Supplementary Materials for "Gene-environment Interaction Identification via Penalized Robust Divergence"

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Table S1 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under linear regression and S1

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	200	LS	0.733(0.103)	0.515(0.181)	0.615(0.166)	3.55(1.40)	4.18(1.64)	0.780(0.112)	0.591(0.203)	0.693(0.185)	5.41(2.24)	6.87(2.65)
		LAD	0.781(0.103)	0.591(0.160)	0.659(0.150)	4.50(1.49)	5.19(1.68)	0.826(0.121)	0.683(0.189)	0.747(0.179)	6.16(2.75)	8.78(3.37)
		DPD	0.873(0.049)	0.721(0.082)	0.774(0.073)	5.65(1.12)	6.29(1.09)	0.939(0.035)	0.839(0.074)	0.879(0.059)	8.70(2.50)	11.79(2.66)
		γ	0.870(0.063)	0.727(0.106)	0.779(0.101)	5.41(1.21)	6.16(1.24)	0.933(0.054)	0.838(0.104)	0.877(0.093)	8.43(2.32)	11.16(2.70)
	400	LS	0.768(0.096)	0.596(0.191)	0.715(0.163)	4.33(1.91)	5.12(2.20)	0.806(0.104)	0.700(0.220)	0.817(0.183)	8.09(2.94)	10.94(3.59)
		LAD	0.865(0.071)	0.702(0.139)	0.768(0.115)	5.92(1.72)	6.74(1.77)	0.919(0.066)	0.818(0.147)	0.873(0.117)	9.48(3.22)	12.46(4.09)
		DPD	0.927(0.040)	0.842(0.060)	0.873(0.054)	7.51(0.72)	7.92(0.75)	0.976(0.018)	0.938(0.036)	0.954(0.028)	11.15(1.34)	16.47(1.79)
		γ	0.924(0.041)	0.844(0.059)	0.874(0.056)	7.52(0.85)	7.92(0.76)	0.975(0.017)	0.940(0.034)	0.956(0.026)	11.05(1.40)	16.04(1.76)
AR2	200	LS	0.713(0.095)	0.470(0.147)	0.569(0.143)	3.48(1.10)	3.87(1.25)	0.752(0.111)	0.542(0.167)	0.642(0.170)	3.68(2.15)	5.39(2.20)
		LAD	0.787(0.093)	0.580(0.152)	0.660(0.139)	4.07(1.46)	4.88(1.66)	0.831(0.110)	0.671(0.184)	0.752(0.165)	3.98(2.21)	6.33(2.59)
		DPD	0.884(0.052)	0.738(0.076)	0.795(0.067)	5.12(1.09)	6.21(1.06)	0.941(0.032)	0.838(0.066)	0.882(0.053)	5.81(2.31)	8.83(2.43)
		γ	0.881(0.049)	0.731(0.075)	0.795(0.066)	4.92(0.96)	5.99(1.08)	0.937(0.031)	0.831(0.066)	0.882(0.051)	5.82(2.24)	8.29(2.70)
	400	LS	0.783(0.099)	0.627(0.189)	0.742(0.161)	4.35(1.75)	5.29(2.21)	0.823(0.097)	0.677(0.207)	0.791(0.168)	5.71(2.43)	7.63(3.13)
		LAD	0.862(0.082)	0.691(0.147)	0.762(0.127)	5.36(1.67)	6.35(1.79)	0.910(0.078)	0.770(0.154)	0.835(0.127)	6.52(2.61)	9.10(3.27)
		DPD	0.918(0.039)	0.820(0.053)	0.857(0.053)	6.52(0.95)	7.54(0.73)	0.973(0.018)	0.923(0.035)	0.945(0.029)	7.65(2.16)	11.46(2.58)
		γ	0.916(0.038)	0.819(0.053)	0.857(0.052)	6.24(0.85)	7.61(0.78)	0.971(0.019)	0.925(0.034)	0.946(0.030)	7.39(2.25)	11.08(2.41)
Band1	200	LS	0.728(0.112)	0.491(0.170)	0.585(0.172)	3.59(1.16)	4.16(1.39)	0.761(0.121)	0.568(0.196)	0.663(0.192)	5.13(1.89)	6.37(2.23)
		LAD	0.783(0.100)	0.573(0.162)	0.648(0.147)	4.33(1.70)	5.01(1.87)	0.826(0.107)	0.687(0.183)	0.741(0.161)	6.05(2.78)	8.23(3.53)
		DPD	0.882(0.052)	0.739(0.093)	0.792(0.079)	5.51(1.30)	6.35(1.25)	0.944(0.035)	0.846(0.076)	0.888(0.060)	8.06(2.73)	10.84(3.26)
		γ	0.879(0.046)	0.743(0.080)	0.795(0.068)	5.50(1.20)	6.35(1.08)	0.942(0.033)	0.854(0.069)	0.894(0.055)	8.06(2.57)	11.04(2.81)
	400	LS	0.780(0.093)	0.621(0.187)	0.733(0.160)	4.59(1.82)	5.46(2.03)	0.822(0.096)	0.730(0.207)	0.841(0.173)	7.92(2.72)	11.01(3.76)
		LAD	0.866(0.076)	0.709(0.140)	0.772(0.119)	6.03(1.83)	6.94(1.75)	0.922(0.058)	0.825(0.137)	0.878(0.108)	9.77(3.43)	12.53(4.32)
		DPD	0.926(0.047)	0.844(0.061)	0.871(0.062)	7.59(0.79)	8.02(0.72)	0.979(0.018)	0.945(0.034)	0.959(0.028)	11.18(1.72)	16.35(1.69)
		γ	0.928(0.046)	0.849(0.065)	0.879(0.063)	7.56(0.81)	8.01(0.73)	0.979(0.016)	0.947(0.034)	0.961(0.026)	10.84(1.43)	16.34(1.86)
Band2	200	LS	0.714(0.110)	0.476(0.172)	0.574(0.168)	3.67(1.30)	4.14(1.68)	0.756(0.119)	0.551(0.195)	0.651(0.187)	5.52(2.42)	6.85(2.72)
		LAD	0.777(0.118)	0.565(0.183)	0.640(0.173)	4.09(1.68)	4.89(1.89)	0.815(0.131)	0.667(0.204)	0.733(0.187)	5.89(2.64)	8.36(3.55)
		DPD	0.874(0.075)	0.717(0.123)	0.773(0.123)	5.37(1.30)	6.18(1.29)	0.933(0.063)	0.824(0.117)	0.867(0.113)	7.50(2.50)	9.66(2.95)
		γ	0.879(0.069)	0.731(0.113)	0.793(0.101)	5.18(1.29)	6.06(1.35)	0.935(0.063)	0.835(0.112)	0.881(0.099)	6.93(2.26)	9.78(2.67)
	400	LS	0.780(0.091)	0.622(0.183)	0.736(0.158)	4.42(1.86)	5.42(2.25)	0.823(0.096)	0.731(0.209)	0.843(0.173)	6.80(2.42)	10.84(3.67)
		LAD	0.857(0.077)	0.695(0.141)	0.758(0.121)	5.70(1.86)	6.74(1.77)	0.907(0.077)	0.802(0.155)	0.855(0.128)	8.15(3.33)	11.29(4.18)
		DPD	0.925(0.041)	0.830(0.060)	0.865(0.058)	7.23(0.79)	7.80(0.72)	0.973(0.021)	0.931(0.039)	0.949(0.032)	9.99(2.15)	14.69(1.93)
		γ	0.927(0.043)	0.839(0.062)	0.872(0.060)	7.34(0.77)	7.88(0.76)	0.974(0.020)	0.936(0.037)	0.953(0.031)	9.86(1.69)	14.47(2.09)

Table S2 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under linear regression and S2

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	200	LS	0.706(0.118)	0.496(0.182)	0.606(0.173)	3.34(1.49)	3.96(1.74)	0.766(0.130)	0.590(0.231)	0.692(0.205)	6.55(3.25)	8.66(3.79)
		LAD	0.769(0.094)	0.572(0.137)	0.632(0.133)	3.92(1.48)	4.66(1.54)	0.846(0.086)	0.698(0.161)	0.741(0.144)	8.45(3.53)	10.66(3.90)
		DPD	0.860(0.063)	0.693(0.091)	0.755(0.088)	5.19(0.96)	5.89(0.99)	0.947(0.043)	0.860(0.081)	0.894(0.070)	11.01(2.13)	13.04(2.33)
		γ	0.860(0.060)	0.703(0.088)	0.763(0.087)	5.23(0.99)	5.85(1.02)	0.943(0.041)	0.865(0.077)	0.898(0.066)	10.81(2.04)	13.10(2.38)
	400	LS	0.820(0.085)	0.569(0.156)	0.662(0.138)	5.01(1.80)	5.71(1.91)	0.894(0.076)	0.703(0.180)	0.774(0.139)	9.83(4.35)	11.76(4.89)
		LAD	0.860(0.069)	0.679(0.108)	0.740(0.102)	5.69(1.16)	6.42(1.24)	0.934(0.052)	0.828(0.101)	0.866(0.088)	11.68(2.69)	13.76(2.72)
		DPD	0.931(0.039)	0.819(0.074)	0.866(0.063)	6.70(1.00)	7.35(1.05)	0.978(0.024)	0.937(0.049)	0.953(0.039)	13.02(1.25)	16.62(1.86)
		γ	0.930(0.041)	0.831(0.074)	0.873(0.065)	6.74(1.07)	7.34(1.05)	0.976(0.025)	0.942(0.048)	0.958(0.038)	12.98(0.79)	16.66(2.01)
AR2	200	LS	0.714(0.098)	0.491(0.179)	0.605(0.159)	2.69(1.57)	3.25(1.74)	0.776(0.118)	0.582(0.243)	0.694(0.205)	4.81(2.77)	7.30(3.59)
		LAD	0.781(0.079)	0.583(0.120)	0.651(0.111)	3.51(1.22)	4.36(1.31)	0.868(0.081)	0.725(0.149)	0.781(0.131)	5.74(2.71)	8.88(3.58)
		DPD	0.864(0.048)	0.681(0.074)	0.752(0.067)	4.72(0.95)	5.60(1.06)	0.949(0.039)	0.862(0.080)	0.898(0.065)	7.86(2.29)	10.60(2.47)
		γ	0.866(0.050)	0.701(0.079)	0.766(0.074)	4.86(1.10)	5.75(1.11)	0.946(0.040)	0.868(0.079)	0.902(0.067)	7.62(2.18)	10.53(2.54)
	400	LS	0.839(0.063)	0.604(0.114)	0.692(0.101)	3.45(1.46)	4.46(1.42)	0.922(0.053)	0.768(0.128)	0.825(0.097)	6.26(3.36)	8.94(3.82)
		LAD	0.883(0.054)	0.694(0.088)	0.770(0.078)	4.88(1.09)	5.80(1.19)	0.956(0.036)	0.865(0.072)	0.901(0.059)	7.47(2.35)	10.64(2.61)
		DPD	0.921(0.040)	0.788(0.070)	0.847(0.062)	5.42(0.79)	6.65(0.94)	0.979(0.016)	0.936(0.036)	0.954(0.029)	9.33(2.11)	12.23(2.12)
		γ	0.927(0.037)	0.807(0.067)	0.862(0.059)	5.45(0.82)	6.74(0.94)	0.977(0.018)	0.939(0.035)	0.957(0.028)	9.31(2.11)	12.30(2.12)
Band1	200	LS	0.710(0.106)	0.491(0.186)	0.605(0.174)	3.10(1.56)	3.69(1.72)	0.769(0.119)	0.573(0.238)	0.685(0.206)	6.56(3.28)	8.53(4.27)
		LAD	0.770(0.083)	0.575(0.134)	0.638(0.124)	3.82(1.42)	4.52(1.49)	0.856(0.088)	0.707(0.169)	0.761(0.146)	8.27(3.67)	10.92(4.40)
		DPD	0.872(0.058)	0.697(0.097)	0.767(0.088)	5.07(1.09)	5.77(1.20)	0.945(0.043)	0.848(0.093)	0.888(0.072)	10.87(2.35)	13.18(2.68)
		γ	0.874(0.057)	0.707(0.097)	0.777(0.088)	5.13(1.05)	5.81(1.20)	0.939(0.048)	0.852(0.090)	0.890(0.075)	10.40(2.32)	13.07(2.70)
	400	LS	0.821(0.087)	0.562(0.169)	0.660(0.149)	4.84(1.80)	5.45(1.79)	0.886(0.080)	0.688(0.184)	0.758(0.144)	9.63(4.23)	11.80(5.02)
		LAD	0.866(0.065)	0.687(0.107)	0.751(0.095)	5.69(1.09)	6.39(1.29)	0.942(0.044)	0.838(0.090)	0.875(0.074)	11.91(2.07)	13.96(2.68)
		DPD	0.937(0.038)	0.827(0.078)	0.876(0.063)	6.72(1.06)	7.39(1.02)	0.980(0.019)	0.940(0.045)	0.956(0.035)	13.13(1.27)	16.53(1.94)
		γ	0.938(0.037)	0.841(0.076)	0.885(0.061)	6.73(0.96)	7.40(1.18)	0.978(0.022)	0.945(0.045)	0.960(0.037)	13.07(1.01)	16.33(1.97)
Band2	200	LS	0.709(0.104)	0.492(0.174)	0.605(0.161)	3.18(1.52)	3.72(1.70)	0.769(0.122)	0.583(0.226)	0.691(0.199)	6.36(2.88)	8.01(3.50)
		LAD	0.766(0.089)	0.555(0.134)	0.626(0.123)	3.68(1.36)	4.45(1.51)	0.844(0.090)	0.683(0.161)	0.743(0.137)	7.81(3.05)	10.34(3.46)
		DPD	0.865(0.058)	0.685(0.088)	0.754(0.081)	4.87(1.00)	5.67(1.14)	0.949(0.036)	0.859(0.070)	0.896(0.058)	9.80(2.20)	12.17(2.26)
		γ	0.864(0.059)	0.699(0.080)	0.766(0.076)	4.80(0.93)	5.63(1.05)	0.947(0.038)	0.868(0.068)	0.903(0.057)	9.43(2.38)	12.35(2.36)
	400	LS	0.815(0.085)	0.545(0.159)	0.645(0.141)	4.55(1.77)	5.30(1.74)	0.877(0.078)	0.669(0.168)	0.744(0.134)	9.31(4.00)	10.95(4.82)
		LAD	0.865(0.066)	0.673(0.104)	0.746(0.098)	5.42(1.20)	6.15(1.18)	0.922(0.056)	0.807(0.086)	0.847(0.080)	10.30(2.76)	12.68(3.06)
		DPD	0.922(0.051)	0.799(0.085)	0.849(0.077)	6.27(1.00)	7.04(1.06)	0.974(0.025)	0.924(0.052)	0.944(0.044)	11.64(2.03)	14.83(2.08)
		γ	0.920(0.049)	0.812(0.084)	0.857(0.074)	6.28(1.05)	7.04(1.11)	0.971(0.027)	0.927(0.053)	0.947(0.045)	11.50(1.87)	14.74(2.16)

Table S3 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under linear regression and S0

