Lista 2 - Calculo I

1) a) $(1+i)-(2-3i) = (1-2)+(i+3i) = -1+4i$

$|z| = \sqrt{(-1)^2+(4)^2} = \sqrt{17}$ e $\bar{z} = -1-4i$

b) $\left(4-\frac{1}{2}i\right)-\left(9+\frac{5}{2}i\right) = (4-9)+\left(-\frac{1}{2}i-\frac{5}{2}i\right) = -5-3i$

$|z| = \sqrt{(-5)^2+(-3)^2} = \sqrt{34}$ e $\bar{z} = -5+3i$

c) $(4-7i)(1+3i) = 4+12i-7i-21i^{2^{-1}} = 4+21+5i = 25+5i$

$|z| = \sqrt{25^2+5^2} = \sqrt{125+25} = \sqrt{150} = 5\sqrt{6}$ e $\bar{z} = 25-5i$

d) $\frac{5-i}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{(15-20i-3i+4i^2)}{(9-12i+12i-16i^2)} = \frac{11}{25} - \frac{23}{25}i$

$|z| = \sqrt{\left(\frac{11}{25}\right)^2+\left(-\frac{23}{25}\right)^2} = \sqrt{\frac{121+529}{125}} = \frac{\sqrt{650}}{25} = \frac{10\sqrt{65}}{25} = \frac{2\sqrt{65}}{5}$

$\bar{z} = \frac{11}{25} + \frac{23}{25}i$

e) $\frac{3}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{12+9i}{16+9} = \frac{12}{25} + \frac{9}{25}i$

$|z| = \sqrt{\left(\frac{12}{25}\right)^2+\left(\frac{9}{25}\right)^2} = \sqrt{\frac{144+81}{125}} = \sqrt{\frac{225}{125}} = \frac{15}{25} = \frac{3}{5}$

$\bar{z} = \frac{12}{25} - \frac{9i}{25}$

2) a) $(3+4i)^2 - 2\bar{z} = z$: Seja $z = a+bi$

$\Rightarrow 9+24i+16i^2-2\bar{z} = z \Rightarrow 9-16+24i = z+2\bar{z}$

$\Rightarrow -7+24i = z+2\bar{z} \rightarrow -7+24i = a+bi+2a-2bi$

$\Rightarrow -7+24i = 3a-bi \Rightarrow \frac{7}{3} - 24i = a+bi$

$\therefore z = \frac{7}{3} - 24i$

b) $iz + 3\bar{z} = 5-2i$ Seja $z = a+bi$

$i(a+bi)+3(a+bi) = 5-2i = ai-b+3a+3bi = 5-2i$

$\begin{cases} 3a-b=5 \\ a+3b=-2 \end{cases}$

$a = -2-3b$

$-6-9b-b=5$

$-10b = 11$

$b = -\frac{11}{10}$

$a = -2-3\left(\frac{11}{10}\right)$

$a = \frac{-20}{10}+\frac{33}{10}$

$a = \frac{13}{10}$

$z = \frac{13}{10} - \frac{11}{10}i$

c) $\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{z} = 1+i$: Seja $z = a+bi$

$\Rightarrow \left(\frac{(1+i)^2}{(1-i)^2}\right) + \frac{1}{z} = 1+i \Rightarrow \left(\frac{1+2i-1}{1-2i-1}\right) + \frac{1}{z} = 1+i$

$\frac{2i}{-2i} + \frac{1}{z} = 1+i \Rightarrow -1 + \frac{1}{z} = 1+i \Rightarrow \frac{1}{z} = 2+i$

$\Rightarrow \frac{1}{2+i} = z \Rightarrow \frac{1 \cdot (2-i)}{(2+i)(2-i)} = \frac{2-i}{4+1} = \frac{2}{5} - \frac{1}{5}i = z$

d) $z^2 = 4+2i\sqrt{5}$ = Seja $z = (a+bi)$

$(a+bi)^2 = 4+2i\sqrt{5} \Rightarrow a^2+2abi-b^2 = 4+2i\sqrt{5}$

$\begin{cases} a^2-b^2=4 \\ 2ab = 2\sqrt{5} \end{cases}$ $b = \frac{\sqrt{5}}{a} \Rightarrow a^2 - \left(\frac{\sqrt{5}}{a}\right)^2 = 4 \Rightarrow a^2 - \frac{5}{a^2} = 4$

$\frac{a^4-5}{a^2} = 4 \Rightarrow a^4-4a^2-5=0$: Seja $a^2 = x$

$x^2-4x-5=0 \Rightarrow (x+1)\cdot(x-5)$ 0

$\therefore x_1 = -1$ e $x_2 = 5 \rightarrow a^2 = x \therefore$ $\cancel{a_1 = \pm\sqrt{-1}}$, $a_2 = \pm\sqrt{5}$

$b = \frac{\sqrt{5}}{a} \Rightarrow b_1 = \frac{\sqrt{5}}{-\sqrt{5}} = -1$ e $b_2 = \frac{\sqrt{5}}{\sqrt{5}} = 1 \therefore b = \pm 1$

