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Lista ~~8~~ 7 - Cálculo I

1) a) $y = 2x - 3x^2$, $P = (2, -8)$ $\qquad \bullet\ y - f(2) = f'(2)\cdot(x-2)$

• Coef. Ang: $f'(2) = \lim\limits_{\Delta x \to 0} \dfrac{f(2+\Delta x) - f(2)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{[2\cdot(2+\Delta x) - 3(2+\Delta x)^2] - (-8)}{\Delta x}$

$= \dfrac{\cancel{4} + 2\Delta x - \cancel{12} - 12\Delta x - 3(\Delta x)^2 + \cancel{8}}{\Delta x} = \dfrac{\cancel{\Delta x}(-10 - 3\Delta x)}{\cancel{\Delta x}} = \lim\limits_{\Delta x \to 0}(-10 - 3\Delta x) =$

$= -10 - 3\cdot 0 = \boxed{-10}$ $\qquad \therefore\ y + 8 = -10(x-2) \rightarrow$ Eq. da Reta

b) $y = x^3 - 3x + 1$, $P = (2, 3)$

• Coef Ang: $f'(2) = \lim\limits_{\Delta x \to 0} \dfrac{f(2+\Delta x) - f(2)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(2+\Delta x)^3 - 3(2+\Delta x) + 1 - 3}{\Delta x}$

$\lim\limits_{\Delta x \to 0} \dfrac{(\Delta x)^3 + 6(\Delta x)^2 + 12\Delta x + 9 - 6 - 3\Delta x - 3}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{(\Delta x)^3 + 6(\Delta x)^2 + 9\Delta x}{\Delta x} =$

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta x((\Delta x)^2 + 6\Delta x + 9)}{\cancel{\Delta x}} = \lim\limits_{\Delta x \to 0} (\cancel{(\Delta x)^2} + \cancel{6\Delta x} + 9) = 9$

$\therefore\ y - 3 = 9(x-2) \rightarrow$ Eq. geral.

c) $y = 2\sqrt{x}$, $P = (1, 2)$

• Coef Ang: $f'(1) = \lim\limits_{\Delta x \to 0} \dfrac{f(1+\Delta x) - f(1)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{2\sqrt{1+\Delta x} - 2}{\Delta x} =$

$\lim\limits_{\Delta x \to 0} \dfrac{2\sqrt{1+\Delta x} - 2}{\Delta x} \cdot \dfrac{2\sqrt{1+\Delta x} + 2}{2\sqrt{1+\Delta x} + 2} = \lim\limits_{\Delta x \to 0} \dfrac{\cancel{4} + 4\Delta x - \cancel{4}}{\Delta x(2\sqrt{1+\Delta x} + 2)} = \lim\limits_{\Delta x \to 0} \dfrac{4\cancel{\Delta x}}{\cancel{\Delta x}(2\sqrt{1+\Delta x} + 2)}$

$= \lim\limits_{\Delta x \to 0} \dfrac{4}{2\sqrt{1+\Delta x} + 2} = \dfrac{4}{2\sqrt{1} + 2} = \dfrac{4}{4} = 1$ $\qquad \therefore\ y - 2 = 1(x - 1)$

d) $f(x) = \dfrac{1}{x^2}$, $P = \left(-2, \dfrac{1}{4}\right)$

• Coef. Ang: $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(-2+\Delta x) - f(-2)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{\frac{1}{(-2+\Delta x)^2} - \frac{1}{4}}{\Delta x} =$

$= \lim\limits_{\Delta x \to 0} \dfrac{\frac{-(\Delta x)^2 + 4\Delta x}{4(-2+\Delta x)^2}}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{-(\Delta x)^2 + 4\Delta x}{4(-2+\Delta x)^2} \cdot \dfrac{1}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{-\Delta x + 4}{4(-2+\cancel{\Delta x})^2} =$

$\dfrac{4}{16} = \dfrac{1}{4}$ $\qquad \therefore\ y - \dfrac{1}{4} = \dfrac{1}{4}(x + 2)$

