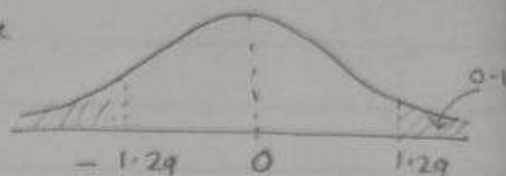Q1) A CAT test population std-dev is known to be 100. A sample of 25 test takers has a mean of 520. Construct a 80% confidence interval about the mean. Find upper & lower fence.

Sol
given $\sigma = 100$, $n = 25$, $\bar{x} = 520$, $CI = 80\% \rightarrow 0.80$

Since σ is given $\Rightarrow$ z test is applied here
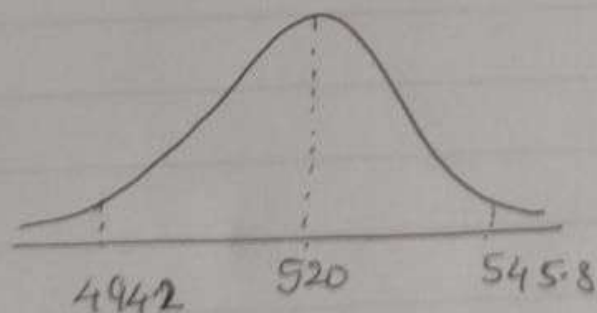
CI = 0.8 means $\alpha = 1 - 0.8 = 0.2$

$\Rightarrow \alpha/2 = 0.1 \Rightarrow$ [curve: 0.9, 0.1]

corresponding to $0.9 \Rightarrow$ z table give $z_{\alpha/2} = 1.29$

$\Rightarrow$ Upper fence $= \bar{x} + z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$ $= 520 + 1.29 \times \dfrac{100}{\sqrt{25}} = 520 + 1.29 \times \dfrac{100}{5}$

$= 520 + 1.29 \times 20 = 520 + 25.8 = 545.8$

$\Rightarrow$ Lower fence $= \bar{x} - z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}} = 520 - 1.29 \times \dfrac{100}{\sqrt{25}}$

$= 520 - 1.29 \times \dfrac{100}{5}^{20} = 520 - 1.29 \times 20 = 494.2$

494.2   520   545.8

Q A company has 100K employee, among a sample of 500 people 300 people ordered large & 200 people ordered XL.
How many people want large & XL in the whole company or how many L & XL must be ordered for whole company?

Sol

$n = 500$

Suppose $\hat{p} = \dfrac{300}{500} = 3/5$ (proportion of people who ordered large)

$$95\% \text{ confidence Interval} = \hat{p} \pm Z_{critical} \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$$

$$= \dfrac{3}{5} \pm 1.96 \sqrt{\dfrac{3/5 \cdot 2/5}{500}}$$

$$= 0.6 \pm 1.96 \ast 0.0219$$

$$= 0.6 \pm 0.0429$$

$$= (0.6429, 0.5571)$$