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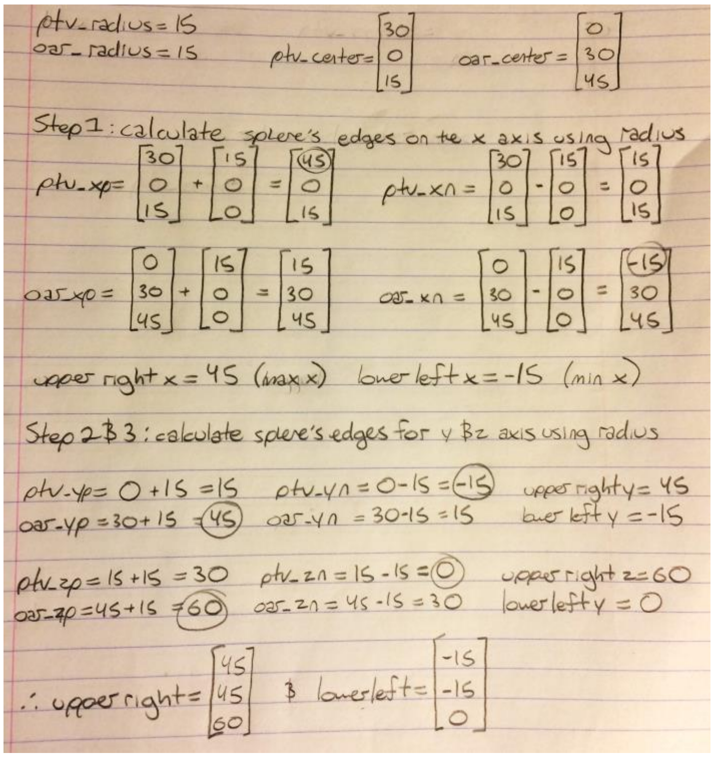
CISC 330- Assignment 4

1. Compute Dose Box

Hand Written Calculations



ptv_radius = 15
oar_radius = 15

$ptv\_center = \begin{bmatrix} 30 \\ 0 \\ 15 \end{bmatrix}$ $oar\_center = \begin{bmatrix} 0 \\ 30 \\ 45 \end{bmatrix}$

Step 1: calculate splere's edges on te x axis using radius

$ptv\_xp = \begin{bmatrix} 30 \\ 0 \\ 15 \end{bmatrix} + \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \\ 15 \end{bmatrix}$ $ptv\_xn = \begin{bmatrix} 30 \\ 0 \\ 15 \end{bmatrix} - \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 15 \end{bmatrix}$

$oar\_xp = \begin{bmatrix} 0 \\ 30 \\ 45 \end{bmatrix} + \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \\ 45 \end{bmatrix}$ $oar\_xn = \begin{bmatrix} 0 \\ 30 \\ 45 \end{bmatrix} - \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -15 \\ 30 \\ 45 \end{bmatrix}$

upper right x = 45 (max x) lower left x = -15 (min x)

Step 2 & 3: calculate splere's edges for y & z axis using radius

ptv_yp = 0 + 15 = 15 ptv_yn = 0 - 15 = -15 upper right y = 45
oar_yp = 30 + 15 = 45 oar_yn = 30 - 15 = 15 lower left y = -15

ptv_zp = 15 + 15 = 30 ptv_zn = 15 - 15 = 0 upper right z = 60
oar_zp = 45 + 15 = 60 oar_zn = 45 - 15 = 30 lower left y = 0

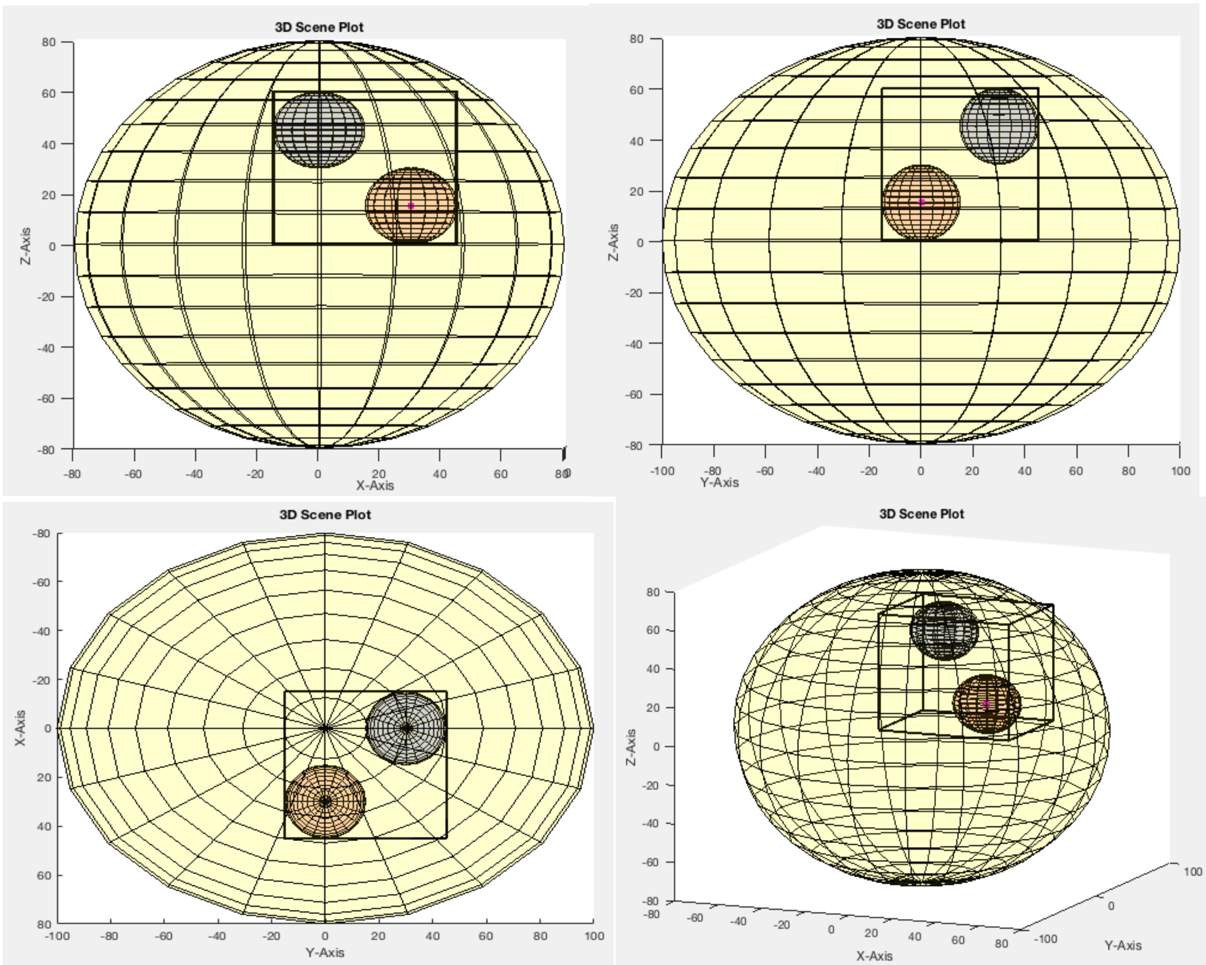
$\therefore$ upper right = $\begin{bmatrix} 45 \\ 45 \\ 60 \end{bmatrix}$ & lower left = $\begin{bmatrix} -15 \\ -15 \\ 0 \end{bmatrix}$