2) a) $f(x) = 3x^2 - 4x + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 - 4(x+\Delta x) + 1 - 3x^2 + 4x - 1}{\Delta x}$$

$$= \lim \frac{\cancel{3x^2} + 6x\Delta x + 3(\Delta x)^2 - \cancel{4x} - 4\Delta x - \cancel{3x^2} + \cancel{4x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(6x + 3\Delta x - 4)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (6x + \cancel{3\Delta x} - 4) = 6x - 4$$

$Dom(f(x)) = \mathbb{R}$ e $Dom(f'(x)) = \mathbb{R}$

b) $f(x) = \dfrac{2x+1}{x+3} \Rightarrow f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x} =$

$$\lim_{\Delta x \to 0} \frac{\frac{2(x+\Delta x)+1}{(x+\Delta x)+3} - \frac{2x+1}{x+3}}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{2x + 2\Delta x + 1}{x + \Delta x + 3} - \frac{2x+1}{x+3} \right) \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \left(\frac{(2x + 2\Delta x + 1)(x+3) - (2x+1)(x + \Delta x + 3)}{(x + \Delta x + 3)(x+3)} \right) \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \left(\frac{\cancel{2x^2} + \cancel{2x\Delta x} + \cancel{x} + \cancel{6x} + 6\Delta x + \cancel{3} - \cancel{2x^2} - \cancel{2x\Delta x} - \cancel{6x} - \cancel{x} - \Delta x - \cancel{3}}{x^2 + x\Delta x + 3x + 3x + 3\Delta x + 9} \right) \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{5\cancel{\Delta x}}{x^2 + 6x + 4x\Delta x + 9} \frac{1}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{5}{x^2 + 6x + \cancel{4x\Delta x} + 9} = \frac{5}{x^2 + 6x + 9}$$

$$= \frac{1}{(x+3)^2}$$

$Dom(f(x)) = \mathbb{R} - \{-3\}$ e $Dom(f'(x)) = \mathbb{R} - \{-3\}$

c) $f(x) = \sqrt{1-2x}^{\frac{1}{2}} \Rightarrow f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{\sqrt{1 - 2(x+\Delta x)} - \sqrt{1-2x}}{\Delta x} \cdot \frac{\sqrt{1-2(x+\Delta x)} + \sqrt{1-2x}}{\sqrt{1-2(x+\Delta x)} + \sqrt{1-2x}} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{1} - \cancel{2x} - 2\Delta x - \cancel{1} + \cancel{2x}}{\sqrt{1-2x-2\Delta x} + \sqrt{1-2x}} = \lim_{\Delta x \to 0} \frac{-2\cancel{\Delta x}}{\sqrt{1-2x-2\cancel{\Delta x}} + \sqrt{1-2x}} = \frac{-\cancel{2}}{\cancel{2}\sqrt{1-2x}}$$

$$= \frac{-1}{\sqrt{1-2x}}$$

$Dom(f(x)): \{x \in \mathbb{R} \mid x \leq \frac{1}{2}\}$

$Dom(f(x)) = \{x \in \mathbb{R} \mid x < \frac{1}{2}\}$

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3) $S = S(t)$, $v(t) = S'(t)$, onde $t = 2$

a) $S(t) = t^2 - 6t - 5$, $S'(2) = \dfrac{f(2+\Delta x) - f(2)}{\Delta x}$

$4 - 12 - 5$

$$\lim_{\Delta x \to 0} \frac{(2+\Delta x)^2 - 6(2+\Delta x) - 5 - 13}{\Delta x} = \lim_{\Delta x \to 0} \frac{4 + 4\Delta x + (\Delta x)^2 - \cancel{12} - 6\Delta x - \cancel{5} + 13}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x(-2 + \Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (-2 + \cancel{\Delta x}) = -2 \text{ m/s}$$

