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Felipe Augusto L. Reis, Raquel Almeida
Silvio Jamil F. Guimarães, Zenilton K. G. do Patrocínio Jr
Audio-Visual Information Processing Laboratory
Pontifical Catholic University of Minas Gerais
Belo Horizonte, Minas Gerais, Brazil
{falreis, raquel.almeida.685026}@sga.pucminas.br
{sjamil, zenilton}@pucminas.br

Simon Malinowski
Linkmedia, IRISA
Université de Rennes 1
Rennes, France
simon.malinowski@irisa.fr

***Abstract*—**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

Image segmentation refers to the partition of an image into a set of regions representing meaningful areas. It is considered a challenging semantic task, aiming to determine and group uniform regions for analysis. According to [1], to create an adequate segmented image it is necessary that the output presents some fundamental characteristics, such as: (i) region uniformity and homogeneity in its features, such as gray level, color or texture; (ii) region continuity, without holes; (iii) significant difference to adjacency regions; and (iv) spacial accuracy with smooth boundaries and without raggedness.

Image segmentation is an active topic of research and in a typical approach the procedure could be divided in two stages [2]: (i) low-level analysis, which evaluate the pixel characteristics, neighboring relation and it is ideally uncommitted in terms of position, orientation, size and contrast; and (ii) high-level analysis, which maps the low-level characteristics to fulfill the task.

Recently, the deep learning approach drastically changed the computational paradigm for visual tasks. The main advantage of deep learning algorithms is that it does not require an engineered model to operate, meaning that they are capable of learning not only the features to represent the data but also the models to describe it [3]. Facing this new paradigm, researches initially replaced hand-engineered features in the low-level analysis by the features learned in deep models [4]–[6], which mostly achieve the desirable characteristics. More recently, there is many proposals exploring the learned model for the high-level analysis, creating maps from the outputs of different layers in a deep learning network [7]–[10].

One challenge on the latter strategy is how to combine the output from distinct layers, considering that they are presented with different sizes and could represent different concepts. In this work, it is presented strategies to combine the outputs from different layers, by using simple merging functions that explore useful behavior in the learning process. It is also studied the amount of combined outputs necessary to create a viable region proposition for the task of image segmentation. In addition, it is also presented a post-processing filtering step using mathematical morphology idempotent functions [11] to better cope with the fundamental characteristics of an ideal segmented image.

The remainder of this work is organized as it follows: In Section II it is characterized the hierarchy of concepts in deep models related work exploring theses models for high-level tasks. In Section III it is presented the reasoning and strategies proposed in this work. In Section VI it is presented the dataset description, the experimental setup and results for the experiments. And, finally, in Section VII the conclusions are laid out.

II. Related work

Deep learning approaches were initially described as black-box methods, meaning that not much were known about the reasoning and decisions of the created models. Much exertion have been applied to investigate the networks operation, whether by methodical experimentation [12]–[15] or visualization methods [16], [17]. Those efforts provided more clarity of the deep models and characterized the learned features as complex concepts build from simpler ones. Also it demonstrate the learning progression from detailed to coarser representations as the scale and resolutions reduce through the network. When applied for object recognition task for instance, the raw pixel on the input layer is learned as segments and parts until the composition of an object concept on posterior layers, while the input scale reduces to a single feature vector on the output.

The knowledge of the concept abstraction and learning progression allowed new research endeavors to explore

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them in high-level tasks. For instance, three main architectures standout in recent years, namely: (i) Holistically-nested Edge Detection (HED) [18]; (ii) Convolutional Oriented Boundaries (COB) [9]; and (iii) Rich Convolutional Features (RCF) [10]. These architectures explicit explore the models of a traditional deep network to perform a certain high-level task, in which all extract side-outputs of the network and each present a different strategy to combine them .

The HED network create a side-output layer at each stage of the VGG16 network [5] as boundary maps. In HED, each side-output is associated with a classifier in a deeply supervised scheme [6] for the task of edge detection. This association insert the fusion process in the network, attributing weights for each side-output that will be learned individually and determine its contribution on the final evaluation. The evaluation is performed by a cost-sensitive function to balance the bias towards non-edge pixels. The HED network significantly improved the performance in multiple datasets and the extended version [7] also applied the network for the segmentation task.