Output from MATLAB

```
lowerLeft =

   -15
   -15
     0


upperRight =

    45
    45
    60
```

The handwritten calculations and the MATLAB calculations for the dose box are equal. This result means that the calculations done by MATLAB are correct and the correct dose box values are (-15,-15,0) for the lower left of the box and (45,45,0) for the upper right of the box.

2. Draw 3D Scene



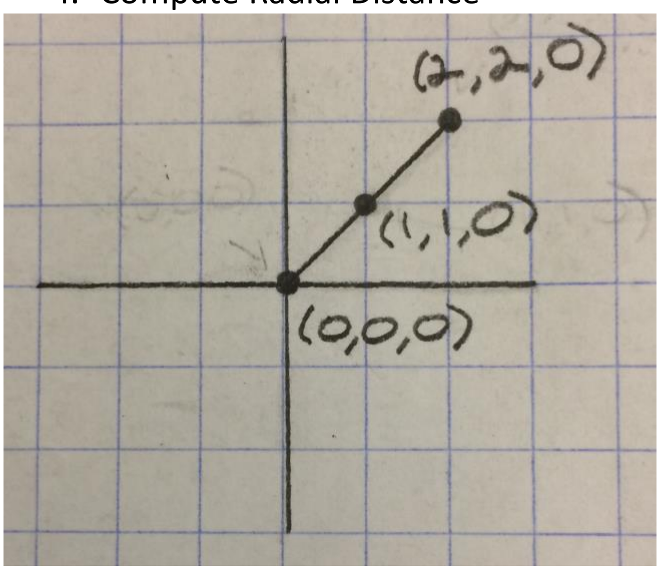
Observing the 4 pictures above it can be seen that the dose box (drawn in black) tightly fits around the PTV (red partially transparent sphere) and the OAR (blue partially transparent sphere) along the x, y, and z axis. The iso center point (magenta point) can also be seen plotted in the middle of the PTV, and the PTV and OAR are placed correctly within the head (drawn in yellow).

3. Compute Linear Function

x_value	y_value
0	0
1	1
2	2
3	3
4	4
5	5
6	6

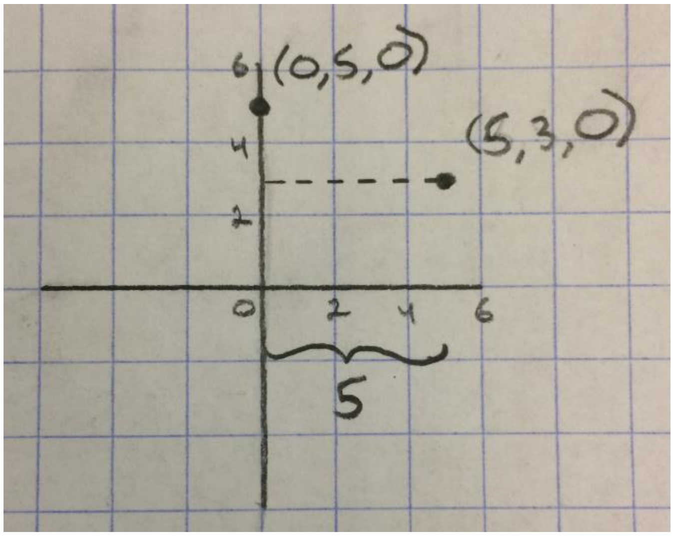
The image above is the x and y values for points along the line generated by points (1,1) and (5,5). This makes sense as the slope of the line is (5-1)/(5-1) = 1, and the y intersect is 0. Therefore, the equation of this line is y = 1*x, aka y=x.