b) $S(t) = \dfrac{1}{t} - t$, $S'(2) = \dfrac{f(2+\Delta x) - f(2)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{\frac{1}{2+\Delta x} - 2 + \Delta x + \frac{3}{2}}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{2 - (2+\Delta x)(4+2\Delta x) + 6 + 3\Delta x}{4 + 2\Delta x} \right) \cdot \frac{1}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \left(\frac{\cancel{2} - \cancel{8} - 8\Delta x - 2(\Delta x)^2 + \cancel{6} + 3\Delta x}{2(2+\Delta x)} \right) \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\cancel{\Delta x}(-2\Delta x - 5)}{2(2+\Delta x)} \right) \cdot \frac{1}{\cancel{\Delta x}} =$$

$$\lim_{\Delta x \to 0} \left(\frac{-2\cancel{\Delta x} - 5}{4 + 2\cancel{\Delta x}} \right) = \frac{-5}{4} \text{ m/s}$$

c) $S(t) = \dfrac{t+1}{t-1}$, $S'(2) = \lim\limits_{\Delta x \to 0} \dfrac{f(2+\Delta x) - f(2)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{\frac{2+\Delta x+1}{2+\Delta x-1} - 3}{\Delta x} = \left(\frac{3+\Delta x}{1+\Delta x} - \frac{3}{1} \right) \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\cancel{3} + \Delta x - \cancel{3} - 3\Delta x}{1 + \Delta x} \right) \frac{1}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{-2\cancel{\Delta x}}{\cancel{\Delta x}(1+\Delta x)} = \lim_{\Delta x \to 0} \frac{-2}{1 + \cancel{\Delta x}} = -2 \text{ m/s}$$

4) a) $f(x) = \begin{cases} y = x^2 \\ y = x \end{cases}$

• $f'_-(x) = \lim\limits_{\Delta x \to 0^-} \dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \lim\limits_{\Delta x \to 0^-} \dfrac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim\limits_{\Delta x \to 0^-} \dfrac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{x^2}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0^-} \dfrac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = \lim\limits_{\Delta x \to 0^-} (2x + \cancel{\Delta x}) = 2x = 0$

• $f'_+(x) = \lim\limits_{\Delta x \to 0^+} \dfrac{f(\Delta x + x) - f(x)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \dfrac{\Delta x + \cancel{x} - \cancel{x}}{\Delta x} = \lim\limits_{\Delta x} \dfrac{\cancel{\Delta x}}{\cancel{\Delta x}} = \boxed{1}$

$\therefore f'_-(0) \neq f'_+(0)$, $\nexists f'(0)$

b) $f(x) \begin{cases} y = 2 \\ y = 2x \end{cases}$ $\quad P = (1, 2)$

• $f'_-(x) = \lim\limits_{\Delta x \to 0^-} \dfrac{f(\Delta x + x) - f(x)}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{2 - 2}{\Delta x} = 0$

• $f'_+(x) = \lim\limits_{\Delta x \to 0^+} \dfrac{f(\Delta x + x) - f(x)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \dfrac{2(\Delta x + x) - 2x}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \dfrac{2\Delta x + \cancel{2x} - \cancel{2x}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0^+} \dfrac{2\cancel{\Delta x}}{\cancel{\Delta x}} = 2$

$\therefore f'_-(1) \neq f'_+(1)$, $\nexists f'(1)$.

4) $f(x) = |3x - 6|$ $\qquad f(x) = \begin{cases} 3x - 6, & x \leq 0 \\ -3x + 6, & x \geq 0 \end{cases}$

a) $x = 2$

• $f'_-(2) = \lim\limits_{\Delta x \to 0^-} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim\limits_{\Delta x \to 0^-} \dfrac{-3(2 + \Delta x) + 6 - (-3 \cdot 2 + 6)}{\Delta x}$

