Lista 4 Cálculo I

1) a) $f(x) = x^9 + x^4 + 2$
↳ Função Polinomial
↳ Grau 9
↳ Algébrico

b) $g(x) = \sqrt[5]{x}$
↳ Função raiz
↳ grau $\frac{1}{5}$
↳ algébrico

c) $h(x) = \sqrt{1-x^2}$
↳ Função raiz
↳ grau 1
↳ Algébrico.

d) $r(x) = \dfrac{x^2+3x+1}{x^3+x}$
↳ Função racional
↳ grau 2 e 3
↳ Algébrico.

e) $s(x) = \tan 2x$
↳ Função Trigonométrica
↳ Função algébrica
↳ grau 1

f) $t(x) = \log_{10} x$
↳ função logarítmica
↳ função algébrica

g) $f(x) = \dfrac{x-6}{x+6}$
↳ Função racional
↳ Algébrico
↳ grau 1

h) $g(x) = x + \dfrac{x^2}{\sqrt{x-1}}$
↳ Função racional
↳ Algébrica

i) $h(x) = 10^x$
↳ função exponencial
↳ função algébrica
↳ grau x

j) $r(x) = x^{10}$
↳ Função potência
↳ Algébrica
↳ grau 10

k) $s(t) = 2t + \pi$
↳ função afim
↳ função algébrica
↳ grau 1

l) $t(\theta) = \cos\theta + \operatorname{sen}\theta$
↳ Trigonométrica
↳ algébrica
↳ grau 1.

2) $f \circ g$, $g \circ f$, $f \circ f$, $g \circ f$

a) $f(x) = 2x^2 - x$, $g(x) = 3x + 2$

• $f \circ g = f(g(x)) = 2(g(x))^2 - (g(x))$
$= 2\cdot(3x+2)^2 - (3x+2)$
$= 2\cdot(9x^2+6x+4) - 3x - 2$
$= 18x^2 + 10x + 6$
$\mathrm{Dom}(f \circ g) = \mathbb{R}$

• $g \circ f = g(f(x)) = 3(f(x)) + 2$
$= 3(2x^2 - x) + 2$
$= 6x^2 - 3x + 2$
$\mathrm{Dom}(g \circ f) = \mathbb{R}$

• $f \circ f = f(f(x)) = 2(f(x))^2 - (f(x))$
$= 2(2x^2 - x) - 2x^2 + x$
$= 2(4x^4 - 4x^3 + x^2) - 2x^2 + x$
$= 8x^4 - 8x^3 + x$
$\mathrm{Dom}(f \circ f) = \mathbb{R}$

• $g \circ g = g(g(x)) = 3(g(x)) + 2$
$= 3(3x+2) + 2$
$= 9x + 8$
$\mathrm{Dom}(g \circ g) = \mathbb{R}$

b) $f(x) = \sqrt{x-1}$, $g(x) = x^2$ $\longrightarrow$ $Dom(f): x \geq 1$ $\quad Dom(g) = \mathbb{R}$ $\quad x \geq 1$

$\cdot\, f \circ g = f(g(x)) = \sqrt{(g(x))-1}$
$= \sqrt{x^2-1}$

$Dom(f \circ g) = \mathbb{R}$

$\cdot\, g \circ f = g(f(x)) = (f(x))^2$
$= (\sqrt{x-1})^2$

$Dom(g \circ f) = \mathbb{R}_+ - \{0\}$

$\cdot\, g \circ g = g(g(x)) = (g(x))^2$
$= (x^2)^2 = x^4$

$Dom(g \circ g) = \mathbb{R}$

$\cdot\, f \circ f = f(f(x)) = \sqrt{(f(x))-1}$
$= \sqrt{\sqrt{x-1}-1} \longrightarrow x > 1$

$Dom(f \circ f) =]1, +\infty[$

c) $f(x) = x + \frac{1}{x}$, $g(x) = x^3 + 2x$ $\longrightarrow$ $Dom(f) = \mathbb{R} - \{0\}$, $Dom(g) = \mathbb{R}$

$\cdot\, f \circ g = f(g(x)) = g(x) + \frac{1}{g(x)}$
$= x^3 + 2x + \frac{1}{x^3+2x} \rightarrow x^3 + 2x \neq 0$
$x(x^2+2) = 0$
$x \neq 0$ e $\cancel{x \neq \sqrt{-2}}$

