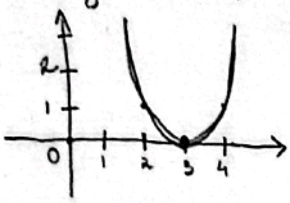
I

Lista 3 - Calculo I

1) uma função é uma relação especial entre 2 conjuntos, onde pontos do 1° conjunto apontam para apenas 1 ponto ao 2° conjunto. Chamamos esse 1° conjunto de Domínio da função (Dom(f)), e o domínio de chegada ou seja, o 2° conjunto de Imagem (Im(f)). Essa relação é feita através da notação: $Y = f(x)$, onde X é fiel a Y, ou seja, o X pode ter apenas um Y.

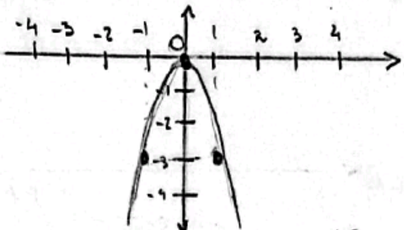
2) a) $y = (x-3)^2$ → Raízes: 3

X	Y
2	1
4	1



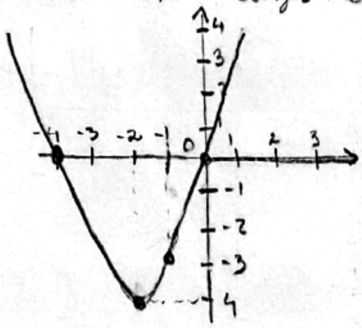
b) $Y = -3x^2 \Rightarrow x(-3x)$ raízes = 0