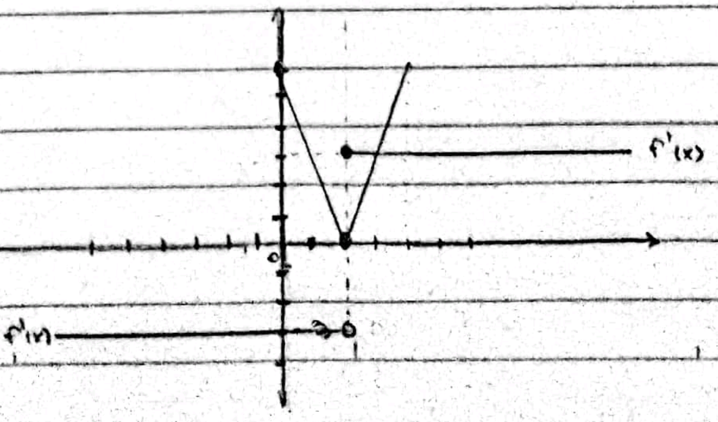
$\lim\limits_{\Delta x \to 0^-} \dfrac{-3\Delta x - 6 + 6 \cancel{+ 6 - 6}}{\Delta x} = \lim\limits_{\Delta x \to 0^-} \dfrac{-3\cancel{\Delta x}}{\cancel{\Delta x}} = \boxed{-3}$

• $f'_+(2) = \lim\limits_{\Delta x \to 0^+} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \dfrac{3 \cdot (2 + \Delta x) - 6 - (3 \cdot 2 - 6)}{\Delta x} =$

$\lim\limits_{\Delta x \to 0^+} \dfrac{\cancel{6} + 3\Delta x - \cancel{6 - 6 + 6}}{\Delta x} = \lim\limits_{\Delta x \to 0^+} \dfrac{3\cancel{\Delta x}}{\cancel{\Delta x}} = \boxed{3}$

$\therefore$ Como as derivadas laterais $f'_+(2) \neq f'_-(2)$, $\nexists f'(2)$.

b) Para $f'(x) = \begin{cases} -3, & x < 2 \\ 3, & x \geq 2 \end{cases}$



6) Em $f'(0)$

a) $\begin{cases} \frac{2\operatorname{sen} x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

- $f'(0) = \lim_{\Delta x \to 0} \frac{f(\Delta x + x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{2\operatorname{sen}\Delta x}{\Delta x} - 2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2\left(\frac{\operatorname{sen}\Delta x}{\Delta x} - 1\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 \cdot \frac{\operatorname{sen}\Delta x - \Delta x}{\Delta x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\operatorname{sen}\Delta x - \Delta x}{(\Delta x)^2} =$

b) $\begin{cases} 2x - 1, & x \geq 0 \\ x^2 + 2x + 7, & x < 0 \end{cases}$

- $f'_+(0) = \lim_{\Delta x \to 0^+} \frac{f(\Delta x + 0) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\Delta x - \cancel{1} \cancel{+1}}{\Delta x} = \lim_{\Delta x \to 0^+} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} = 2$

- $f'_-(0) = \lim_{\Delta x \to 0^-} \frac{f(\Delta x + 0) - f(0)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{(\Delta x)^2 + 2\Delta x + \cancel{7} \cancel{- 7}}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{\cancel{\Delta x}(\Delta x + 2)}{\cancel{\Delta x}} = 2$

Existe, os limites laterais são iguais: $f'_+(0) = f'_-(0)$.

c) $f(x) = \begin{cases} x^{2/3}, & x \geq 0 \\ x^{1/3}, & x < 0 \end{cases}$

- $f'_+(0) = \lim_{\Delta x \to 0^+} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\Delta x)^{2/3}}{\Delta x} = \lim_{\Delta x \to 0} (\Delta x)^{2/3 - 1} = \lim_{\Delta x \to 0} \Delta x^{-\frac{1}{3}}$

não derivável no ponto 0.

d) $f(x) = \begin{cases} 2x + \tan x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$

- $f'_+(0) = \lim_{\Delta x \to 0} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} + \lim_{\Delta x \to 0} \frac{\tan x}{\Delta x} = 2 + 1 = 3$

