure (or a curve) which divides the white pixels into 2 groups, interior (inside the enclosure) and exterior (outside the enclosure). This division is possible when the white structures follow d-4 connectivity. Since white structures cannot connect with each other diagonally, it will separate out the interior with exterior. The length of the curve is 17 structural units.

2. Filtering: (25) Given the following 1-D filter and 1-D pixel image:
a) Apply a 3 pixel average box filter and add the resulting values into the empty boxes below. (9)

(1/3)	1	1	1

Box Filter

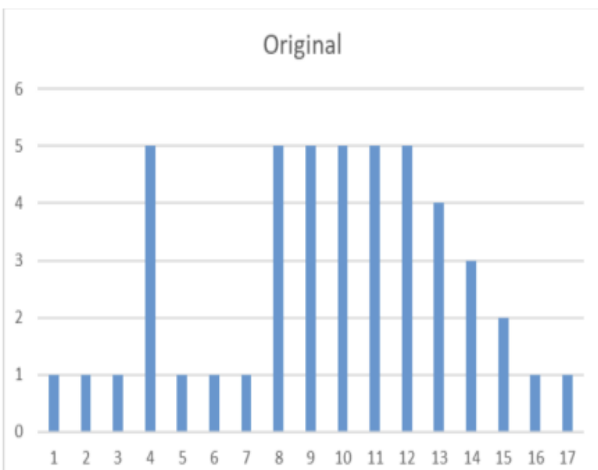
Image Signal

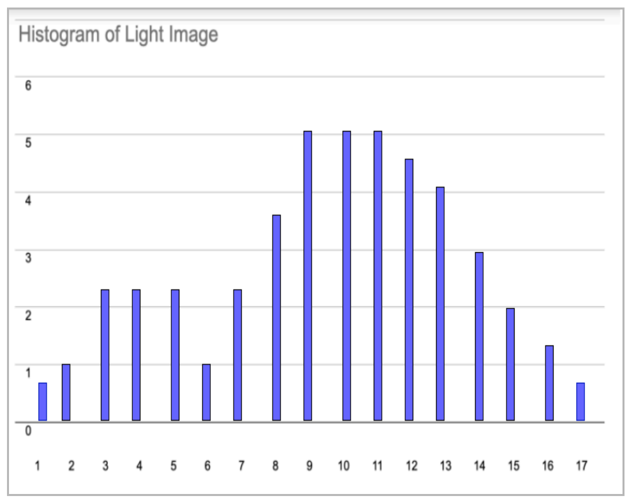
0	1	1	1	5	1	1	1	5	5	5	5	5	4	3	2	1	1	0

Filtered Signal

	0.66	1	2.33	2.33	2.33	1	2.33	3.66	5	5	5	4.66	4	3	2	1.33	0.66	

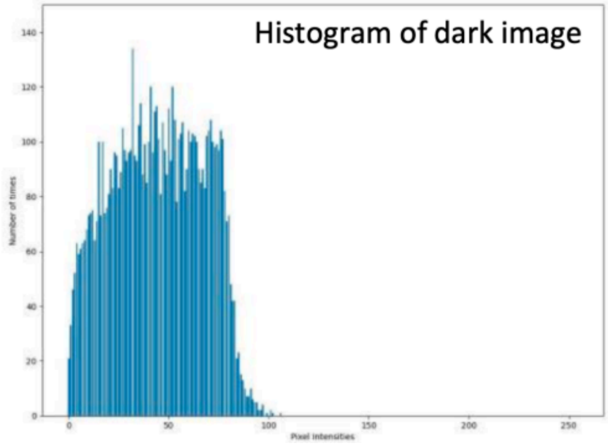
b) Plot the original image signal into the left graph, and the filtered signal into the right one. The horizontal axis is the pixel location, and the vertical axis the intensity value (9)

