Binary Trees

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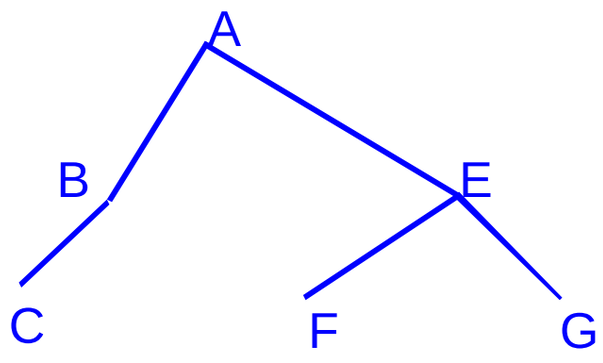
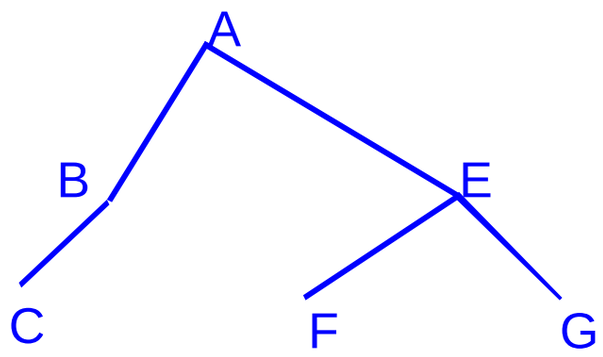
Binary Tree

A binary tree is a tree with each node having at most 2 children.
We often call the children of an inner nodes the 'left' or 'right' child.



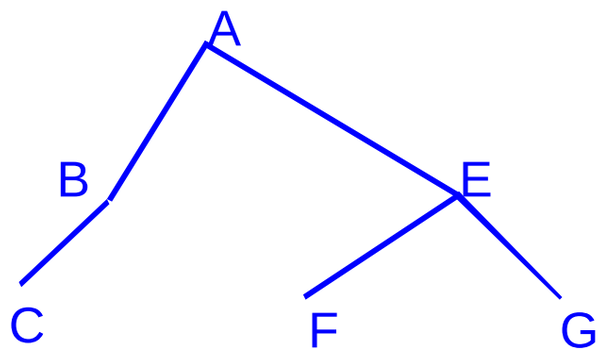
DFT of a Binary Tree

- DFT of a binary tree can be done recursively.
- A tree can be traversed by:
  - visit the root,
  - traverse the left subtree,
  - traverse the right subtree,
- This is pre-order traversal.
- The order traversed is : A, B, C, E, F, G.



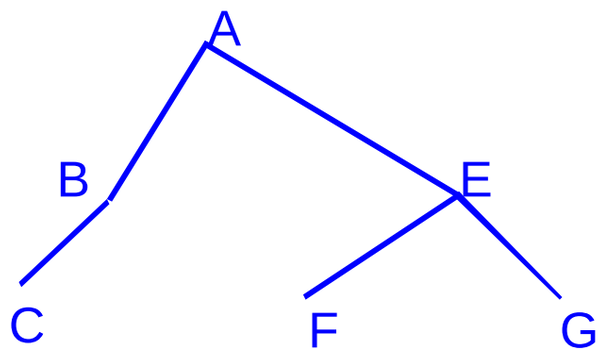
DFT of a Binary Tree

- There is also in-order traversal:
  - traverse the left subtree,
  - visit the root,
  - traverse the right subtree.
- The order traversed in the below tree is : C, B, A, F, E, G.



DFT of a Binary Tree

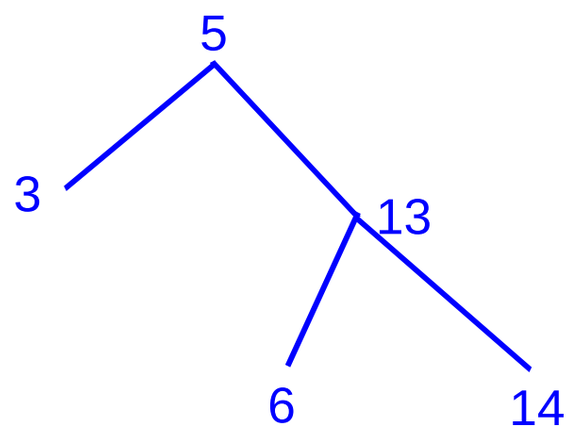
- There is also post-order traversal.
  - traverse the left subtree,
  - traverse the right subtree,
  - visit the root,
- The order traversed in the below tree is : C, B, F, G, E, A.