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	200	LS	0.947(0.041)	0.847(0.083)	0.893(0.067)	6.53(1.23)	7.44(1.26)	0.958(0.028)	0.867(0.063)	0.899(0.048)	8.48(2.32)	11.88(2.84)
		LAD	0.910(0.045)	0.757(0.089)	0.820(0.076)	5.55(1.16)	6.43(1.08)	0.928(0.036)	0.793(0.082)	0.848(0.065)	7.38(2.50)	9.91(2.72)
		DPD	0.936(0.039)	0.815(0.080)	0.870(0.067)	6.13(1.13)	7.03(1.22)	0.952(0.028)	0.851(0.068)	0.894(0.052)	8.55(1.94)	11.44(2.62)
		γ	0.938(0.037)	0.833(0.076)	0.882(0.061)	6.27(1.10)	7.13(1.18)	0.952(0.029)	0.871(0.066)	0.909(0.051)	8.53(2.12)	11.65(2.50)
	400	LS	0.987(0.020)	0.962(0.040)	0.974(0.034)	8.78(0.97)	9.28(0.70)	0.990(0.011)	0.954(0.026)	0.966(0.022)	10.09(1.74)	16.2(1.71)
		LAD	0.976(0.025)	0.927(0.051)	0.949(0.042)	8.25(0.97)	8.82(0.80)	0.983(0.015)	0.937(0.038)	0.956(0.029)	9.44(1.99)	14.71(2.33)
		DPD	0.981(0.025)	0.949(0.044)	0.964(0.038)	8.52(0.92)	9.07(0.76)	0.987(0.013)	0.954(0.029)	0.966(0.024)	10.41(1.88)	15.73(1.90)
		γ	0.982(0.024)	0.955(0.042)	0.967(0.038)	8.68(0.89)	9.16(0.69)	0.986(0.014)	0.964(0.027)	0.974(0.024)	10.18(1.29)	16.17(1.65)
AR2	200	LS	0.935(0.038)	0.805(0.075)	0.865(0.062)	5.41(0.94)	6.78(1.12)	0.946(0.030)	0.830(0.064)	0.877(0.052)	5.80(2.15)	8.67(2.29)
		LAD	0.900(0.048)	0.731(0.080)	0.801(0.072)	4.69(1.13)	5.86(1.20)	0.918(0.038)	0.760(0.081)	0.827(0.064)	4.88(2.05)	7.37(2.46)
		DPD	0.921(0.046)	0.776(0.085)	0.841(0.071)	4.95(1.07)	6.43(1.14)	0.939(0.034)	0.817(0.069)	0.869(0.056)	5.82(1.94)	8.04(2.30)
		γ	0.923(0.047)	0.799(0.081)	0.857(0.070)	5.13(1.00)	6.54(1.08)	0.940(0.035)	0.840(0.067)	0.887(0.057)	6.03(1.85)	8.34(2.41)
	400	LS	0.975(0.019)	0.917(0.044)	0.946(0.034)	6.85(1.14)	8.42(0.96)	0.980(0.013)	0.924(0.032)	0.946(0.025)	6.98(1.91)	10.67(2.41)
		LAD	0.958(0.028)	0.873(0.057)	0.911(0.048)	6.53(1.09)	7.82(1.00)	0.969(0.020)	0.899(0.045)	0.929(0.037)	6.51(2.15)	10.28(2.51)
		DPD	0.968(0.025)	0.901(0.052)	0.934(0.041)	6.75(1.12)	8.08(1.02)	0.977(0.016)	0.922(0.036)	0.946(0.028)	6.94(2.04)	10.74(2.57)
		γ	0.965(0.028)	0.906(0.051)	0.934(0.044)	6.84(0.97)	8.21(0.96)	0.974(0.022)	0.930(0.039)	0.951(0.035)	7.03(2.09)	10.94(2.55)
Band1	200	LS	0.936(0.045)	0.832(0.088)	0.875(0.076)	6.62(1.30)	7.37(1.28)	0.949(0.036)	0.854(0.075)	0.889(0.064)	8.49(2.19)	11.64(2.55)
		LAD	0.909(0.048)	0.763(0.079)	0.823(0.071)	5.43(1.31)	6.46(1.27)	0.927(0.041)	0.796(0.077)	0.851(0.065)	7.03(2.65)	9.51(2.86)
		DPD	0.926(0.043)	0.807(0.084)	0.860(0.069)	5.93(1.16)	6.96(1.15)	0.945(0.034)	0.842(0.074)	0.884(0.059)	8.34(2.49)	10.83(2.85)
		γ	0.926(0.046)	0.826(0.082)	0.870(0.070)	6.21(1.14)	7.16(1.17)	0.946(0.033)	0.866(0.070)	0.902(0.057)	8.46(2.20)	11.21(2.95)
	400	LS	0.988(0.016)	0.960(0.043)	0.974(0.032)	8.97(0.97)	9.23(0.86)	0.990(0.010)	0.955(0.028)	0.965(0.022)	10.27(1.79)	16.01(2.00)
		LAD	0.974(0.026)	0.915(0.060)	0.942(0.046)	8.06(0.97)	8.66(0.87)	0.979(0.019)	0.927(0.043)	0.950(0.033)	9.45(1.92)	14.49(2.32)
		DPD	0.982(0.022)	0.946(0.050)	0.964(0.038)	8.57(0.96)	9.08(0.85)	0.987(0.013)	0.951(0.033)	0.966(0.026)	10.21(1.71)	15.27(2.15)
		γ	0.980(0.028)	0.950(0.052)	0.963(0.047)	8.59(0.90)	9.09(0.85)	0.984(0.017)	0.960(0.032)	0.971(0.028)	10.3(1.77)	15.52(2.06)
Band2	200	LS	0.934(0.043)	0.822(0.081)	0.870(0.070)	6.01(1.24)	7.05(1.10)	0.944(0.037)	0.841(0.074)	0.880(0.061)	7.28(2.43)	10.15(2.45)
		LAD	0.907(0.049)	0.755(0.088)	0.818(0.076)	5.37(1.06)	6.31(1.24)	0.927(0.043)	0.792(0.097)	0.849(0.075)	6.32(2.43)	8.72(2.49)
		DPD	0.923(0.044)	0.798(0.080)	0.853(0.066)	5.62(1.18)	6.63(1.31)	0.939(0.036)	0.829(0.078)	0.875(0.059)	7.37(2.38)	9.93(2.66)
		γ	0.926(0.044)	0.816(0.076)	0.867(0.064)	5.76(1.14)	6.82(1.18)	0.940(0.038)	0.852(0.074)	0.894(0.058)	7.34(2.26)	9.91(2.77)
	400	LS	0.981(0.020)	0.940(0.045)	0.959(0.036)	8.23(0.93)	8.94(0.88)	0.985(0.013)	0.941(0.030)	0.955(0.026)	9.08(2.14)	13.75(2.19)
		LAD	0.968(0.027)	0.906(0.052)	0.934(0.043)	7.68(1.01)	8.47(0.90)	0.975(0.019)	0.918(0.042)	0.942(0.033)	8.47(2.24)	12.72(2.57)
		DPD	0.974(0.027)	0.927(0.051)	0.950(0.043)	7.98(0.93)	8.71(0.92)	0.982(0.016)	0.942(0.033)	0.958(0.027)	8.94(2.10)	13.46(2.36)
		γ	0.973(0.029)	0.934(0.049)	0.951(0.046)	8.04(0.96)	8.82(0.83)	0.980(0.018)	0.949(0.032)	0.963(0.029)	9.27(1.79)	13.65(2.15)

Table S4 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under logistic regression and S1

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	300	logistic	0.748(0.075)	0.449(0.111)	0.557(0.104)	2.04(1.09)	2.83(1.08)	0.783(0.075)	0.501(0.124)	0.604(0.111)	2.44(2.18)	3.41(2.41)
		constant	0.790(0.077)	0.504(0.128)	0.611(0.119)	2.17(1.13)	3.08(1.41)	0.823(0.067)	0.546(0.127)	0.656(0.110)	2.53(2.13)	3.98(2.56)
		DPD	0.840(0.070)	0.619(0.111)	0.707(0.097)	3.55(1.31)	4.49(1.45)	0.881(0.064)	0.688(0.110)	0.767(0.089)	4.77(2.88)	6.36(2.94)
		γ	0.840(0.069)	0.640(0.109)	0.724(0.096)	3.59(1.37)	4.56(1.46)	0.876(0.065)	0.699(0.103)	0.774(0.086)	4.81(2.74)	6.37(3.04)
	500	logistic	0.796(0.074)	0.571(0.109)	0.655(0.103)	3.38(1.34)	4.28(1.44)	0.845(0.078)	0.646(0.124)	0.722(0.113)	4.48(2.23)	6.08(2.72)
		constant	0.834(0.053)	0.575(0.104)	0.670(0.087)	3.97(1.23)	4.81(1.49)	0.860(0.060)	0.656(0.132)	0.734(0.107)	5.43(2.33)	6.77(2.31)
		DPD	0.915(0.044)	0.782(0.087)	0.837(0.072)	5.75(1.29)	6.66(1.23)	0.943(0.040)	0.841(0.083)	0.879(0.065)	7.86(2.49)	10.94(3.01)
		γ	0.913(0.048)	0.784(0.088)	0.839(0.072)	5.83(1.23)	6.60(1.24)	0.956(0.058)	0.843(0.081)	0.877(0.065)	7.88(2.53)	10.80(3.06)
AR2	300	logistic	0.734(0.079)	0.420(0.108)	0.534(0.105)	1.70(1.02)	2.43(1.18)	0.772(0.077)	0.477(0.114)	0.587(0.103)	1.70(1.90)	2.57(2.15)
		constant	0.779(0.073)	0.486(0.127)	0.597(0.114)	2.00(1.15)	3.08(1.34)	0.806(0.072)	0.514(0.136)	0.626(0.116)	1.88(2.03)	2.71(2.09)
		DPD	0.833(0.061)	0.601(0.114)	0.687(0.102)	3.50(1.14)	4.45(1.17)	0.872(0.063)	0.675(0.122)	0.749(0.104)	3.89(2.22)	5.99(2.81)
		γ	0.834(0.062)	0.619(0.114)	0.703(0.100)	3.49(1.19)	4.42(1.23)	0.876(0.064)	0.684(0.115)	0.755(0.096)	4.14(1.92)	5.90(2.45)
	500	logistic	0.796(0.064)	0.554(0.100)	0.649(0.091)	2.94(1.17)	3.79(1.30)	0.846(0.066)	0.628(0.120)	0.717(0.101)	3.36(2.34)	5.26(2.79)
		constant	0.816(0.058)	0.563(0.101)	0.665(0.087)	3.12(1.12)	3.85(1.42)	0.848(0.053)	0.651(0.115)	0.721(0.097)	4.33(1.85)	6.14(2.23)
		DPD	0.881(0.054)	0.726(0.089)	0.790(0.081)	4.66(1.19)	5.81(1.27)	0.928(0.044)	0.809(0.077)	0.857(0.067)	5.91(2.19)	8.41(2.74)
		γ	0.887(0.054)	0.730(0.090)	0.797(0.080)	4.63(1.22)	5.72(1.30)	0.938(0.041)	0.805(0.073)	0.854(0.058)	5.96(2.05)	8.20(2.76)
Band1	300	logistic	0.735(0.077)	0.418(0.117)	0.528(0.112)	1.86(1.06)	2.63(1.32)	0.766(0.083)	0.460(0.140)	0.571(0.126)	2.55(1.99)	3.12(2.21)
		constant	0.787(0.068)	0.502(0.107)	0.611(0.098)	2.36(1.36)	3.21(1.29)	0.826(0.067)	0.550(0.126)	0.662(0.106)	2.94(2.06)	3.93(2.43)
		DPD	0.849(0.058)	0.629(0.104)	0.718(0.092)	3.71(1.23)	4.65(1.37)	0.891(0.051)	0.704(0.108)	0.779(0.086)	5.03(2.60)	6.94(3.01)
		γ	0.848(0.061)	0.649(0.104)	0.733(0.090)	3.65(1.21)	4.68(1.32)	0.887(0.050)	0.716(0.100)	0.786(0.079)	5.20(2.88)	6.98(3.01)
	500	logistic	0.825(0.060)	0.612(0.106)	0.666(0.091)	3.52(1.53)	4.57(1.52)	0.817(0.072)	0.677(0.121)	0.723(0.106)	4.64(2.62)	6.14(3.19)
		constant	0.831(0.059)	0.632(0.119)	0.692(0.098)	3.94(1.25)	4.84(1.71)	0.858(0.058)	0.686(0.132)	0.742(0.103)	5.88(1.44)	7.26(1.98)
		DPD	0.907(0.052)	0.784(0.084)	0.836(0.078)	5.44(1.34)	6.49(1.17)	0.944(0.041)	0.850(0.072)	0.888(0.065)	8.12(2.66)	10.70(3.05)
		γ	0.912(0.054)	0.782(0.098)	0.839(0.091)	5.49(1.35)	6.52(1.29)	0.941(0.044)	0.859(0.079)	0.874(0.072)	8.27(2.80)	10.83(2.90)
Band2	300	logistic	0.729(0.074)	0.426(0.112)	0.527(0.107)	1.90(1.04)	2.69(1.21)	0.764(0.086)	0.471(0.130)	0.572(0.123)	2.48(2.06)	3.23(2.17)
		constant	0.772(0.080)	0.479(0.112)	0.591(0.106)	2.12(1.22)	3.02(1.34)	0.811(0.083)	0.535(0.130)	0.644(0.120)	2.96(1.93)	3.88(2.52)
		DPD	0.828(0.066)	0.600(0.109)	0.684(0.098)	3.59(1.30)	4.54(1.36)	0.874(0.061)	0.678(0.114)	0.753(0.100)	4.45(2.32)	6.47(3.12)
		γ	0.830(0.066)	0.618(0.106)	0.701(0.096)	3.59(1.26)	4.51(1.37)	0.867(0.061)	0.687(0.108)	0.757(0.095)	4.55(2.33)	6.43(3.11)
	500	logistic	0.822(0.066)	0.604(0.093)	0.692(0.092)	3.41(1.29)	4.38(1.22)	0.820(0.061)	0.676(0.101)	0.758(0.088)	4.69(2.63)	6.29(2.90)
		constant	0.835(0.057)	0.638(0.105)	0.718(0.092)	3.88(1.17)	4.42(1.47)	0.857(0.059)	0.695(0.121)	0.769(0.101)	5.82(1.46)	7.87(2.14)
		DPD	0.905(0.050)	0.778(0.080)	0.832(0.075)	5.46(1.00)	6.50(1.14)	0.943(0.039)	0.843(0.069)	0.883(0.060)	7.72(2.45)	10.36(2.71)
		γ	0.911(0.047)	0.784(0.081)	0.841(0.072)	5.44(1.03)	6.47(1.17)	0.940(0.041)	0.838(0.069)	0.877(0.058)	7.52(2.24)	10.44(2.63)

Table S5 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under logistic regression and S2