X	Y
1	-3
-1	-3



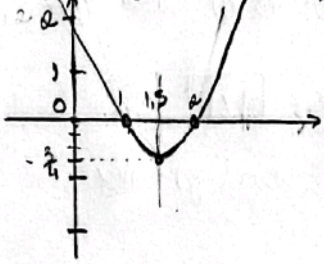
c) $Y = x(x+4)$ → raízes: 0 e -4

X	Y
-2	-4
-1	-3



d) $Y = x^2 - 3x + 2 \Rightarrow (x-2)\cdot(x-1)$

X	Y
1,5	-0,25
0	2

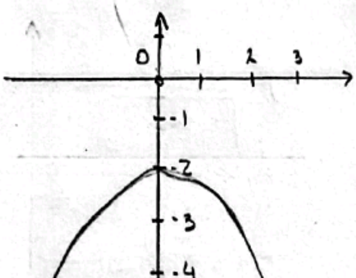


e) $Y = \frac{-x^2}{5} - 2$

$\Delta = 0 - 4\left(-\frac{1}{5}\right)\cdot(-2)$

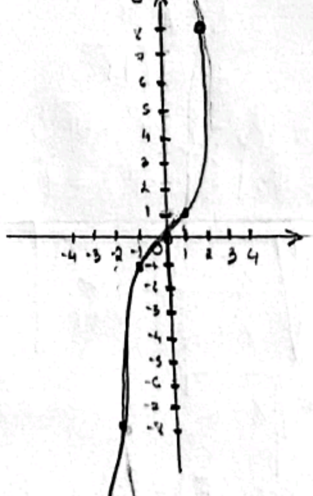
$\Delta = -\frac{8}{5}$

X	Y
0	-2
-3	



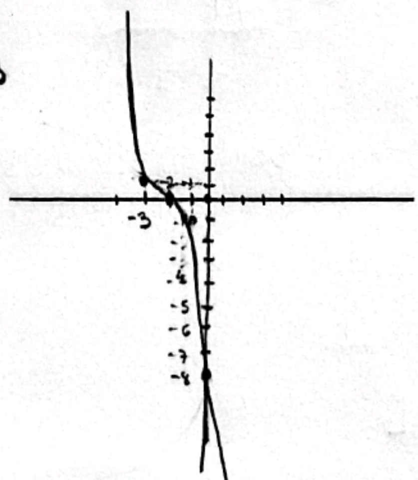
f) $y = x^3$ raiz = 0

X	Y
-1	-1
1	1
-2	-8
2	8



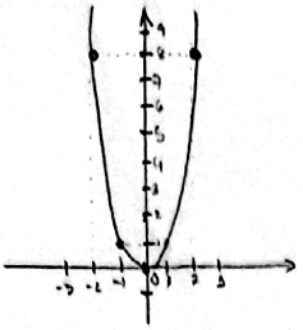
g) $Y = -(x+2)^3$

X	Y
0	-8
-2	0
-1	-1
-3	1
1	



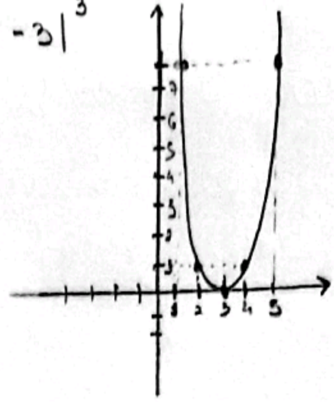
h) $y = |x|^3$

x	y
0	0
-1	1
1	1
-2	8
2	8



i) $y = |x-3|^3$

x	y
3	0
2	1
4	1
1	8
5	8



3) a) $f(x) = -4x+3$

$Dom(f) = \mathbb{R}$

b) $f(x) = -x^2 - 3x + 1$

$Dom(f) = \mathbb{R}$

c) $f(x) = \sqrt{3x+9}$

restrição $3x+9 \geq 0$

$3x \geq -9$

$\boxed{x \geq -3}$

$Dom(f) = [-3, +\infty[$

d) $f(t) = \dfrac{4}{3-t}$ restrição : $t \neq 3$

$Dom(f) = \mathbb{R} - \{3\} =]-\infty, 3[\cap]3, +\infty[$

e) $g(x) = 1 - \sqrt{x}$

restrição : $x \geq 0$

$Dom(g) = [0, +\infty[$

f) $g(x) = \sqrt{x^2-3x+2}$

rest. $x^2-3x+2 \geq 0$

$\therefore (x-1)\cdot(x-2)$

	1		2	
x-1	-	+		+
x-2	-	-		+
R	+	-		+

$\therefore Dom(g) =]-\infty, 1] \cup [2, +\infty[$

g) $g(x) = \sqrt{|x|}$

$Dom(g) = \mathbb{R}$

h) $g(x) = \sqrt{-x}$

restrição : $-x \geq 0 \therefore Dom(g) =]\infty, 0]$

i) $h(t) = \dfrac{1}{|t^2 - \pi|}$

restrição : $t^2 - \pi \neq 0$

$t^2 \neq \pi$

$t \neq \pm\sqrt{\pi}$

$Dom(h) = \mathbb{R} \setminus \{\sqrt{\pi}, -\sqrt{\pi}\}$

ou $t \in \mathbb{R} \mid t \neq \sqrt{\pi}$ e $t \neq -\sqrt{\pi}\}$

j) $h(t) = \dfrac{t}{|t|}$ Restrição : $t \neq 0$

$Dom(h) = \mathbb{R} \setminus \{0\}$

k) $F(x) = \sqrt{\dfrac{x^2-4}{x+2}} \underset{simp}{\Rightarrow} \sqrt{\dfrac{\cancel{(x+2)}\cdot(x-2)}{\cancel{(x+2)}}}$