$z = a+bi \Rightarrow z = \pm\sqrt{5} \pm i$

3) a) $9z^2+16=0$ $x = \frac{\pm\sqrt{-576}}{2\cdot 9} = \frac{\pm\sqrt{576}\cdot\sqrt{-1}}{18} = \frac{\pm 24i}{18}$ $\frac{\pm 4i}{3}$

$\Delta = -4\cdot 9\cdot 16$

$\Delta = -576$ $x_1 = \frac{4i}{3}$ $x_2 = \frac{-4i}{3}$

b) $z^4-1=0 \Rightarrow z^4 = 1 \Rightarrow z = \sqrt[4]{1}$

$x_1 = \sqrt[4]{1} = 1$ $x_2 = \sqrt[4]{1} = -1$

$x_3^2 = \sqrt{1} = -1$ $= x = \sqrt{-1} \Rightarrow \boxed{x_3 = i}$

$x_4^2 = \sqrt{1} = 1 \rightarrow (-1)\cdot(-1) \Rightarrow x_4^2 = \sqrt{(-1^2)} = \boxed{x_4 = -i}$

c) $2z^2-2z+1=0$ $\quad x=\frac{-(-2)\pm\sqrt{-4}}{2\cdot 2}=\frac{2\pm\sqrt{4}\sqrt{-1}}{4}=\frac{2\pm 2i}{4}$

$\Delta=(-2)^2-4\cdot 2\cdot 1$

$\Delta=4-8$ $\quad x_1=\frac{1}{2}+\frac{i}{2}$ / $x_2=\frac{1}{2}-\frac{i}{2}$

$\Delta=-4$

d) $z^2+z+2=0$ $\quad x=\frac{-1\pm\sqrt{-7}}{2}=\frac{-1\pm\sqrt{7}\sqrt{-1}}{2}=\frac{-1\pm\sqrt{7}i}{2}$

$\Delta=1-4\cdot 1\cdot 2$

$\Delta=-7$ $\quad x_1=\frac{-1}{2}+\frac{\sqrt{7}}{2}i$ $\quad x_2=\frac{-1}{2}-\frac{\sqrt{7}}{2}$

e) $z^4+3z^2+2=0$ seja $x=z^2$

$x^2+3x+2=0 \Rightarrow (x+1)\cdot(x+2) \Rightarrow x_1=-1$ e $x_2=-2$

$z^2=x \Rightarrow z_1^2=-1=\pm\sqrt{-1}=\boxed{z_1=i}$ $\boxed{z_3=-i}$

$z_2^2=-2=\pm\sqrt{-2}=\boxed{z_2=\sqrt{2}i}$ $\boxed{z_4=-\sqrt{2}i}$

f) $z^4-2z^2+4=0$ seja $x=z^2$

$x^2-2x+4=0$ $\quad x=\frac{2\pm\sqrt{-12}}{2}=\frac{\cancel{2}\pm\cancel{2}\sqrt{3}\sqrt{-1}}{\cancel{2}}=1\pm\sqrt{3}i$

$\Delta=4-4\cdot 1\cdot 4$

$\Delta=-12$ $\quad x_1=1+\sqrt{3}i$ $\quad x_2=1-\sqrt{3}i$

$z^2=1\pm\sqrt{3}i \Rightarrow z=\pm\sqrt{1\pm\sqrt{3}i}$

$z_1=\sqrt{1+\sqrt{3}i}$, $z_2=\sqrt{1-\sqrt{3}i}$, $z_3=-\sqrt{1+\sqrt{3}i}$, $z_4=-\sqrt{1-\sqrt{3}i}$.

4) Definimos $z=a+bi$, $a,b\in\mathbb{R}$ e $i^2=-1$

a) $z+\bar{z}=2Re(z)$

$(a+bi)+\overline{(a+bi)}=a+bi+a-bi=2a$, $a\in\mathbb{R}$

b) $z-\bar{z}=2Im$

$(a+bi)-(a-bi)=a+bi-a+bi=2bi$, $\therefore 2Im$

c) $|Re(z)|\leq|z| \Rightarrow |a|\leq|a+bi| \Rightarrow |a|\leq\sqrt{a^2+b^2}$

d) $|z_1+z_2|^2=|z_1|^2+|z_2|^2+2Re(z_1\bar{z}_2)$

$|a+bi+c+di|^2=|a+bi|^2+|c+di|^2+2R((a+bi)\cdot(c-di))$

$(\sqrt{(a+c)^2+(b+d)^2})^2=(\sqrt{a^2+b^2})^2+(\sqrt{c^2+d^2})^2+2R(ac-adi+cbi+bd)$