The authors in [8] use the edge maps created by the HED network alongside with other features such as brightness, colors, gradient and variance to describe images. The goal of their proposal was to create an efficient framework to be used as real-time segmentation system, focused on a fusion strategy to update region features.

In the COB network, the authors also create edge maps from side activations, differing mainly from HED by the attribution to candidate contours orientation information and weights representing the contour strength. The contour orientations are estimated by approximation to known polygon segments and the segments weights are computed based on the candidate contour neighboring region used as a confidence measure. To combine the side outputs it is used a non-linear function to regress both the segment weights and the orientation maps, creating region hierarchical trees by thresholding the contour strength. The network perform well in multiple tasks such object proposal, object detection, semantic contour and segmentation.

Finally, the RCF network, that not only create multiple side-outputs, but also uses multiple scales of the images in the input layer. Differently from the HED network, RCF extract one side-output at each convolutional layer of VGG, arguing that this could create more detailed representations and improve the network accuracy. The merging process is performed by a series of operations, comprising grouping by convolutions, element-wise sums, up-samplings, local loss functions and concatenation.

III. Convolutional side-outputs for image segmentation

Hierarchies are long associated with the image segmentation task [19]–[23], to a degree that it improves a coherent organization of nested regions. The main motivation for using well-defined hierarchies is that different hierarchical level contains different detail level. In this work, instead of using a well-defined hand-engineered hierarchical structure, it is proposed to explore the concept abstraction resultant of the deep network dynamics, extracting side-outputs at different layers that ideally would contain different level of details.

The idea is to combine the side-output maps into a single proposition to be evaluated in the image segmentation task, driving the learning flow towards creating adequate regions for the task. In an optimal scenario, the side-outputs would contain enough details to cope with the task, whilst creating coherent region proposals.

Amongst the many strategies for deep models, convolutional networks are well-know by the concept abstraction due the multiple stages of convolution and have been successfully used for the object recognition task. They are usually characterized by three nested functions in multiple layers, namely: (i) convolution; (ii) spatial pooling; and (iii) non-linear activation. Formally, let a convolutional network f composed by L layers be defined as:

$$f(\mathbf{X}) = \mathbf{W}_L \mathbf{H}_{L-1} \tag{1}$$

in which:

- $\mathbf{W}_l$ is the associated weights for the layer l;
- $\mathbf{H}_l$ is the output of the hidden layer l, defined as:

$$\mathbf{H}_l = pooling(activation(\mathbf{W}_l \mathbf{H}_{l-1}))\ \forall l \in \{1, ..., L-1\} \tag{2}$$

For consistency, consider $\mathbf{H}_0 = \mathbf{X} = \{X_1, X_2, ..., X_n\}$ the set of N input images I.

The VGG network [5] is one of the first attempts to create deeper models following the convolutional scheme. The core of the layers in VGG is defined by a convolution immediately followed by a rectified linear unit, as follows:

$$C_l = ReLU(\mathbf{W}_l \mathbf{H}_{l-1})\ \forall l \in \{1, .., L-1\} \tag{3}$$

in which $ReLU(\cdot) = max(0, \cdot)$. There is also two types of stages, $S^{(1)}$ and $S^{(2)}$, that could formally defined as:

$$S^{(1)} = ReLU(\mathbf{W}_l(ReLU(\mathbf{W}_{l-1}\mathbf{H}_{l-2}))) \tag{4}$$

$$S^{(2)} = ReLU(\mathbf{W}_l(ReLU(\mathbf{W}_{l-1}(ReLU(\mathbf{W}_{l-2}\mathbf{H}_{l-3})))) \tag{5}$$

The output of a hidden layer is computed as $maxpool(S^{(1)})$ or $maxpool(S^{(2)})$ for all S stages in the network.