$F(x) = \sqrt{x-2}$ restrição : $x - 2 \geq 0$

$\boxed{x \geq 2}$

$Dom(F) = [2, +\infty[$

l) $\dfrac{2}{\sqrt{9t^2-25}} = G(t)$

Rest: $(3t+5)\cdot(3t-5) > 0$

		$-\frac{5}{3}$		$\frac{5}{3}$	
3t+5	-		+		+
3t-5	-		-		+
R	+				+

$Dom(G) =]-\infty, -\frac{5}{3}[\cap]\frac{5}{3}, +\infty[$

m) $H(x) = \log_{10}(2x^2+5x-3)$

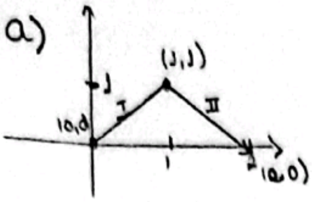
restrição : $(2x-1)\cdot(x+3) > 0$

		-3		1/2	
2x-1	-		-		+
x+3	-		+		+
R	+		-		+

$Dom(H) =]-\infty, -3[\cap]\frac{1}{2}, +\infty[$

II

6) a)



I) $y - 0 = 1 \cdot (x - 0)$

$y = x$

II) $y - 1 = \frac{-1}{1}(x + 1)$

$y - 1 = -x + 1$

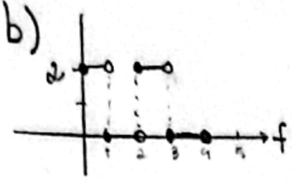
$y = -x$

$\therefore f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x, & 1 < x \leq 2 \end{cases}$

$Dom(f) = [0, 2]$

$Im(f) = [0, 1]$

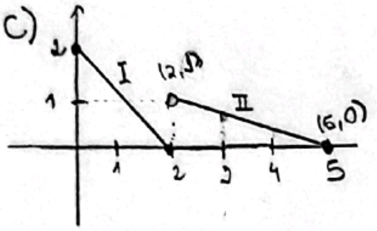
b)



$f(t) = \begin{cases} 0, & \text{se } 1 \leq t < 2 \text{ ou } 3 \leq t < 4 \\ 2, & \text{caso contrario.} \end{cases}$

$Don(f) = [0, 4]$ $Im(f) = \{0, 2\}$

c)



I. $y - 2 = \frac{-2}{2}(x - 0)$

$y = -x + 2$

II. $y - 1 = \frac{-1}{5-2}(x - 2)$

$y - 1 = -\frac{1}{3}x + \frac{2}{3}$

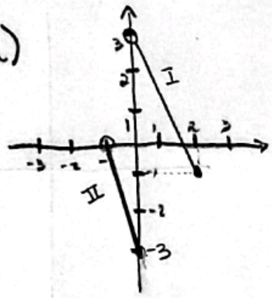
$y = -\frac{1}{3}x + \frac{5}{3}$

$f(x) = \begin{cases} -x + 2, & \text{para } 0 \leq x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & \text{para } 2 < x \leq 5 \end{cases}$

$Don(f) = [0, 2]$

$Im(f) =]2, 5]$

d)



I: $y - 3 = \frac{-1-3}{2-0}(x - 0)$

$y - 3 = -2x$

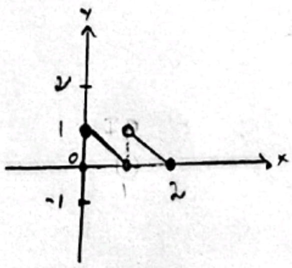
$y = -2x + 3$

II. $y = \frac{-3}{1}(x - (-1))$

$y = -3x - 3$

$f(x) = \begin{cases} -2x + 3, & \text{se } 0 < x \leq 2; \\ -3x - 3, & \text{se } -1 < x \leq 0; \end{cases}$

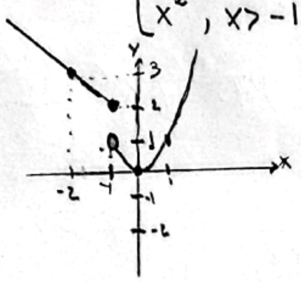
7) a) $g(x) = \begin{cases} 1 - x, & 0 \leq x \leq 1 \quad I \\ 2 - x, & 1 < x \leq 2 \quad II \end{cases}$