$(a+c)^2+(b+d)^2=a^2+b^2+c^2+d^2+2(ac+bd)$

$\Rightarrow (a+c)^2 + (b+d)^2 = a^2+b^2+c^2+d^2+2ac+2bd$

$a^2+2ac+c^2+b^2+2bd+d^2 = a^2+b^2+c^2+d^2+2ac+2bd$

$a^2+b^2+c^2+d^2+2ac+2bd = a^2+b^2+c^2+d^2+2ac+2bd$

$\therefore$ Verdadeiro

e) $|z_1+z_2| \leq |z_1|+|z_2|$

$|(a+bi)+(c+di)| \leq |(a+bi)|+|(c+di)|$

$\sqrt{(a+b)^2+(c+d)^2} \leq \sqrt{(a+b)^2}+\sqrt{(c+d)^2}$

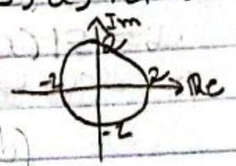
$\sqrt{a^2+b^2+2ab+c^2+d^2+2cd} \leq a+b+c+d$

Supondo $a, b, c, d = 1$

$\sqrt{1+1+2+1+1+2} \leq 1+1+1+1$

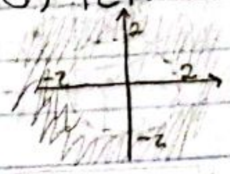
$\sqrt{8} \leq 4 \Rightarrow$ Verdade

5) a) $|z|=2$

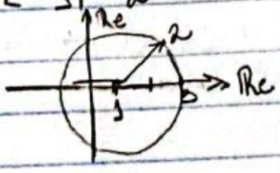


b) $|z|\leq 2$

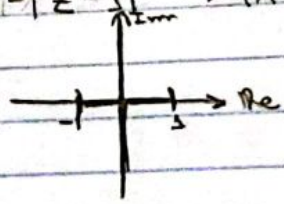
c) $|z|>2$



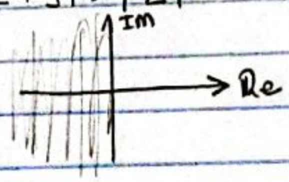
d $|z-1|=2 \to x-1 \quad y$



e) $|z+1|=|z-1| \to (x+1, yi) = (x-1, yi)$



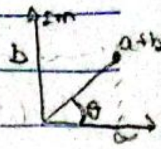
e) $|z+1| \geq |z| \to (x+1, yi) \geq x, yi$



6) $z = r(\cos\theta + i\,sen\,\theta) \Rightarrow \frac{1}{z} = \frac{1}{r}(\cos\theta - i\,sen\,\theta)$

$\frac{1}{z} = \frac{1}{r(\cos\theta + i\,sen\,\theta)} \cdot \frac{(\cos\theta - i\,sen\,\theta)}{(\cos\theta - i\,sen\,\theta)} = \frac{\cos\theta - i\,sen\,\theta}{r(\cos^2\theta + sen^2\theta)}$

$= \frac{\cos\theta - i\,sen\,\theta}{r \cdot 1} = \frac{1}{r} \cdot (\cos\theta - i\,sen\,\theta)$

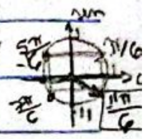


$sen\,\theta = \frac{CO}{h}$ $\cos = \frac{CA}{h}$

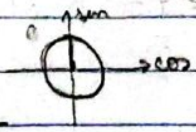
7) zw, z/w, $1/z$

a) $z = 2\sqrt{3} - 2i \Rightarrow r = \sqrt{(2\sqrt{3})^2 + (-2)^2} = \sqrt{(12+4)} = 4$

$\cos\theta_1 = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{6}$ $\quad sen\,\theta = \frac{-2}{4} = -\frac{1}{2} = -\frac{\pi}{6} \Rightarrow \theta = \frac{11\pi}{6}$



$w = 8i \Rightarrow r = \sqrt{8^2 + 0^2} = 8$



$\cos\theta_2 = \frac{0}{4} = 0$ $\quad sen\,\theta_2 = \frac{8}{8} = 1 \Rightarrow \frac{\pi}{2}$

$\therefore$ I) $zw = 4 \cdot 8 \left(\cos\left(\frac{11\pi}{6} + \frac{\pi}{2}\right) + i\,sen\left(\frac{11\pi}{6} + \frac{\pi}{2}\right)\right) =$

$= 32\left(\cos\left(\frac{11\pi}{6} + \frac{3\pi}{6}\right) + i\,sen\left(\frac{11\pi}{6} + \frac{3\pi}{6}\right)\right) = 32\left(\cos\left(\frac{7\pi}{3}\right) + i\,sen\left(\frac{7\pi}{3}\right)\right)$

II) $\frac{z}{w} = \frac{4}{8}\left[\cos\left(\frac{11\pi}{6} - \frac{\pi}{2}\right) + i\,sen\left(\frac{11\pi}{6} - \frac{\pi}{2}\right)\right] =$

$\frac{1}{2}\left[\cos\left(\frac{11\pi}{6} - \frac{3\pi}{6}\right) + i\,sen\left(\frac{11\pi}{6} - \frac{3\pi}{6}\right)\right] = \frac{1}{2}\left(\cos\frac{4\pi}{3} + i\,sen\,\frac{4\pi}{3}\right)$