$Dom(f \circ g) = \mathbb{R}^*$

$\cdot\, g \circ f = g(f(x)) = (f(x))^3 + 2 \cdot f(x)$
$= \left(x + \frac{1}{x}\right)^3 + 2 \cdot \left(\frac{x^2+1}{x}\right)$
$= \left(\frac{x^2+1}{x}\right)^3 + \frac{2x^2+2}{x}$
$= \frac{(x^2+1)^3}{x^3} + \frac{2x^2+2}{x}$

$Dom(g \circ f) = \mathbb{R}^*$

$\cdot\, f \circ f(x) = f(f(x)) = f(x) + \frac{1}{f(x)}$
$= x + \frac{1}{x} + \frac{1}{x + \frac{1}{x}}$

$Dom(f \circ f) = \mathbb{R} - \{0, 1\}$

$\cdot\, g \circ g = g(g(x)) = (g(x))^3 + 2(g(x))$
$= (x^3+2x)^3 + 2(x^3+2x)$
$= (x^3+2x)^3 + 2x^3 + 4x$

$Dom(g \circ g) = \mathbb{R}$.

d) $f(x) = \cos x$, $g(x) = 1 - \sqrt{x}$ $\rightarrow$ $Dom(f) = \mathbb{R}$ e $Dom(g) = \mathbb{R}_+ - \{0\}$

$f \circ g = f(g(x)) = \cos(g(x))$
$= \cos(1 - \sqrt{x})$

$Dom(f \circ g) = \mathbb{R}_+ - \{0\}$

$\cdot\, g \circ f = g(f(x)) = 1 - \sqrt{f(x)}$
$= 1 - \sqrt{\cos x}$

$Dom(g \circ f) = [0, \frac{\pi}{2}[$

$\cdot\, f \circ f = f(f(x)) = \cos(f(x))$
$= \cos(\cos(x))$

$Dom(f \circ f) = \mathbb{R}$

$\cdot\, g \circ g = g(g(x)) = 1 - \sqrt{g(x)}$
$= 1 - \sqrt{1 - \sqrt{x}}$ $\quad x \neq 0$ e $0 < x \leq 1$

$Dom(g \circ g) =]0, 1]$

3) $f \circ g \circ h$

a) $f(x) = 3x - 2$, $g(x) = \text{sen}\, x$, $h(x) = x^2$

$f(g(h(x))) = 3(g(h(x))) - 2 = 3(\text{sen}(h(x))) - 2$

$= 3\,\text{sen}(x^2) - 2$

b) $f(x) = |x-4|$, $g(x) = 2^x$, $h(x) = \sqrt{x}$

$f(g(h(x))) = |g(h(x)) - 4| = |2^{h(x)} - 4| = |2^{\sqrt{x}} - 4|$

c) $f(x) = \sqrt{x-3}$, $g(x) = x^2$, $h(x) = x^3 + 2$

$f(g(h(x))) = \sqrt{g(h(x)) - 3} = \sqrt{h(x)^2 - 3} = \sqrt{(x^3+2)^2 - 3}$

d) $f(x) = \text{tg}(x)$, $g(x) = \dfrac{x}{x-1}$, $h(x) = \sqrt[3]{x}$

$f(g(h(x))) = \text{Tg}(g(h(x))) = \text{Tg}\left(\dfrac{h(x)}{h(x) - 1}\right) = \text{Tg}\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}\right)$

4) $f(x) = \dfrac{1+x}{1-x}$, mostre $f(-x) = \dfrac{1}{f(x)}$ e $f \circ f = -\dfrac{1}{x}$

I) $f(-x) = \dfrac{1 + (-x)}{1 - (-x)} = \dfrac{1}{\frac{1+x}{1-x}} \rightarrow \dfrac{1-x}{1+x} = 1 \cdot \dfrac{1-x}{1+x} \longrightarrow \dfrac{1-x}{1+x} = \dfrac{1-x}{1+x}$

II) $f \circ f = \dfrac{1 + f(x)}{1 - f(x)} = \dfrac{\frac{1}{1} + \frac{1+x}{1-x}}{\frac{1}{1} - \frac{(1+x)}{1-x}} = \dfrac{\frac{1-\cancel{x}+1+\cancel{x}}{1-x}}{\frac{1-x-1-x}{1-x}} = \dfrac{\frac{2}{1-x}}{\frac{-2x}{1-x}} = \dfrac{2}{\cancel{1-x}} \cdot \dfrac{\cancel{1-x}}{-2x}$