$[0, 1]$: Decrescente

$]1, 2]$: Crescente

b) $f(x) = \begin{cases} 1 - x, & x \leq -1 \\ x^2, & x > -1 \end{cases}$

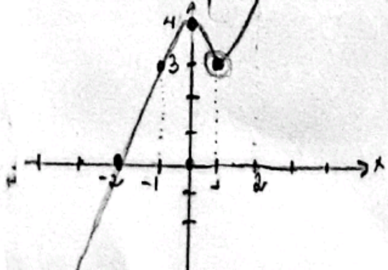


$]-\infty, -1]$: Decrescente

$]-1, 0]$: Decrescente

$]0, +\infty[$: Crescente

c) $f(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ x^2 + 2x, & x > 1 \end{cases}$

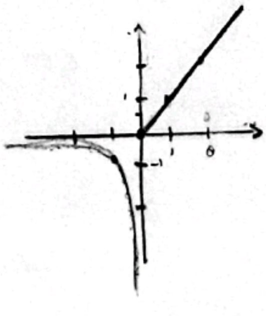


$]-\infty, 0]$: Crescente

$]0, 1]$: Decrescente

$]1, +\infty[$: crescente.

d) $g(x) = \begin{cases} 1/x, & x < 0 \\ x, & x \geq 0 \end{cases}$



$]-\infty, 0[$: Decrescente

$[0, +\infty[$: crescente

4) a) $f(x) = \dfrac{x+3}{4-\sqrt{x^2-25}}$

Restrições: I. $(x+5)\cdot(x-5) \geq 0$

II. $x^2 - 25 \neq 4^2$

$x^2 = 16 + 25$

$x = \pm\sqrt{41}$

	-5		5	
$x+5$	$-$	$+$		$+$
$x-5$	$-$	$-$		$+$
R	$+$	$-$		$+$

$\therefore \mathrm{Dom}(f) =]-\infty, -5] \cap [5, +\infty[- \{\pm\sqrt{41}\}$

$S = \{x \in \mathbb{R} \mid x \neq \pm\sqrt{41} \text{ e } x \geq 5 \text{ e } x \leq -5\}$

b) $f(x) = 2 + \sqrt{16 + x^2}$

Para $f(0) = 6$

$\therefore$

$\mathrm{Im}(f) = [6, +\infty[$

restrição: $16 + x^2 \geq 0$

$\boxed{x^2 = -16}$

Impossível.

5) a) $\begin{cases} 20 = 2x + 2y \rightarrow 10 = x + y \\ A = x \cdot y \end{cases}$ $\quad y = 10 - x$

$A = x \cdot (10 - x)$ → rest $= 10 - x > 0$

$A = 10x - x^2$ $\quad x < 10$

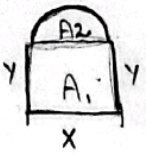
- $f(x) = 10x - x^2$
- $\mathrm{Dom}(f) =]0, 10[= \{x \in \mathbb{R} \mid 0 < x < 10\}$

b) 

$\operatorname{sen} 60 = \dfrac{h}{l} = \dfrac{\sqrt{3}}{2} \Rightarrow h = \dfrac{l \cdot \sqrt{3}}{2}$

$A = \dfrac{l \cdot \dfrac{l\sqrt{3}}{2}}{2} \Rightarrow \dfrac{l^2\sqrt{3}}{4}$

- $f(x) = \dfrac{x^2\sqrt{3}}{4}$
- $\mathrm{Dom}(f) = \mathbb{R}_+ - \{0\} =]0, +\infty[$

c) 

$\begin{cases} A_2 = \pi\left(\dfrac{x}{2}\right)^2 \cdot \dfrac{1}{2} = \dfrac{\pi x^2}{4} \cdot \dfrac{1}{2} = \dfrac{\pi x^2}{8} \\ A_1 = x \cdot \left(\dfrac{10 - x - \frac{\pi x}{2}}{2}\right) = \dfrac{10x - x^2 - \frac{\pi x^2}{2}}{2} = 5x - \dfrac{x^2}{2} - \dfrac{\pi x^2}{4} \end{cases}$