III) $\frac{1}{z} = z^{-1} \Rightarrow \frac{1}{z} = \frac{1}{4}\left(\cos\frac{11\pi}{6} - i\,sen\,\frac{11\pi}{6}\right)$

b) • $z = \sqrt{3} + i \Rightarrow r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad sen\,\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{6}$

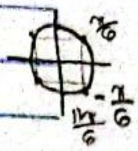
• $w = 1 + \sqrt{3}\,i \Rightarrow r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\cos\theta = \frac{1}{2}$ $\quad sen\,\theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{3}$

I) $z \cdot w = 2 \cdot 2\left[\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) + i\,sen\left(\frac{\pi}{6} + \frac{\pi}{3}\right)\right] \rightarrow \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6}$

$z \cdot w = 4\left(\cos\frac{\pi}{2} + i\,sen\,\frac{\pi}{2}\right)$

II) $\frac{z}{w} = \frac{2}{2} \cdot \left[\cos\left(\frac{\pi}{6} - \frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{6} - \frac{\pi}{3}\right)\right]$ $\quad \frac{\pi}{6} - \frac{2\pi}{6} = -\frac{\pi}{6}$



$\therefore \frac{z}{w} = \left(\cos\frac{11\pi}{6} + i \operatorname{sen}\frac{11\pi}{6}\right)$

III) $\frac{1}{z} = \frac{1}{2}\left(\cos\frac{\pi}{6} - i \operatorname{sen}\frac{\pi}{6}\right)$

c) • $z = 5\sqrt{3} + 5i \Rightarrow r = \sqrt{(5\sqrt{3})^2 + 5^2} = \sqrt{100} = 10$

$\cos\theta = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ $\quad \operatorname{sen}\theta = \frac{5}{10} = \frac{1}{2}$ $\quad \therefore \frac{\pi}{6}$

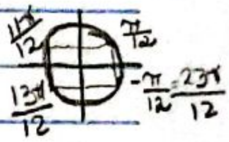
• $w = -3(1+i) \Rightarrow r = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\quad \frac{x \cdot 180}{12}$ 165

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \operatorname{sen}\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \frac{\pi}{4}$

I) $zw = 10\sqrt{2}\left[\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) + i \operatorname{sen}\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\right] \rightarrow \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$

$= 10\sqrt{2}\left(\cos\frac{5\pi}{12} + i \operatorname{sen}\frac{5\pi}{12}\right)$ $\quad 165$

II) $\frac{z}{w} = \frac{10}{\sqrt{2}}\left[\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) + i \operatorname{sen}\left(\frac{\pi}{6} - \frac{\pi}{4}\right)\right] = -\frac{\pi}{12}$



$= 5\sqrt{2}\left(\cos\frac{23\pi}{12} + i \operatorname{sen}\frac{23\pi}{12}\right)$

III) $\frac{1}{z} = \frac{1}{10}\left(\operatorname{sen}\frac{\pi}{6} - i \operatorname{sen}\frac{\pi}{6}\right)$

d) • $z = 4\sqrt{3} - 4i \Rightarrow r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \operatorname{sen} = \frac{-1}{2} = \frac{\pi}{6} \Rightarrow \frac{11\pi}{6}$

• $w = -1 + i \Rightarrow r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad \operatorname{sen}\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \frac{\pi}{4} \therefore \boxed{\frac{3\pi}{4}}$

I) $z \cdot w = 2\sqrt{2}\left[\cos\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right)\right]$ $\quad \frac{22\pi + 9\pi}{12} = \frac{31\pi}{12} \equiv \frac{7\pi}{12}$

$= 2\sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) + i \operatorname{sen}\left(\frac{7\pi}{12}\right)\right)$

II) $\frac{z}{w} = \frac{2}{\sqrt{2}}\left[\cos\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right)\right]$ $\quad \frac{22\pi - 9\pi}{12} = \frac{13\pi}{12}$

$= \sqrt{2}\left(\cos\frac{13\pi}{12} + i \operatorname{sen}\frac{13\pi}{12}\right)$

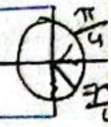
III) $\frac{1}{z} = \frac{1}{2}\left(\cos\frac{11\pi}{6} - i \operatorname{sen}\frac{11\pi}{6}\right)$

8) a) $(1-\sqrt{3}\,i)^6 \Rightarrow r=\sqrt{1^2+(-\sqrt{3})^2} = 2$

• $\cos\theta = \frac{1}{2}$ $\quad \text{sen}\,\theta = -\frac{\sqrt{3}}{2}$ $\quad ∴ \frac{\pi}{3} = \frac{5\pi}{3}$

$z^6 = 2^6\left(\cos(6\cdot\frac{5\pi}{3}) + i\,\text{sen}(\cancel{6}\cdot\frac{5\pi}{\cancel{3}})\right) \to \boxed{2\pi}$