4. Compute Radial Distance




```
The Distance between the line (0,0,0) to (2,2,0) and the point (1,1,0) is 0.0000
```

The MATLAB function calculated that the distance between the line with the point (0,0,0) and a direction vector of (2,2,0), and the point (1,1,0) is 0. Looking at the plotted image above we can see that this is correct as the point is on the line and therefore the distance from the point to the line is 0.

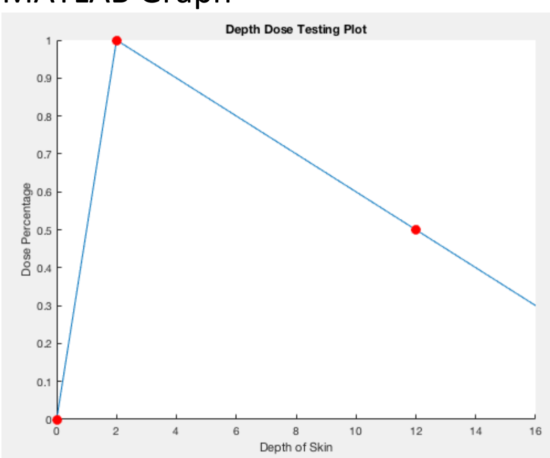


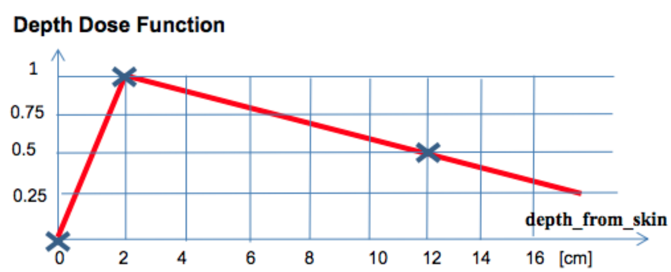

```
The Distance between the line (0,0,0) to (0,5,0) and the point (5,3,0) is 5.0000
```

The MATLAB function calculated that the distance between the line with point (0,0,0) and a direction vector (0,5,0) and the point (5,3,0) is 5. Looking at the plotted image above we can see that this is correct as the point is exactly 5 units away from the line.

5. Compute Depth Dose

MATLAB Graph | Assignment graph



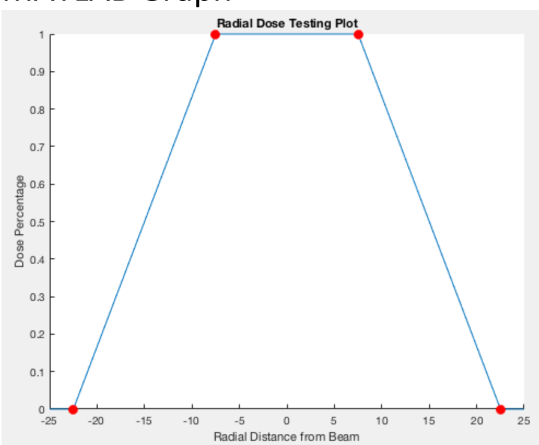


In these graphs, the three marked points (red dots in MATLAB graph, blue x's in assignment graph) are at the same spots and the lines connecting them are of the same slope. This proves that the depth dose function is correctly computing the linear equations associated with this depth dose function and is able to plot the data correctly as well. It can also be seen in the table output below that the marked x value points are equivalent by looking for the x axis (0, 2, and 12) and seeing the associated y axis value (0, 1, and 0.5 respectively).

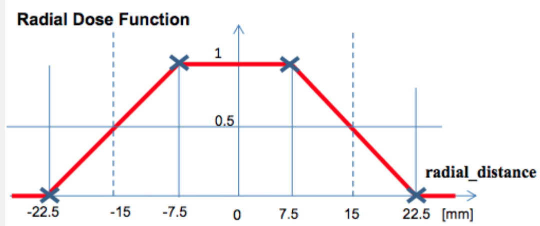
Depth	Dose_Percentage														
_____	_______________	1.9	0.95	4.1	0.895	6.3	0.785	8.5	0.675	10.7	0.565	12.9	0.455		
		2	1	4.2	0.89	6.4	0.78	8.6	0.67	10.8	0.56	13	0.45		
		2.1	0.995	4.3	0.885	6.5	0.775	8.7	0.665	10.9	0.555	13.1	0.445		
0	0	2.2	0.99	4.4	0.88	6.6	0.77	8.8	0.66	11	0.55	13.2	0.44		
0.1	0.05	2.3	0.985	4.5	0.875	6.7	0.765	8.9	0.655	11.1	0.545	13.3	0.435		
0.2	0.1	2.4	0.98	4.6	0.87	6.8	0.76	9	0.65	11.2	0.54	13.4	0.43		
0.3	0.15	2.5	0.975	4.7	0.865	6.9	0.755	9.1	0.645	11.3	0.535	13.5	0.425		
0.4	0.2	2.6	0.97	4.8	0.86	7	0.75	9.2	0.64	11.4	0.53	13.6	0.42		
0.5	0.25	2.7	0.965	4.9	0.855	7.1	0.745	9.3	0.635	11.5	0.525	13.7	0.415		
0.6	0.3	2.8	0.96	5	0.85	7.2	0.74	9.4	0.63	11.6	0.52	13.8	0.41		
0.7	0.35	2.9	0.955	5.1	0.845	7.3	0.735	9.5	0.625	11.7	0.515	13.9	0.405		
0.8	0.4	3	0.95	5.2	0.84	7.4	0.73	9.6	0.62	11.8	0.51	14	0.4	15.1	0.345
0.9	0.45	3.1	0.945	5.3	0.835	7.5	0.725	9.7	0.615	11.9	0.505	14.1	0.395	15.2	0.34
1	0.5	3.2	0.94	5.4	0.83	7.6	0.72	9.8	0.61	12	0.5	14.2	0.39	15.3	0.335
1.1	0.55	3.3	0.935	5.5	0.825	7.7	0.715	9.9	0.605	12.1	0.495	14.3	0.385	15.4	0.33
1.2	0.6	3.4	0.93	5.6	0.82	7.8	0.71	10	0.6	12.2	0.49	14.4	0.38	15.5	0.325
1.3	0.65	3.5	0.925	5.7	0.815	7.9	0.705	10.1	0.595	12.3	0.485	14.5	0.375	15.6	0.32
1.4	0.7	3.6	0.92	5.8	0.81	8	0.7	10.2	0.59	12.4	0.48	14.6	0.37	15.7	0.315
1.5	0.75	3.7	0.915	5.9	0.805	8.1	0.695	10.3	0.585	12.5	0.475	14.7	0.365	15.8	0.31
1.6	0.8	3.8	0.91	6	0.8	8.2	0.69	10.4	0.58	12.6	0.47	14.8	0.36	15.9	0.305
1.7	0.85	3.9	0.905	6.1	0.795	8.3	0.685	10.5	0.575	12.7	0.465	14.9	0.355	16	0.3
1.8	0.9	4	0.9	6.2	0.79	8.4	0.68	10.6	0.57	12.8	0.46	15	0.35		