Binary Search Tree

- If the data stored in the nodes (or a key which forms part of the data) can be ordered then the nodes of the binary tree can be organized as a binary search tree.
- A Binary Search Tree can be used to store a set of elements.

e.g. the set $\{13, 3, 5, 6, 14\}$ can be store via the below binary search tree:



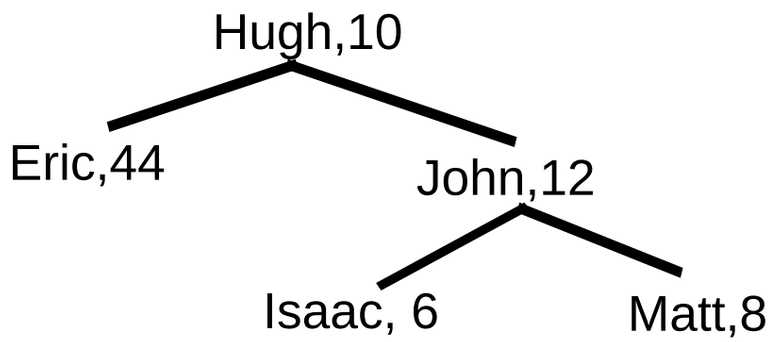
Binary Search Tree

- A Binary Search Tree can be an efficient way of storing a table.

So the table:

Hugh	10
Matt	8
Isaac	6
Eric	44
John	12

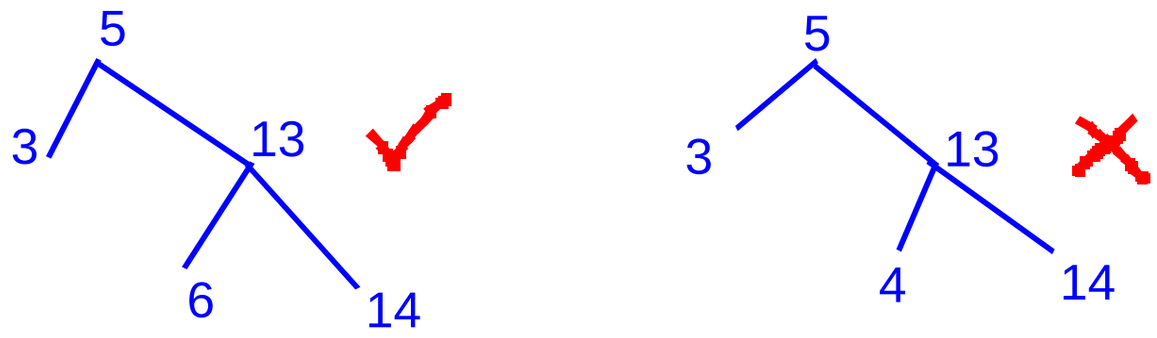
could be stored using the binary search tree:



Binary Search Tree

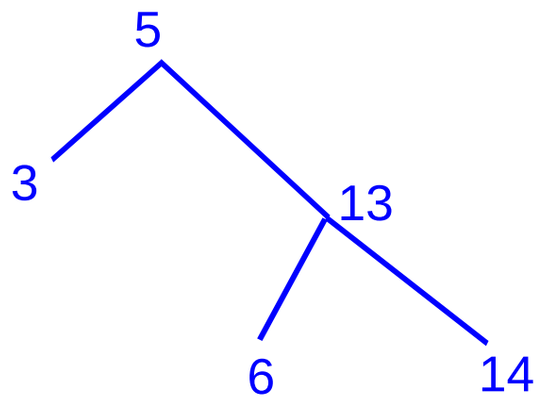
A **Binary Search Tree** is a binary tree where either:

- the root has no children, or
- the following three conditions hold:
  - all the nodes in the left subtree are less than the root node,
  - all the nodes in the right subtree are greater than the root node,
  - both children are binary search trees.