$A_T = 5x - \dfrac{x^2}{2} - \dfrac{\pi x^2}{4} + \dfrac{\pi x^2}{8} = 5x - \dfrac{x^2}{2} - \dfrac{\pi x^2}{8} > 0$

$x\left(5 - \dfrac{x}{2} - \dfrac{\pi x}{8}\right) > 0$

$5 - \dfrac{x}{2} - \dfrac{\pi x}{8} > 0 \rightarrow \dfrac{x}{2} + \dfrac{\pi x}{8} < 5$

$\dfrac{4x + \pi x}{8} < 5$

$4x + \pi x < 40$

$x(4 + \pi) < 40$

$x(7{,}14) < 40$

$x \cong 5{,}6$

$\pi \approx 3{,}14$

$2{,}8 \mid 2$
$1{,}4 \mid 2$

- $f(x) = 5x - \dfrac{x^2}{2} - \dfrac{\pi x^2}{8}$
- $\mathrm{Dom}(f) =]0; 5{,}6[$

ou $\mathrm{Dom}(f) = \mathbb{R}_+ - \{0\}$

8) a) $f(x) = 4$

Para $f(-x) = 4$

Para $f(x) = 4$

Ou seja: Par

b) $f(x) = 3x^2 + 1$

- $f(x) = 3x^2 + 1$
- $f(-x) = 3 \cdot (-x)^2 + 1$
  $= 3 \cdot x^2 + 1$

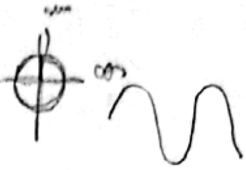
∴ É par

c) $f(x) = x^2 + x$

- $f(x) = x^2 + x$

$f(-x) = (-x)^2 + (-x)$
$= x^2 - x$

∴ Sem paridade



d) $g(x) = x^3 + x$

- $g(x) = x^3 + x$
- $g(-x) = (-x^3) + (-x)$
  $= -x^3 - x$
  $= -g(x)$

∴ Impar.

e) $g(x) = x^4 + 3x^2 - 1$

- $g(x) = x^4 + 3x^2 - 1$
- $g(-x) = (-x)^4 + 3 \cdot (-1)^2 - 1$
  $= x^4 + 3x^2 - 1$

∴ É par

f) $g(x) = \frac{x}{x^2 - 1}$

- $g(x) = \frac{x}{x^2-1}$
- $g(-x) = \frac{-x}{(-x)^2 - 1}$

$\rightarrow g(-x) = -\frac{x}{x^2-1}$
$= -g(x)$

∴ Impar

g) $h(t) = \frac{1}{t-1}$

- $h(t) = \frac{1}{t-1}$
- $h(-t) = \frac{1}{-t-1}$

∴ Sem paridade

h) $h(t) = 2t + 1$

- $h(t) = 2t + 1$
- $h(-t) = 2(-t) + 1$
  $= -2t + 1$

$h(t) \neq h(-t)$
$h(t) \neq -h(t)$

∴ Sem paridade.

i) $h(t) = 2|t| + 1$

- $h(t) = 2t + 1$
- $h(-t) = 2|-t| + 1$
  $= 2t + 1$

∴ É par

j) $f(x) = \text{sen}\, 2x$

- $f(x) = \text{sen}\, 2x$
- $f(-x) = \text{sen}(-2x)$
  $= -\text{sen}(2x)$

É impar.

k) $f(x) = \text{sen}\,(x^2)$

- $f(x) = \text{sen}\, x^2$
- $f(-x) = \text{sen}\,(-x)^2$
  $= \text{sen}\, x^2$

∴ É par

l) $f(x) = \cos 3x$

- $f(x) = \cos 3x$
- $f(x) = \cos -3x$
  $= \cos 3x$

É par

m) $f(x) = 1 + \text{sen}^3 x$

- $f(x) = 1 + \text{sen}^3 x$
- $f(-x) = 1 + \text{sen}^3 (-x)$

Sem paridade.