$= \dfrac{2}{-2x} = \boxed{-\dfrac{1}{x}}$

5) a) $f(3) = 7$, $f(7)^{-1} = ?$

$y = f^{-1}(x) \rightarrow f(y) = x \Rightarrow y = f^{-1}(7) \rightarrow f(3) = 7$

$\therefore f^{-1}(7) = 3$

b) $f(x) = 3 + x^2 + \text{tg}\left(\frac{\pi x}{2}\right)$, $f^{-1}(3) = ?$

$y = f^{-1}(x) \rightarrow f(y) = x \Rightarrow y = f^{-1}(3) \rightarrow 3 = 3 + y^2 + \text{tg}\left(\frac{\pi y}{2}\right)$

$y^2 + \text{Tg}\left(\frac{\pi y}{2}\right) = 0 \rightarrow y^2 = -\text{Tg}\left(\frac{\pi y}{2}\right)$

$\hookrightarrow y = 0$ satisfaz

$\therefore f^{-1}(3) = 0$

c) $f(x) = 3 + x + e^x$, $f^{-1}(4) = ?$

$y = f^{-1}(x) \rightarrow x = f(y) \Rightarrow y = f^{-1}(4) \rightarrow 4 = f(y)$

$4 = 3 + y + e^y \Rightarrow y + e^y = 1 \quad \therefore \quad y = 0 \rightarrow f^{-1}(4) = 0$

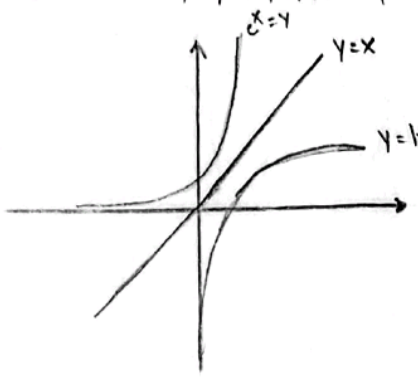
d) $f(x) = 2x + \ln x$, $f^{-1}(2)$

$y = f^{-1}(x) \rightarrow x = f(y) \Rightarrow y = f^{-1}(2) \rightarrow 2 = 2x + \ln x$

Se $x = 1$, então $2 = 2 \cdot 1 + \ln(1)$ $\quad \therefore f^{-1}(2) = 1$

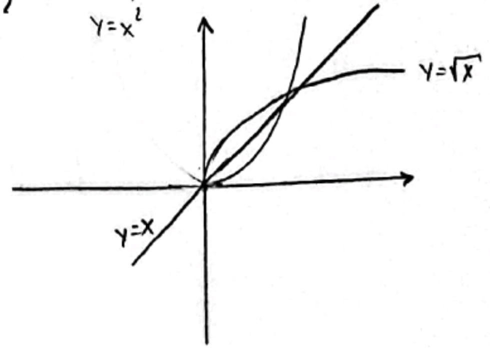
$2 = 2 + 0$

6) a) $y = e^x$, $y = \ln x$, $y = x$



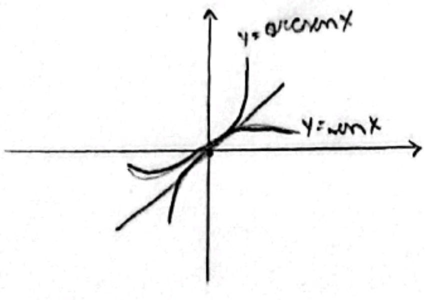
As funções $y = e^y$ e $y = \ln x$ são uma inversa da outra, e a função $f(x) = y$ é o eixo de espelhamento.

b) $y = x^2$, $y = \sqrt{x}$, $y = x$

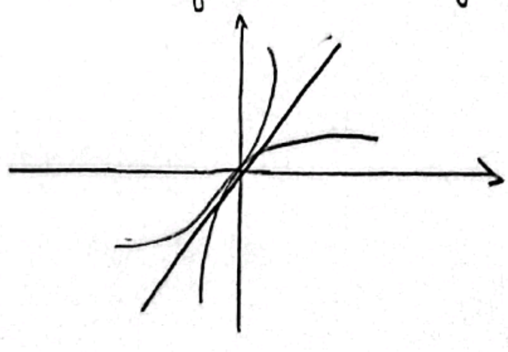


A mesma coisa da anterior

c) $y = \operatorname{sen} x$, $y = \operatorname{arcsen} x$, $y = x$



d) $y = \operatorname{Tg}(x)$, $y = \operatorname{arctg}(x)$, $y = x$



Reparamos que todas as funções acima são inversas uma da outra.