- $f'_-(0) = \lim_{\Delta x \to 0^-} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \Delta x = 0$

não existe, derivadas são diferentes

7) a) $h(x) = 5x - 1$

$$\frac{dh}{dx} = \frac{d}{dx}(5x-1) = \frac{d}{dx}(5x) + \frac{d}{dx}(-1) = 5$$

b) $F(x) = -4x^{10}$

$$\frac{dF}{dx} = \frac{d}{dx}(-4x^{10}) = -4 \cdot 10x^{10-1} = -40x^{9}$$

c) $f(x) = x^3 + 6x - 4$

$$\frac{df}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(6x) + \frac{d}{dx}(-4) = 3x^2 + 6$$

d) $g(x) = 5x^8 - 2x^5 + 6$

$$\frac{dg}{dx} = \frac{d}{dx}(5x^8) + \frac{d}{dx}(-2x^5) + \frac{d}{dx}(6) = 40x^7 - 10x^4$$

e) $y = x^{-2/5}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{-2/5}) = \frac{-2}{5}x^{-\frac{7}{5}}$$

f) $V(r) = \frac{4}{3}\pi r^3$

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$$

g) $y(t) = 7t^{-9}$

$$\frac{dy}{dx} = \frac{d}{dx}(7t^{-9}) = -63t^{-10}$$

h) $R(x) = \sqrt{10} \cdot x^{-7}$

$$\frac{dR}{dx} = \frac{d}{dx}(\sqrt{10} \cdot x^{-7}) = -7\sqrt{10}\,x^{-6}$$

i) $y = \sqrt[3]{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{\frac{1}{3}}) = \frac{1}{3}x^{\frac{-2}{3}}$$

j) $g(x) = x^2 + x^{-2}$

$$\frac{dg}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(x^{-2}) = 2x - 2x^{-3} = 2x - \frac{2}{x^3}$$

k) $y = x^{4/3} - x^{2/3}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{4/3}) - \frac{d}{dx}(x^{2/3}) = \frac{4}{3}\sqrt[3]{x} - \frac{2}{3}x^{\frac{-1}{3}} = \frac{4\sqrt[3]{x} - 2}{3\sqrt[3]{x}}$$

l) $y = x\sqrt{x} + \frac{1}{x^2\sqrt{x}} \Rightarrow x^{\frac{3}{2}} + x^{\frac{-5}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^{\frac{3}{2}}) + \frac{d}{dx}(x^{\frac{-5}{2}}) = \frac{3}{2}\sqrt[3]{x} - \frac{5}{2}x^{\frac{-7}{2}} = \frac{3\sqrt[3]{x} - 5}{-14\sqrt[2]{x^7}}$$

m)...

9. $\frac{d}{dx}(\operatorname{sen} x) = \cos x$, $\frac{d}{dx}(\cos x) = -\operatorname{sen} x$

a) $\frac{d}{dx}(\tan x) = \sec^2 x$

$$\frac{d}{dx}\left(\frac{\operatorname{sen} x}{\cos x}\right) = \frac{\frac{d}{dx}\operatorname{sen} x \cdot \cos x - \operatorname{sen} x \cdot \frac{d}{dx}\cos x}{\cos^2 x} = \frac{\cos x \cdot \cos x - \operatorname{sen} x \cdot (-\operatorname{sen} x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

b) $\frac{d}{dx}(\sec x) = \sec x \tan x$

$$\frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{\frac{d}{dx}1 \cdot \cos x - 1 \cdot \left(\frac{d}{dx}(\cos x)\right)}{(\cos x)^2} = \frac{\operatorname{sen} x \cos x}{\cos^3 x} =$$

$$\frac{\operatorname{sen} x}{\cos^2 x} = \frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \tan x$$