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	300	logistic	0.721(0.079)	0.419(0.105)	0.525(0.105)	1.92(1.08)	2.69(1.22)	0.803(0.092)	0.546(0.153)	0.643(0.140)	2.68(2.79)	3.87(2.91)
		constant	0.751(0.068)	0.447(0.106)	0.558(0.102)	1.96(1.13)	2.73(1.35)	0.809(0.080)	0.564(0.144)	0.667(0.128)	2.80(2.17)	3.97(2.57)
		DPD	0.810(0.063)	0.572(0.101)	0.657(0.093)	3.40(1.18)	4.23(1.29)	0.883(0.068)	0.719(0.114)	0.780(0.100)	6.28(2.92)	7.79(3.39)
		γ	0.821(0.064)	0.595(0.10)	0.676(0.092)	3.41(1.08)	4.21(1.14)	0.889(0.069)	0.732(0.109)	0.788(0.096)	6.36(2.96)	7.88(3.28)
	500	logistic	0.778(0.068)	0.591(0.098)	0.652(0.096)	3.51(1.17)	4.37(1.16)	0.831(0.053)	0.691(0.102)	0.696(0.083)	6.26(3.46)	7.72(3.83)
		constant	0.795(0.068)	0.598(0.116)	0.673(0.102)	3.67(1.25)	4.72(1.49)	0.840(0.069)	0.706(0.150)	0.710(0.120)	6.69(2.36)	8.14(2.48)
		DPD	0.872(0.051)	0.707(0.079)	0.770(0.076)	5.31(1.11)	6.07(1.02)	0.947(0.032)	0.851(0.070)	0.887(0.053)	10.04(3.12)	12.78(2.64)
		γ	0.867(0.057)	0.708(0.082)	0.773(0.072)	5.37(1.14)	6.09(0.99)	0.944(0.045)	0.844(0.070)	0.888(0.055)	10.07(2.96)	12.77(2.93)
AR2	300	logistic	0.729(0.076)	0.416(0.101)	0.521(0.100)	1.88(0.88)	2.61(1.06)	0.798(0.074)	0.510(0.127)	0.615(0.114)	2.72(2.56)	3.78(2.69)
		constant	0.751(0.074)	0.438(0.098)	0.557(0.095)	1.96(1.11)	2.71(1.19)	0.810(0.071)	0.511(0.132)	0.631(0.111)	2.75(2.10)	4.26(2.68)
		DPD	0.812(0.073)	0.569(0.099)	0.662(0.095)	3.17(1.06)	3.99(1.14)	0.886(0.055)	0.700(0.105)	0.774(0.084)	4.99(3.10)	6.77(3.03)
		γ	0.807(0.069)	0.586(0.097)	0.674(0.092)	3.19(1.11)	4.04(1.15)	0.884(0.053)	0.712(0.099)	0.782(0.080)	4.89(2.95)	6.82(3.15)
	500	logistic	0.764(0.067)	0.542(0.101)	0.645(0.094)	3.20(1.02)	4.10(1.13)	0.826(0.07)	0.610(0.125)	0.703(0.106)	4.73(3.33)	6.93(3.62)
		constant	0.782(0.071)	0.569(0.118)	0.663(0.112)	3.33(1.06)	4.21(1.30)	0.835(0.074)	0.627(0.147)	0.711(0.131)	5.33(0.82)	7.18(1.76)
		DPD	0.857(0.056)	0.683(0.096)	0.754(0.085)	4.47(1.13)	5.5(1.10)	0.930(0.049)	0.822(0.085)	0.866(0.074)	7.67(3.04)	10.11(2.92)
		γ	0.865(0.052)	0.690(0.092)	0.764(0.078)	4.54(1.10)	5.5(1.04)	0.931(0.046)	0.820(0.081)	0.864(0.069)	7.47(3.23)	10.04(2.80)
Band1	300	logistic	0.734(0.076)	0.445(0.107)	0.545(0.101)	2.23(1.16)	2.93(1.23)	0.806(0.083)	0.549(0.149)	0.644(0.129)	2.92(3.04)	4.80(3.60)
		constant	0.746(0.077)	0.456(0.11)	0.558(0.102)	2.31(1.14)	3.09(1.12)	0.816(0.076)	0.550(0.132)	0.657(0.114)	2.96(2.23)	4.86(2.72)
		DPD	0.806(0.07)	0.584(0.109)	0.662(0.103)	3.75(1.27)	4.55(1.21)	0.889(0.067)	0.730(0.117)	0.790(0.106)	6.61(3.18)	8.17(3.27)
		γ	0.805(0.069)	0.598(0.107)	0.675(0.100)	3.76(1.25)	4.58(1.26)	0.883(0.066)	0.737(0.114)	0.793(0.102)	6.66(3.20)	8.46(3.21)
	500	logistic	0.763(0.074)	0.567(0.119)	0.581(0.109)	3.41(1.14)	4.12(1.23)	0.825(0.068)	0.708(0.127)	0.733(0.104)	6.11(3.50)	8.07(3.75)
		constant	0.784(0.069)	0.559(0.119)	0.608(0.107)	3.56(1.18)	4.41(1.54)	0.833(0.073)	0.726(0.152)	0.754(0.128)	6.37(1.00)	8.25(2.13)
		DPD	0.865(0.069)	0.710(0.116)	0.768(0.105)	5.04(1.11)	5.81(1.30)	0.937(0.051)	0.844(0.092)	0.879(0.080)	9.87(3.10)	12.18(3.19)
		γ	0.869(0.068)	0.714(0.113)	0.776(0.100)	5.06(1.11)	5.84(1.28)	0.937(0.050)	0.840(0.091)	0.876(0.076)	9.78(2.99)	12.26(3.06)
Band2	300	logistic	0.718(0.074)	0.407(0.120)	0.507(0.110)	1.94(1.06)	2.73(1.32)	0.789(0.090)	0.521(0.157)	0.618(0.142)	3.09(3.00)	3.91(3.39)
		constant	0.755(0.076)	0.461(0.117)	0.566(0.112)	1.96(1.11)	2.95(1.34)	0.797(0.078)	0.542(0.151)	0.651(0.130)	3.94(2.50)	4.31(3.31)
		DPD	0.804(0.068)	0.569(0.104)	0.655(0.095)	3.49(1.1)	4.37(1.19)	0.885(0.058)	0.721(0.100)	0.783(0.087)	6.11(3.44)	7.85(3.56)
		γ	0.800(0.068)	0.585(0.102)	0.667(0.093)	3.59(1.11)	4.45(1.21)	0.880(0.057)	0.727(0.096)	0.786(0.084)	6.14(3.40)	7.91(3.52)
	500	logistic	0.796(0.069)	0.580(0.098)	0.658(0.096)	3.42(1.31)	4.50(1.22)	0.838(0.066)	0.719(0.113)	0.750(0.099)	5.92(3.03)	7.55(3.72)
		constant	0.793(0.058)	0.591(0.104)	0.699(0.092)	3.63(1.18)	4.73(1.38)	0.842(0.065)	0.724(0.140)	0.772(0.116)	6.57(1.49)	7.98(2.20)
		DPD	0.870(0.055)	0.718(0.089)	0.777(0.082)	5.15(1.07)	5.95(1.19)	0.946(0.036)	0.855(0.070)	0.893(0.057)	8.82(2.97)	11.75(2.75)
		γ	0.873(0.053)	0.724(0.087)	0.786(0.076)	5.20(1.11)	5.93(1.16)	0.948(0.034)	0.853(0.068)	0.899(0.052)	8.91(2.90)	11.48(2.66)

Table S6 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions under logistic regression and S0

			Main effect					Interaction				
Correlation	n	method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
AR1	300	logistic	0.906(0.048)	0.778(0.074)	0.837(0.065)	5.97(1.26)	6.59(1.12)	0.930(0.059)	0.792(0.084)	0.854(0.076)	7.29(2.88)	10.23(2.85)
		constant	0.893(0.060)	0.762(0.088)	0.795(0.077)	5.89(1.17)	6.44(1.16)	0.915(0.068)	0.778(0.089)	0.823(0.080)	7.12(2.48)	9.87(3.10)
		DPD	0.909(0.046)	0.765(0.076)	0.824(0.069)	5.96(1.15)	6.66(1.17)	0.927(0.044)	0.797(0.083)	0.845(0.073)	7.24(2.31)	10.16(2.33)
		γ	0.909(0.045)	0.777(0.076)	0.834(0.068)	6.05(1.15)	6.63(1.23)	0.922(0.047)	0.798(0.082)	0.845(0.072)	7.31(2.09)	10.14(2.48)
	500	logistic	0.950(0.039)	0.862(0.087)	0.914(0.076)	7.46(1.00)	8.03(0.95)	0.953(0.048)	0.900(0.072)	0.916(0.064)	9.14(2.07)	13.85(2.73)
		constant	0.934(0.047)	0.860(0.067)	0.895(0.063)	7.28(1.21)	8.05(1.24)	0.939(0.052)	0.842(0.072)	0.884(0.069)	8.69(1.81)	13.34(1.92)
		DPD	0.949(0.038)	0.870(0.071)	0.900(0.062)	7.33(0.96)	8.07(0.92)	0.961(0.038)	0.888(0.067)	0.910(0.057)	8.98(1.72)	13.47(2.45)
		γ	0.947(0.042)	0.877(0.072)	0.905(0.064)	7.40(0.93)	8.11(0.92)	0.955(0.042)	0.881(0.067)	0.900(0.059)	8.83(1.50)	13.42(2.50)
AR2	300	logistic	0.919(0.051)	0.794(0.079)	0.851(0.072)	6.01(1.18)	7.06(1.16)	0.944(0.055)	0.836(0.079)	0.879(0.068)	5.65(2.33)	8.76(2.70)
		constant	0.907(0.054)	0.789(0.083)	0.827(0.078)	5.88(1.25)	6.68(1.27)	0.920(0.054)	0.897(0.089)	0.844(0.080)	5.08(2.43)	8.27(2.55)
		DPD	0.910(0.054)	0.785(0.082)	0.835(0.076)	6.00(1.06)	6.93(1.09)	0.936(0.050)	0.828(0.083)	0.869(0.071)	5.52(2.14)	8.59(2.47)
		γ	0.912(0.054)	0.795(0.084)	0.845(0.077)	5.89(1.02)	6.95(1.09)	0.932(0.052)	0.828(0.085)	0.867(0.072)	5.56(1.85)	8.66(2.29)
	500	logistic	0.952(0.033)	0.831(0.066)	0.919(0.062)	6.85(0.95)	7.93(0.98)	0.965(0.047)	0.897(0.060)	0.923(0.059)	6.13(2.40)	9.72(2.68)
		constant	0.934(0.038)	0.849(0.066)	0.909(0.061)	6.74(1.42)	7.72(1.27)	0.957(0.032)	0.854(0.059)	0.892(0.057)	5.73(1.92)	9.19(1.86)
		DPD	0.949(0.037)	0.859(0.070)	0.897(0.064)	6.88(0.83)	7.84(0.94)	0.969(0.027)	0.897(0.055)	0.922(0.049)	5.81(1.72)	9.56(2.03)
		γ	0.948(0.039)	0.868(0.069)	0.905(0.061)	6.88(0.81)	7.88(0.96)	0.964(0.031)	0.890(0.056)	0.932(0.049)	5.96(1.94)	9.64(2.18)
Band1	300	logistic	0.908(0.052)	0.790(0.075)	0.840(0.065)	5.78(1.35)	6.51(1.13)	0.931(0.058)	0.796(0.077)	0.869(0.065)	7.51(2.64)	9.69(2.81)
		constant	0.889(0.069)	0.775(0.096)	0.804(0.085)	5.71(1.22)	6.52(1.17)	0.906(0.068)	0.785(0.093)	0.836(0.083)	6.98(2.44)	9.46(2.83)
		DPD	0.908(0.051)	0.769(0.089)	0.825(0.078)	5.71(1.19)	6.67(1.12)	0.930(0.045)	0.803(0.082)	0.851(0.070)	7.26(2.28)	9.68(2.49)
		γ	0.907(0.051)	0.782(0.087)	0.835(0.076)	5.71(1.14)	6.65(1.07)	0.925(0.049)	0.806(0.079)	0.850(0.069)	7.31(2.19)	9.98(2.50)
	500	logistic	0.949(0.042)	0.880(0.077)	0.915(0.068)	7.45(0.98)	7.96(1.00)	0.972(0.056)	0.889(0.06)	0.938(0.055)	9.02(2.19)	13.85(2.44)
		constant	0.928(0.045)	0.868(0.067)	0.891(0.064)	7.24(1.31)	9.92(1.17)	0.949(0.042)	0.845(0.067)	0.893(0.062)	8.74(1.90)	12.88(1.84)
		DPD	0.952(0.034)	0.876(0.063)	0.908(0.053)	7.37(1.05)	8.01(0.95)	0.964(0.038)	0.893(0.058)	0.920(0.049)	8.89(1.57)	13.58(2.23)
		γ	0.951(0.035)	0.884(0.060)	0.912(0.051)	7.46(1.05)	8.05(0.91)	0.961(0.036)	0.886(0.059)	0.924(0.052)	8.93(1.50)	13.53(2.25)
Band2	300	logistic	0.902(0.050)	0.774(0.080)	0.828(0.073)	5.69(1.11)	6.63(1.07)	0.929(0.064)	0.792(0.076)	0.843(0.067)	6.06(2.43)	9.14(2.43)
		constant	0.876(0.061)	0.740(0.085)	0.799(0.077)	5.48(1.10)	6.50(1.16)	0.881(0.065)	0.732(0.092)	0.798(0.082)	5.89(2.45)	8.75(2.73)
		DPD	0.900(0.051)	0.756(0.087)	0.813(0.080)	5.58(1.01)	6.63(1.12)	0.917(0.051)	0.785(0.089)	0.834(0.078)	6.10(2.14)	9.06(2.37)
		γ	0.899(0.051)	0.769(0.087)	0.823(0.078)	5.60(0.95)	6.66(1.02)	0.910(0.054)	0.788(0.086)	0.833(0.076)	6.02(1.94)	9.10(2.28)
	500	logistic	0.947(0.036)	0.877(0.064)	0.913(0.062)	6.96(0.97)	7.92(0.78)	0.949(0.043)	0.884(0.057)	0.923(0.057)	7.67(2.61)	11.67(2.23)
		constant	0.927(0.045)	0.852(0.065)	0.889(0.059)	6.66(1.36)	7.75(1.22)	0.945(0.039)	0.838(0.053)	0.886(0.053)	7.32(1.83)	11.17(1.81)
		DPD	0.947(0.035)	0.860(0.063)	0.897(0.055)	7.06(0.83)	7.83(0.88)	0.962(0.030)	0.883(0.049)	0.911(0.043)	7.58(1.90)	11.59(2.08)
		γ	0.948(0.035)	0.870(0.062)	0.905(0.052)	7.09(0.82)	7.84(0.86)	0.956(0.034)	0.879(0.05)	0.914(0.045)	7.60(1.98)	11.62(2.15)

Table S7 : Simulation of marginal analysis: mean(sd) of AUC, pAUC and Top20,40 for main effects and interactions based on the TNBC data

