

Two Perspectives to Learning with Circuits

Motivation

Given a selection of sushi...



...and people's preferences...

Alice:     

Bob:     

Carol:     

...how can we model this as a probability distribution...

$$p(1^{\text{st}} = \text{salmon nigiri}, 3^{\text{rd}} = \text{salmon nigiri})$$

$$p(2^{\text{nd}} = \text{salmon nigiri} \mid 1^{\text{st}} = \text{white rice ball (maki)})$$

$$\arg \max p(1^{\text{st}} = ?, 2^{\text{nd}} = ?, 3^{\text{rd}} = ?, 4^{\text{th}} = \text{white rice ball (maki)}, 5^{\text{th}} = \text{maki roll with red sauce})$$

$$p((3^{\text{rd}} = \text{salmon nigiri} \rightarrow 1^{\text{st}} = \text{white rice ball (maki)}) \vee 2^{\text{nd}} = \text{salmon nigiri})$$

...and extract meaningful queries from it?

Motivation

Given a selection of sushi...



...and people's preferences...

Alice:     

Bob:     

Carol:     

...how can we model this as a probability distribution...

$$p(1^{\text{st}} = \text{salmon nigiri}, 3^{\text{rd}} = \text{tuna nigiri})$$

$$p(2^{\text{nd}} = \text{tuna nigiri} \mid 1^{\text{st}} = \text{white rice ball})$$

$$\arg \max p(1^{\text{st}} = ?, 2^{\text{nd}} = ?, 3^{\text{rd}} = ?, 4^{\text{th}} = \text{white rice ball}, 5^{\text{th}} = \text{maki roll})$$

$$p((3^{\text{rd}} = \text{salmon nigiri} \rightarrow 1^{\text{st}} = \text{white rice ball}) \vee 2^{\text{nd}} = \text{tuna nigiri})$$

Marginals

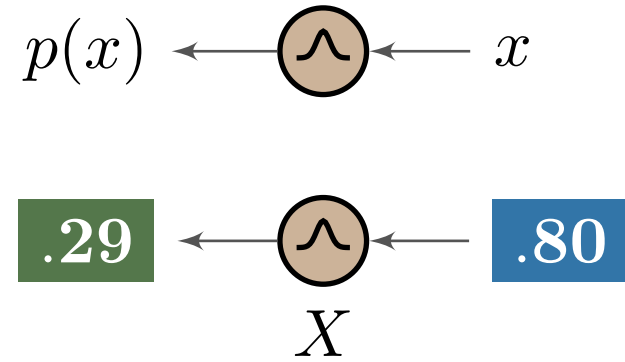
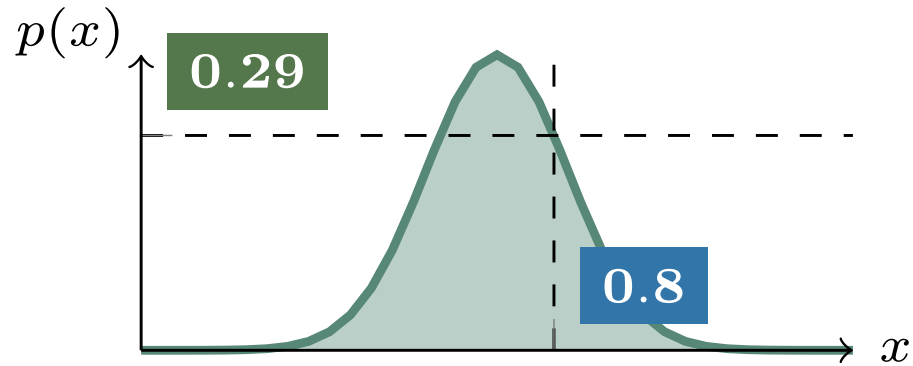
Conditionals