6. Compute Radial Dose

MATLAB Graph



Assignment Graph



In these graphs, the four marked points (-22,5,0), (-7.5, 1), (7.5,1) and (22.5,0) are at the same spots (red dots in MATLAB graph, blue x's in assignment graph) and the lines connecting them are of the same slope. This proves that the depth dose function is correctly computing the linear equations associated with this radial dose function and is able to plot the data correctly as well. It can also be seen in the table output generated that the marked points are equivalent by looking for the x axis and seeing the associated y axis value. The screenshots below show the first 10 values for each linear equation. The trends and slopes can be observed in their y values.

Depth	Dose_Percentage								
-25	0	-22.4	0.0066667	-7.5	1	7.6	0.99333	22.5	0
-24.9	0	-22.3	0.013333	-7.4	1	7.7	0.98667	22.6	0
-24.8	0	-22.2	0.02	-7.3	1	7.8	0.98	22.7	0
-24.7	0	-22.1	0.026667	-7.2	1	7.9	0.97333	22.8	0
-24.6	0	-22	0.033333	-7.1	1	8	0.96667	22.9	0
-24.5	0	-21.9	0.04	-7	1	8.1	0.96	23	0
-24.4	0	-21.8	0.046667	-6.9	1	8.2	0.95333	23.1	0
-24.3	0	-21.7	0.053333	-6.8	1	8.3	0.94667	23.2	0
-24.2	0	-21.6	0.06	-6.7	1	8.4	0.94	23.3	0
-24.1	0	-21.5	0.066667	-6.6	1	8.5	0.93333	23.4	0
								23.5	0

7. Compute Beam Direction Vector

longitude = 0, latitude = 0, a=80, b=100, c=80

$x = a * \cos(\text{latitude}) * \cos(\text{longitude}) = 80*1*1 = 80$
$y = b * \cos(\text{latitude}) * \sin(\text{longitude}) = 100*1*0 = 0$
$z = c * \sin(\text{latitude}) = 100*0 = 0$

$$\text{point} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \end{bmatrix} \qquad \text{vector} = \begin{bmatrix} 30 \\ 0 \\ 15 \end{bmatrix} - \begin{bmatrix} 80 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -50 \\ 0 \\ 15 \end{bmatrix}$$

$$\text{unit vector} = \begin{bmatrix} -50 \\ 0 \\ 15 \end{bmatrix} / \|\text{vector}\| \qquad \|\text{vector}\| = \sqrt{(-50)^2 + 0^2 + 15^2} = \sqrt{2725} \cong 52.2$$

$$= \begin{bmatrix} -50/52.2 \\ 0/52.2 \\ 15/52.2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.9578 \\ 0 \\ 0.28735 \end{bmatrix} \Leftarrow \text{unit direction vector for point @ (longitude = 0, latitude = 0)}$$

Longitude	Latitude	Direction_Vectors_xyz		
0	0	-0.95783	0	0.28735
0	90	0.41906	0	-0.90796
90	0	0.28443	-0.94809	0.14221
90	90	0.41906	0	-0.90796
180	0	0.99083	0	0.13511
180	90	0.41906	0	-0.90796
270	0	0.28443	0.94809	0.14221
270	90	0.41906	0	-0.90796

The MATLAB output and the hand calculation done for latitude = 0 and longitude = 0 is equal. This shows that the MATLAB function is correct as the values match the hand calculations. The same method of calculation is used for all points and so verifying the first calculation also verifies the later calculations.