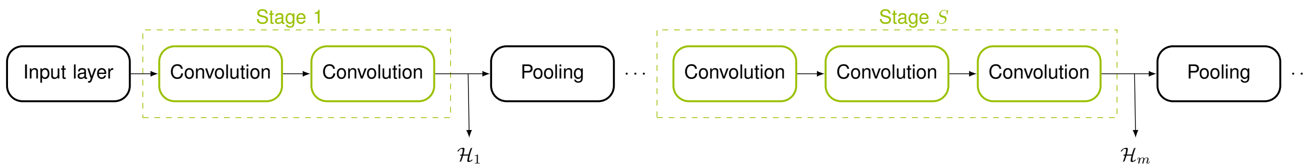
Questions on which and how many side-outputs would be adequate for the image segmentation task, are assessed using two different extraction strategies, both applied in the VGG network. Namely: (i) Stage Layer Outputs (**SLO**), inspired by the HED model, creating one side-output for each VGG stage; and (ii) All Layers Outputs (**ALO**), inspired by the RCF model, creating one side-output for each convolutional layer.

Formally, the set $\mathcal{H}$ of M side outputs maps in each strategy is defined as:

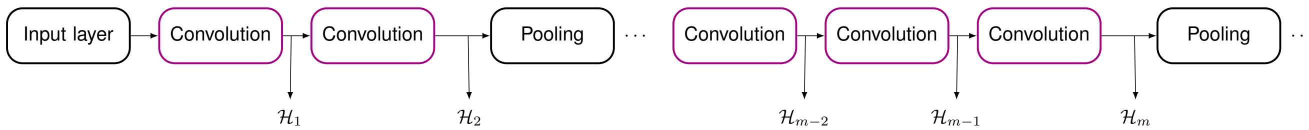
$$\mathcal{H}_{SLO} = \{\mathcal{H}_1, ..., \mathcal{H}_m | m \in [1, S] \text{ and } \mathcal{H}_m \in \{S^{(1)}, S^{(2)}\}\} \tag{6}$$

$$\mathcal{H}_{ALO} = \{\mathcal{H}_1, ..., \mathcal{H}_m | m \in [1, L-1] \text{ and } H_m = C_l\ \forall l \in \{1, .., L-1\}\} \tag{7}$$

In the case of **SLO**, the number of side-outputs amounts to the number of pooling layers in the network and for **ALO**, it



(a) Stage Layer Outputs (SLO)



(b) All Layers Outputs (ALO)

Fig. 1. Illustration for the two side-outputs extraction strategies: (a) side-outputs extracted at each stage of the network and (b) side-outputs extracted at each convolutional layer

is equal to the number of convolutional layers. An illustration for both strategies is presented in Figure 1.

IV. Merging strategies

The main question in dealing with side-outputs in convolutional networks is how to combine them, considering that they are presented in different scales and could represent different concepts. The goal is to produce a single proposition to be evaluated in the task, but retain the useful information presented at different layers.

In this work, the strategy to overcome those challenges is to combine the side-outputs by exploring the knowledge of the learning process. To achieve that, it is proposed to apply simple merging functions that would enhance different desirable behavior, as described in the following:

- *ADD*: Aims to balance negative and positive weights;
- *AVG*: Aims to create a proposition representing the whole network learning;
- *MAX*: Aims to represent confident values.

Formally, the single proposition Z to be evaluated in the task, under each strategy could be defined as:

$$Z_{ADD} = \sum_{i=1}^{M}(\mathcal{H}_i) \tag{8}$$

$$Z_{AVG} = \frac{\sum_{i=1}^{M}(\mathcal{H}_i)}{M} \tag{9}$$

$$Z_{MAX} = \max_{1 \leq i \leq M}(\mathcal{H}_i) \tag{10}$$

The operations are performed element-wise on each side-output after they are re-scaled for the input size, while maintaining the connectivity pattern.

Once the combined map is created, it is evaluated on the segmentation task which aims to provide partition of an image into a set of regions representing meaningful areas. This could be reduced to a binary problem aiming to distinguish each pixel of the image as belonging to a region of interest or the background. If confronted with multiple regions of interest this minimal formulation could be executed individually and paired later.

