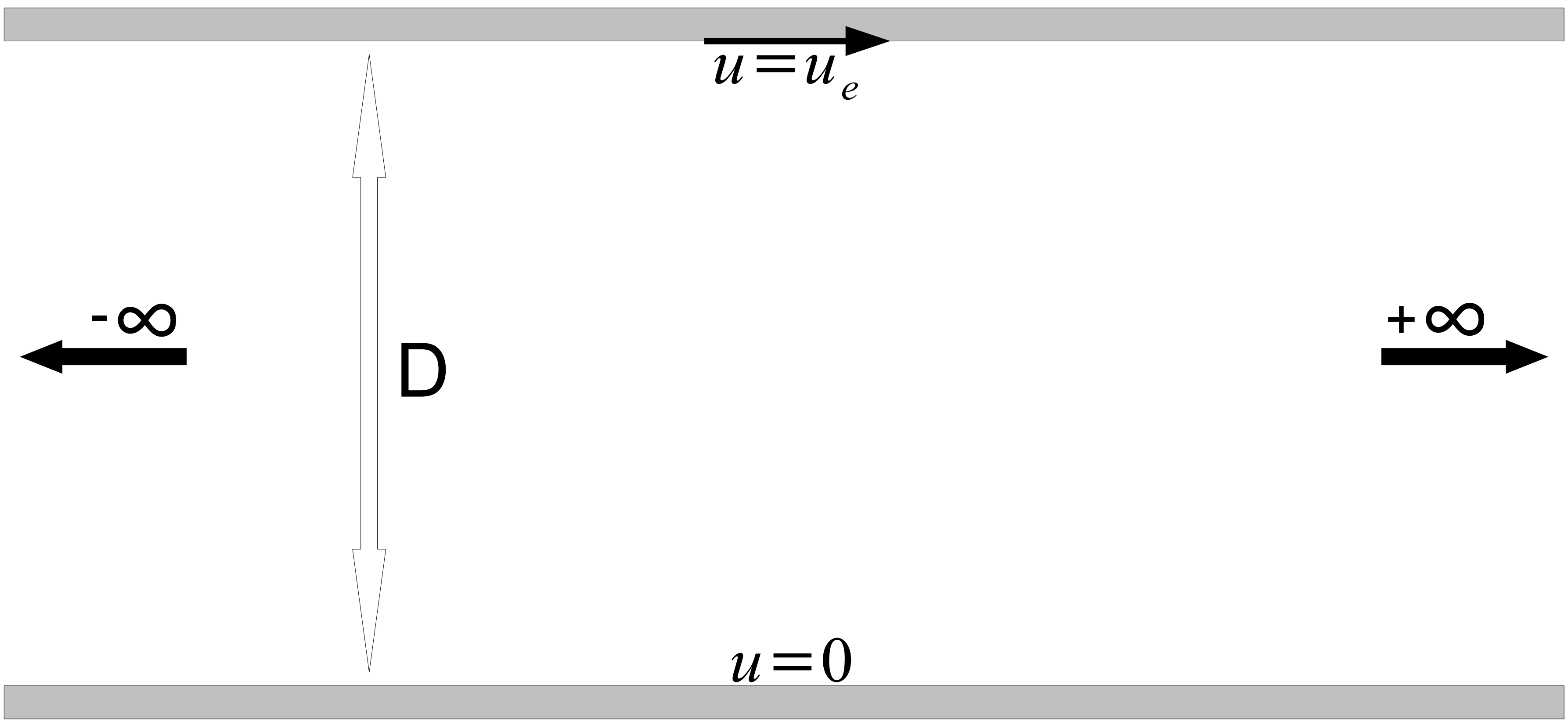
Escoamento de Couette



Equação da Continuidade

$$\frac{\partial u}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

$$mas \quad \frac{\partial}{\partial t}=0 \quad e \quad \frac{\partial}{\partial x}=0$$

$$logo \quad \frac{\partial(\rho v)}{\partial y}=0$$

Equação da Continuidade

$$ou \quad \rho\frac{\partial v}{\partial y}+v\frac{\partial \rho}{\partial y}=0$$

avaliando na parede inferior onde y=0 e v=0

$$\left(\rho\frac{\partial v}{\partial y}\right)_{y=0}=0 \quad logo \quad \left(\frac{\partial v}{\partial y}\right)_{y=0}=0$$

Expandindo-se v em série de Taylor em torno do ponto y=0

$$v(y) = v(0) + \left(\frac{\partial v}{\partial y}\right)_{y=0} y + \left(\frac{\partial^2 v}{\partial y^2}\right)_{y=0} \frac{y^2}{2} + \ldots$$

E avaliando na parede superior onde y=D

$$v(D) = v(0) + \left(\frac{\partial v}{\partial y}\right)_{y=0} D + \left(\frac{\partial^2 v}{\partial y^2}\right)_{y=0} \frac{D^2}{2} + \ldots$$

$$mas \quad v(D)=0, \quad v(0)=0 \quad e \quad \left(\frac{\partial v}{\partial y}\right)_{y=0}=0$$

$$logo \quad \left(\frac{\partial^n v}{\partial y^n}\right)_{y=0}=0 \quad para\,todo\,n$$

$$resultando \;\; para\,todo\,o\,domínio\,em$$

$$\boxed{v \;=\; 0}$$

Equação do Momento em y

$$\frac{\partial \rho v}{\partial t}+\frac{\partial(\rho u v)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}$$

$$\frac{\partial(\rho v^2)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yy}}{\partial y}$$

$$2\rho v\frac{\partial(v)}{\partial y}+v^2\frac{\partial(\rho)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yy}}{\partial y}$$

Equação do Momento em y

$$logo \quad 0 = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yy}}{\partial y}$$

$$mas \quad \tau_{yy} = -\frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial v}{\partial y} = 0$$

$$resultando\ para\ todo\ o\ domínio\ em$$

$$\boxed{\frac{\partial p}{\partial y} = 0}$$

Equação do Momento em x

$$\frac{\partial \rho u}{\partial t}+\frac{\partial\left(\rho u^2\right)}{\partial x}+\frac{\partial(\rho u v)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}$$

$$mas \quad \tau_{xx} = -\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial u}{\partial x} = 0$$

$$e \quad \tau_{xy} = \mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) = \mu\frac{\partial u}{\partial y}$$

Equação do Momento em x

substitiundo na equação do momento

$$0 = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

para escoamento incompressível e a temperatura constate

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 0}$$

Solução Analítica

$$u = c_1 y + c_2$$

$$em \quad y=0 \quad temos \quad u=0, \quad logo \quad c_2=0$$

$$em \quad y=D \quad temos \quad u=u_e, \quad logo \quad c_1=\frac{u_e}{D}$$

$$\boxed{\frac{u}{u_e} = \frac{y}{D}}$$

Perfil Linear de Velocidade