7) a) $f(x)=x^2+1 \rightarrow y=f^{-1}(x) \Rightarrow x=f(y)$

$y=x^2+1 \rightarrow y-1=x^2 \rightarrow x=\sqrt{y-1} \Rightarrow x=f^{-1}(y) \rightarrow y=f(x)$

$= f^{-1}(x)=\sqrt{x-1}$

b) $f(x): x^2-2x+1 \rightarrow y=x^2-2x+1 \Rightarrow y=(x-1)^2 \Rightarrow y^2=x-1 \Rightarrow y^2-1=x$

$=f^{-1}(x)=x^2-1$

c) $f(x)=x^3-1 \rightarrow y=x^3-1 \Rightarrow y+1=x^3 \Rightarrow x=\sqrt[3]{y+1}$

$= f^{-1}(x)=\sqrt[3]{x+1}$

d) $f(x)=x^{2/3} \rightarrow y=x^{2/3} \rightarrow y=\sqrt[3]{x^2} \rightarrow y^3=x^2 \rightarrow \sqrt{y^3}=x$

$=f^{-1}(x)=x^{3/2}$.

8) a) $f(x)=1+\sqrt{2+5x} \rightarrow y=1+\sqrt{2+5x} \rightarrow \frac{(y-1)^2+2}{5} = \frac{y^2-2y+3}{5}=x$

$\cdot f^{-1}(x)=\frac{x^2-2x+3}{5}$

Dom$(f) = 2+5x \geq 0 \Rightarrow x \geq -\frac{2}{5}$

Dom$(f) = [\frac{2}{5}, +\infty[$ e Im$(f) = [1,+\infty[$

Dom$(f^{-1}) = [1,+\infty[$ e Im$(f^{-1}) = [-\frac{2}{5},+\infty[$.

b) $f(x) = \frac{4x-1}{2x+3} \rightarrow 2x+3 \neq 0 \rightarrow x = -\frac{3}{2}$ $y = \frac{4x-1}{2x+3} \Rightarrow y(2x+3) = 4x+1$

$= 2xy+3y = 4x+1 \rightarrow 2xy-4x = 1-3y \rightarrow x(2y-4) = 1-3y \Rightarrow x = \frac{1-3y}{2y-4}$

• Dom$(f) = \mathbb{R} - \{-\frac{3}{2}\}$ e Im$(f) = \mathbb{R} - \{2\}$

• $f^{-1}(x) = \frac{1-3x}{2x-4}$

• Dom$(f^{-1}) = \mathbb{R} - \{2\}$ e Im$(f) = \mathbb{R} - \{-\frac{3}{2}\}$.

c) $f(x) = e^{2x-1} \rightarrow y = e^{2x-1} \rightarrow \ln(y) = 2x-1 \rightarrow \ln+1 = 2x \rightarrow \frac{\ln(y)+1}{2} = x$

• Dom$(f) = \mathbb{R}$ e Im$(f) = [0,+\infty[$

• $f^{-1}(x) = \frac{\ln(y)+1}{2}$

• Dom$(f^{-1}) = [0,+\infty[$ e Im$(f^{-1}) = \mathbb{R}$

d) $f(x) = 2^{10^x} \rightarrow y = 2^{10^x} \rightarrow \log_2(y) = 10^x \rightarrow x = \log_{10}(\log_2 y)$

• $f^{-1}(x) = \log_{10}(\log_2 x)$ • Dom$(f) = \mathbb{R}$ • Im$(f) = \mathbb{R}_+$

• Dom$(f^{-1}) = \mathbb{R}_+$ • Im$(f^{-1}) = \mathbb{R}$.

e) $f(x) = \ln(\overset{\to -3}{x+3}) \to y = \ln(x+3) \to e^y = x+3 \to x = e^y - 3$

- $f^{-1}(x) = e^y - 3$
  - $Dom(f) =]-3, +\infty[$ e $Im(f) = \mathbb{R}$.
  - $Dom(f^{-1}) = \mathbb{R}$ e $Im(f^{-1}) =]-3, +\infty[$.