			Main effect					Interaction				
		method	AUC	pAUC1	pAUC2	Top20	Top40	AUC	pAUC1	pAUC2	Top20	Top40
linear	S0	LS	0.841(0.045)	0.689(0.102)	0.718(0.129)	5.21(1.41)	6.09(1.24)	0.881(0.068)	0.726(0.124)	0.780(0.112)	5.61(1.71)	6.62(1.82)
		LAD	0.844(0.051)	0.691(0.111)	0.722(0.125)	5.24(1.15)	6.11(1.52)	0.895(0.069)	0.732(0.131)	0.789(0.134)	5.67(1.85)	6.70(1.93)
		DPD	0.857(0.021)	0.697(0.086)	0.747(0.116)	5.49(1.54)	6.19(1.38)	0.904(0.057)	0.743(0.104)	0.804(0.091)	5.76(1.82)	6.86(1.71)
		γ	0.854(0.024)	0.698(0.123)	0.742(0.121)	5.37(1.42)	6.21(1.25)	0.912(0.032)	0.741(0.129)	0.799(0.095)	5.70(1.91)	6.89(1.65)
	S1	LS	0.673(0.032)	0.528(0.038)	0.576(0.035)	2.70(1.16)	3.98(0.90)	0.717(0.034)	0.555(0.059)	0.614(0.049)	2.99(1.37)	4.37(1.49)
		LAD	0.722(0.021)	0.584(0.026)	0.626(0.025)	3.68(1.19)	4.92(0.94)	0.760(0.047)	0.595(0.072)	0.673(0.060)	3.94(1.33)	5.20(1.44)
		DPD	0.808(0.018)	0.656(0.024)	0.715(0.023)	4.59(1.20)	5.95(0.94)	0.842(0.044)	0.677(0.046)	0.734(0.034)	4.94(1.34)	6.13(1.53)
		γ	0.798(0.020)	0.667(0.027)	0.715(0.048)	4.68(1.18)	5.86(0.92)	0.850(0.044)	0.686(0.071)	0.744(0.035)	4.85(1.40)	6.21(1.52)
	S2	LS	0.679(0.032)	0.530(0.039)	0.580(0.039)	2.91(1.08)	4.07(0.94)	0.738(0.041)	0.574(0.057)	0.634(0.041)	3.59(1.11)	4.57(1.27)
		LAD	0.730(0.019)	0.585(0.052)	0.634(0.027)	3.82(1.04)	4.96(0.97)	0.790(0.029)	0.626(0.044)	0.687(0.055)	4.51(1.14)	5.46(1.22)
		DPD	0.825(0.044)	0.671(0.050)	0.731(0.024)	5.00(1.05)	6.14(0.92)	0.877(0.052)	0.723(0.068)	0.777(0.029)	5.69(1.08)	6.67(1.24)
		γ	0.820(0.020)	0.679(0.026)	0.728(0.026)	4.90(1.11)	6.14(0.99)	0.877(0.027)	0.719(0.043)	0.787(0.026)	5.65(1.14)	6.56(1.23)
logistic	S0	logistic	0.775(0.052)	0.609(0.041)	0.632(0.045)	3.23(1.11)	3.94(1.18)	0.786(0.086)	0.637(0.067)	0.687(0.076)	3.89(1.23)	4.71(1.87)
		constant	0.778(0.069)	0.613(0.054)	0.638(0.061)	3.20(0.92)	3.91(1.22)	0.798(0.062)	0.643(0.078)	0.693(0.081)	3.80(1.46)	4.68(2.10)
		DPD	0.785(0.035)	0.624(0.021)	0.656(0.055)	3.35(1.18)	4.02(1.15)	0.822(0.046)	0.655(0.032)	0.703(0.022)	3.91(1.38)	4.87(1.86)
		γ	0.783(0.031)	0.628(0.063)	0.658(0.047)	3.31(0.97)	4.05(0.92)	0.824(0.025)	0.659(0.051)	0.705(0.025)	3.83(1.54)	4.98(1.97)
	S1	logistic	0.631(0.061)	0.441(0.054)	0.503(0.053)	2.20(1.02)	3.09(0.97)	0.651(0.063)	0.473(0.094)	0.527(0.074)	2.66(1.33)	3.47(1.36)
		constant	0.672(0.046)	0.490(0.067)	0.545(0.039)	2.50(0.98)	3.35(1.05)	0.699(0.074)	0.516(0.108)	0.567(0.061)	3.07(1.37)	3.72(1.32)
		DPD	0.749(0.075)	0.555(0.069)	0.623(0.038)	3.01(0.98)	3.93(1.01)	0.764(0.052)	0.584(0.107)	0.637(0.085)	3.49(1.29)	4.23(1.39)
		γ	0.744(0.050)	0.560(0.040)	0.623(0.039)	3.04(1.05)	3.89(1.12)	0.768(0.074)	0.589(0.104)	0.642(0.087)	3.45(1.36)	4.26(1.33)
	S2	logistic	0.610(0.054)	0.432(0.060)	0.489(0.056)	2.13(1.01)	3.18(1.04)	0.633(0.055)	0.463(0.101)	0.512(0.076)	2.44(1.24)	3.56(1.62)
		constant	0.653(0.041)	0.482(0.074)	0.538(0.044)	2.45(0.97)	3.46(1.07)	0.678(0.043)	0.509(0.088)	0.560(0.090)	2.79(1.27)	3.83(1.58)
		DPD	0.716(0.065)	0.536(0.072)	0.598(0.041)	2.93(0.98)	3.73(1.01)	0.736(0.066)	0.571(0.112)	0.617(0.065)	3.24(1.21)	4.36(1.59)
		γ	0.713(0.042)	0.539(0.048)	0.596(0.043)	2.86(1.04)	3.73(1.08)	0.739(0.041)	0.569(0.088)	0.622(0.061)	3.22(1.20)	4.29(1.49)

Table S8 : Simulation of marginal analysis: mean(sd) of TPR and FPR for main effects and interactions under AR ($\rho = 0.25$) correlation, with $n = 400$ for linear regression and $n = 500$ for logistic regression.

			Main effect		Interaction	
		method	TPR	FPR	TPR	FPR
linear	S0	LS	0.844(0.079)	0.083(0.090)	0.886(0.079)	0.085(0.089)
		LAD	0.804(0.076)	0.019(0.034)	0.812(0.076)	0.022(0.032)
		DPD	0.816(0.123)	0.003(0.001)	0.843(0.117)	0.004(0.001)
		γ	0.826(0.087)	0.008(0.016)	0.840(0.071)	0.011(0.015)
	S1	LS	0.635(0.107)	0.035(0.065)	0.648(0.115)	0.036(0.054)
		LAD	0.646(0.070)	0.024(0.033)	0.674(0.081)	0.025(0.032)
		DPD	0.764(0.123)	0.013(0.018)	0.804(0.121)	0.016(0.019)
		γ	0.789(0.100)	0.018(0.028)	0.796(0.073)	0.020(0.025)
	S2	LS	0.624(0.093)	0.070(0.084)	0.663(0.097)	0.073(0.083)
		LAD	0.673(0.077)	0.014(0.018)	0.692(0.080)	0.017(0.025)
		DPD	0.786(0.085)	0.011(0.013)	0.806(0.121)	0.014(0.011)
		γ	0.802(0.070)	0.010(0.011)	0.828(0.065)	0.016(0.014)
logistic	S0	logistic	0.827(0.151)	0.057(0.069)	0.829(0.119)	0.079(0.059)
		constant	0.817(0.092)	0.068(0.044)	0.793(0.091)	0.065(0.041)
		DPD	0.805(0.088)	0.027(0.012)	0.787(0.067)	0.030(0.012)
		γ	0.797(0.094)	0.024(0.012)	0.792(0.080)	0.027(0.011)
	S1	logistic	0.495(0.098)	0.065(0.053)	0.607(0.097)	0.067(0.053)
		constant	0.516(0.092)	0.085(0.045)	0.616(0.091)	0.084(0.044)
		DPD	0.652(0.107)	0.035(0.012)	0.712(0.119)	0.037(0.012)
		γ	0.670(0.093)	0.037(0.016)	0.718(0.096)	0.039(0.015)
	S2	logistic	0.512(0.105)	0.054(0.043)	0.610(0.103)	0.056(0.042)
		constant	0.529(0.081)	0.085(0.043)	0.624(0.095)	0.084(0.042)
		DPD	0.629(0.088)	0.035(0.016)	0.709(0.103)	0.037(0.016)
		γ	0.641(0.080)	0.039(0.017)	0.724(0.095)	0.041(0.016)

Table S9 : Simulation of joint analysis: mean(sd) of TPR and FPR for main effects and interactions under linear regression and S0

			Main effect		Interaction	
Correlation	n	method	TPR	FPR	TPR	FPR
AR1	200	LS	0.947(0.072)	0.045(0.010)	0.929(0.067)	0.049(0.010)
		LAD	0.949(0.069)	0.063(0.019)	0.944(0.076)	0.065(0.018)
		DPD	0.913(0.119)	0.047(0.029)	0.898(0.145)	0.054(0.031)
		γ	0.923(0.086)	0.041(0.010)	0.908(0.098)	0.048(0.013)
	400	LS	1.000(0.000)	0.068(0.024)	0.994(0.018)	0.071(0.023)
		LAD	1.000(0.000)	0.079(0.025)	0.999(0.007)	0.079(0.023)
		DPD	1.000(0.000)	0.075(0.065)	0.998(0.011)	0.075(0.061)
		γ	1.000(0.000)	0.062(0.020)	0.999(0.007)	0.064(0.019)
AR2	200	LS	0.879(0.108)	0.036(0.019)	0.883(0.109)	0.040(0.018)
		LAD	0.946(0.067)	0.064(0.017)	0.940(0.061)	0.065(0.016)
		DPD	0.870(0.109)	0.048(0.013)	0.891(0.137)	0.056(0.017)
		γ	0.904(0.089)	0.045(0.010)	0.900(0.095)	0.051(0.011)
	400	LS	0.960(0.064)	0.011(0.003)	0.939(0.063)	0.017(0.003)
		LAD	1.000(0.000)	0.028(0.017)	0.998(0.010)	0.032(0.016)
		DPD	0.998(0.014)	0.071(0.057)	0.996(0.013)	0.073(0.054)
		γ	1.000(0.000)	0.036(0.015)	1.000(0.000)	0.040(0.014)
Band1	200	LS	0.949(0.072)	0.045(0.010)	0.928(0.077)	0.050(0.009)
		LAD	0.950(0.069)	0.066(0.018)	0.949(0.062)	0.067(0.017)
		DPD	0.928(0.106)	0.045(0.015)	0.917(0.122)	0.051(0.022)
		γ	0.922(0.084)	0.042(0.010)	0.904(0.095)	0.049(0.012)
	400	LS	1.000(0.000)	0.059(0.023)	0.997(0.012)	0.063(0.022)
		LAD	1.000(0.000)	0.078(0.024)	1.000(0.000)	0.078(0.023)
		DPD	0.999(0.010)	0.068(0.052)	0.997(0.014)	0.069(0.049)
		γ	0.999(0.010)	0.064(0.019)	1.000(0.000)	0.066(0.017)
Band2	200	LS	0.894(0.110)	0.048(0.070)	0.906(0.115)	0.054(0.066)
		LAD	0.954(0.069)	0.067(0.017)	0.940(0.076)	0.068(0.017)
		DPD	0.883(0.110)	0.049(0.017)	0.892(0.131)	0.055(0.019)
		γ	0.901(0.085)	0.046(0.012)	0.886(0.099)	0.053(0.013)
	400	LS	0.973(0.049)	0.038(0.024)	0.962(0.055)	0.047(0.024)
		LAD	1.000(0.000)	0.081(0.028)	1.000(0.000)	0.081(0.026)
		DPD	0.978(0.042)	0.068(0.078)	0.968(0.057)	0.073(0.080)
		γ	0.999(0.010)	0.070(0.026)	0.999(0.010)	0.073(0.024)

Table S10 : Simulation of joint analysis: mean(sd) of TPR and FPR for main effects and interactions under linear regression and S1

			Main effect		Interaction	
Correlation	n	method	TPR	FPR	TPR	FPR
AR1	200	LS	0.573(0.129)	0.010(0.007)	0.548(0.146)	0.011(0.007)
		LAD	0.626(0.139)	0.029(0.018)	0.574(0.129)	0.029(0.018)
		DPD	0.803(0.160)	0.052(0.040)	0.855(0.136)	0.057(0.042)
		γ	0.815(0.170)	0.043(0.016)	0.871(0.141)	0.048(0.017)
	400	LS	0.744(0.122)	0.011(0.008)	0.818(0.102)	0.015(0.008)
		LAD	0.785(0.118)	0.055(0.039)	0.844(0.096)	0.058(0.038)
		DPD	0.994(0.028)	0.080(0.068)	0.995(0.021)	0.084(0.066)
		γ	0.994(0.031)	0.075(0.023)	0.996(0.019)	0.080(0.023)
AR2	200	LS	0.473(0.147)	0.011(0.007)	0.490(0.159)	0.012(0.007)
		LAD	0.600(0.141)	0.028(0.016)	0.557(0.148)	0.028(0.016)
		DPD	0.834(0.118)	0.052(0.027)	0.886(0.089)	0.058(0.030)
		γ	0.846(0.117)	0.047(0.012)	0.900(0.088)	0.052(0.013)
	400	LS	0.642(0.140)	0.011(0.007)	0.656(0.142)	0.013(0.007)
		LAD	0.775(0.107)	0.036(0.029)	0.720(0.090)	0.037(0.029)
		DPD	0.988(0.036)	0.066(0.035)	0.995(0.017)	0.072(0.035)
		γ	0.992(0.031)	0.063(0.020)	0.996(0.013)	0.068(0.019)
Band1	200	LS	0.466(0.149)	0.013(0.025)	0.541(0.157)	0.014(0.025)
		LAD	0.515(0.158)	0.033(0.031)	0.580(0.161)	0.034(0.031)
		DPD	0.828(0.151)	0.044(0.027)	0.874(0.124)	0.049(0.030)
		γ	0.845(0.153)	0.048(0.017)	0.900(0.124)	0.052(0.017)
	400	LS	0.765(0.117)	0.013(0.008)	0.822(0.104)	0.016(0.008)
		LAD	0.791(0.116)	0.046(0.035)	0.845(0.099)	0.049(0.034)
		DPD	0.997(0.017)	0.064(0.035)	0.997(0.012)	0.068(0.034)
		γ	0.998(0.014)	0.076(0.032)	0.999(0.007)	0.080(0.031)
Band2	200	LS	0.527(0.147)	0.012(0.013)	0.526(0.172)	0.013(0.013)
		LAD	0.615(0.155)	0.027(0.017)	0.555(0.130)	0.027(0.016)
		DPD	0.813(0.131)	0.050(0.034)	0.874(0.108)	0.055(0.033)
		γ	0.830(0.140)	0.048(0.016)	0.894(0.109)	0.052(0.017)
	400	LS	0.710(0.118)	0.016(0.010)	0.703(0.108)	0.019(0.011)
		LAD	0.783(0.112)	0.049(0.042)	0.746(0.107)	0.050(0.042)
		DPD	0.997(0.022)	0.075(0.036)	0.998(0.016)	0.079(0.035)
		γ	0.996(0.024)	0.070(0.023)	0.996(0.018)	0.075(0.023)

Table S11 : Simulation of joint analysis: mean(sd) of TPR and FPR for main effects and interactions under linear regression and S2