MPE

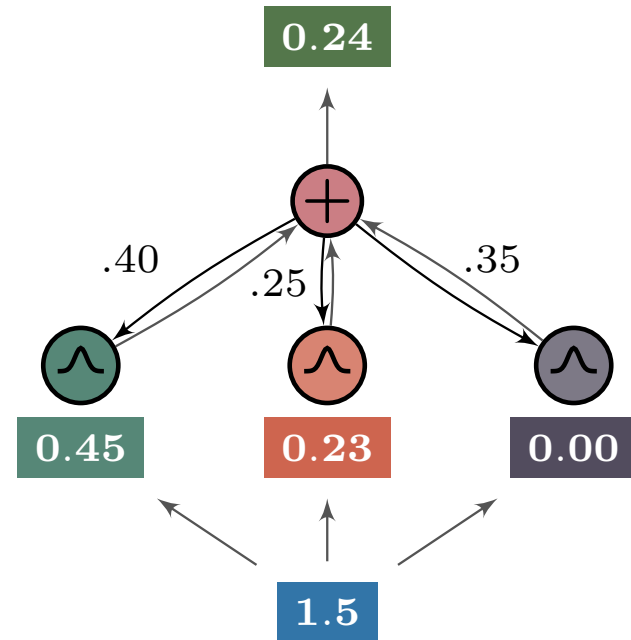
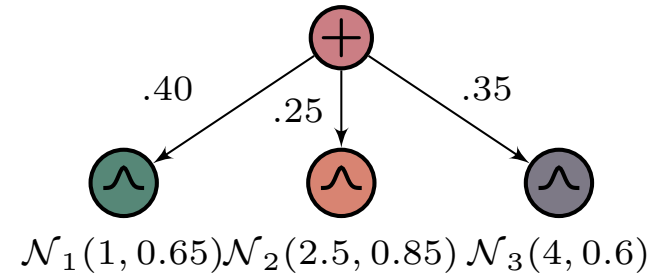
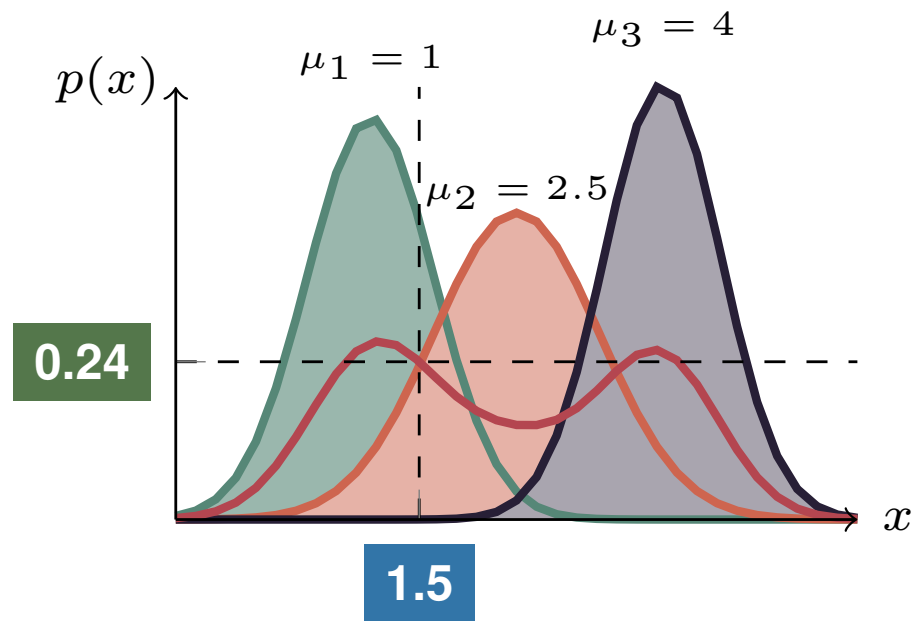
Logical events

...and extract meaningful queries from it?