Formally, consider once again the set of N training images $\mathbf{X}$ and alike $\mathbf{Y} = \{Y_1, Y_2, ..., Y_n\}$ the set of ground-truth images in which each pixel is labeled. The ground-truth images are used to calculate the pixel accuracy measuring the rate that a pixel is correctly predicted to belong to the region of interest or the background.

Insert evaluation function and network cross-entropy

V. Post-processing

Mathematical morphology is consistent with the non-linear image analysis, presenting solid theoretical foundation and idempotent functions. The formulations are presented in the complete lattice geometric space, in which the functions are performed considering whole sets operating over another whole set. In mathematical morphology, the operators are known a priori and defined using the sets of structuring elements.

In this work, it is proposed to use mathematical morphology as post-processing step, meaning that this step is not inserted in the learning stage. The main goal is to better cope with the fundamental properties of a well-segmented image, particularly, region uniformity and continuity. To achieve that, it is proposed to use a function filter, called area opening, which tend to destroy the small, thin and conspicuous areas.

Formally, let $\hat{Y} \in \mathbb{R}^2$ be the output of a testing image consistent with the representation created by the parameters learned in the network. Consider B a structuring element and γ_B the morphological opening produced by it. Consider also λ the threshold parameter which will determine how small a certain area must be to be purged. In this case, $\gamma_B \subseteq \gamma_\lambda$ if and only if B is a finite union of connected components of area greater or equal to λ. It is expected that this simple additional step could reduce possible noises on the final result, and in this way improve the results.

VI. Experiments

Experiments were conducted in the KITTI Road/Lane Dataset, part of KITTI Vision Benchmarking Suite [24]. The dataset contains images for road and lane estimation for the

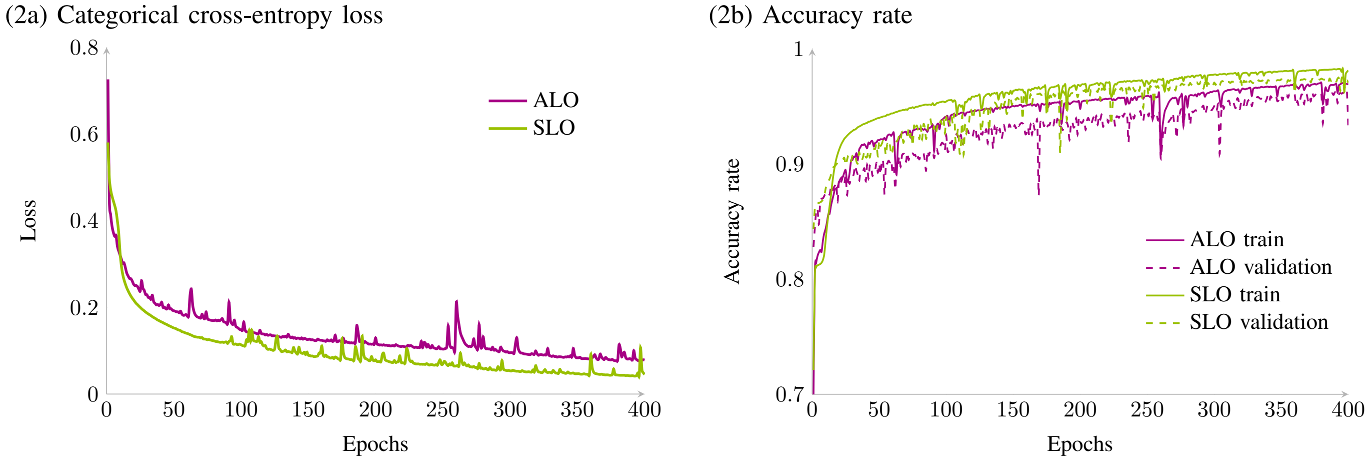


Fig. 2. Learning curves for the compared approaches. Left panel displays the the cross-entropy objective function during the learning step for the validation set. Right panel displays accuracy rate obtained on the training and validations sets during the learning step.

task of image segmentation. It is consisted of 289 training and 290 test images. The ground-truth is manually annotated for two different road types: (i) road, road area composing all lanes; and (ii) lane, lane the vehicle is currently driving on. It is important to notice that the ground-truth is only available for training set and the test evaluation should be performed using KITTI Server.