			Main effect		Interaction	
Correlation	n	method	TPR	FPR	TPR	FPR
AR1	200	LS	0.619(0.123)	0.009(0.006)	0.588(0.142)	0.011(0.006)
		LAD	0.669(0.111)	0.026(0.016)	0.626(0.135)	0.028(0.015)
		DPD	0.849(0.113)	0.048(0.027)	0.886(0.089)	0.052(0.028)
		γ	0.894(0.098)	0.045(0.013)	0.930(0.061)	0.051(0.015)
	400	LS	0.789(0.128)	0.010(0.006)	0.850(0.119)	0.014(0.006)
		LAD	0.819(0.105)	0.048(0.027)	0.876(0.086)	0.051(0.027)
		DPD	0.999(0.010)	0.066(0.073)	0.997(0.012)	0.068(0.070)
		γ	0.999(0.010)	0.065(0.021)	0.999(0.007)	0.068(0.020)
AR2	200	LS	0.502(0.123)	0.010(0.006)	0.490(0.149)	0.011(0.006)
		LAD	0.606(0.133)	0.024(0.014)	0.566(0.142)	0.025(0.014)
		DPD	0.794(0.114)	0.053(0.022)	0.848(0.109)	0.059(0.027)
		γ	0.847(0.100)	0.048(0.013)	0.904(0.069)	0.054(0.014)
	400	LS	0.662(0.125)	0.012(0.007)	0.725(0.136)	0.014(0.007)
		LAD	0.721(0.100)	0.040(0.027)	0.732(0.124)	0.041(0.027)
		DPD	0.994(0.024)	0.083(0.025)	0.996(0.014)	0.088(0.026)
		γ	0.996(0.020)	0.068(0.019)	0.998(0.009)	0.072(0.019)
Band1	200	LS	0.558(0.136)	0.009(0.006)	0.620(0.161)	0.011(0.005)
		LAD	0.599(0.133)	0.031(0.016)	0.660(0.147)	0.033(0.016)
		DPD	0.840(0.121)	0.047(0.025)	0.886(0.100)	0.051(0.024)
		γ	0.880(0.102)	0.046(0.015)	0.922(0.072)	0.052(0.017)
	400	LS	0.796(0.129)	0.010(0.006)	0.858(0.116)	0.014(0.007)
		LAD	0.834(0.106)	0.050(0.027)	0.888(0.090)	0.052(0.027)
		DPD	1.000(0.000)	0.084(0.073)	0.998(0.010)	0.086(0.072)
		γ	1.000(0.000)	0.074(0.023)	0.999(0.007)	0.077(0.021)
Band2	200	LS	0.536(0.131)	0.010(0.007)	0.573(0.161)	0.012(0.007)
		LAD	0.649(0.109)	0.028(0.016)	0.610(0.134)	0.029(0.015)
		DPD	0.806(0.134)	0.054(0.021)	0.844(0.128)	0.060(0.022)
		γ	0.877(0.104)	0.049(0.012)	0.923(0.078)	0.055(0.014)
	400	LS	0.719(0.116)	0.014(0.009)	0.720(0.112)	0.017(0.010)
		LAD	0.745(0.099)	0.054(0.028)	0.746(0.116)	0.055(0.027)
		DPD	0.997(0.017)	0.091(0.115)	0.996(0.016)	0.092(0.112)
		γ	0.999(0.010)	0.073(0.020)	1.000(0.005)	0.077(0.019)

Table S12 : Simulation of joint analysis: mean(sd) of TPR and FPR for main effects and interactions based on the TNBC data

			Main effect		Interaction	
		method	TPR	FPR	TPR	FPR
linear	S0	LS	0.926(0.074)	0.023(0.018)	0.960(0.055)	0.027(0.013)
		LAD	0.929(0.071)	0.025(0.028)	0.972(0.042)	0.024(0.026)
		DPD	0.935(0.061)	0.013(0.023)	0.985(0.032)	0.018(0.019)
		γ	0.943(0.056)	0.014(0.022)	0.991(0.029)	0.021(0.020)
	S1	LS	0.805(0.114)	0.041(0.044)	0.540(0.089)	0.029(0.025)
		LAD	0.863(0.090)	0.062(0.053)	0.714(0.103)	0.054(0.048)
		DPD	0.885(0.090)	0.014(0.046)	0.848(0.097)	0.016(0.095)
		γ	0.908(0.077)	0.020(0.039)	0.898(0.079)	0.020(0.036)
	S2	LS	0.877(0.097)	0.031(0.032)	0.613(0.073)	0.024(0.016)
		LAD	0.893(0.072)	0.049(0.034)	0.716(0.092)	0.034(0.028)
		DPD	0.951(0.080)	0.012(0.022)	0.904(0.079)	0.011(0.019)
		γ	0.968(0.053)	0.016(0.029)	0.965(0.055)	0.016(0.025)
logistic	S0	logistic	0.714(0.132)	0.036(0.019)	0.882(0.138)	0.038(0.017)
		constant	0.719(0.189)	0.043(0.025)	0.898(0.201)	0.035(0.012)
		DPD	0.726(0.101)	0.031(0.014)	0.902(0.123)	0.029(0.016)
		γ	0.721(0.106)	0.038(0.021)	0.914(0.130)	0.036(0.019)
	S1	logistic	0.453(0.131)	0.023(0.007)	0.438(0.129)	0.022(0.007)
		constant	0.465(0.124)	0.048(0.080)	0.453(0.132)	0.029(0.057)
		DPD	0.507(0.134)	0.032(0.010)	0.515(0.147)	0.032(0.010)
		γ	0.518(0.134)	0.035(0.011)	0.539(0.155)	0.035(0.011)
	S2	logistic	0.538(0.148)	0.019(0.007)	0.501(0.170)	0.019(0.007)
		constant	0.568(0.129)	0.055(0.104)	0.541(0.164)	0.054(0.049)
		DPD	0.641(0.155)	0.029(0.012)	0.656(0.191)	0.030(0.011)
		γ	0.639(0.148)	0.028(0.009)	0.663(0.181)	0.029(0.009)

Table S13 : Marginal analysis of TNBC: stability analysis of identified main effects and interactions.

Gene	DPD			γ		
	Main	Interaction		Main	Interaction	
		age	race		age	race
AGR2	1.00	0.63	1.00	0.99	0.99	0.99
AGR3	1.00	1.00	1.00	0.99	0.99	0.99
AR	1.00	0.06	1.00	0.99		0.99
B3GNT5	1.00	1.00	1.00	0.99	0.99	0.99
BCL11A	1.00	1.00	1.00	0.99	0.85	0.99
C5AR2	0.72	0.72	0.72			
CA12	1.00	0.71	1.00	0.99	0.15	0.99
CHODL	0.88	0.88	0.38	0.59	0.59	0.11
CLCN4	0.98	0.98	0.98	0.77	0.73	0.77
CXXC5	1.00	0.09	1.00	0.99	0.40	0.99
DLI1	0.81	0.17	0.81			
EN1	1.00	1.00	1.00	0.99	0.99	0.99
ESR1	1.00	1.00	1.00	0.99	0.99	0.99
FAM171A1	1.00	0.97	1.00			
FBP1	1.00		1.00	0.94	0.94	0.94
FOXA1	1.00	0.80	1.00	0.99	0.95	0.99
FOXC1	1.00	1.00	1.00	0.99	0.99	0.99
GATA3	1.00	0.88	1.00	0.99	0.30	0.99
HAPLN3	0.97	0.97	0.97	0.95	0.86	0.95
HORMAD1	0.99	0.99	0.99	0.98	0.98	0.98
MLPH	1.00	0.94	1.00	0.99	0.13	0.99
PPP1R14C	1.00	0.74	1.00	0.99	0.99	0.59
PRR15	1.00	0.13	1.00	0.99	0.69	0.99
PSAT1	1.00	1.00	1.00	0.99	0.94	0.99
RGMA	0.99	0.91	0.99			
RHOB	0.84	0.48	0.84	0.80		0.80
ROPN1	0.71	0.71	0.71	0.85	0.85	0.85
SLC44A4	1.00	0.08	1.00	0.99	0.16	0.99
SLC7A8	1.00	0.12	1.00	0.99	0.07	0.99
SPDEF	1.00	0.22	1.00	0.99	0.66	0.99
SRSF12	1.00	1.00	0.11	0.99	0.99	0.17
TBC1D9	1.00	0.18	1.00	0.99	0.11	0.99
TFF3	1.00	0.18	1.00	0.99	0.01	0.99
UGT8	1.00	1.00	1.00	0.99	0.99	0.99
VGLL1	1.00	1.00	0.99	0.99	0.99	0.29
SFT2D2				0.70	0.20	0.70

Table S14 : Marginal analysis of TNBC: identified main effects and interactions using the alternatives.

logistic				constant			
Gene	Main	Interaction		Gene	Main	Interaction	
		age	race			age	race
FOXC1	0.2361	-0.0183	0.0709	FOXC1	0.0017	-0.0017	
ROPN1	0.2252	-0.0174	0.0635	CA12	-0.0022	-0.0010	-0.0023
YBX1	0.2222	-0.0196	0.0308	ESR1	-0.0021	-0.0010	-0.0022
CLCN4	0.2147	-0.0101	0.0607	VGLL1	0.0017	-0.0017	
SLC7A8	-0.2212		-0.0546	AGR2	-0.0021		-0.0022
CCNE1	0.2166	-0.0122	0.0448	GATA3	-0.0024	-0.0011	-0.0024
AGR2	-0.2318	-0.0020	-0.0659	TBC1D9	-0.0020		-0.0021
TBC1D9	-0.2469		-0.0745	MLPH	-0.0024		-0.0023
MYB	-0.2105		-0.0531	MYB	-0.0015		-0.0017
BCL11A	0.2297	-0.0128	0.0617	BCL11A	0.0016	-0.0017	
SPDEF	-0.2614		-0.1024	SPDEF	-0.0019		-0.0020
C5AR2	-0.2182	-0.0116	-0.0550	FOXA1	-0.0027		-0.0027
PSAT1	0.2267	-0.0106	0.0527	VAV3	-0.0013		-0.0017
HORMAD1	0.2273	-0.0136	0.0699	TTLL4	0.0015	-0.0017	
RHOB	-0.2362	-0.0028	-0.0823	LMX1B	-0.0015		-0.0016
STAC	0.2169	-0.0100	0.0628	SRSF12	0.0017	-0.0017	
FAM171A1	0.2293	-0.0107	0.0559	TFF3	-0.0016		-0.0018
TFF3	-0.2570		-0.0987	SMIM14	-0.0016		-0.0018
EN1	0.2331	-0.0197	0.0747	DLI1	-0.0017		-0.0019
DLI1	-0.2339		-0.0859	CMBL	-0.0017		-0.0019
CMBL	-0.2362		-0.0859	FBP1	-0.0016		-0.0018
FBP1	-0.2243		-0.0693	DEGS2	-0.0015		-0.0016
DEGS2	-0.2201	-0.0245	-0.0602	AR	-0.0017		-0.0019
AR	-0.2264		-0.0598	CXXC5	-0.0016		-0.0017
UGT8	0.2330	-0.0176	0.0639	AGR3	-0.0021	-0.0010	-0.0022
PRR15	-0.2580		-0.0841	PRR15	-0.0019		-0.0020
ZNF552	-0.2266		-0.0681	ZNF552	-0.0016		-0.0018
RGMA	0.2205	-0.0022	0.0686	THSD4	-0.0016		-0.0018
SLC44A4	-0.2419		-0.0753	SLC44A4	-0.0020		-0.0021
SFT2D2	0.2185	-0.0081	0.0663				

Table S15 : Joint analysis of TNBC: stability analysis of identified main effects and interactions.

Gene	DPD			γ		
	Main	Interaction		Main	Interaction	
		age	race		age	race
AADAT	1.00	1.00	1.00	1.00	1.00	1.00
ASPN	1.00	1.00	1.00	1.00	1.00	1.00
CCL13	0.97	0.92	0.94			
CENPW	1.00	1.00	1.00	1.00	1.00	1.00
COL9A3	1.00	1.00	1.00	1.00	1.00	1.00
CPA4	1.00	1.00	1.00	1.00	1.00	1.00
CXXC5	1.00	1.00	1.00	1.00	1.00	1.00
DCLRE1C	1.00	1.00	1.00	1.00	1.00	1.00
DJB11	1.00	1.00	1.00	1.00	1.00	1.00
DPF3	0.79	0.76	0.79			
ERBB2	1.00	0.82	1.00	1.00		1.00
ESR1	1.00	1.00	1.00	1.00	1.00	1.00
HAPLN3	1.00	1.00	0.98			
HDAC2	1.00	1.00	1.00	0.81	0.77	0.80
ITGB5	1.00	1.00	1.00	1.00	1.00	1.00
LYPD1	1.00	1.00	1.00	1.00	1.00	0.89
MISP3	0.44	0.24	0.36	1.00	1.00	1.00
MMP12	1.00	1.00		1.00	1.00	
MMS22L	1.00	1.00	1.00	1.00	1.00	1.00
PCDHB9	1.00	1.00	1.00	1.00	1.00	1.00
PGAP3	1.00		1.00	1.00	1.00	1.00
PTCHD1	1.00	1.00	1.00	1.00	1.00	1.00
RPL39L	1.00	1.00	1.00	1.00	1.00	1.00
SIX3	0.69	0.67	0.67	1.00	1.00	
SLC15A1	1.00		1.00	1.00		1.00
SLC38A3	1.00	1.00	1.00	1.00	1.00	
SLC6A11	1.00	1.00	1.00	1.00	1.00	1.00
TLX1	1.00		1.00	1.00		1.00
TMEM217	1.00	1.00	1.00	1.00	1.00	1.00
TRPV6	0.94	0.94	0.93			
ZIC1	1.00	1.00	1.00	1.00	1.00	1.00
ASB12				1.00	1.00	1.00
GZMB				1.00	1.00	1.00
IL22RA2				1.00	1.00	1.00
KCNS1				1.00	1.00	1.00
LYAR				0.57	0.53	
PDIA5				1.00	1.00	1.00
PLCG2				1.00	1.00	1.00
PML				1.00	1.00	1.00
PPP1R14C				1.00		1.00
RAD51AP2				1.00	0.85	1.00
UBASH3B				1.00	1.00	1.00
ZNF883				1.00	1.00	1.00

Table S16 : Joint analysis of TNBC data: identified main effects and interactions after removing outliers.

