CISC 330 - Assignment 1

Ellipsoid & Line Intersection

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} + \frac{(z-z_0)^2}{c^2} = 1$$

Line $= P + \vec{v}t$

$L_x = P_x + v_x t$ $\quad x = L_x$

$L_y = P_y + v_y t$ $\quad y = L_y$ $\Big\}$ sub into ellipsoid eqn

$L_z = P_z + v_z t$ $\quad z = L_z$

$$\frac{(P_x+v_xt-x_0)^2}{a^2} + \frac{(P_y+v_yt-y_0)^2}{b^2} + \frac{(P_z+v_zt-z_0)^2}{c^2} = 1$$

Let $Q_x = P_x - x_0$, $Q_y = P_y - y_0$, $Q_z = P_z - z_0$

$$1 = \frac{(Q_x+v_xt)(Q_x+v_xt)}{a^2} + \frac{(Q_y+v_yt)(Q_y+v_yt)}{b^2} + \frac{(Q_z+v_zt)(Q_z+v_zt)}{c^2}$$

$$1 = \frac{v_x^2t^2+2Q_xv_xt+Q_x^2}{a^2} + \frac{v_y^2t^2+2Q_yv_yt+Q_y^2}{b^2} + \frac{v_z^2t^2+2Q_zv_zt+Q_z^2}{c^2}$$

$$1 = t^2v_x^2a^{-2} + t^2v_y^2b^{-2} + t^2v_z^2c^{-2} + t2Q_xv_xa^{-2} + t2Q_yv_yb^{-2} + t2Q_zv_zc^{-2} + Q_x^2a^{-2} + Q_y^2b^{-2} + Q_z^2c^{-2}$$

$$0 = t^2(v_x^2a^{-2}+v_y^2b^{-2}+v_z^2c^{-2}) + t(2Q_xv_xa^{-2}+2Q_yv_yb^{-2}+2Q_zv_zc^{-2}) + Q_x^2a^{-2}+Q_y^2b^{-2}+Q_z^2c^{-2}-1$$

Quadratic Formula: $x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$ in $0 = Ax^2+Bx+C$

$A = v_x^2a^{-2}+v_y^2b^{-2}+v_z^2c^{-2}$, $B = 2(Q_xv_xa^{-2}+Q_yv_yb^{-2}+Q_zv_zc^{-2})$

$C = Q_x^2a^{-2}+Q_y^2b^{-2}+Q_z^2c^{-2}-1$

Use A B and C to solve for t (there are 0, 1 or 2 intersections)
↳ check the determinant to see how many intersections exist