c) $\frac{d}{dx}(\cot x) = -\operatorname{cossec}^2 x$

$$\frac{d}{dx}\left(\frac{1}{\frac{\operatorname{sen} x}{\cos x}}\right) = \frac{d}{dx}\left(\frac{\cos x}{\operatorname{sen} x}\right) = \frac{\frac{d}{dx}\cos x \cdot \operatorname{sen} x - \cos x \cdot \frac{d}{dx}\operatorname{sen} x}{\operatorname{sen}^2 x} =$$

$$= \frac{-\operatorname{sen} x \cdot \operatorname{sen} x - \cos x \cdot \cos x}{\operatorname{sen}^2 x} = \frac{-(\operatorname{sen}^2 x + \cos^2 x)}{\operatorname{sen}^2 x} = \frac{-1}{\operatorname{sen}^2 x} = -\operatorname{cossec}^2 x.$$

d) $\frac{d}{dx}(\operatorname{cossec} x) = -\operatorname{cossec} x \tan x$

$$\frac{d}{dx}\left(\frac{1}{\operatorname{sen} x}\right) = \frac{\frac{d}{dx}(1) \cdot \operatorname{sen} x - 1 \cdot \frac{d}{dx}(\operatorname{sen} x)}{\operatorname{sen}^2 x} = \frac{-1 \cdot \cos x}{-\operatorname{sen}^2 x} = \frac{-1}{\operatorname{sen} x} \cdot \frac{\cos x}{\operatorname{sen} x}$$

$= \operatorname{cossec} x \tan x$

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10) a) $f(x) = x\,\text{sen}\,x$

$$\frac{d}{dx}(x\,\text{sen}\,x) = \frac{d}{dx}x \cdot \text{sen}\,x + x \cdot \frac{d}{dx}\text{sen}\,x = \text{sen}\,x + x\cos x$$

b) $y = \cos x - 2\tan(x)$

$$\frac{dy}{dx} = \frac{d}{dy}\cos x - \frac{d}{dx}2\tan(x) = -\text{sen}\,x - \left(\cancel{\frac{d}{dx}(2)\tan} + 2 \cdot \frac{d}{dx}\tan x\right) =$$

$$= -\text{sen}\,x - 2\sec^2 x = -\text{sen}\,x - \frac{2}{\cos^2 x}$$

c) $g(t) = t^3\cos t$

$$\frac{d}{dt}(t^3\cos t) = \frac{d}{dt}(t^3) \cdot \cos t + t^3 \frac{d}{dt}\cos t = 3t^2\cos t - t^3\,\text{sen}\,x$$

d) $g(t) = 4\sec t + \tan t$

$$\frac{d}{dt}(4\sec t + \tan t) = \cancel{\frac{d}{dt}4\sec t} + 4 \cdot \frac{d}{dt}\sec t + \frac{d}{dx}\tan t =$$

$$= 4 \cdot \sec t \cdot \tan t + \sec^2 t$$

e) $y = \frac{\tan x}{x}$

$$\frac{d}{dt}\left(\frac{\tan x}{x}\right) = \frac{\frac{d}{dx}\tan x \cdot x - \tan x \cdot \frac{d}{dx}x}{x^2} = \frac{x\sec^2 x - \tan x}{x^2}$$

f) $y = \frac{x}{\text{sen}\,x + \cos x}$

$$\frac{d}{dx}\left(\frac{x}{\text{sen}\,x + \cos x}\right) = \frac{\frac{d}{dx}x \cdot (\text{sen}\,x + \cos x) - x \cdot \frac{d}{dx}(\text{sen} + \cos x)}{(\text{sen}\,x + \cos x)^2} =$$

$$= \frac{\text{sen}\,x + \cos x - x\left(\frac{d}{dx}\text{sen}\,x + \frac{d}{dx}\cos x\right)}{\text{sen}^2 x + \cos^2 x + 2\,\text{sen}\,x\cos x} = \frac{\text{sen}\,x + \cos x - x\cos x - x\,\text{sen}\,x}{2\,\text{sen}\,x\cos x + 1}$$