DPD				γ			
Gene	Main	Interaction		Gene	Main	Interaction	
		age	race			age	race
AADAT	0.1744	0.0541	-0.0041	AADAT	0.0387	-0.0013	-0.0165
ASB12	0.0468	-0.1027	-0.0014	AGR3	-0.2173	-0.0259	-0.0463
C11orf91	0.0375		-0.0062	AL137860.1	0.0015		
CCL13	-0.0017	-0.0065	-0.0045	ATP6V1C2	0.0312	-0.0015	-0.0122
CENPW	0.0999	-0.0379	-0.1143	B3GNT5	0.0901		-0.0059
COL9A3	0.1478	0.0082	-0.0220	BCAS1	-0.1214	-0.0047	-0.0659
CPA4	0.0019	-0.0039	-0.0072	C1QL4	0.0023	-0.0012	-0.0013
CXXC5	-0.5980	-0.1411	-0.2600	C5AR2	-0.0303	-0.0014	-0.0062
DNAJB11	0.3012	-0.1412	-0.4242	CA12	-0.3239	-0.0495	-0.0489
ERBB2	-0.0116	-0.0023	-0.0108	CD300E	0.0312	-0.0198	-0.0105
ESR1	-0.3092	-0.1500	-0.0888	CDCA7	0.1012	-0.0361	
FAAH2	0.0014	-0.0046	-0.0014	COL9A3	0.0926	0.0300	-0.0041
FOSL1	-0.0028	-0.0189	-0.0102	DNAJB11	0.0979	-0.0485	-0.0252
GABRE	-0.0012	-0.0034	-0.0014	DPF3	0.0899		-0.0203
GATA3	-0.1225	-0.0298	-0.0500	EGFR	0.0824		0.1453
IL1R2	0.1325	-0.0297	-0.0792	ERBB2	-0.1622		-0.0499
IL22RA2	0.0329	-0.0471	-0.0015	ESR1	-0.3682	-0.0229	-0.0414
ITGB5	-0.0100		-0.0102	FABP7	0.1701	0.0708	-0.0013
LYPD1	0.0710	-0.0404	-0.0106	FOXA1	-0.2879		-0.0476
MYLK4	0.0472	-0.0065	-0.0016	FZD9	0.0285		-0.0107
NASP	0.0037	-0.0017	-0.0011	GATA3	-0.2764	-0.0216	
PGAP3	-0.2549	0.0115	-0.0490	GSTM3	-0.0040		-0.0015
RAD51AP2	0.1353	-0.0460	0.1468	GZMB	0.0877	-0.0195	-0.0016
RPIA	0.0390	-0.0021	-0.0103	LANCL3	0.0945	-0.0288	0.0645
SIX3	0.0060		-0.0022	MICALL1	0.0773	-0.0112	
SLC15A1	0.0407	-0.0045	-0.0294	PGAP3	-0.3522	-0.0038	0.0509
SLC38A3	0.0162	-0.0094	-0.0021	POU5F1	0.0113	-0.0015	-0.0044
SUV39H2	0.5173	-0.1317	-0.2585	PPP1R14C	0.0306	-0.0012	-0.0017
TLX1	0.0714	-0.0020	-0.0082	RAD51AP2	0.0952	-0.0039	
TMEM217	0.1643	-0.0678	-0.0383	SEPHS1	0.0095	-0.0034	-0.0046
VGLL1	0.0169	-0.0017	-0.0065	SH2D2A	0.0598	-0.0162	
WNT6	0.0581	-0.0023	-0.0486	SLC15A1	0.0084	-0.0017	-0.0057
ZIC1	0.1135	-0.0389	-0.0732	SLC44A4	-0.0297	-0.0030	-0.0162
				SPDEF	-0.1990	-0.0024	-0.0624
				TMSB15A	0.4018	-0.0750	-0.0941
				UPP1	-0.0013	-0.0138	-0.0100
				VGLL1	0.1146		-0.0198

Table S17 : Joint analysis of TNBC: identified main effects and interactions using the alternatives.

logistic				constant			
Gene	Main	Interaction		Gene	Main	Interaction	
		age	race			age	race
PRKCQ	0.1175	-0.0185		PDIA5	0.0861		-0.0570
SLC15A1	0.0767	-0.0037	-0.0434	PRKCQ	0.0979		
DJB11	0.0774	-0.0300	-0.0724	ITGB5	-0.0537		-0.0935
ESR1	-0.2553	-0.1650	-0.0955	SLC15A1	0.0994		
COL9A3	0.0734	-0.0301	-0.0242	DJB11	0.0777		-0.0723
ASPN	-0.0233	-0.0044	-0.0143	ESR1	-0.3344		-0.0929
TLX1	0.0081		-0.0031	COL9A3	0.0897		
PLA2G4A	-0.0099	-0.0149	-0.0217	GZMB	0.1632		
CPA4	0.0430	-0.0458	-0.0297	OCA2	0.0714		0.1115
SLC6A11	0.1375	0.0881	-0.0074	AADAT	0.1822	0.0841	
HAPLN3	0.1221	-0.0127		KCNS1	0.0668		0.0507
ERBB2	-0.0653	-0.0051	-0.0440	SLC6A11	0.1378	0.0881	
MMS22L	0.0807	-0.0497	-0.0445	KCTD3	-0.0928		
LYPD1	0.1862	-0.0645	-0.0019	TMOD1	0.0634		
DCLRE1C	0.0739	-0.0035	-0.0492	SIX3	0.0526		
ZIC1	0.2329	-0.0863	-0.1413	PML	0.1152	-0.0594	
PGAP3	-0.1331		-0.0344	ERBB2	-0.0826		
RPL39L	0.0071	-0.0014	-0.0093	MISP3	-0.0533		
IL22RA2	-0.0052	-0.0194	-0.0209	MMS22L	0.0648		-0.0702
TRPV6	-0.0044	-0.0045	-0.0013	LYPD1	0.1998		
PTCHD1	0.0601	-0.0095	-0.0598	DCLRE1C	0.0741		
CXXC5	-0.6289	-0.1554	-0.4036	ZIC1	0.1834	-0.0562	-0.1518
TMEM217	0.0551	-0.0195	-0.0063	UBASH3B	0.0517		
PCDHB9	0.0010	-0.0027	-0.0026	PGAP3	-0.1343		
SLC38A3	0.0804	-0.0426	-0.0066	DEFB1	0.0581		
HDAC2	0.0020	-0.0021	-0.0021	TRPV6	0.0685		
CENPW	0.0798	-0.0257	-0.0784	PTCHD1	0.0572		-0.0638
MMP12	0.0079	-0.0061	-0.0011	CXXC5	-0.1784		-0.2421
				TMEM217	0.0554		
				SLC38A3	0.0995		
				PLCG2	0.1069		
				PPP1R14C	0.0585		
				KIF1BP	-0.0536		-0.0806
				CENPW	0.0802		-0.0782
				RAD51AP2	0.0932		0.1069
				ZNF883	0.0606		

Table S18 : Marginal analysis of Diabetes Data: identified main effects and interactions.

SNP	Gene	DPD							γ						
		Main	Interaction						Main	Interaction					
			age	famdb	act	trans	ceraf	heme		age	famdb	act	trans	ceraf	heme
rs1398547	RPS23P3								-0.0124	0.0799	-0.0127	0.0412	-0.0108	-0.0024	-0.0112
rs13111778	RP11-584P21.2	-0.0045	-0.0102	-0.0158	-0.0096	-0.0157	-0.0044	-0.0157	-0.0189	0.0484	-0.0063	0.0263	-0.0052		-0.0054
rs1484186	UBA6-AS1								0.0033	0.0432	-0.0063	0.0232	-0.0053		-0.0055
rs17088752	UBA6-AS1	0.2096	-0.0933	-0.0875	-0.0847	-0.0877	-0.0523	-0.0876	0.4816	0.0258	-0.0032	0.0132	-0.0025		-0.0027
rs353172	UBA6-AS1	-0.0131	-1.4590	-0.1261	-0.8442	-0.1421	-0.2059	-0.1382							
rs17088851	UBA6-AS1								-0.0015		-0.0015		-0.0015	-0.0014	-0.0015
rs10033058	YTHDC1								-0.0027	0.0228	-0.0032	0.0118	-0.0026		-0.0027
rs2293595	YTHDC1	-0.0069	-0.0449	-0.0167	-0.0308	-0.0173	-0.0195	-0.0171	0.0352	0.0905	-0.0127	0.0442	-0.0105	-0.0013	-0.0110
rs6600893	UGT2B7	-0.0025	-0.0086	-0.0085	-0.0060	-0.0086		-0.0085	-0.0086	0.0227	-0.0032	0.0122	-0.0026		-0.0027
rs4327554	RP13-644M16.1	-0.1667	-1.3560	-0.0918	-0.7950	-0.1055	-0.1620	-0.1023	0.0018	0.0056		0.0031			
rs13109616	RP11-593F5.2	-0.0024	-0.0136	-0.0167	-0.0115	-0.0167	-0.0010	-0.0167	-0.0095	0.0439	-0.0063	0.0235	-0.0053		-0.0055
rs4563469	RUFY3	0.0018	-0.0210	-0.0175	-0.0184	-0.0175	-0.0178	-0.0175	-0.1277	-0.0018	-0.0280	-0.0139	-0.0274	-0.0251	-0.0276
rs1878756	ADAMTS3	0.0020	-0.0099	-0.0084	-0.0065	-0.0085		-0.0085	0.0089	0.0213	-0.0032	0.0117	-0.0027		-0.0028
rs1841587	HNRNPA1P67	-0.0131	-1.4541	-0.1261	-0.8486	-0.1416	-0.2055	-0.1383							
rs6446910	RNU4ATAC9P	-0.0011	-0.0359	-0.0229	-0.0278	-0.0232	-0.0242	-0.0232	0.0219	0.0844	-0.0127	0.0421	-0.0107	-0.0020	-0.0111
rs11730667	IL8	-0.0017	-0.0099	-0.0084	-0.0066	-0.0085		-0.0084	-0.0066	0.0217	-0.0032	0.0118	-0.0026		-0.0028
rs6853774	RP11-542G1.1	-0.0025	-0.0430	-0.0223	-0.0312	-0.0227	-0.0243	-0.0226	0.0213	0.0856	-0.0127	0.0417	-0.0106	-0.0017	-0.0111
rs10009464	SEPT11	0.0019	-0.0153	-0.0165	-0.0125	-0.0166	-0.0011	-0.0166	0.0103	0.0425	-0.0063	0.0222	-0.0053		-0.0055
rs2866442	CXCL13	-0.1629	-1.3349	-0.0711	-0.7634	-0.0845	-0.1361	-0.0812	0.0019	0.0060		0.0032			
rs895087	RASGEF1B								-0.0035	0.0383	-0.0064	0.0200	-0.0054	-0.0013	-0.0056
rs545073	RP11-689K5.3	0.0063	-0.0109	-0.0171	-0.0103	-0.0170	-0.0010	-0.0170	0.0227	0.0445	-0.0063	0.0228	-0.0053		-0.0055
rs1003176	RP11-689K5.3	0.0022	-0.0151	-0.0166	-0.0121	-0.0166	-0.0011	-0.0166	0.0112	0.0427	-0.0063	0.0227	-0.0053		-0.0055
rs4320128	HPSE	0.0034	-0.0062	-0.0087	-0.0046	-0.0087		-0.0087	0.0103	0.0249	-0.0032	0.0134	-0.0026		-0.0027
rs6823482	Y-R	-0.0104	-1.3397	-0.0642	-0.7663	-0.0771	-0.1273	-0.0741							
rs3100249	ARHGAP24	-0.0102	-1.3316	-0.0640	-0.7543	-0.0769	-0.1258	-0.0739							
rs7660693	RP11-10L7.1	-0.0045	-0.0511		-0.0294	-0.0014	-0.0054	-0.0012	0.0118	0.0908	-0.0127	0.0466	-0.0105	-0.0011	-0.0110
rs6817766	RP11-84C13.2								-0.0012		-0.0011		-0.0011	-0.0011	-0.0011
rs6839114	GPRIN3	0.0039	-0.0129	-0.0168	-0.0112	-0.0168	-0.0010	-0.0168	0.0165	0.0435	-0.0063	0.0231	-0.0053		-0.0055
rs17018060	CCSER1	0.0010	-0.0095	-0.0084	-0.0064	-0.0085		-0.0085	0.0049	0.0225	-0.0032	0.0121	-0.0026		-0.0027
rs4475115	GRID2								-0.0040	0.0104	-0.0016	0.0055	-0.0013		-0.0014
rs10050093	R5SP164	-0.0022	-0.0087	-0.0085	-0.0060	-0.0086		-0.0085	-0.0077	0.0230	-0.0032	0.0121	-0.0026		-0.0027
rs10856909	HPGDS								0.0073	0.0809	-0.0127	0.0401	-0.0108	-0.0023	-0.0112
rs7658830	RP11-398J16.1								0.0018	0.0049		0.0026			
rs1472962	RP11-554D13.1	0.0013	-0.0175	-0.0163	-0.0135	-0.0164	-0.0165	-0.0164	0.0085	0.0408	-0.0063	0.0214	-0.0054	-0.0011	-0.0056
rs1996334	RP11-145G20.1	-0.0011	-0.0115	-0.0082	-0.0076	-0.0083		-0.0083	-0.0057	0.0204	-0.0032	0.0107	-0.0027		-0.0028
rs11737416	RP11-145G20.1								-0.0023	0.0475	-0.0063	0.0250	-0.0052		-0.0054
rs10516427	Y-R								0.0091	0.0014					
rs1483292	TECRL	-0.0102	-1.3282	-0.0709	-0.7542	-0.0837	-0.1336	-0.0807							
rs17251102	BANK1	-0.0063	-0.0102	-0.0162	-0.0098	-0.0161	-0.0038	-0.0161	-0.0242	0.0470	-0.0063	0.0248	-0.0052		-0.0055
rs1425385	RP11-63H19.1	0.0043	-0.0471	-0.0043	-0.0285	-0.0051	-0.0084	-0.0049	-0.0136	0.0916	-0.0127	0.0467	-0.0105	-0.0010	-0.0110
rs227361	AF224669.3	-0.0053	-0.0441	-0.0171	-0.0304	-0.0176	-0.0198	-0.0175	0.0287	0.0897	-0.0127	0.0444	-0.0105	-0.0013	-0.0110
rs2636739	PPA2	0.0021	-0.0105	-0.0084	-0.0070	-0.0084		-0.0084	0.0102	0.0205	-0.0032	0.0106	-0.0027		-0.0028
rs17261094	ARHGEF38	0.0429	-0.0563	-0.0411	-0.0453	-0.0414	-0.0428	-0.0413	-0.2538	0.0124	-0.0262	-0.0041	-0.0254	-0.0219	-0.0256
rs11722403	SGMS2	-0.0151	-0.0710	-0.0516	-0.0570	-0.0521	-0.0537	-0.0520	0.2555	0.0149	-0.0262	-0.0020	-0.0253	-0.0218	-0.0255
rs13126645	LEF1-AS1								0.0058	0.0941	-0.0126	0.0474	-0.0104		-0.0109
rs13130005	RP11-498E11.2								0.0010						
rs11569126	EGF	-0.0015	-0.0011						-0.0024	-0.0018	-0.0013	-0.0015	-0.0013	-0.0014	-0.0013
rs11098070	ELOVL6	-0.0057	-0.0103	-0.0166	-0.0099	-0.0166	-0.0024	-0.0166	-0.0206	0.0465	-0.0063	0.0243	-0.0052		-0.0055

Table S19 : Marginal analysis of Diabetes Data: stability analysis of identified main effects and interactions.

