This project is inspired by the gravitational interactions of celestial bodies. Using a 2D orthonormal basis and Newton's law of universal gravitation here are the calculations involved:

Contents

1 Movement

Initial Positions and Velocities

Let the initial positions of the two celestial bodies be:

$$\mathbf{r}_1 = (x_1, y_1)$$

$$\mathbf{r}_2 = (x_2, y_2)$$

Their initial velocities be:

$$\mathbf{v}_1 = (v_{1x}, v_{1y})$$

$$\mathbf{v}_2 = (v_{2x}, v_{2y})$$

And their masses be $\mathbf{m_1}$ and $\mathbf{m_2}$

Calculate the Distance and Unit Vector

Calculate the distance r between the two bodies:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the unit vector $\hat{\mathbf{r}}$ pointing from body 1 to body 2:

$$\hat{\mathbf{r}} = \left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r} \right)$$

Calculate the Gravitational Force

With G the gravitational constant. Using Newton's law of universal gravitation, the force on body 1 due to body 2 is:

$$\mathbf{F}_1 = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}} = G\frac{m_1 m_2}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r} \right)$$

By Newton's third law, the force on body 2 due to body 1 is:

$$\mathbf{F}_2 = -\mathbf{F}_1 = -G\frac{m_1 m_2}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r} \right)$$

Calculate the Accelerations

Using Newton's second law, $\mathbf{F} = m\mathbf{a}$

The acceleration of body 1 due to the gravitational force is:

$$\mathbf{a}_1 = \frac{\mathbf{F}_1}{m_1} = G\frac{m_2}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}\right)$$

Similarly, the acceleration of body 2 is:

$$\mathbf{a}_2 = \frac{\mathbf{F}_2}{m_2} = -G\frac{m_1}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}\right)$$

Update the Velocities

Update the velocities of the bodies after 1 second (assuming $\Delta t = 1$ second):

$$\mathbf{v}_1' = \mathbf{v}_1 + \mathbf{a}_1 \cdot \Delta t = (v_{1x}, v_{1y}) + G\frac{m_2}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}\right)$$

$$\mathbf{v}_2' = \mathbf{v}_2 + \mathbf{a}_2 \cdot \Delta t = (v_{2x}, v_{2y}) - G\frac{m_1}{r^2}\left(\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}\right)$$

This gives:

$$v_{1x}' = v_{1x} + G\frac{m_2}{r^2}\frac{x_2 - x_1}{r}$$

$$v_{1y}' = v_{1y} + G\frac{m_2}{r^2}\frac{y_2 - y_1}{r}$$

$$v_{2x}' = v_{2x} - G\frac{m_1}{r^2}\frac{x_2 - x_1}{r}$$

$$v_{2y}' = v_{2y} - G\frac{m_1}{r^2}\frac{y_2 - y_1}{r}$$

Update the Positions

Update the positions of the bodies after 1 second:

$$\mathbf{r}_1' = \mathbf{r}_1 + \mathbf{v}_1' \cdot \Delta t = (x_1, y_1) + (v_{1x}', v_{1y}') \cdot 1$$

$$\mathbf{r}_2' = \mathbf{r}_2 + \mathbf{v}_2' \cdot \Delta t = (x_2, y_2) + (v_{2x}', v_{2y}') \cdot 1$$

This gives:

$$x_1' = x_1 + v_{1x}'$$

$$y_1' = y_1 + v_{1y}'$$

$$x_2' = x_2 + v_{2x}'$$

$$y_2' = y_2 + v_{2y}'$$

Sum Accelerations

The net acceleration for body i is the sum of the accelerations due to all other bodies:

$$\mathbf{a}_i = \sum_{j \neq i} \mathbf{a}_{ij}$$

Separating the x and y components:

$$a_{ix} = \sum_{j \neq i} G \frac{m_j}{r_{ij}^2} \frac{x_j - x_i}{r_{ij}}$$

$$a_{iy} = \sum_{j \neq i} G \frac{m_j}{r_{ij}^2} \frac{y_j - y_i}{r_{ij}}$$

where:
- m_j is the mass of body j,
- r_{ij} is the distance between body i and body j,
- (x_i, y_i) and (x_j, y_j) are the coordinates of bodies i and j respectively.

2 Collision

A collision between two bodies will generate a new body whose mass is the sum of the masses, its velocity and position the weighted average.

Velocity

The weighted average of the velocity components, given the velocities (v_{1x}, v_{1y}) and (v_{2x}, v_{2y}) and the masses m_1 and m_2, is:

$$v_{x,\text{avg}} = \frac{m_1 \cdot v_{1x} + m_2 \cdot v_{2x}}{m_1 + m_2}$$

$$v_{y,\text{avg}} = \frac{m_1 \cdot v_{1y} + m_2 \cdot v_{2y}}{m_1 + m_2}$$

Position

The weighted average of the coordinates, given the coordinates (x_1, y_1) and (x_2, y_2) and the masses m_1 and m_2, is:

$$x_{\text{avg}} = \frac{m_1 \cdot x_1 + m_2 \cdot x_2}{m_1 + m_2}$$

$$y_{\text{avg}} = \frac{m_1 \cdot y_1 + m_2 \cdot y_2}{m_1 + m_2}$$