g) $y = \frac{\tan x - 1}{\sec}$

$$\frac{d}{dx}\left(\frac{\tan x - 1}{\sec x}\right) = \frac{\frac{d}{dx}(\tan x - 1) \cdot \sec x - (\tan x - 1) \cdot \frac{d}{dx}\sec x}{\sec^2 x} =$$

$$= \frac{\sec^2 x \cdot \cancel{\sec x} - \cancel{\sec x}\tan^2 x - \cancel{\sec x}\tan x}{} = \frac{\sec^2 x - \tan^2 x - \tan x}{\sec x}$$

h) $y = \frac{\text{sen}\,x}{x^2}$

$$\frac{d}{dx}\left(\frac{\text{sen}\,x}{x^2}\right) = \frac{\frac{d}{dx}\text{sen}\,x \cdot x^2 - \text{sen}\,x\frac{d}{dx}(x^2)}{x^4} = \frac{x^2\cos x - 2x\,\text{sen}\,x}{x^4} = \frac{x\cos x - 2\,\text{sen}\,x}{x^3}$$

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i) $Y = \tan\theta(\operatorname{sen}\theta + \cos\theta)$

$$\frac{dY}{dx} = \frac{d}{dx}\tan\theta(\operatorname{sen}\theta + \cos\theta) = \frac{d}{dx}\left(\frac{\operatorname{sen}\theta}{\cos\theta}(\operatorname{sen}\theta + \cos\theta)\right) =$$

$$\frac{d}{dx}\left(\frac{\operatorname{sen}^2\theta + \operatorname{sen}\theta\cos\theta}{\cos\theta}\right) = \frac{d}{dx}\left(\frac{\operatorname{sen}^2\theta}{\cos\theta} + \frac{\operatorname{sen}\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}\right) =$$

$$\frac{d}{dx}\left(\frac{\operatorname{sen}^2\theta}{\cos\theta} + \operatorname{sen}\theta\right) = \frac{d}{dx}\left(\frac{\operatorname{sen}^2\theta}{\cos\theta}\right) + \frac{d}{dx}(\operatorname{sen}\theta) =$$

$$\frac{\frac{d}{dx}\operatorname{sen}^2\theta \cdot \cos\theta - \operatorname{sen}^2\theta \cdot \frac{d}{dx}\cos\theta}{\cos^2\theta} + \cos\theta = \frac{2\operatorname{sen}\theta\cos\theta - \operatorname{sen}^2 \cdot (-\operatorname{sen}\theta)}{\cos^2\theta} + \cos\theta$$

$$= \frac{2\operatorname{sen}\theta\cos\theta + \operatorname{sen}^3\theta}{\cos^2\theta} + \frac{\cos\theta}{1} = \frac{2\operatorname{sen}\theta\cos\theta + \operatorname{sen}^3\theta + \cos^3\theta}{\cos^2\theta}$$

j) $Y = \operatorname{cossec}x \cot x = \frac{1}{\operatorname{sen}x} \cdot \frac{\cos x}{\operatorname{sen}x} = \frac{\cos x}{\operatorname{sen}^2 x}$

$$\frac{d}{dx}\left(\frac{\cos x}{\operatorname{sen}^2 x}\right) = \frac{\frac{d}{dx}\cos x \cdot \operatorname{sen}^2 x - \cos x \cdot \frac{d}{dx}\operatorname{sen}^2 x}{\operatorname{sen}^4 x} =$$

$$= \frac{-\operatorname{sen}x \cdot \operatorname{sen}^2 x - \cos x \cdot 2\operatorname{sen}x\cos x}{\operatorname{sen}^4 x} = \frac{-\operatorname{sen}^3 x - 2\operatorname{sen}x\cos^2 x}{\operatorname{sen}^4 x}$$

$$= \frac{-\operatorname{sen}^2 x - 2\cos^2 x}{\operatorname{sen}^3 x} = -\frac{1 + \cos x}{\operatorname{sen}^3 x}$$