SNP	Gene	DPD							γ						
		Main	Interaction						Main	Interaction					
			age	famdb	act	trans	ceraf	heme		age	famdb	act	trans	ceraf	heme
rs1398547	RPS23P3								0.95	0.92	0.95	0.94	0.95	0.95	0.95
rs13111778	RP11-584P21.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs1484186	UBA6-AS1								1.00	1.00	1.00	1.00	1.00		1.00
rs17088752	UBA6-AS1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs353172	UBA6-AS1	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
rs17088851	UBA6-AS1								1.00		0.91		1.00	0.96	1.00
rs10033058	YTHDC1								1.00	1.00	1.00	1.00	1.00		1.00
rs2293595	YTHDC1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs6600893	UGT2B7	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs4327554	RP13-644M16.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00			
rs13109616	RP11-593F5.2	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs4563469	RUFY3	0.97	0.97	0.95	0.94	0.97	0.93	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs1878756	ADAMTS3	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs1841587	HNRNPA1P67	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
rs6446910	RNU4ATAC9P	0.81	0.80	0.81	0.76	0.81	0.78	0.81	0.92	0.90	0.92	0.92	0.89	0.88	0.90
rs11730667	IL8	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs6853774	RP11-542G1.1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs10009464	SEPT11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs2866442	CXCL13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00			
rs895087	RASGEF1B								0.86	0.85	0.86	0.82	0.80	0.82	0.86
rs545073	RP11-689K5.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs1003176	RP11-689K5.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs4320128	HPSE	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs6823482	Y-R	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
rs3100249	ARHGAP24	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
rs7660693	RP11-10L7.1	1.00	1.00	0.39	1.00	0.93	1.00	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs6817766	RP11-84C13.2	0.01							1.00		1.00		1.00	1.00	1.00
rs6839114	GPRIN3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs17018060	CCSER1	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs4475115	GRID2								1.00	1.00	1.00	1.00	1.00		1.00
rs10050093	R5SP164	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs10856909	HPGDS								1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs7658830	RP11-398J16.1								1.00	0.86		0.92			
rs1472962	RP11-554D13.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs1996334	RP11-145G20.1	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs11737416	RP11-145G20.1								1.00	1.00	1.00	1.00	1.00		1.00
rs10516427	Y-R								1.00	1.00					
rs1483292	TECRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
rs17251102	BANK1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs1425385	RP11-63H19.1	0.94	0.94	0.94	0.94	0.94	0.91	0.90	0.96	0.96	0.93	0.96	0.91	0.90	0.95
rs227361	AF224669.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs2636739	PPA2	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00
rs17261094	ARHGEF38	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs11722403	SGMS2	0.92	0.91	0.90	0.92	0.90	0.91	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs13126645	LEF1-AS1								1.00	1.00	1.00	1.00	1.00		1.00
rs13130005	RP11-498E11.2								1.00						
rs11569126	EGF	1.00	1.00						1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs11098070	ELOVL6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		1.00

Table S20 : Marginal analysis of Diabetes Data: numbers of main G effects and interactions identified by different methods and their overlaps.

Diabetes	Main effects				Interactions			
	LS	LAD	DPD	γ	LS	LAD	DPD	γ
LS	27	5	3	3	162	6	17	16
LAD		46	13	18		72	25	33
DPD			35	30			196	149
γ				43				209

Table S21 : Marginal analysis of Diabetes Data: identified main effects and interactions using the alternatives.

LS									LAD								
SNP	Gene	Main	Interaction						SNP	Gene	Main	Interaction					
			age	famdb	act	trans	ceraf	heme				age	famdb	act	trans	ceraf	heme
rs17090286	RP11-593F5.2	0.0218	-0.0272	-0.0081	-0.0403	-0.0084	-0.0103	-0.0083	rs13129688	Y-RNA	0.1014	0.0590		0.0332			
rs2172802	LPHN3	0.0053	-0.0116	-0.0061	-0.0165	-0.0062	-0.0068	-0.0062	rs6551565	AC095061.1	0.0703	0.0909		0.0485			
rs17085296	RP11-63H19.1	0.0130	-0.0243	-0.0079	-0.0404	-0.0082	-0.0099	-0.0081	rs1489568	RP11-257A22.1	0.1117	0.0628		0.0316			
rs1397755	MIR1269A	-0.0080	-0.0171	-0.0072	-0.0275	-0.0073	-0.0083	-0.0073	rs2714934	TECRL	0.0370						
rs1511286	RPS23P3	-0.0102	-0.0220	-0.0077	-0.0371	-0.0079	-0.0095	-0.0079	rs1425385	RP11-63H19.1	-0.0942	0.0603		0.0358			
rs11936928	RPS23P3	-0.0124	-0.0172	-0.0073	-0.0284	-0.0075	-0.0086	-0.0074	rs1543095	RPS23P3	0.1100	0.0644		0.0319			
rs6838523	RPS23P3	-0.0056	-0.0105	-0.0060	-0.0157	-0.0061	-0.0066	-0.0060	rs1398547	RPS23P3	0.0603	0.0543		0.0321			
rs17088752	UBA6-AS1	0.0058	-0.0085	-0.0092	-0.0086	-0.0092	-0.0091	-0.0092	rs4032301	RPS23P3	-0.0867	0.0617		0.0322			
rs17089267	YTHDC1	0.0028	-0.0170	-0.0071	-0.0285	-0.0072	-0.0083	-0.0072	rs13111778	RP11-584P21.2	-0.0476	0.0589		0.0358			
rs11249477	CSN1S2AP	0.0066	-0.0113	-0.0061	-0.0148	-0.0062	-0.0067	-0.0062	rs1484186	UBA6-AS1	-0.0343	0.0319					
rs10003790	DCK	0.0017		-0.0035		-0.0035			rs2293595	YTHDC1	0.1120	0.0613		0.0345			
rs6446910	RNU4ATAC9P	0.0063	-0.0269	-0.0082	-0.0496	-0.0084	-0.0103	-0.0083	rs7656541	UGT2A1	0.0489	0.0500					
rs12649753	RN7SL218P	-0.0018	-0.0096	-0.0054	-0.0143	-0.0054			rs1847366	SULT1B1	-0.0862	0.0591		0.0333			
rs10028707	SHROOM3	0.0015	-0.0108	-0.0056	-0.0156	-0.0057	-0.0062		rs1405376	SLC4A4	0.0912	0.0521					
rs12500486	AC104687.1	-0.0141	-0.0120	-0.0068	-0.0179	-0.0069	-0.0074	-0.0068	rs6446910	RNU4ATAC9P	0.0982	0.0579		0.0332			
rs7681755	LINC01088	-0.0031	-0.0119	-0.0061	-0.0161	-0.0062	-0.0068	-0.0062	rs1534450	ANKRD17	-0.0570	0.0513					
rs10004901	C4orf22	0.0021	-0.0085	-0.0053	-0.0111	-0.0053			rs422140	RN7SL218P	-0.1187	0.0616		0.0353			
rs7672440	RP11-689K5.3	0.0032	-0.0115	-0.0059	-0.0171	-0.0060	-0.0065		rs12649753	RN7SL218P	0.1625	0.0998		0.0533			
rs676592	RP11-689K5.3	-0.0025	-0.0226	-0.0077	-0.0365	-0.0079	-0.0094	-0.0078	rs17242392	RP11-727M10.1	0.0741	0.0537					
rs11722328	RP11-689K5.3	-0.0031	-0.0125	-0.0063	-0.0191	-0.0064	-0.0071	-0.0063	rs6853774	RP11-542G1.1	0.0975	0.0587		0.0329			
rs6842681	RP11-218C23.1	0.0023	-0.0097	-0.0054	-0.0134	-0.0054			rs6812261	RP11-567N4.2	0.0816	0.0513					
rs13143182	AFF1	-0.0087	-0.0095	-0.0064	-0.0125	-0.0065	-0.0068	-0.0064	rs10028707	SHROOM3	-0.0871	0.0506					
rs3775373	FAM13A	0.0013	-0.0161	-0.0069	-0.0252	-0.0071	-0.0081	-0.0071	rs10009464	11-Sep	0.0373	0.0537		0.0316			
rs4699606	Y-RNA	0.0012	-0.0098	-0.0052	-0.0148	-0.0053			rs904032	11-Sep	-0.0310						
rs2726516	PPA2	-0.0111	-0.0167	-0.0072	-0.0274	-0.0073	-0.0083	-0.0073	rs7672440	RP11-689K5.3	-0.0325	0.0459					
rs2636739	PPA2	-0.0014	-0.0105	-0.0056	-0.0158	-0.0057	-0.0062		rs676592	RP11-689K5.3	0.0376						
rs2686293	RP13-612N21.1	0.0020	-0.0129	-0.0062	-0.0188	-0.0063	-0.0070	-0.0063	rs545073	RP11-689K5.3	0.0513	0.0549		0.0317			
									rs1003176	RP11-689K5.3	0.0510	0.0321					
									rs4334746	RP11-778J15.1	-0.0328	0.0862	-0.0352	0.0389	-0.0327		-0.0332
									rs6833161	SPP1	0.0303						
									rs7660693	RP11-10L7.1	-0.0665	0.0592		0.0351			
									rs6839114	GPRIN3	0.0444	0.0542		0.0322			
									rs10856909	HPGDS	0.0821	0.0559		0.0317			
									rs11737416	RP11-145G20.1	0.0386	0.0314					
									rs6834324	RP11-145G20.1	0.0887	0.0573		0.0316			
									rs9994641	Y-RNA	-0.0393	0.0466					
									rs4306962	EMCN	-0.0569	0.0875		0.0475			
									rs11936939	EMCN	-0.0492	0.0481					
									rs17251102	BANK1	-0.0528	0.0578		0.0341			
									rs6844449	RP11-498M5.2	-0.0385	0.0462					
									rs151414	SLC39A8	-0.0440	0.0506					
									rs227361	AF224669.3	0.1092	0.0579		0.0335			
									rs17033609	RP11-328K4.1	-0.0353	0.0440					
									rs2636751	PPA2	-0.1079	0.0574		0.0332			
									rs13126645	LEF1-AS1	0.0841	0.0605		0.0357			
									rs11098070	ELOVL6	-0.0488	0.0576		0.0338			

Table S22 : Joint analysis of Diabetes Data: identified main effects and interactions.

SNP	Gene	DPD							γ						
		Main	Interaction						Main	Interaction					
			age	famdb	act	trans	ceraf	heme		age	famdb	act	trans	ceraf	heme
rs1346077	RP11-584P21.2	0.3121	-0.0039	0.0227		-0.0224		-0.0270	0.9363	-0.0116	0.0680		-0.0672	0.0017	-0.0809
rs17088752	UBA6-AS1	0.1933	-0.0023	-0.0326		0.0235	-0.0018	-0.0020	0.5799	-0.0070	-0.0978		0.0706	-0.0055	-0.0061
rs11249478	CSN1S2AP	0.1035	-0.0034	-0.0099	-0.0028	-0.0016	-0.0048	0.0063	0.3104	-0.0102	-0.0297	-0.0084	-0.0048	-0.0145	0.0188
rs17090286	RP11-593F5.2	-0.1575	-0.0013	0.0127	-0.0049		-0.0051	-0.0108	-0.4830	0.0039	0.0459	-0.0073		-0.0079	-0.0256
rs10012631	DCK	0.3679	-0.0039	-0.0436		0.0266		-0.0940	1.1038	-0.0117	-0.1309		0.0798	-0.0020	-0.2821
rs1951703	AC112518.3	0.2451	-0.0033	0.0147		-0.0297	-0.0035	0.0073	0.7352	-0.0100	0.0440		-0.0890	-0.0104	0.0219
rs16850078	PPBPP1								0.2474	-0.0020	0.0370		-0.0043	-0.0089	-0.0227
rs4694666	MTHFD2L	0.0814		-0.0083		-0.0057	-0.0038	-0.0194	0.2442	-0.0021	-0.0249		-0.0171	-0.0115	-0.0583
rs7810	CCNI	-0.1441		-0.0166		0.0700	-0.0024	-0.0197	-0.4369	0.0034	-0.0470		0.2138	-0.0032	-0.0565
rs6827052	LINC01088	0.2414	-0.0017	0.0485		0.0153	-0.0114	-0.0371	0.7241	-0.0051	0.1456	-0.0022	0.0458	-0.0343	-0.1112
rs17226412	LPHN3								0.0022					0.0011	
rs10004901	C4orf22	-0.0992	-0.0029	-0.0190	-0.0056	-0.0064	-0.0049	0.0016	-0.2975	-0.0087	-0.0569	-0.0167	-0.0190	-0.0146	0.0049
rs6819983	RASGEF1B	0.0770	-0.0017	-0.0086	-0.0038	-0.0220	-0.0096		0.2309	-0.0052	-0.0259	-0.0115	-0.0659	-0.0289	-0.0020
rs12500169	SCD5	0.0818	-0.0189	-0.0110	-0.0188		-0.0218	-0.0206	0.3027	-0.0359	-0.0035	-0.0353	0.0355	-0.0478	-0.0428
rs4364254	HPSE	0.0990	-0.0015	-0.0229	-0.0067		-0.0160	-0.0239	0.3231	0.0114	-0.0606	-0.0051		-0.0369	-0.0640
rs12648917	RP11-8L2.1	-0.1471	-0.0158		-0.0198	0.0572	-0.0212	-0.0024	-0.4599	-0.0345		-0.0477	0.1870	-0.0524	0.0084
rs2452554	PDLIM5	0.1660		0.0326	-0.0013		-0.0079	-0.0296	0.4980	-0.0030	0.0977	-0.0039	-0.0024	-0.0237	-0.0887
rs2170781	UNC5C	0.1186	-0.0039	-0.0065	-0.0094		-0.0178	-0.0347	0.3557	-0.0117	-0.0194	-0.0281		-0.0535	-0.1042
rs34987374	RP11-145G20.1	0.3179	-0.0031	0.0227		-0.0261	-0.0069	-0.0217	0.9537	-0.0094	0.0680		-0.0783	-0.0207	-0.0652
rs10021842	RP11-681L8.1	-0.1455		-0.0434	-0.0061	-0.0085	-0.0027	0.0490	-0.4549		-0.1249	-0.0030	-0.0107	0.0074	0.1624
rs1230151	FAM177A1P1	-0.0023							0.1666	-0.0139		-0.0175	-0.0367	-0.0255	-0.0183
rs7375429	RP11-696N14.1	0.0662	-0.0080	-0.0012		-0.0078	-0.0078	-0.0093	0.2452	-0.0029	0.0217		-0.0020	-0.0020	-0.0081
rs1789888	ADH1B	-0.2345		-0.0149	-0.0042		-0.0063	0.0401	-0.7118	0.0114	-0.0347	-0.0010	0.0132	-0.0075	0.1295
rs4510490	TECRL	0.0988		-0.0496		-0.0015		-0.0136	0.2965	-0.0011	-0.1489	-0.0013	-0.0045	-0.0013	-0.0408
rs12505043	BANK1								0.0026						
rs4607255	RP11-63H19.1	0.2091		-0.0470	-0.0013	-0.0118	-0.0066	-0.0221	0.6273	-0.0018	-0.1411	-0.0038	-0.0355	-0.0197	-0.0664
rs17085296	RP11-63H19.1	-0.2539	0.0016	-0.0280	-0.0022	-0.0278		0.0715	-0.7639	0.0093	-0.0807	-0.0023	-0.0802	0.0044	0.2170
rs4616758	NPNT								-0.0015						
rs17041386	ELOVL6	-0.1687	-0.0020	0.0507	-0.0056	-0.0246	-0.0074	0.0038	-0.5232	0.0094	0.1662	-0.0012	-0.0631	-0.0073	0.0268

Table S23 : Joint analysis of Diabetes Data: stability analysis of identified main effects and interactions.