8. Compute Skin Entry Point

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$$

$Line = P + \vec{v}t$

$L_x = P_x + v_x t \qquad x = L_x$

$L_y = P_y + v_y t \qquad y = L_y$ sub into ellipsoid eqn

$L_z = P_z + v_z t \qquad z = L_z$

$$\frac{(P_x+v_x t-x_0)^2}{a^2}+\frac{(P_y+v_y t-y_0)^2}{b^2}+\frac{(P_z+v_z t-z_0)^2}{c^2}=1$$

Let $Q_x = P_x - x_0$, $Q_y = P_y - y_0$, $Q_z = P_z - z_0$

$$1=\frac{(Q_x+v_x t)(Q_x+v_x t)}{a^2}+\frac{(Q_y+v_y t)(Q_y+v_y t)}{b^2}+\frac{(Q_z+v_z t)(Q_z+v_z t)}{c^2}$$

$$1=\frac{v_x^2 t^2+2Q_x v_x t+Q_x^2}{a^2}+\frac{v_y^2 t^2+2Q_y v_y t+Q_y^2}{b^2}+\frac{v_z^2 t^2+2Q_z v_z t+Q_z^2}{c^2}$$

$$1=t^2 v_x^2 a^{-2}+t^2 v_y^2 b^{-2}+t^2 v_z^2 c^{-2}+t\,2Q_x v_x a^{-2}+t\,2Q_y v_y b^{-2}+t\,2Q_z v_z c^{-2}+Q_x^2 a^{-2}+Q_y^2 b^{-2}+Q_z^2 c^{-2}$$

$$0=t^2(v_x^2 a^{-2}+v_y^2 b^{-2}+v_z^2 c^{-2})+t(2Q_x v_x a^{-2}+2Q_y v_y b^{-2}+2Q_z v_z c^{-2})+Q_x^2 a^{-2}+Q_y^2 b^{-2}+Q_z^2 c^{-2}-1$$

Quadratic Formula: $x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$ in $0=Ax^2+Bx+C$

$A=v_x^2 a^{-2}+v_y^2 b^{-2}+v_z^2 c^{-2}$, $B=2(Q_x v_x a^{-2}+Q_y v_y b^{-2}+Q_z v_z c^{-2})$

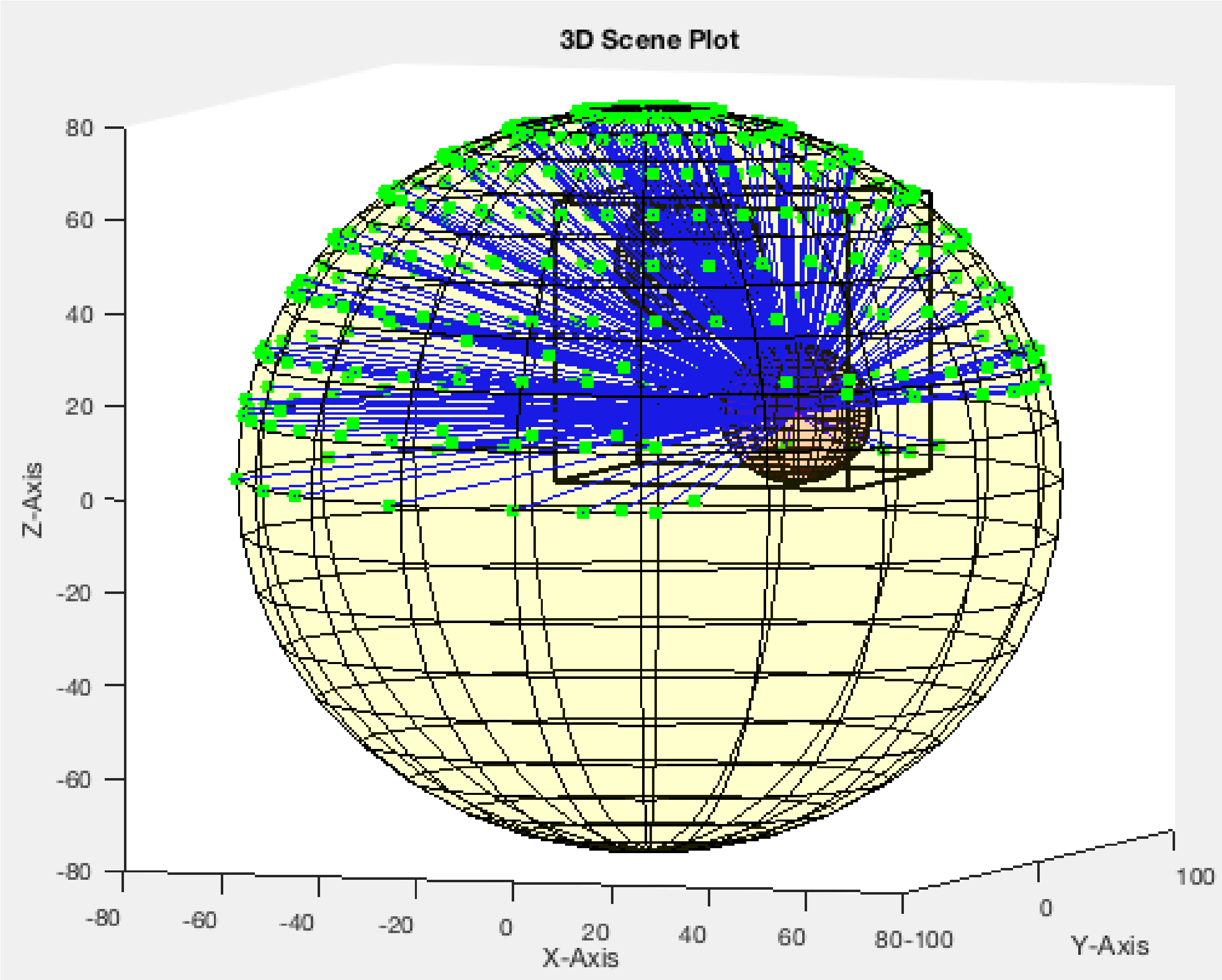
$C=Q_x^2 a^{-2}+Q_y^2 b^{-2}+Q_z^2 c^{-2}-1$

Use A B and C to solve for t (there are 0, 1 or 2 intersections)

$\hookrightarrow$ check the determinant to see how many intersections exist

The skin entry point for the beam passing through the head to the PTV can be calculated by determining the point of intersection of the beam and the ellipsoid's surface. Above is the mathematical quadratic formula derivation that was implemented in this MATLAB function to calculate the skin entry point of the beam on the head.