k) $x \cdot \operatorname{sen}x \cdot \cos x$

$$\frac{d}{dx}x \cdot \operatorname{sen}x\cos x = \frac{d}{dx}(x \cdot \operatorname{sen}x) \cdot \cos x + x\operatorname{sen}x \cdot \frac{d}{dx}(\cos x) =$$

$$\left(\frac{d}{dx}x \cdot \operatorname{sen}x + x\frac{d}{dx}\operatorname{sen}x\right) \cdot \cos x + x\operatorname{sen}x \cdot (-\operatorname{sen}x) =$$

$$= \operatorname{sen}x\cos x + x\cos^2 x - x\operatorname{sen}^2 x =$$

11) a) $f(x) = (x^3 + 2x) \cdot e^x$

$$\frac{d}{dx}(x^3+2x)e^x = \frac{d}{dx}(x^3+2x) \cdot e^x + (x^3+2x)\frac{d}{dx}e^x = \left(\frac{d}{dx}(x^3) + \frac{d}{dx}(2x)\right)e^x +$$

$$e^x(x^3+2x) = 3x^2e^x + 2e^x + x^3e^x + 2xe^x$$

b) $g(x) = (e^x + 3x^2)\sqrt{x}$

$\frac{d}{dx}(e^x+3x^2)\sqrt{x} = \frac{d}{dx}(e^x+3x^2)\cdot\sqrt{x} + (e^x+3x^2)\cdot\frac{d}{dx}\sqrt{x} =$

$e^x\sqrt{x} + 6x\sqrt{x} + (e^x+3x^2)\cdot\frac{1}{2x} = e^x\sqrt{x} + 6x\sqrt{x} + \frac{e^x}{2x} + \frac{3x}{2}$

c) $f(z) = (1-e^z)\cdot(z+e^z) =$

$\frac{d}{dz}(1-e^z)\cdot(z+e^z) = \frac{d}{dz}(1-e^z)\cdot(z+e^z) + (1-e^z)\cdot\frac{d}{dz}(z+e^z) =$

$= -e^z(z+e^z) + (1+e^z)\cdot(1+e^z) = -e^z z - \cancel{e^{2z}} + 1 + 2e^z + \cancel{e^{2z}}$

$= 1 + 2e^z - e^z z$

d) $y = \frac{e^x}{x^2} = \frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{\frac{d}{dx}e^x\cdot x^2 - e^x\cdot\frac{d}{dx}(x^2)}{x^4} = \frac{e^x x^2 - e^x 2x}{x^4}$

$= \frac{xe^x - 2e}{x^3}$

e) $y = \frac{e^x}{1+x} \Rightarrow \frac{d}{dx}\left(\frac{e^x}{1+x}\right) = \frac{\frac{d}{dx}e^x(1+x) - e^x\frac{d}{dx}(1+x)}{(1+x)^2} =$

$= \frac{\cancel{e^x} + xe^x - \cancel{e^x}}{(1+x)^2} = \frac{xe^x}{(1+x^2)}$

f) $y = \frac{1-xe^x}{x+e^x} = \frac{d}{dx}\left(\frac{1-xe^x}{x+e^x}\right) = \frac{\frac{d}{dx}(1-xe^x)\cdot(x+e^x) - (1-xe^x)\frac{d}{dx}(x+e^x)}{(x+e^x)^2}$

$= \frac{\frac{d}{dx}(-xe^x)\cdot(x+e^x) - (1-xe^x)\cdot(1+e^x)}{(x+e^x)^2} = \frac{(-e^x - xe^x)\cdot(x+e^x) - (1+e^x - xe^x - xe^{2x})}{(x+e^x)^2}$

$= \frac{-\cancel{xe^x} - e^{2x} - x^2e^x - \cancel{xe^{2x}} - 1 - e^x + \cancel{xe^x} + \cancel{xe^{2x}}}{(x+e^x)^2} = \frac{-(e^{2x} + x^2e^x + 1)}{(x+e^x)^2}$

g) ...