9) a) $h(x) = f(x) - f(-x)$ É impar.

Impar: $h(-x) = -h(x)$

$h(-x) = f(-x) - f(-(-x))$
$= f(-x) - f(x)$
$= -(f(x) - f(-x))$
$= -h(x)$

b) $g(x) = \frac{f(x) + f(-x)}{2}$ e $h(x) = \frac{f(x) - f(-x)}{2}$

$f(x) = g(x) + h(x) = \frac{f(x) + f(-x) + f(x) - f(-x)}{2}$

$= \frac{2f(x)}{2} = f(x)$

10) $f+g$, $f-g$, $f\cdot g$ e f/g e domínios

a) $f(x) = x^3 + 2x^2$ e $g(x) = 3x^2 - 1$

• $(f+g)(x) = f(x) + g(x)$
$= x^3 + 2x^2 + 3x^2 - 1$
$= x^3 + 5x^2 - 1$ → Lei de Assoc.

$Dom(f+g) = \mathbb{R}$.

• $(f-g)x = f(x) - g(x)$
$= x^3 + 2x^2 - (3x^2 - 1)$
$= x^3 - x^2 + 1$ → L.A

$Dom(f-g) = \mathbb{R}$

• $(f\cdot g)(x) = f(x)\cdot g(x)$
$= (x^3+2x^2)\cdot(3x^2-1)$
$= 3x^5 - x^3 + 6x^4 - 2x^2$
$= 3x^5 + 6x^4 - x^3 - 2x^2$ → L.A

$Dom(f g) = \mathbb{R}$

• $(f/g) = \dfrac{f(x)}{g(x)}$
$= \dfrac{x^3+2x^2}{3x^2-1}$ → LA

Restrição $3x^2 - 1 \neq 0$
$3x^2 \neq 1$
$x \neq \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} = \boxed{\pm\frac{\sqrt{3}}{3}}$

$Dom(f/g) = \mathbb{R} - \left\{\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right\}$

b) $f(x) = 2$, $g(x) = x^2 + 1$

• $(f+g)(x) = f(x) + g(x)$
$= 2 + x^2 + 1$
$= x^2 + 3$ → LA

$Dom(f+g) = \mathbb{R}$

• $(f-g)(x) = f(x) - g(x)$
$= 2 - x^2 - 1$
$= 1 - x^2$ → LA

$Dom(f-g) = \mathbb{R}$

• $(f\cdot g)(x) = f(x)\cdot g(x)$
$= 2\cdot(x^2+1)$
$= 2x^2 + 2$ → LA

$Dom(f g) = \mathbb{R}$.

• $(f/g)(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{2}{x^2+1}$ → LA

Restrição $x^2 + 1 \neq 0$
$x^2 \neq -1$
$x \neq \sqrt{-1}$
$\therefore \mathbb{C}$

$Dom(f/g) = \mathbb{R}$

c) $f(x) = \overset{\to \mathbb{R}}{1}$, $g(x) = \overset{\to \mathbb{R}+}{1 + \sqrt{x}}$

• $(f+g)(x) = f(x) + g(x)$
$= 1 + 1\sqrt{x}$
$= 2 + \sqrt{x}$ → LD

$Dom(f+g) = \mathbb{R}+$

• $(f-g)(x) = f(x) - g(x)$
$= 1 - (1 + \sqrt{x})$
$= -\sqrt{x}$ → LD

$Dom(f-g) = \mathbb{R}+$

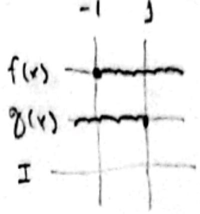
• $(f\cdot g)(x) = f(x)\cdot g(x)$
$= 1\cdot(1+\sqrt{x})$
$= 1 + \sqrt{x}$ → LD