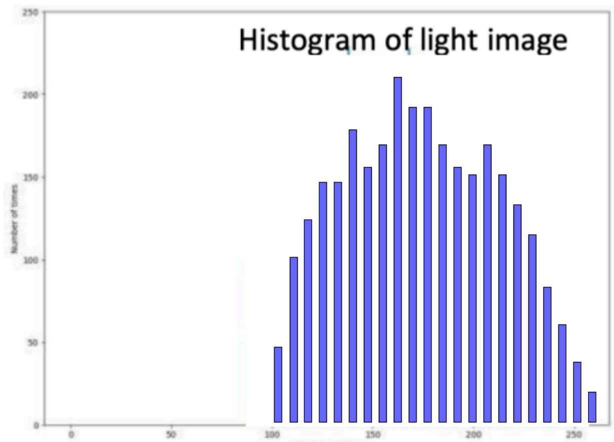


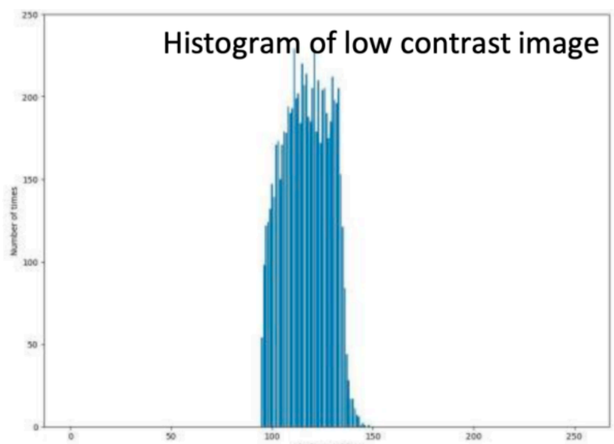


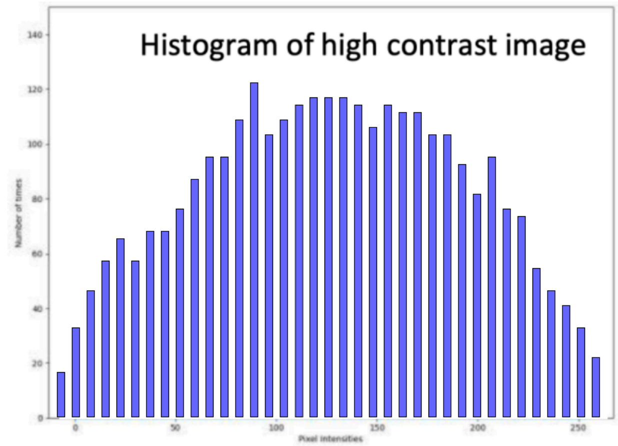
c) Briefly discuss the result after linear filtering when comparing the two plots. (7)
We can clearly observe the blurring effect the averaging kernel has on the original image. The sharp transition in pixel intensity value at pixel 4 and 8 in the original image are made smooth by distributing pixel intensity to the neighboring pixels in the filtered image, hence blurring the image (at edges). We can also observe that the contrast is significantly reduced as the transition from low intensity to high intensity pixels is not sharp anymore.

3. Histograms (25) Below are a few examples of the histograms of the images. Roughly sketch the histogram for the a) light image and b) high contrast image and c)briefly explain your reasoning.(10)(10)(5)









In the light image as displayed in part b), there will be larger number of high intensity pixels and the distribution appears to shift to the right (towards the higher intensity pixels). CDF in the darker image increases quicker at low intensity pixels and becomes equal to 1 around pixel intensity 100 as given in the original histogram but in the lighter image, the CDF remains low till we reach high intensity pixels and rapidly increases to 1.
In the low contrast image, we have pixels from only a certain range of pixel intensity (around 100 to 150) but in the high contrast image, the distribution is spread out throughout the intensity spectrum thus having a significant number of pixels from every intensity rage, providing a high contrast image.

4. Non-Linear Filtering and Shift Invariance (25) The median, z, of a set of numbers is such that half the values in the set are below z and the other half are above it. With filtering such as cross-correlation and convolution, we discussed properties such as: Shift invariance: Recall the definition, if g(x) is the response of the system to f(x), then g(x-x0) is the response of the system to f(x-x0):

$$g = w \circ f \qquad g(x) = \sum_{s=-a}^{s=a} w(s) f(x+s) = w \circ f(x) \qquad g(x-x_0) = \sum_{s=-a}^{s=a} w(s) f(x - x_0 + s) = w \circ f(x - x_0)$$

a) Discuss in a few words why or why not a median filter fulfills the shift invariance properties:(5)
Let f(x) be a 1-D discrete signal image such that;
f(x) = [104, 206, 52, 173, 255]
By applying median filter to the signal, we get:
g(x) = [104, 173, 173] (where, g(x) is the output filtered signal)

Now, lets shift the input signal of the image by a constant integer (eg: 2)
f(x+2) = [173, 255, 104, 206, 52]
And g(x+2) = [173, 173, 104]
By applying median filter on f(x+2), we get:
Median (f(x+2)) = [173, 206, 104]
We can observe that g(x+2) ≠ Median(f(x+2))