$z^6 = 64(\cos 2\pi + i\,\text{sen}\,2\pi) = 64$



b) $(1-i)^8 \Rightarrow r=\sqrt{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\text{sen}\,\theta = -\frac{\sqrt{2}}{2}$

∴ $\theta = \frac{7\pi}{4}$

$z^8 = (\sqrt{2})^{\cancel{8}\,4}\cdot\left(\cos(\cancel{8}\cdot\frac{7\pi}{\cancel{4}}) + i\,\text{sen}(\cancel{8}\cdot\frac{7\pi}{\cancel{4}})\right) \to 14\pi \to \boxed{2\pi}$

$z^8 = 16(\cos 2\pi + i\,\text{sen}\,2\pi) = 16$

c) $(2\sqrt{3}+2i)^7 \Rightarrow r = \sqrt{(\sqrt{3})^2+(1^2)} = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \text{sen}\,\theta = \frac{1}{2}$ $\quad ∴ \frac{\pi}{6} = \theta$

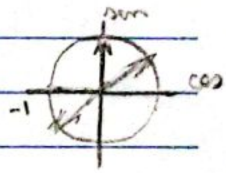
$z^7 = 2^7\left[\cos(7\cdot\frac{\pi}{6}) + i\,\text{sen}(7\frac{\pi}{6})\right] = 128\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

$z^7 = 64\sqrt{3} + 64i = 64(\sqrt{3}+i)$

d) $(1+i)^{40} \Rightarrow r = \sqrt{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\text{sen}\,\theta = \frac{\sqrt{2}}{2}$ $\quad ∴ \theta = \frac{\pi}{4}$

$z^{40} = \sqrt{2}^{40}\cdot\left(\cos(\cancel{40}\cdot\frac{10\pi}{\cancel{4}}) + i\,\text{sen}(\cancel{40}\cdot\frac{10\pi}{\cancel{4}})\right) \to 10\pi = 2\pi$

∴ $= 2^{20}\cdot(\cos 2\pi + i\,\cancel{\text{sen}\,2\pi}) \to \boxed{2^{20}}$



9) a) raízes de i → $r = \sqrt{1} = 1$

• $\cos\theta = \frac{0}{1} = 0$ $\quad$ • $\text{sen}\,\theta = \frac{1}{1} = 1$ $\quad ∴ \theta = \frac{\pi}{2}$

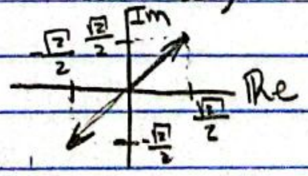
$z = \cos\frac{\pi}{2} + \text{sen}\,\frac{\pi}{2}$

$w_0 = 1^{\frac{1}{2}}\left[\cos\left(\frac{\frac{\pi}{2}+2\pi 0}{2}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{2}+2\pi 0}{2}\right)\right] = \frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$ $\quad 9\pi$

$w_0 = \left(\cos\frac{\pi}{4} + i\,\text{sen}\,\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

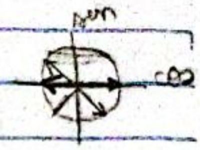
$w_1 = 1\left[\cos\left(\frac{\frac{\pi}{2}+2\pi 1}{2}\right) + i\,\text{sen}\left(\frac{\frac{\pi}{2}+2\pi 1}{2}\right)\right] = \frac{\frac{\pi}{2}+\frac{4\pi}{2}}{2} = \frac{5\pi}{2}\cdot\frac{1}{2} = \frac{5\pi}{4}$

$w_1 = \left(\cos\frac{5\pi}{2} + i\,\text{sen}\,\frac{5\pi}{2}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$



D S T Q Q S S
D L M M J V S

b) Raízes cúbicas de 1. $\rightarrow r = \sqrt{1^2} = 1$



• $\cos\theta = \frac{1}{1} = 1$ • $sen\theta = \frac{0}{1} = 0$ $\therefore 0$

$\therefore z = 1 \cdot (\cos 0 + sen 0)$

$n = 3$, $k = 0, 1, 2$

$\rightarrow w_0 = 1\left[\cos\left(\frac{0+0 \cdot 2\pi}{3}\right) + i\, sen\left(\frac{0+2\pi \cdot 0}{3}\right)\right] \rightarrow 0$

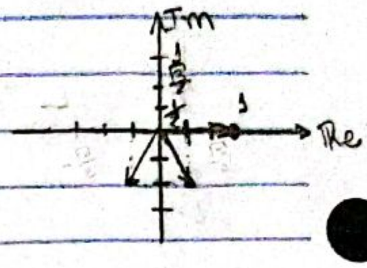
$= \cos 0 + i\, sen\, 0 = 1 + 0 = \boxed{1}$

$\rightarrow w_1 = 1\left[\cos\left(\frac{0+2\pi \cdot 1}{3}\right) + i\, sen\left(\frac{0+2\pi \cdot 1}{3}\right)\right] \rightarrow \frac{2\pi}{3}$