9. Compute Skin Entry Point Table



Although difficult to see, the iso center point located in the middle of the PTV (the magenta colored point in the middle of the red partially transparent sphere) is the point where all of the beams (blue lines) are intersecting in the head. This is correct, as the helmet has been centered at that point so the beams are all targeting and hitting the PTV from different angles. The green markers along the surface of the head (yellow sphere) are the skin entry points from the beam to the iso center point. Again, although it is difficult to see it appears visually that the beams do intersect these skin entry points separately (each beam has its own skin entry point, no skin entry point has two beams intersecting with it).

10. Compute Beam Safety

To compute beam safety, the distance between the center line of the beam and the center point of the OAR is calculated. If the calculated distance is less than or equal to 30mm (the radius of the OAR (15mm) + the radius of the beam (15mm)) then the OAR is being hit by the beam (either by the center of the beam or within the radius of the beam) and the beam is marked as unsafe. Although there is still radiation leakage outside of the beam as well and the OAR may still receive some radiation, the rule for defining a beam unsafe was defined as the beam intersecting the point, not the OAR receiving any radiation from outside of the beam as well. The function returns 0 if the beam is determined unsafe and 1 if it is safe.

11. Compute Beam Safety Table

Pictured below is the first 30 values in the beam safety table. The values in the right most column specify whether or not the beam is safe (1 = safe, 0 = unsafe).

Longitude	Latitude	Unsafe
0	0	1
0	10	1
0	20	1
0	30	1
0	40	1
0	50	1
0	60	1
0	70	1
0	80	1
0	90	1
0	100	1
0	110	1
0	120	1
0	130	1
0	140	1
0	150	1
0	160	1
0	170	1
0	180	1
10	0	1
10	10	1
10	20	1
10	30	1
10	40	1
10	50	1
10	60	1
10	70	1
10	80	1
10	90	1
10	100	1

12. Compute Point Dose from Beam

Total Dose from Beam Calculated

Longitude	Latitude	Dosage
0	0	0.7
0	90	0.7
90	0	0.6
90	90	0.7

Depth dose from beam

```
depth_y =

    0.7000


depth_y =

    0.7000


depth_y =

    0.6000


depth_y =

    0.7000
```

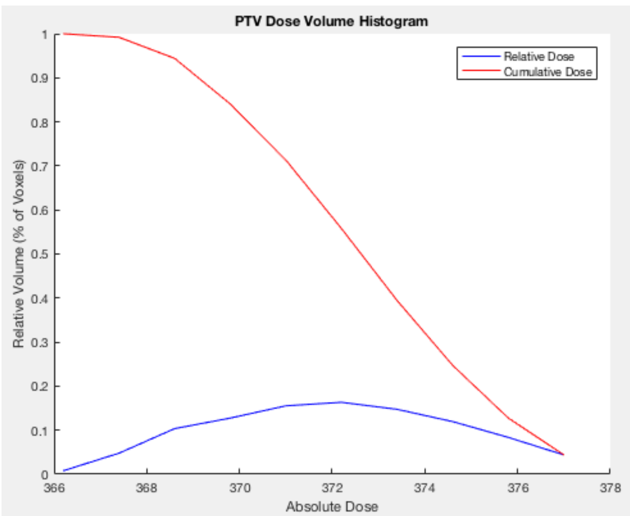
To calculate the point dose from a specific beam, the beam direction vector, skin entry point, radial dose value and depth dose value is calculated. In this test case with setting the iso-center point and point of interest at the center of the head (0,0,0) the total dosage from each beam is equal to the depth dosage of each beam. This is correct, as the radial dosage from each beam is equal to 1 since the point is being hit by the beam's center line. Since the radial dosage is 1, and the total dosage is equal to radial dosage x depth dosage, these total values make sense.

13. Compute Point Dose

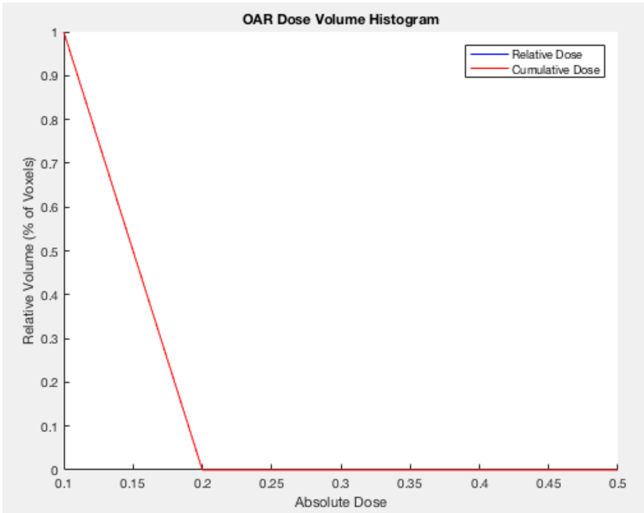
This function calculates the total point dose for a specified point using all of the beams. This function calls the Compute_Point_Dose_from_Beam function and inputs the point of interest and one of the beams and adds the calculated dosage of that beam to the total dosage for the point. The function continues to call Compute_Point_Dose_from_Beam until all of the beams and their contribution to the total point dose has been calculated.

