CSC263 Fall 2017 Assignment 1

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Q1. SOLUTION

(a) $T(n)$ is $O(n^2)$.
Since $T(n)$ is the worst case time complexity of executing procedure, the first loop (i.e. for $i = 2$ to n do) runs in $n-1$ times and each time, the inner loop (i.e. for $j = i$ to n do) has size $n - i$.
The worst case complexity $T(n)$ can be expressed as:

$$T(n) = (n-1) + (n-2) + ... + 1 = \frac{n^2}{2} \in O(n^2)$$

(b) $T(n)$ is $O(n^2)$.
As discussed in part (a), $T(n)$ can be express as:

$$T(n) = (n-1) + (n-2) + ... + 1 = \frac{n^2}{2} \in \Omega(n^2)$$

Q2. SOLUTION

(a) In the best case, Line #2 ("if $A[i] = k$") will be executedd only once.
the first value assigned to i is $n-1$ so that in the first time that Line #2 is being executed, $A[i] = A[n-1]$ and the algorithm is checking whether the last integer in array A is equal to k. Since the integer stored in $A[n-1]$ can be any inumber between 0 and n. Also, we assume that k is an integer whose value satisfies $1 \leq k \leq n$. Therefore, the best case is that $A[n-1] == k$ and Line #2 is only excuted once.

(b) The probability that the best case occurs is $\dfrac{1}{n+1}$.
Justify my work?

(c) In the worst case, Line #2 ("if $A[i] = k$") will be executedd n times.
the worst case happens when k does not appear in A. In this case, Line #2 is excuted n times and the function returns -1.

(d) since for $A[i]$ we pick an integer from 0 to $i+1$ uniformly, the probability of choosing each integer is equal. Assume a given $1 \leq k \leq n$, then it is impossible for k to appear in $A[i]$ where $i < k-1$.

$$P(A[i] \neq k, i < k-1) = 1$$

The probability of choosing k in $A[i]$ where $k-1 \leq i \leq n-1$ is $\dfrac{i+1}{i+2}$.

$$P(A[i] \neq k, k-1 \leq i \leq n-1) = \frac{i+1}{i+2}$$

The probability for choosing each number in array$A$ is independent. Therefore, for a given k, the probability that worst case occurs is:

$$\begin{aligned} P(worstcase) &= \Pi_{i=k-1}^{n-1} P(A[i] \neq k) \\ &= \Pi_{i=k-1}^{n-1} \frac{i+1}{i+2} \\ &= \frac{k}{n+1} \end{aligned}$$

(e) In the average case, the last appearance of value k is possible to be found at any $A[i]$ with $k-1 \geq i \geq n-1$. In order to find the average case, we calculate the expected last apperarace of k when the value of k is given.
For a given $k(1 \leq k \leq n-1)$ the probability of $A[i](k-1 \neq i \neq n-1)$ being the last appearance of value$k$ is

$$\begin{aligned} P(A[i]) &= \frac{1}{i+1} \times \Pi_{j=n-1}^{i+1} \frac{j+1}{j+2} \\ &= \frac{1}{n+1} \end{aligned}$$

Also if $A[i](k-1 \geq i \geq n-1)$ is the last appearance of value k, we know that the number of times that Line #2 is excuted (assume m) will be

$$m = n - i$$

Therefore, the expected last appearance of a given k is

$$\begin{aligned} E(Line2execucutedtimes) &= \textstyle\sum_{m=1}^{n-k+1} \frac{1}{n+1} \\ &= \frac{1}{n+1} \end{aligned}$$

Q3. SOLUTION

(a) $T(n)$ is $O(n^2)$.

Q4. SOLUTION