Exercise 1. In the following definition of a probabilistic model :

$$y_i \sim Normal\,(\mu, \sigma)$$
$$\mu \sim Normal\,(20, 10)$$
$$\sigma \sim HalfCauchy\,(20)$$

We know that the formule bayes : $P(B|A) = \frac{P(A|B)P(B)}{P(B)}$

(Here we have$P(B) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$.

If we want to use a sample data y to research the parameter θ who is about the about the population, we can use Bayes frmular like :$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(\theta)}$, so we can know that :

$P(\theta|y) \propto P(y|\theta)P(\theta)$.

Here, $P(\theta|y)$ is our posterior,

$P(y|\theta)$ is our likelihood,

$P(\theta)$ is our piror.

From exercise 1, we know the distribution is the distribution normal with the expectation is μ, and variance δ.

The distribution of μ is also distributon normal, →Normal(20,10)

The distribution of δ is distribution halfcauchy, →HalfCauchy(20)

So we can know that : the parameters of the posterior are : μ and δ, according to 3 distribution above.

Now, we can respond to these questions

1. Identify the prior.

   The prior is : (μ, δ) ~ (Normal(20,10), HalfCauchy(20))

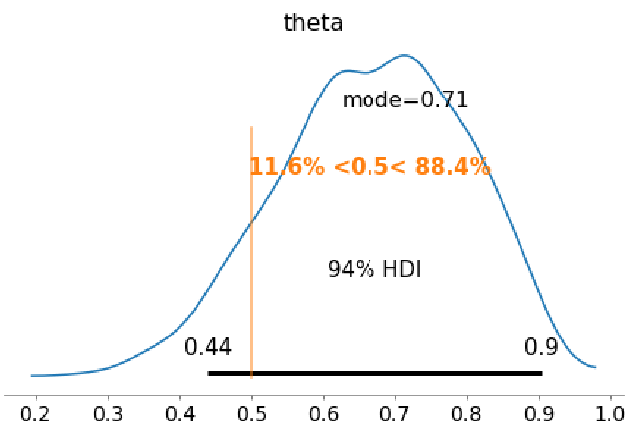
2. Identify the likelihood:

   Our likelihood is y_i ~ Normal(μ,δ).

3. How many parameters will the posterior have ?

   We have 2 parameters, they are μ and δ

Exercise 2. After a Bayesian analysis we have obtained the following plot of the posterior distribution.



This is a picture of the posterior, we can know:

(1) The mode of posterior is 0.71(the parameter with that we can have the density of probability maximum)

(2) From the picture, the line yellow is the point of $\theta = 0.5$(It's in the interval HDI). And in the distribution of posterior, the probability of $\theta >= 0.5$ is 88.4%, the probability of $\theta <= 0.5$ is 11.6%.

(3) The interval of 94% HDI is [0.44, 0.9]

1. a) What's the MAP estimator for the parameter theta ?

   The MAP can provide a point estimate for the amount that cannot be directly observed in the experimental data, and the value estimated is the mode of the distribution of posterior, for our example is 0.71.

2. Say True or False. Justify the false ones.

   b) There's at least a 94% chance that theta is between 0.3 and 0.9

   False.

   The probability of theta in [0.44, 0.9] is 94%.

   c) The probability that theta is between 0.7 and 1.0 is bigger than 94%.

   False, it's smaller than 94%.

   d) With at least 85% certainty, we can say that theta is bigger than 0.5.

   False, with at least 88.4% certainty, we can say that theta is bigger than 0.5.