14. Compute Dose Volume Histogram

To compute the dose volume of both the PTV and OAR spheres, the dose box was broken down into 1mm voxels and the point dose at the voxels within the PTV or OAR were calculated using the Compute_Point_Dose function. A histogram of both the cumulative and relative doses for the PTV and the OAR is created and illustrated below.



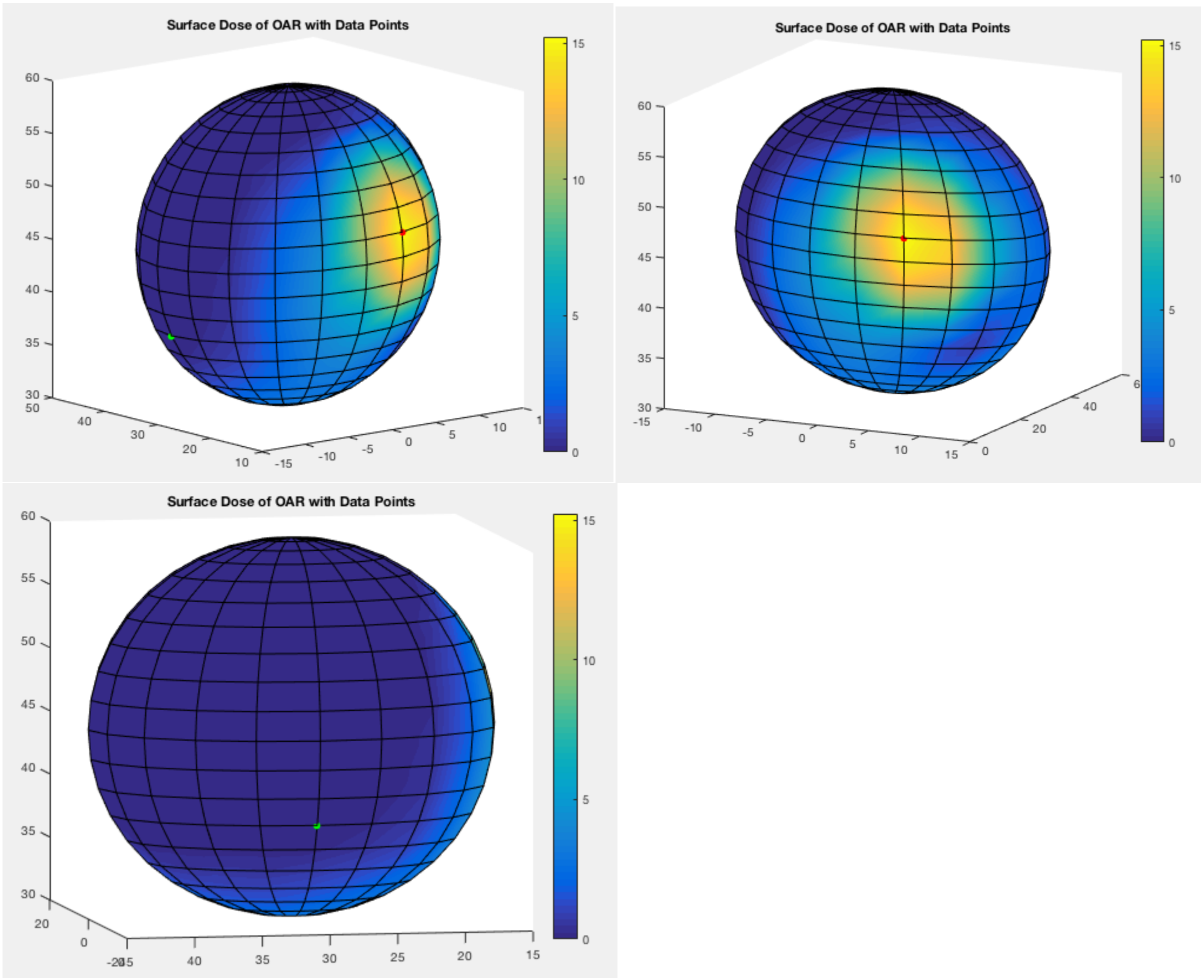
The blue curve for the PTV Dose Volume Histogram is the non-cumulative (relative) dose volume histogram. By observing this plotted relative histogram, we can see that almost 20% of the PTV is receiving a dose of 374, whereas other dosages are less frequent in the PTV. The red curve for the PTV dose Volume Histogram is the cumulative dose volume histogram. This cumulative line illustrates that 100% of the PTV is receiving a dosage of at least 368, and that as the dose value increases, less volume is receiving a higher dose (as the absolute dose increases, the cumulative relative volume percentage decreases).



Both the cumulative and relative (non-cumulative) dose volume histograms for the OAR are equivalent, which is why only a red line is plotted on the graph as the blue relative histogram is covered by it. This tells us that 100% of the voxels in the OAR are receiving a dosage between 0-0.1, and none of the voxels are receiving a dosage that is greater than or equal to 0.2. This very small dose for 100% of the OAR is expected as all of the beams that would have given the OAR a higher dosage were removed since they were marked as unsafe and therefore were plugged and not used in this calculation to ensure that the PTV has as high a dosage as possible while also minimizing the dosage for the OAR.