In this work, it is used only the road ground-truths and the lane annotations are ignored, due the fact that not all the images contains ground-truths for both categories. Then, we prefer to use the road estimation and build the classifier on a binary problem (road and background). The road type is divided in three different categories of road scenes, namely: (i) uu_road, urban unmarked; (ii) um_road, urban marked; and (ii) umm_road, urban multiple marked lanes.

To increase the number of images in the training set, it is performed some data augmentation procedures. It was added images with pepper/salt noise, horizontal flipping (mirror) and changes in contrast and brightness. Were avoided procedures that would create aberrations, such as the road in the sky and distortions that would change the nature of the objects in the scene, such as cars and pedestrians. These procedures resulted in 1445 images, divided in 1228 samples for training and 217 samples for validation (about 15%).

A. Experimental setup

Our network were build using using Keras [25] with Tensorflow [26] and trained for 400 epochs. We used a pre-trained VGG16 model to initialize the weights. Also, we use SGD optimization with learning rate set to 1e-4, decay of 1e-6 and momentum of 0.95. The default batch size contains 8 images. Other experiments with different values will be discussed in next sections. All training experiments were performed in GeForce GTX 1080 8GB GPU. HED and RCF projects provided custom functions to balance the number of pixels of edges from the non-edges pixels. Once our problem is not as unbalanced as the edge detection, we decided to use the *categorical cross-entropy* loss function.

For simplicity, in the remaining of this work, the network using the side outputs extracted at each stage of the VGG will be called Stage Layer Outputs (**SLO**) and it is composed by $n = 5$ side outputs. Similarly, for the side outputs extracted at each convolutional layer, it will be called All Layers Outputs (**ALO**) and it is composed by $n = 13$ side outputs.

B. Training results

In Figure 2 it is presented the relevant curves obtained during the learning step for the proposed approaches. As one could see in Figure 2a, both compared approaches presents an expected loss curve and there is no significant difference between both approaches in terms of losses values, although the SLO model appears to be more stable and presents a faster decay than the ALO model.

Regarding the accuracy rate, illustrated in Figure 2b, it is possible to see that the SLO model presents a better performance than the ALO model. The accuracy rate achieved by SLO was 0.974 while ALO it was 0.963 on the validation set (about 1.2% worse). It is also possible to notice that the gap between the accuracy achieved in the training set and the accuracy achieved in validations set is smaller for the SLO model, which indicates that the ALO model is more prone to over-fit the data.

For the performance regarding time the average to process SLO model is 12.2% smaller than the ALO network, and could process 33.60 images per second in training time, while the ALO model process 29.48 images per second.

In summary, for all the metrics in the leaning step, the SLO model presented a slighted superior and more desirable behavior than the ALO model. It is believed that these results are consequence of the considerably larger amount of side outputs in the ALO model, which create more possibilities of interchangeability between confidant values.

In order to improve the results a new set of tests were performed using 2000 training epochs. The best accuracy rate achieve after the new training procedures by the SLO models was **0.980**. As for the ALO model, a more careful design of parameters were tested, particularly, we defined the learning rate as 1e-4, the decay as 1e-6 and used the Nesterov optimization in the process. After 46 epochs the model achieved the best accuracy rate of **0.982**. But the instable behavior persisted, in which in some epochs were close to this top accuracy but in many others the values were close to 0.86 accuracy. Some visual results are presented in Table II.

C. Evaluation results and comparison with the state-of-the-art

After the training procedure, we create a post processing step to reduce possible noises in results proposition. For this, we used the mathematical morphology operation of Opening [11]. This procedure removes small noises created by the foreground (the road)in the background. We defined a set of kernels with the sizes of 5×5, 7×7, 9×9, 11×11 and 13×13 and applied in the images to reduce different sizes of noises. Results using this strategy are under the label **ALO-mm** and **SLO-mm**.

