***Covariance and correlation between the payout outcome of time-overlapping binary options with differing strike prices on the same underlying asset***

Consider two binary option contracts on the same underlying asset :

i. A cash or nothing call option with strike price and time to expiry

ii. A cash or nothing call option with strike price and time to expiry .

The options are simultaneously active for a time length of, and option(ii) starts a time length of ahead of option (i).

Let X and Y be the payout outcomes (including purchase prices) of both options respectively. One can model X and Y as random variables which take the value & with probability & and value & with probability & respectively.

**Assuming follows a geometric brownian motion (GBM) process with drift rate (equivalent to the risk-free rate) and volatility , what is the correlation between the payout outcomes?**

Using the gaussian increment property of a GBM process, one can model the variation of the log returns of for a specific timeframe as a sum of normal iid random variables. Let (1). and (2). describe the log returns of during the active timeframe of options (i) and (ii) respectively, where ,, and are standard normal random variables. is the variation of the log returns when both options are active simultaneously, and thus present in both (1) and (2). The covariance and correlation between (1) and (2) are as follow:

(3).

(4).

With the knowledge of (3) which is a constant, one is able to model , the pair of log returns of during the active timeframe of option (i) and (ii) perfectly with a bivariate normal distribution with mean vector , and covariance matrix . Thus, the covariance and correlation of X & Y are as follow:

(5).

To evaluate (5), one needs to calculate the probability weights for all outcomes. Let be the price of when option (i) becomes active. Also, let be and be which are implied strike prices of the options given their probabilities of receiving the payout (and ). Then

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where .

Thus, resuming from (5):

(6).

(7).