$5 - 2x \neq 0$, $x \neq \frac{5}{2}$

f) $f(x) = \dfrac{1+3x}{5-2x} \to y = \dfrac{1+3x}{5-2x} \to y(5-2x) = 1+3x \to 5y - 2xy = 1 + 3x$

$-2xy - 3x = 1 - 5y \Rightarrow x(-2y-3) = 1 - 5y \Rightarrow x = \dfrac{1-5y}{-2y-3}$

$-2y - 3 \neq 0$, $2y \neq -3$, $y \neq -\frac{3}{2}$

- $f^{-1}(x) = \dfrac{1-5x}{-2y-3}$
  - $Dom(f) = \mathbb{R} - \{\frac{5}{2}\}$ $Im(f) = \mathbb{R} - \{-\frac{3}{2}\}$
  - $Dom(f^{-1}) = \mathbb{R} - \{-\frac{3}{2}\}$ $Im(f^{-1}) = \mathbb{R} - \{\frac{5}{2}\}$

$x \neq 0$

g) $f(x) = \dfrac{1+e^x}{1-e^x} \to y = \dfrac{1+e^x}{1-e^x} \to y(1-e^x) = 1 + e^x = y - ye^x = 1 + e^x$

$y - 1 = ye^x + e^x \to y - 1 = e^x(y+1) \to e^x = \dfrac{y-1}{y+1} \to x = \ln\left(\dfrac{y-1}{y+1}\right)$

	-1		1	
$x-1$	$-$	$-$		$+$
$x+1$	$-$		$+$	$+$
I	$+$	$-$		$+$

- $f^{-1}(x) = \ln\left(\dfrac{x-1}{y+1}\right)$
  - $Dom(f) = \mathbb{R} - \{0\}$ $Im(f) =]-\infty, -1[\cap]1, +\infty[$
  - $Dom(f^{-1}) =]-\infty, -1[\cap]1, +\infty[$ $Im(f^{-1}) = \mathbb{R} - \{0\}$

h) $f(x) = \dfrac{e^x}{1+2e^x} \longrightarrow y = \dfrac{e^x}{1+2e^x} \to y + 2ye^x = e^x$

$y = e^x - 2ye^x \to y = e^x(1-2y) \to e^x = \dfrac{y}{1-2y} \to x = \ln\left(\dfrac{y}{1-2y}\right)$

$1 - 2x > 0$, $x < \frac{1}{2}$, $x \neq 0$

- $f^{-1}(x) = \ln\left(\dfrac{x}{1-2x}\right)$
  - $Dom(f) = \mathbb{R}$ $Im(f) =]0, \frac{1}{2}[$
  - $Dom(f^{-1}) =]0, \frac{1}{2}[$ $Im(f^{-1}) = \mathbb{R}$.

9) a) $e^x = 16$

$x = \ln(16)$

b) $e^{e^x} = 2$

$e^x = \ln(2)$

$x = \ln(\ln 2)$

c) $e^{2x+3} - 7 = 0$

$e^{2x+3} = 7$

$2x + 3 = \ln(7)$

$2x = \ln 7 - 3$

$x = \dfrac{\ln(7) - 3}{2}$

d) $\ln x = -1$

$x = e^{-1}$

e) $\ln(2x-1) = 3$

$2x - 1 = e^3$

$2x = e^3 + 1$

$x = \dfrac{e^3+1}{2}$

f) $\ln(x) + \ln(x-1) = 0$

$\ln(x \cdot (x-1)) = 0$

$\ln(x^2 - x) = 0$

$e^{\ln(x^2-1)} = e^0$

$x(x-1) = 1$

$x^2 - x - 1 = 0$

$\Delta = 1 - 4 \cdot 1 \cdot (-1)$

$\Delta = 5$

$x = \dfrac{1 \pm \sqrt{5}}{2}$ → não neg

$\therefore x = \dfrac{1+\sqrt{5}}{2}$

g) $\ln(\ln x) = 1$

$\ln x = e$

$x = e^e$

h) $2^{x-5} = 3$

$x - 5 = \log_2 3$

$x = \log_2 3 + 5$

i) $1 + \arctg(x) = \sqrt{x}$