SNP	Gene	DPD							γ						
		Main	Interaction						Main	Interaction					
			age	famdb	act	trans	ceraf	heme		age	famdb	act	trans	ceraf	heme
rs1346077	RP11-584P21.2	1.00	1.00	1.00		1.00		1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs17088752	UBA6-AS1	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs11249478	CSN1S2AP	0.89	0.83	0.83	0.81	0.80	0.85	0.82	0.86	0.81	0.78	0.76	0.80	0.80	0.82
rs17090286	RP11-593F5.2	1.00	1.00	0.85	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00
rs10012631	DCK	1.00	1.00	1.00		1.00		1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs1951703	AC112518.3	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs16850078	PPBPP1								1.00	1.00	1.00		1.00	1.00	1.00
rs4694666	MTHFD2L	1.00		1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs7810	CCNI	1.00		1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs6827052	LINC01088	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs17226412	LPHN3								0.86					0.84	
rs10004901	C4orf22	1.00	0.76	1.00	0.91	0.87	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs6819983	RASGEF1B	1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs12500169	SCD5	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs4364254	HPSE	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00
rs12648917	RP11-8L2.1	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00
rs2452554	PDLIM5	1.00		1.00	1.00		1.00	1.00	1.00	0.64	1.00	1.00	1.00	1.00	1.00
rs2170781	UNC5C	1.00	0.68	0.65	0.76		0.61	0.78	1.00	1.00	1.00	1.00		1.00	1.00
rs34987374	RP11-145G20.1	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs10021842	RP11-681L8.1	1.00		0.89	0.92	0.81	0.82	0.93	1.00		1.00	1.00	1.00	1.00	1.00
rs1230151	FAM177A1P1	1.00							1.00	1.00		1.00	1.00	1.00	1.00
rs7375429	RP11-696N14.1	1.00	1.00	1.00		0.93	0.96	1.00	1.00	1.00	1.00		1.00	1.00	1.00
rs1789888	ADH1B	1.00		1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs4510490	TECRL	0.95		0.92		0.85		1.00	0.92	0.90	0.91	0.88	0.85	0.84	0.86
rs12505043	BANK1								1.00						
rs4607255	RP11-63H19.1	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs17085296	RP11-63H19.1	1.00	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
rs4616758	NPNT								1.00						
rs17041386	ELOVL6	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table S24 : Joint analysis of Diabetes Data: numbers of main G effects and interactions identified by different methods and their overlaps.

Diabetes	Main effects				Interactions			
	LS	LAD	DPD	γ	LS	LAD	DPD	γ
LS	23	11	12	13	56	23	30	34
LAD		25	15	15		91	55	68
DPD			25	25			115	115
γ				29				142

Table S25 : Joint analysis of Diabetes Data: identified main effects and interactions using the alternatives.

LS									LAD								
SNP	Gene	Main	Interaction						SNP	Gene	Main	Interaction					
			age	famdb	act	trans	ceraf	heme				age	famdb	act	trans	ceraf	heme
rs7693763	LPHN3	0.0125	0.0258	-0.0171	0.0074	-0.0170	-0.0125	-0.0155	rs17090286	RP11-593F5.2	-0.0093	-0.0304	-0.0305	-0.0310	-0.0313	-0.0314	-0.0318
rs17672564	LARP1BP1	-0.0034							rs7693763	LPHN3	0.0106	0.0064	0.0068	0.0067	0.0061	0.0067	0.0076
rs4510490	TECRL	-0.0039							rs4510490	TECRL	0.0162		-0.0114				-0.0055
rs1397755	MIR1269A	0.0031							rs17085296	RP11-63H19.1	-0.0341		-0.0066		-0.0066		0.0066
rs1346077	RP11-584P21.2	0.0058							rs34801717	RP11-707A18.1	0.0227						
rs17088752	UBA6-AS1	-0.0037							rs4860208	RPS23P3	0.0054						
rs17148800	SMR3A	-0.0030							rs2175327	RPS23P3	-0.0078						
rs10012631	DCK	-0.0217	-0.0125	-0.0107	-0.0143	-0.0148	-0.0132	-0.0077	rs1346077	RP11-584P21.2	0.1346	-0.0207	-0.0095	-0.0192	-0.0290	-0.0190	-0.0310
rs4694666	MTHFD2L	0.0067		-0.0042		-0.0040	-0.0037	-0.0051	rs17088752	UBA6-AS1	-0.0235	-0.0066		-0.0069	-0.0099	-0.0068	-0.0067
rs10004901	C4orf22	-0.0030							rs11249478	CSN1S2AP	0.0277	0.0079	0.0069	0.0084	0.0092	0.0082	0.0112
rs12648917	RP11-8L2.1	0.0049							rs17148800	SMR3A	-0.0055						
rs10516713	RP11-8L2.1	-0.0032							rs13108630	NPFFR2	0.0160	-0.0062	-0.0065	-0.0060	-0.0092	-0.0057	
rs2452554	PDLIM5	-0.0081		-0.0122	-0.0039	-0.0104	-0.0090	-0.0089	rs12500486	AC104687.1	0.0159						
rs265030	UNC5C	0.0080							rs7810	CCNI	0.0185	-0.0066		-0.0065	-0.0156	-0.0064	
rs34987374	RP11-145G20.1	-0.0409	-0.0069	-0.0135	-0.0083	-0.0071	-0.0093	-0.0077	rs10004901	C4orf22	0.0181	-0.0180	-0.0161	-0.0180	-0.0185	-0.0186	-0.0198
rs10021842	RP11-681L8.1	0.0128				-0.0031	-0.0035	-0.0079	rs6819983	RASGEF1B	0.0089	-0.0092	-0.0095	-0.0092	-0.0112	-0.0097	-0.0086
rs1789888	ADH1B	0.0303	0.0207	0.0099	0.0188	0.0081	0.0102	0.0031	rs12500169	SCD5	-0.0061	-0.0101	-0.0112	-0.0103	-0.0118	-0.0103	-0.0104
rs12651056	EMCN	0.0130	0.0111		0.0054				rs7675247	GRID2	0.0081	-0.0100	-0.0080	-0.0100	-0.0106	-0.0102	-0.0089
rs12499640	EMCN	0.0034	0.0327	-0.0176	0.0120	-0.0176	-0.0134	-0.0165	rs2452554	PDLIM5	-0.0173	-0.0107	-0.0150	-0.0111	-0.0117	-0.0109	-0.0086
rs2636739	PPA2	0.0035	0.0354	-0.0229	0.0124	-0.0209	-0.0157	-0.0210	rs34987374	RP11-145G20.1	0.0346	-0.0125	-0.0098	-0.0122	-0.0151	-0.0130	-0.0146
rs4616758	NPNT	-0.0034							rs1230151	FAM177A1P1	0.0100	-0.0085	-0.0072	-0.0085	-0.0094	-0.0087	-0.0082
rs11569126	EGF	-0.0043							rs7375429	RP11-696N14.1	-0.0165						
rs17041386	ELOVL6	-0.0100	-0.0035	-0.0033	-0.0049	-0.0077	-0.0064	-0.0060	rs1789888	ADH1B	-0.0277	-0.0133	-0.0153	-0.0138	-0.0134	-0.0142	-0.0088
									rs12499640	EMCN	0.0066						
									rs2636739	PPA2	-0.0072						

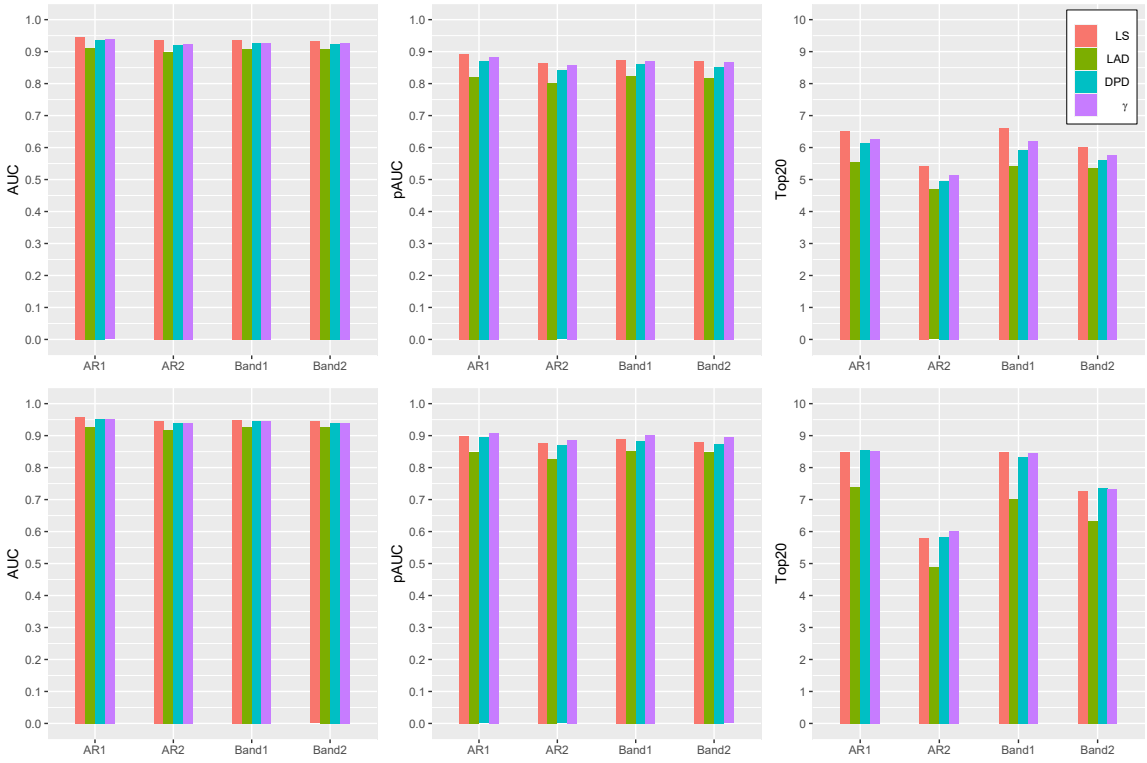

Figure S1 : Simulation results for linear model under S0 and $n = 200$. Three subgraphs at the top correspond to the main effects and the rest correspond to the interactions.

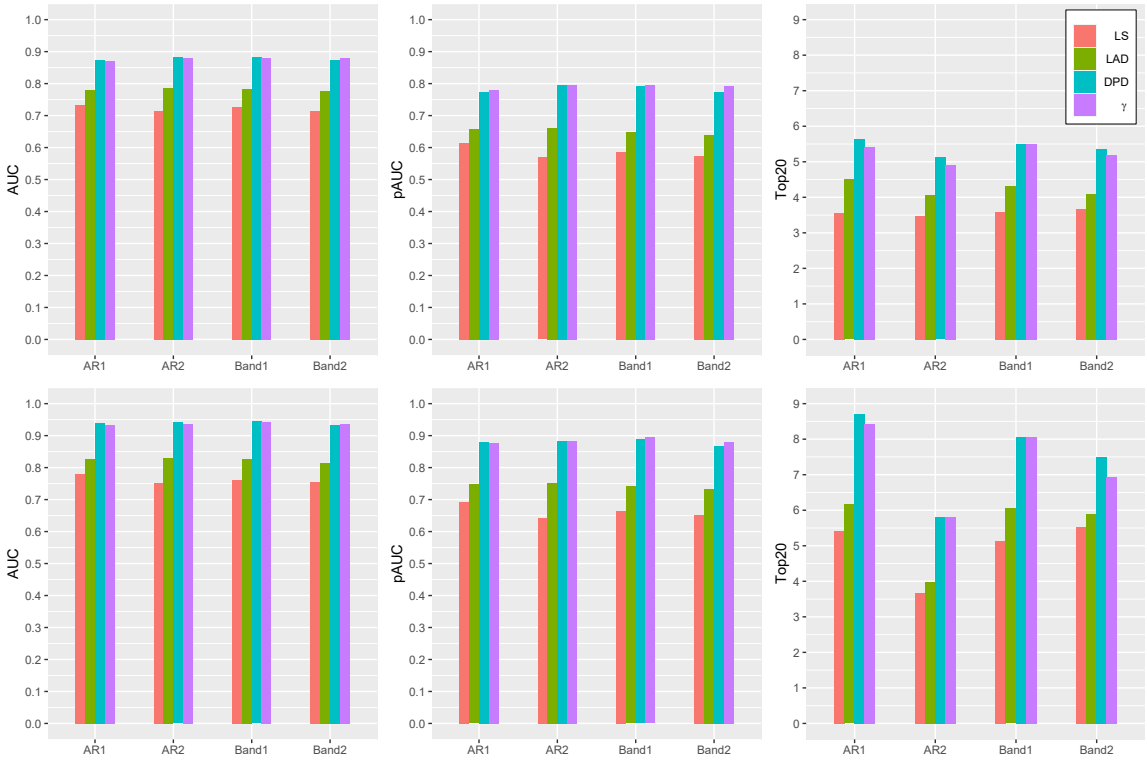

Figure S2 : Simulation results for linear model under S1 and $n = 200$. Three subgraphs at the top correspond to the main effects and the rest correspond to the interactions.

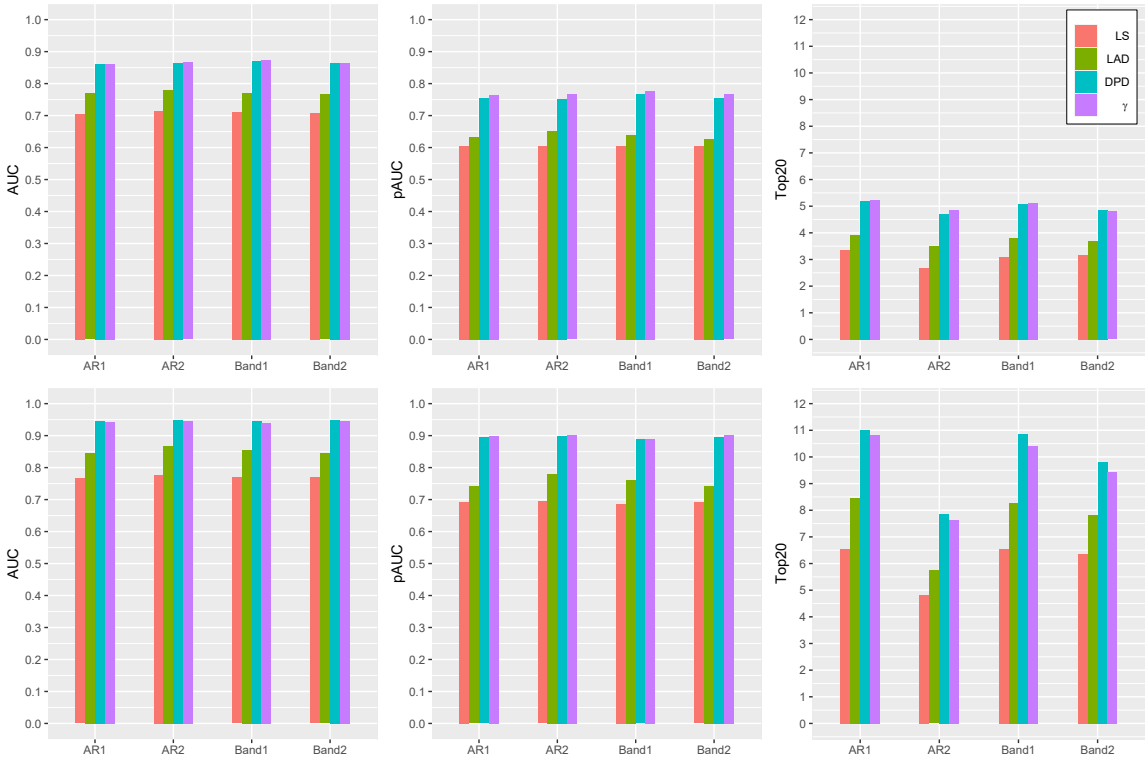

Figure S3 : Simulation results for linear model under S2 and $n = 200$. Three subgraphs at the top correspond to the main effects and the rest correspond to the interactions.