$Dom(f g) = \mathbb{R}_+$

• $f/g = \dfrac{f(x)}{g(x)} = \dfrac{1}{1+\sqrt{x}}$ → LD

$Dom(f/g) = \mathbb{R}^+$

d) $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1-x}$



$f(x) \cap g(x) = [1;1]$

$Dom(f) = \{x \in \mathbb{R} \mid x \geq -1\}$ e $Dom(g) = \{x \in \mathbb{R} \mid x \leq 1\}$

$\cdot (f+g)(x) = f(x) + g(x)$

$= \sqrt{1+x} + \sqrt{1-x} \rightarrow LD$

$Dom(f+g) = \{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$

$\cdot (f-g)(x) = f(x) - g(x)$

$= \sqrt{1+x} - \sqrt{1-x}$

$Dom(f-g) = \{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$

$\cdot (f \cdot g)(x) = f(x) \cdot g(x)$

$= \sqrt{1+x} \cdot \sqrt{1-x}$

$= \sqrt{1-x^2} \rightarrow LD$

$Dom(fg) = \{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$

$\cdot f/g = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{1+x}}{\sqrt{1-x}} \rightarrow LD$

Rest: $1-x > 0$

$x < 1$

$Dom(f/g) = \{x \in \mathbb{R} \mid -1 \leq x < 1\}$

e) $f(x) = x$, $g(x) = 1/x$ $\rightarrow Dom(f) = \mathbb{R}$, $Dom(g) = \mathbb{R} - \{0\}$

$\cdot (f+g)(x) = f(x) + g(x)$

$= \dfrac{x}{1} + \dfrac{1}{x}$

$= \dfrac{x^2+1}{x} \rightarrow LD$

$Dom(f+g) = \mathbb{R} - \{0\}$

$\cdot (f-g)(x) = f(x) - g(x)$

$= x - \dfrac{1}{x}$

$= \dfrac{x^2-1}{x} \rightarrow LD$

$Dom = \mathbb{R} - \{0\}$

$\cdot (f \cdot g)(x) = f(x) \cdot g(x)$

$x \cdot \left(\dfrac{1}{x}\right)$

$Dom(f) = \mathbb{R} - \{0\}$

$\cdot f/g = \dfrac{f(x)}{g(x)} = \dfrac{x}{\frac{1}{x}} \rightarrow LD$

$Dom(f/g) = \mathbb{R} - \{0\}$

f) $f(x) = x$, $g(x) = \sqrt{x-1}$ $\rightarrow Dom(f) = \mathbb{R}$, $Dom(f) = \{x \in \mathbb{R} \mid x \geq 1\}$

$\cdot (f+g)(x) = x + \sqrt{x+1} \rightarrow LD$

$Dom(f+g) = \{x \in \mathbb{R} \mid x \geq 1\}$

$\cdot (f-g)(x) = x - \sqrt{x+1} \rightarrow LD$

$Dom(f-g) = \{x \in \mathbb{R} \mid x \geq 1\}$

$\cdot (f \cdot g)(x) = f(x) \cdot g(x) = x \cdot \sqrt{x+1} \rightarrow LD$

$Dom(fg) = \{x \in \mathbb{R} \mid x \geq 1\}$

$\cdot (f/g)(x) = \dfrac{x}{\sqrt{x-1}}$ $\rightarrow$ Rest: $x > 1$

$Dom(f/g) = \{x \in \mathbb{R} \mid x > 1\}$

11) $\dfrac{f(x)-3}{f(x)+3} = x \rightarrow f(x)-3 = x(f(x)+3)$

$f(x)-3 = xf(x)+3x \rightarrow f(x)-xf(x) = 3x+3 \rightarrow f(x).(1-x) = 3+3x$

$f(x) = \dfrac{3+3x}{1-x} \rightarrow LD$

Restrição: $1-x \neq 0$

$\boxed{x \neq 1}$

$\therefore Dom(f) = \mathbb{R} - \{1\}$