Binary Search Tree - Search

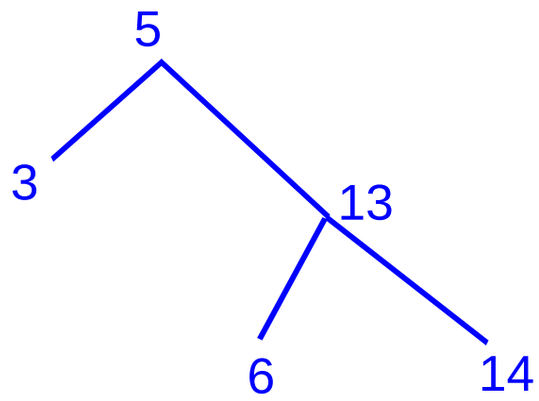
- Elements can be searched for recursively:
  - if the element we are searching for is at the root then we have found it,
  - if the element we are searching for is less than the root then search the left subtree,
  - else (the element must be greater than the root element) search the right subtree.
- If the tree is balanced than this is O(lg n) where n is the number of elements in the tree.



Binary Search Tree - Add

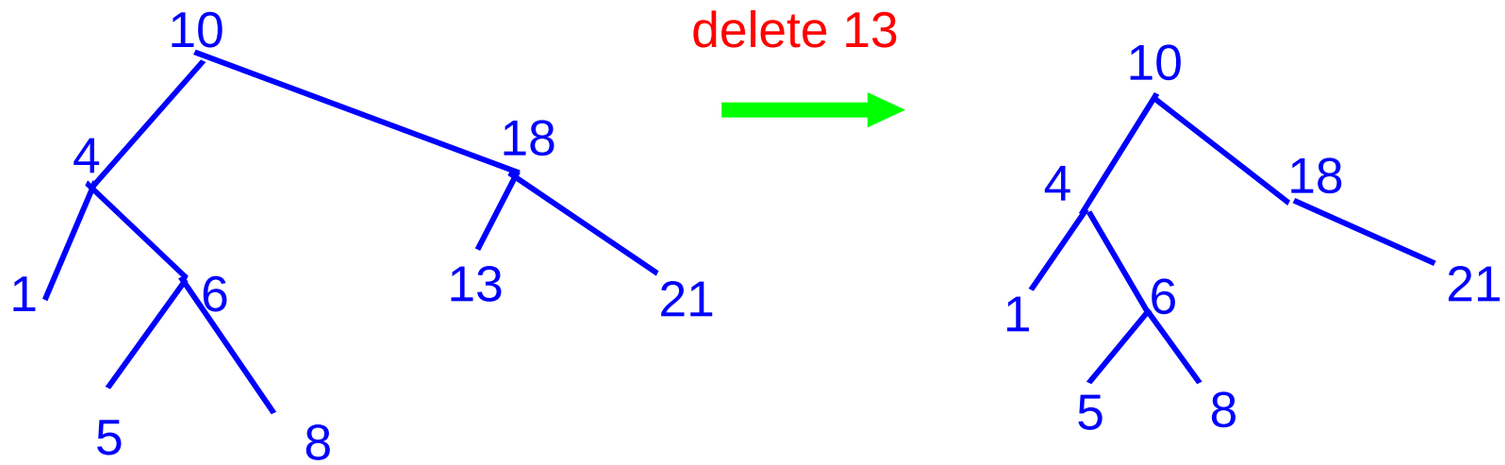
- Elements can be added to the binary search tree by recurring down the tree (basically the same as the search method) and then added as a leaf node at the appropriate position.

- This is also O(lg n) where n is the number of elements in the tree. (assuming the tree is balanced)