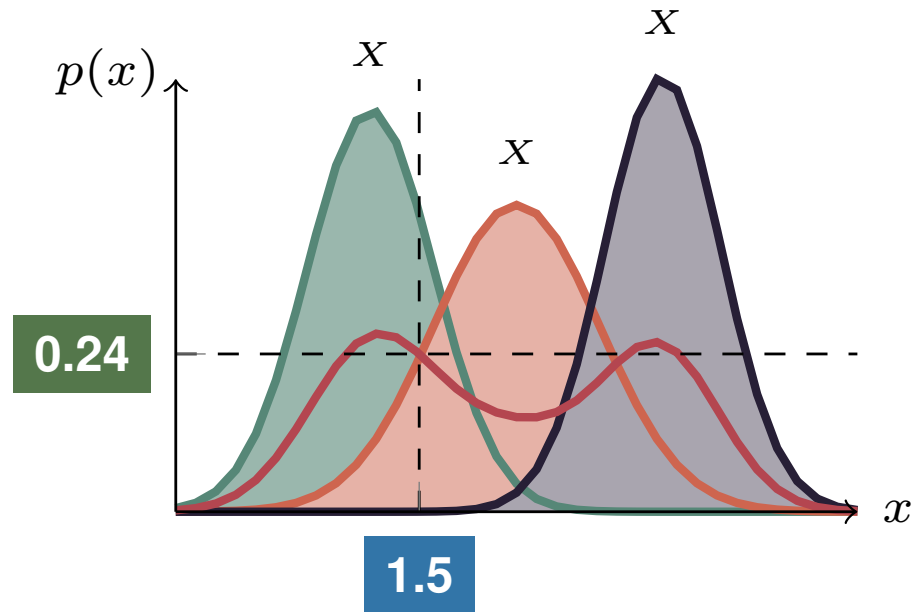
Probabilistic Circuits – Inputs



Probabilistic Circuits – Sums

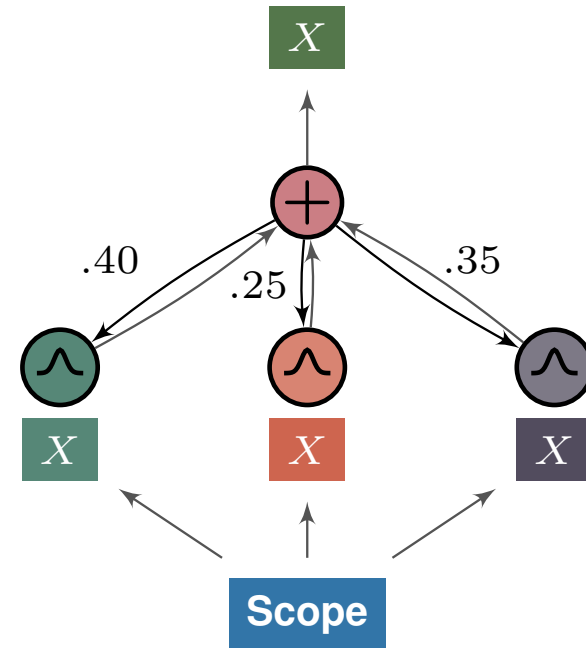
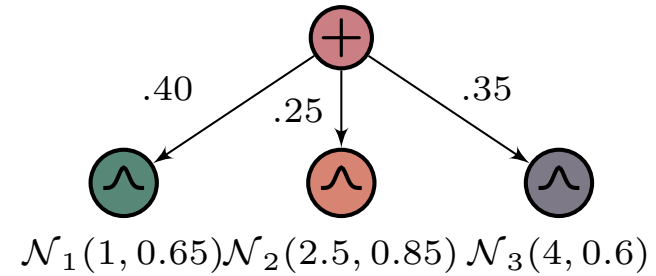