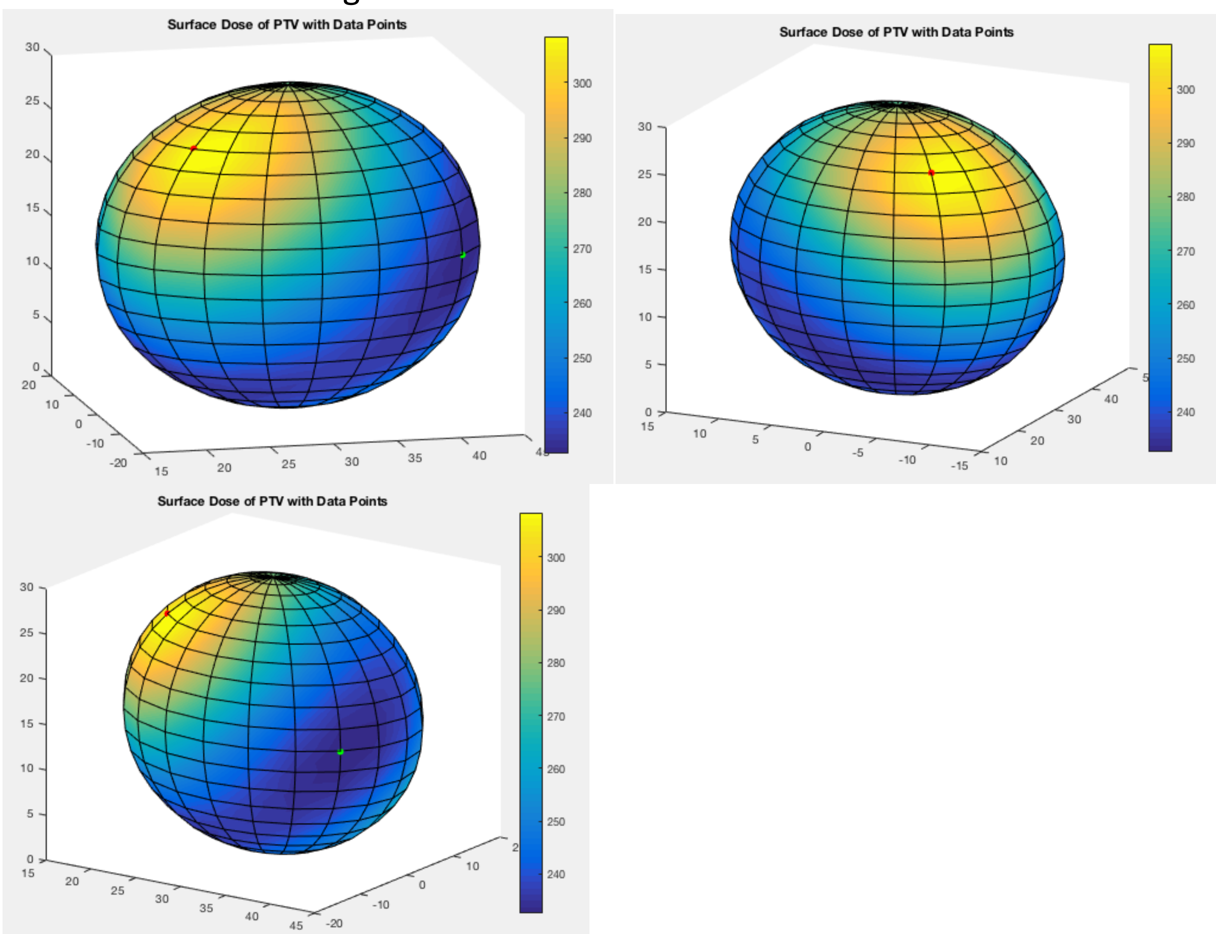
15. Compute Surface Dose

OAR Surface with Dosages



Pictured above are three different images of the OAR surface dose. The hottest dose location is plotted as the red point and the coldest dose location is the green point. Using the color bar attached to the side of the plot the dosage amounts on the surface of the sphere in different locations can be interpreted (blue is less dosage, yellow is more dosage). This plot is an accurate representation of the surface dosages of the OAR as the maximum dosage point is located in the middle of the yellow area (higher dosage) and the minimum dosage point is located in the darker blue area (lower dosage). The majority of the OAR surface is on the darker blue side of the color spectrum, meaning that the majority of it’s surface has little to no radiation dosage, and the max value of the color bar is small indicating that the areas that are yellow aren’t receiving a very high dosage. Since we plugged all of the beams that would intersect with the OAR, this colored surface plot makes sense.

PTV Surface with Dosages



Pictured above are three different images of the PTV surface dose. The hottest dose location is plotted as the red point and the coldest dose location is the green point. Using the color bar attached to the side of the plot the dosage amounts on the surface of the sphere in different locations can be interpreted (blue is less dosage, yellow is more dosage). This plot is an accurate representation of the surface dosages of the PTV as the maximum dosage point is located in the middle of the yellow area (higher dosage) and the minimum dosage point is located in the darker blue area (lower dosage). The color bar for the PTV has a much higher range of values compared to the OAR, which correlates with what we are doing as we want the PTV to have a much higher dosage than the OAR. The PTV surface also contains more yellow and green than the OAR surface and has a small amount of dark blue towards the back bottom of the sphere. This correlates with what we want as well because it means that the majority of the PTV surface is getting a very high dose of radiation whereas the OAR surface is getting a lot less radiation.

16. Compute Dose Surface Histogram

The compute dose surface histogram plots a histogram of the cumulative surface area of the sphere containing certain doses. To calculate the doses of the surface area of the PTV, the surface area was divided into small triangles and the total point dosages for the vertices of all the triangles was calculated. To find the dosage of the inside of the triangle, the average of the three vertices dosages was calculated and assigned as the triangle's dosage. These dosages were converted to relative dose percentages by dividing by the total surface area dose. The percentage of total surface area containing a certain dose was calculated and the cumulative values were plotted in the histogram below. This plot indicates that 100% of the PTV surface area receives a radiation dose of approximately 240 and as the dosage increases, the percentage of surface area receiving that dose (or more) declines.