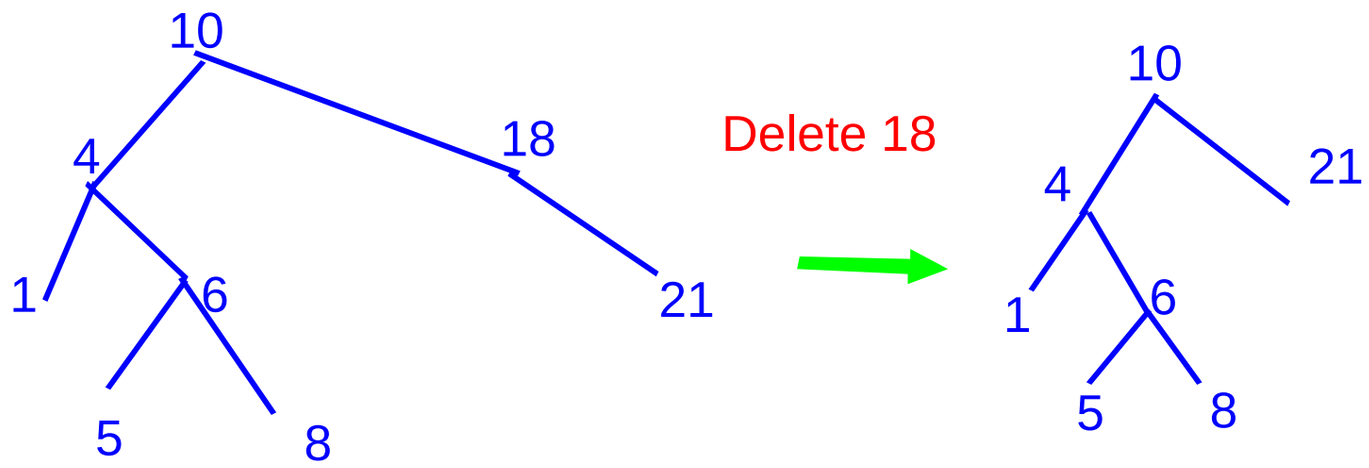
Binary Search Tree - Delete

- If the element to be removed is a leaf node then it can just be removed.



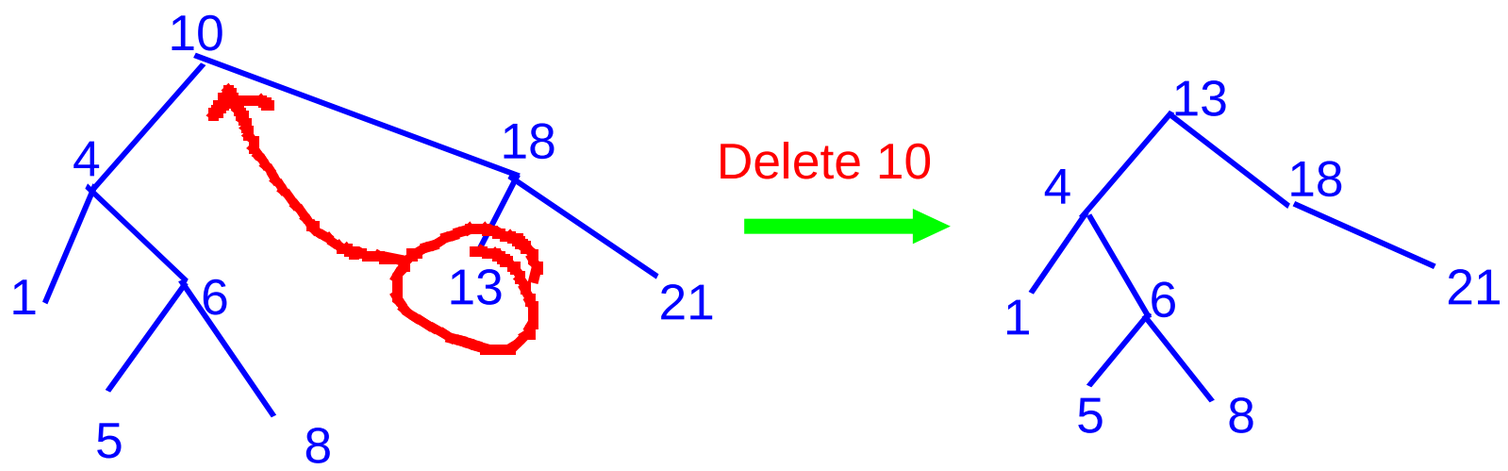
Binary Search Tree - Delete

- If the element to be removed is an inner node with only one child than it can be deleted and the child subtree of the deleted element can be moved into the place of the deleted element's subtree.



Binary Search Tree - Delete

- If the element to be removed is an inner node and it has two children then the minimum element from the right sub-tree can be moved into the deleted elements place. (or the maximum of the left sub-tree)
- Delete is O(lg n) where n is the number of nodes in the tree. (assumes a balanced tree)



Binary Search Tree - Balanced

- The shape of the tree will depend on the order elements are added. If elements are added in ascending (or descending) order than the binary search tree will look (and perform) like a linked list.
- A tree with $2^k - 1$ elements can be placed in a tree with height $k - 1$. If a binary search tree is within some constant range away from this optimum height then it would be considered balanced (note different algorithms will define "balanced" in different ways).
- There is a number of approaches to efficiently maintain a "balanced" tree as elements are added. (AVL Tree's, Red-Black Tree's)

Example Exam Questions

- Define a binary tree. What are some different approaches for traversing a binary tree?
- Write a method that would determine if a binary tree is a binary search tree.