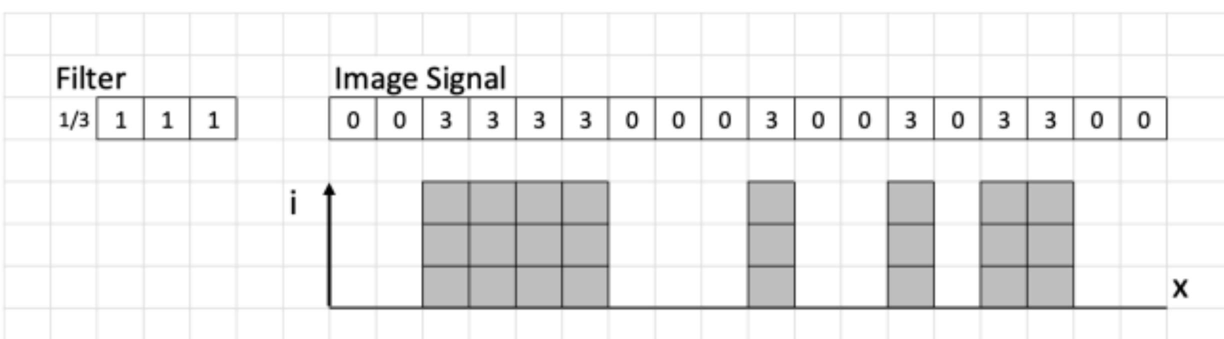
Thus, median filter does not follow shift invariance. This is majorly due to the fact that median filter heavily relies on the relative order of the pixel values within the local neighborhood and shifting the input changes the set of values being considered in each neighborhood. As a result, the obtained output in our case is significantly different after the shift.

b) Show by numerical example (e.g. create two 1-D pixel images with 3 elements each) if a median filter is a linear or nonlinear operator. Remember that the additivity property for linearity requires that OP(A+B) = OP(A)+OP(B), with OP representing an operator, here the median, and A and B are two small 1-D pixel images, for example.(5)

Let image A be a 1-D image such that;
A = [101, 248, 49]
Thus the median of the three pixels in A without any zero padding will be **101.**
Let image B be another 1-D image such that;
B = [58, 20, 127]
Thus the median of the three pixels in B without any zero padding will be **58.**
Let C be another 1-D image such that C = A+B;
C = [101+58, 248+20, 49+127] = [169, 268, 176]
Thus the median of the three pixels in C without any zero padding will be **176.**

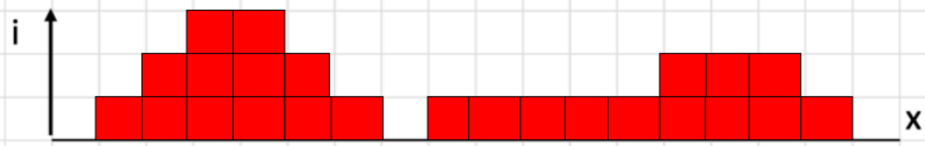
For it to be a linear operation, we want median of C = median of A + median of B;
But, (101 + 58) = 159 ≠ 176
Thus, median filter is not a linear operation.

c) Consider the following 1-D pixel images, shown with numbers and also a plot of x versus i(x).



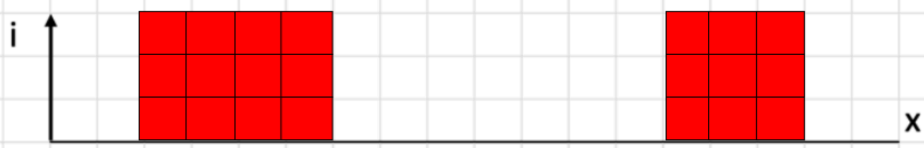
c) Apply a linear filter as shown above to the left to the pixel image. Provide the result as pixel numbers and also a plot of x versus i(x).(5)

Linear Filter Result		1	2	3	3	2	1	0	1	1	1	1	1	2	2	2	1	



d) Apply a median filter of width 3 to the image signal. Provide the result as pixel numbers and also a plot of x versus i(x).(5)

Median Filter Result		0	3	3	3	3	0	0	0	0	0	0	0	3	3	3	0	



e) Briefly compare and discuss the two results.(5)

We can observe that the linear averaging filter has resulted in a blurring effect on the original image. It has also smoothened the image substantially as we can see that pixel 10 and 13 originally having value 3, are smoothened into a pixel with value 1 distributing the intensity to neighboring pixels.

On the other hand, median filter being a non linear filter has effectively eliminated noise from the image i.e. at pixel 10 and 13. Moreover, it has not caused any blurring effects with sharp transitions in pixel intensities.