Reminding that the test evaluation could only be performed using KITTI Server, the metrics provided are maximum F1-measure (MaxF), average precision (AP), precision (PRE), recall (REC), false positive rate (FPR) and false negative rate (FNR).

The results achieved on the test set according to each category in the road scenes are presented in Table I. As expected, the SLO model performed better then the ALO model in all almost all of the cases. Particularly when using the post processing procedure with mathematical morphology. It is also possible to notice that although the post-processing slightly improved the overall performance, it also increased the number of false negatives. This could be an indications that perhaps the applied kernel sizes are not adequate and are removing more of the foreground than the desired.

If compared with the state-of-the-art (anonymous submission on the KITTI Server platform), the proposed method is comparable and sometimes superior, regarding the maximum F1-measure and the recall metrics. This is due the fact that although the reported state-of-the-art on the dataset presents a superior average precision, it also almost always presents a higher rate of false positives an negatives. This indicates that the proposed methods are more precise in delineating the regions to be segmented.

VII. Conclusion

This work addressed the problem of merging side outputs extracted from the convolutional layer model VGG to create region propositions for the task of image segmentation. It was proposed to use a $max()$ function to enhance confident values during training to be evaluated using a cross-entropy loss function. It was also studied the impact that the number of side outputs have on the proposed strategy and if a simple mathematical morphology operation could enhance the performance on the task.

Experiments demonstrated that the $max()$ function is viable for merging maps with different sizes and connotations, and could place the proposed strategy among the state-of-the-art approaches for the task on the Kitti dataset. It was also demonstrated that a large amount of side outputs increases the network confusion during the training step, but could also create jumps that could lead to better performance, in terms of

TABLE I
KITTI benchmark evaluation results for each category

um_road

Method	MaxF	AP	PRE	REC	FPR	FNR
SLO	96.92%	87.36%	94.47%	**99.49%**	1.13%	**0.51%**
SLO-mm	**97.01%**	**87.68%**	**94.83%**	99.30%	**1.05%**	0.70%
ALO	96.39%	86.81%	93.87%	99.05%	1.25%	0.95%
ALO-mm	96.65%	87.51%	94.64%	98.74%	1.08%	1.26%
State-of-the-art	**97.05%**	**93.53%**	**97.18%**	**96.92%**	**1.28%**	**3.08%**

umm_road

Method	MaxF	AP	PRE	REC	FPR	FNR
SLO	97.57%	89.44%	96.05%	**99.15%**	1.24%	**0.85%**
SLO-mm	**97.61%**	**89.67%**	**96.30%**	98.97%	**1.16%**	1.03%
ALO	97.05%	88.83%	95.37%	98.78%	1.46%	1.22%
ALO-mm	97.21%	89.31%	95.90%	98.56%	1.29%	1.44%
State-of-the-art	**97.77%**	**95.64%**	**97.75%**	**97.79%**	**2.48%**	**2.21%**

uu_road

Method	MaxF	AP	PRE	REC	FPR	FNR
SLO	95.16%	**85.73%**	92.94%	97.49%	1.16%	2.51%
SLO-mm	**95.42%**	86.48%	**93.77%**	97.13%	**1.01%**	2.87%
ALO	94.70%	84.87%	92.00%	**97.56%**	1.33%	**2.44%**
ALO-mm	95.20%	86.15%	93.40%	97.08%	1.08%	2.92%
State-of-the-art	**95.95%**	**95.25%**	**95.25%**	**95.65%**	**1.21%**	**4.35%**

TABLE II
Visual illustration of the obtained results

umm_000025.png

Original Image

ALO

SLO

Ground-truth

accuracy. The post-processing strategy slightly improved the performance, but requires further studies.

This research opens novel opportunities for study such as: (i) exploring different merging functions, less susceptible a values fluctuations; (ii) explore regularization techniques to sustain larger amounts of side outputs consistent; and (iii) insert the mathematical morphology kernels on the learning process to search for the best kernel size.

The code and a file containing all dependencies to reproduce the experiments is public available online in https://github.com/falreis/segmentation-eval.

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