$= \cos\frac{2\pi}{3} + i\, sen\frac{2\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\rightarrow w_2 = 1\left[\cos\left(\frac{0+2\pi \cdot 2}{3}\right) + i\, sen\left(\frac{0+2\pi \cdot 2}{3}\right)\right] \rightarrow \frac{4\pi}{3}$

$= \cos\frac{4\pi}{3} + i\, sen\frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$



c) raízes 4ª de $1+i$ $\rightarrow r = \sqrt{1^2+1^2} = \sqrt{2}$

• $\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ • $Sen = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\rightarrow \theta = \frac{\pi}{4}$

$\therefore z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\, sen\frac{\pi}{4}\right)$

$n = 4$, w_0, w_1, w_2, w_3

$\rightarrow w_0 = (\sqrt{2})^{\frac{1}{4}}\left[\cos\left(\frac{\frac{\pi}{4}+0\pi 2}{4}\right) + i\, sen\left(\frac{\frac{\pi}{4}+0\pi 2}{4}\right)\right] \rightarrow \frac{\pi}{4} \cdot \frac{1}{4} = \frac{\pi}{16}$

$w_0 = \sqrt[8]{2}\left(\cos\frac{\pi}{16} + i\, sen\frac{\pi}{16}\right)$

$\rightarrow w_1 = (\sqrt{2})^{\frac{1}{4}}\left[\cos\left(\frac{\frac{\pi}{4}+1\pi 2}{4}\right) + i\, sen\left(\frac{\frac{\pi}{4}+1\pi 2}{4}\right)\right] \rightarrow \left(\frac{\pi}{4} + \frac{8\pi}{4}\right)\frac{1}{4} = \frac{9\pi}{16}$

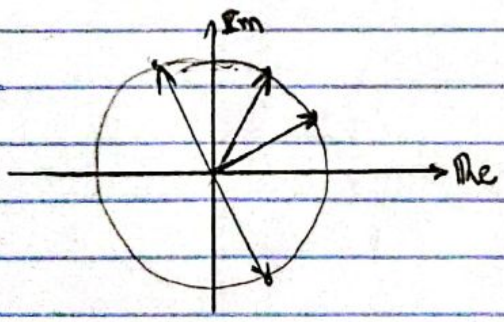
$w_1 = \sqrt[8]{2}\left(\cos\frac{9\pi}{16} + i\, sen\frac{9\pi}{16}\right)$

$\rightarrow w_2 = \sqrt[8]{2}\left[\cos\left(\frac{\frac{\pi}{4}+2\pi 2}{4}\right) + i\, sen\left(\frac{\frac{\pi}{4}+2\pi 2}{4}\right)\right] \rightarrow \left(\frac{\pi}{4} + \frac{16\pi}{4}\right) = \frac{17\pi}{16}$

$w_2 = \sqrt[8]{2}\left(\cos\frac{17\pi}{16} + i\, sen\frac{17\pi}{16}\right)$

$\rightarrow w_3 = \sqrt[8]{2}\left[\cos\left(\frac{\frac{\pi}{4}+3\pi 2}{4}\right) + i\, sen\left(\frac{\frac{\pi}{4}+6\pi 2}{4}\right)\right] \rightarrow \left(\frac{\pi}{4} + \frac{24\pi}{4}\right)\frac{1}{4} = \frac{25\pi}{4}$

$w_3 = \sqrt[8]{2}\left(\cos\frac{25\pi}{4} + i\, sen\frac{25\pi}{4}\right)$



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d) raízes cúbicas de $-8i$ → $r = \sqrt{(-8)^2} = 8$

• $\cos\theta = \frac{0}{8} = 0$ • $\text{sen}\,\theta = \frac{-8}{8} = -1$ $\theta = \frac{3\pi}{2}$

$n = 3$, w_0, w_1, w_2

→ $w_0 = \sqrt[3]{8}\left[\cos\left(\frac{\frac{3\pi}{2} + 0\pi 2}{3}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2} + 0\pi 2}{3}\right)\right] \cdot \frac{3\pi}{2} \cdot \frac{1}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$

$= 2\left(\cos\frac{\pi}{2} + i\,\text{sen}\frac{\pi}{2}\right) = 1 \cdot 2 = 2$

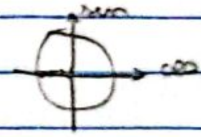
→ $w_1 = \sqrt[3]{8}\left[\cos\left(\frac{\frac{3\pi}{2} + 1\pi 2}{3}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2} + 1\pi 2}{3}\right)\right] \to \left(\frac{3\pi}{2} + \frac{4\pi}{2}\right)\frac{1}{3} = \frac{7\pi}{6}$

$= 2\left(\cos\frac{7\pi}{6} + i\,\text{sen}\frac{7\pi}{6}\right) = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \cdot 2 = -1 + \sqrt{3}\,i$

