11. CONSTRAINT-BASED STRUCTURE LEARNING

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Abstract. This document contains the solutions to the proposed exercises from Lecture 11.

1. Solutions

Exercise 1. *Learn a PDAG using d-separation as oracle in each of the following structures.*

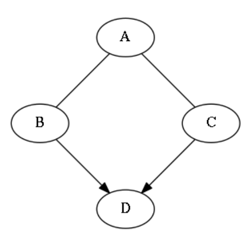
Solution. For the first structure let's consider the following sentences

$$B \perp C | A$$
$$A \perp D | B$$

We know these two to be true from our oracle. Therefore we can remove edges $A-D$ and $B-C$ with witness sets $\mathcal{Z}_{AD} = \{B\}$ and $\mathcal{Z}_{BC} = \{A\}$ respectively. These two sentences are the only relevant (in)dependencies for this first step. Now we must find convergent connections. Our oracle shows that we only have one convergent connection, since

$$B \not\perp C | D$$

From there we have the following resulting PDAG: Edges $A-B$ and $A-C$ are



both undirected since $B-A-C$ can either be a serial or divergent connection, but it cannot be convergent.

For the second structure we have the following independencies

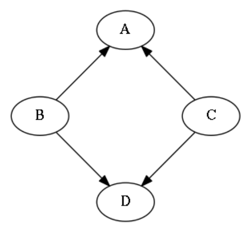
$$A \perp D | C$$
$$B \perp C | \emptyset$$

We then remove edges $A-D$ with $\mathcal{Z}_{AD} = \{C\}$ and $\mathcal{Z}_{BC} = \emptyset$. The next step is to find potential immoralities (convergent connections). From the definition of convergent connections we can then ask the oracle

$$\text{Given connection } B - A - C,\ A \in \mathcal{Z}_{BC}? \tag{1}$$

$$\text{Given connection } B - D - C,\ D \in \mathcal{Z}_{BC}? \tag{2}$$

Who is going to return false to both these questions. We now know that $B \to A \leftarrow C$ from 1 and $B \to D \leftarrow C$ from 2. This gives the following final PDAG:



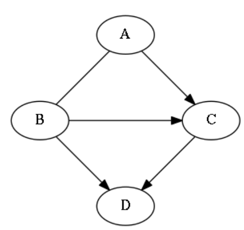
For the last structure:

$$A \perp D|B$$
$$D \perp A|C$$

This removes edges $A - D$ with $\mathcal{Z}_{AD} = \{B, C\}$. Testing immoralities, we have that:

$$B \not\perp C|D \Rightarrow B \to D \leftarrow C$$
$$A \not\perp B|C \Rightarrow A \to C \leftarrow B$$

This gives the final PDAG The undirected edge $A - B$ tells us that there are equiv-



alent networks with $A \to B$ and $A \leftarrow B$ that do not contradict the independencies we assumed. □

Exercise 2. *Answer the following questions:*

(i) Give an example where the PC algorithm reconstructs the wrong structure due to the presence of a single wrong answer of the oracle.
(ii) Give an example where the algorithm reconstructs the correct skeleton but makes a single mistake when extracting the immoralities (and hence learns the wrong structure).

Solution.

(i) Consider the solution to the second structure of Exercise 1. Had our oracle answered the query $A \perp D|C$ as false, we wouldn't have removed the edge $A - D$, which would've caused the resulting PDAG to be different then what we found.

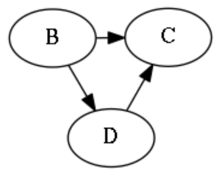


FIGURE 1

(ii) Take the solution to the third structure of Exercise 1 as example. If the query $B \perp C|D$ had returned true, we then wouldn't have had a convergent connection $B \rightarrow D \leftarrow C$. Instead, because of the Acyclicity step of the PC algorithm, we would've had a a different BCD connection, as shown in Figure 1. $\square$