→ $w_2 = \sqrt[3]{8}\left[\cos\left(\frac{\frac{3\pi}{2} + 2\pi 2}{3}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2} + 2\pi 2}{3}\right)\right] \to \left(\frac{3\pi}{2} + \frac{8\pi}{2}\right)\frac{1}{3} = \frac{11\pi}{6}$

$= 2\left(\cos\frac{11\pi}{6} + i\,\text{sen}\frac{11\pi}{6}\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}\,i$

e) raízes 5^a de -32: $r = \sqrt{(32)^2} = 32$



• $\cos\theta = \frac{-32}{32} = -1$ • $\text{Sen}\,\theta = \frac{0}{32} = 0$ $\theta = \pi$

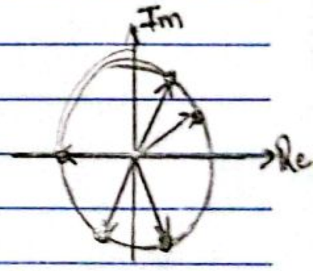
$n = 5$, w_0, w_1, w_2, w_3, w_4

→ $w_0 = \sqrt[5]{32}\left[\cos\left(\frac{\pi + 0\pi 2}{5}\right) + i\,\text{sen}\left(\frac{\pi + 0\pi 2}{5}\right)\right] \to \frac{\pi}{5}$

$= 2\left(\cos\frac{\pi}{5} + i\,\text{sen}\frac{\pi}{5}\right)$

→ $w_1 = \sqrt[5]{32}\left[\cos\left(\frac{\pi + 1\pi 2}{5}\right) + i\,\text{sen}\left(\frac{\pi + 1\pi 2}{5}\right)\right] \to \frac{3\pi}{5}$

$= 2\left(\cos\frac{3\pi}{5} + i\,\text{sen}\frac{3\pi}{5}\right)$



→ $w_2 = \sqrt[5]{32}\left[\cos\left(\frac{\pi + 2\pi 2}{5}\right) + i\,\text{sen}\left(\frac{\pi + 2\pi 2}{5}\right)\right] \to \frac{5\pi}{5} = \pi$

$= 2(\cos\pi + i\,\text{sen}\,\pi) = 2 \cdot (-1) = -2$

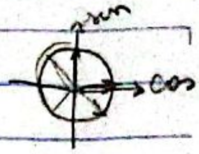
→ $w_3 = \sqrt[5]{32}\left[\cos\left(\frac{\pi + 3\pi 2}{5}\right) + i\,\text{sen}\left(\frac{\pi + 3\pi 2}{5}\right)\right] \to \frac{7\pi}{5}$

$= 2\left(\cos\frac{7\pi}{5} + i\,\text{sen}\frac{7\pi}{5}\right)$

→ $w_4 = \sqrt[5]{32}\left[\cos\left(\frac{\pi + 4\pi 2}{5}\right) + i\,\text{sen}\left(\frac{\pi + 4\pi 2}{5}\right)\right] \to \frac{9\pi}{5}$

$= 2\left(\cos\frac{9\pi}{5} + i\,\text{sen}\frac{9\pi}{5}\right)$

f) raízes 6^a de 64 $\rightarrow r = 64$



$\bullet \cos\theta = \frac{64}{64} = 1 \qquad \bullet \operatorname{sen}\theta = \frac{0}{64} = 0 \qquad \theta = 0$

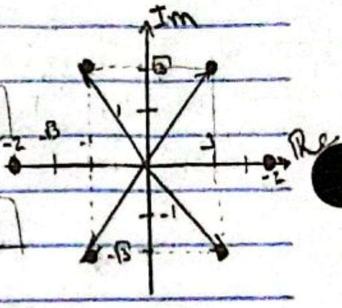
$n = 6$, $w_0 \rightarrow w_5$

$\rightarrow W_0 = \sqrt[6]{64}\left[\cos\left(\frac{0+0\pi 2}{6}\right) + i \cdot \operatorname{sen}\left(\frac{0+0\pi 2}{6}\right)\right] \rightarrow \frac{\pi}{2} \cdot \frac{1}{6} = \frac{\pi}{12}$

$= 2(\cos 0 + i \operatorname{sen} 0) = 1 \cdot 2 = \boxed{2}$

$\rightarrow W_1 = \sqrt[6]{64}\left[\cos\left(\frac{0+1\pi 2}{6}\right) + i \operatorname{sen}\left(\frac{0+1\pi 2}{6}\right)\right] \rightarrow \frac{\pi}{3}$

$2\left(\cos\frac{\pi}{3} + i \operatorname{sen}\frac{\pi}{3}\right) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)2 = \boxed{1 + \sqrt{3}\, i}$



$\rightarrow W_2 = \sqrt[6]{64}\left[\cos\left(\frac{0+2\pi 2}{6}\right) + i \operatorname{sen}\left(\frac{0+2\pi 2}{6}\right)\right] \rightarrow \frac{2\pi}{3}$

$2\left(\cos\frac{2\pi}{3} + i \operatorname{sen}\frac{2\pi}{3}\right) = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) \cdot 2 = \boxed{-1 + \sqrt{3}\, i}$

$\rightarrow W_3 = \sqrt[6]{64}\left[\cos\left(\frac{0+3\pi 2}{6}\right) + i \operatorname{sen}\left(\frac{0+3\pi 2}{6}\right)\right] \rightarrow \pi$

$= 2(\cos\pi + i \operatorname{sen}\pi) = -1 \cdot 2 = \boxed{-2}$

$\rightarrow W_4 = \sqrt[6]{64}\left[\cos\left(\frac{0+4\pi 2}{6}\right) + i \operatorname{sen}\left(\frac{0+4\pi 2}{6}\right)\right] \rightarrow \frac{4\pi}{3}$

$= 2\left(\cos\frac{4\pi}{3} + i \operatorname{sen}\frac{4\pi}{3}\right) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \boxed{-1 - \sqrt{3}\, i}$

$\rightarrow W_5 = \sqrt[6]{64}\left[\cos\left(\frac{0+5\pi 2}{6}\right) + i \operatorname{sen}\left(\frac{0+5\pi 2}{6}\right)\right] \rightarrow \frac{11\pi}{6}$

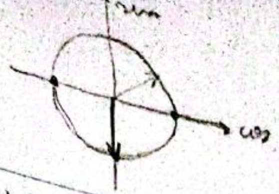
$= 2\left(\cos\frac{11\pi}{6} + i \operatorname{sen}\frac{11\pi}{6}\right) \rightarrow \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)2 = \boxed{1 - \sqrt{3}\, i}$

b) $e^{ix} = \cos x + i \operatorname{sen} x$

$\cancel{1} + ix + \frac{(ix)^2}{2} + \frac{(ix)^3}{6} = \cancel{1} - \frac{x^2}{2} + i\left(x - \frac{x^3}{6}\right)$

$ix + \frac{x^2}{2} - \frac{ix^3}{6} = -\frac{x^2}{2} + ix - \frac{ix^3}{6} \quad = ix - ix - \frac{x^2}{2} + \frac{x^2}{2} - \frac{ix^3}{6} + \frac{ix^3}{6} = 0$

$\rightarrow 0 = 0 \quad \therefore$ verdadeiro



11) a) $e^{-i\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + i\,sen(-\frac{\pi}{2}) = 0 - 1i = -1i$

b) $e^{2\pi i} = \cos 2\pi + i\,sen\,2\pi = 1 + 0 = 1$

c) $e^{i\frac{\pi}{3}} = \cos\frac{\pi}{3} + i\,sen\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

d) $e^{-i\pi} = \cos(-\pi) + i\,sen(-\pi) = -1$

e) $e^{2+i\pi} = e^2 \cdot e^{i\pi} = e^2(\cos\pi + i\,sen\,\pi) = e^2 + 0i = e^2$

f) $e^{\pi+i} = e^i \cdot e^\pi = e^\pi(\cos 1 + i\,sen\,1)$

12) I) $\cos x = \frac{e^{ix} + e^{-ix}}{2}$, $sen\,x = \frac{e^{ix} - e^{ix}}{2}$

• $\cos x = \frac{\cos x + i\,sen\,x + \cos(-x) + i\,sen(-x)}{2}$

$= \frac{2\cos x + i\,sen\,x - i\,sen\,x}{2} = \frac{2\cos x}{2}$ ∴ $\cos x = \cos x$

• $sen\,x = \frac{\cos x + i\,sen\,x - (\cos(-x) + i\,sen(-x))}{2i} =$

$= \frac{\cos x + i\,sen\,x - \cos x + i\,sen\,x}{2} = \frac{\cancel{2}\,sen\,x}{i\cancel{2}} = sen\,x$

∴ $Sen\,x = sen\,x$

→ CAOS
*(JURO QUE NÃO FOI PROPOSITAL)

II) $sen^2 x + \cos^2 x = 1$

$\left(\frac{e^{ix} - e^{-ix}}{2}\right)^2 + \left(\frac{e^{ix} + e^{-ix}}{2}\right)^2 = 1 \Rightarrow \frac{(e^{ix} - e^{-ix})^2}{4} + \frac{(e^{ix} + e^{-ix})}{4} = 1$

$\Rightarrow \frac{e^{2ix} - 2e^{ix}e^{-ix} + 2^{-2ix}}{-4} + \frac{e^{2ix} + 2e^{ix}e^{-ix} + 2^{-2ix}}{4} = 1$

$e^{ix}\cdot\frac{1}{e^{ix}} = 1$

$\Rightarrow \frac{-\cancel{e^{2ix}} + 2e^{ix}e^{-ix} - \cancel{2^{-2ix}} + \cancel{e^{2ix}} + 2e^{ix}e^{-ix} + \cancel{2^{-2ix}}}{4} = 1 \Rightarrow \frac{4e^{ix}e^{-ix}}{4} = 1$

$\Rightarrow \frac{4}{4} = 1 \rightarrow 1 = 1$ ∴ $sen^2 + \cos^2 = 1$ é verdadeiro