Probabilistic Circuits – Smoothness

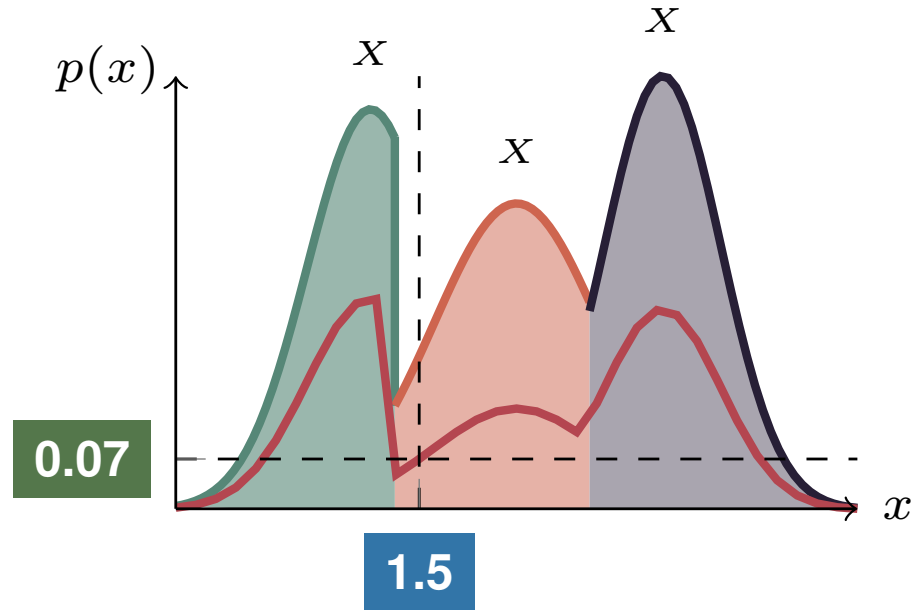


Definition 1 (Smoothness).

Every sum node child mentions the same variables.

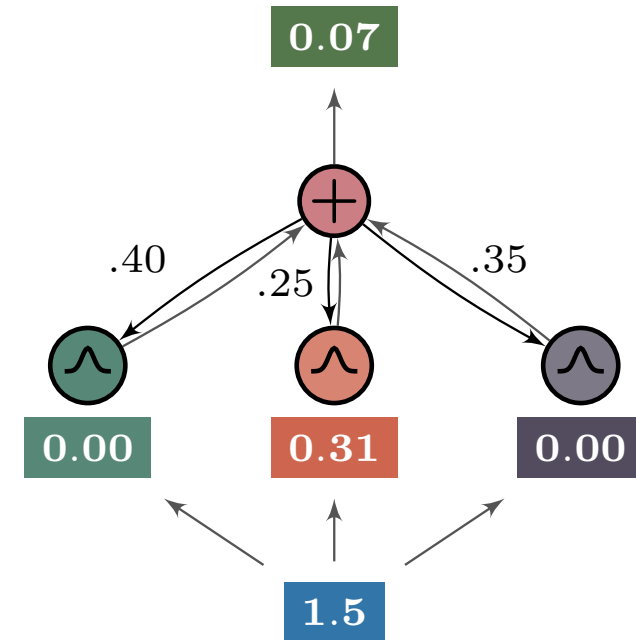
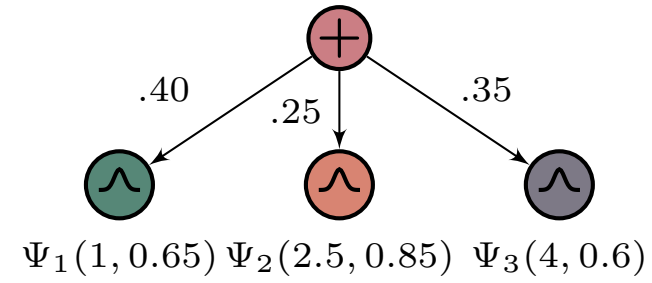


Probabilistic Circuits – Determinism

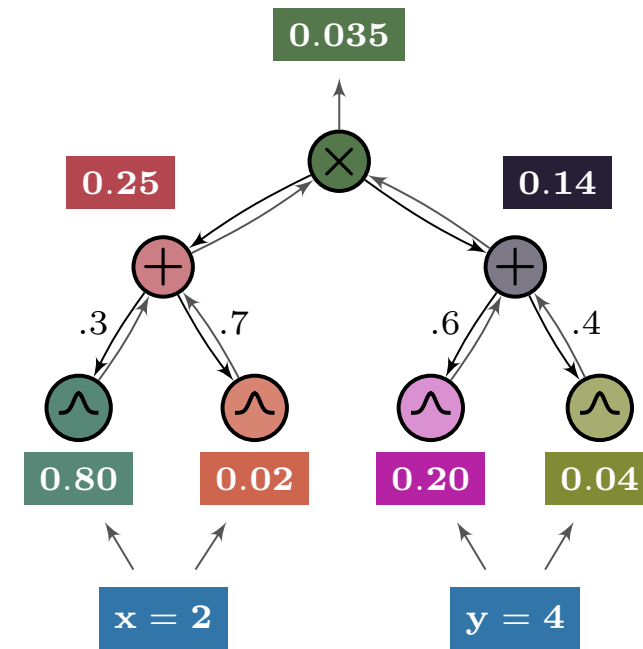
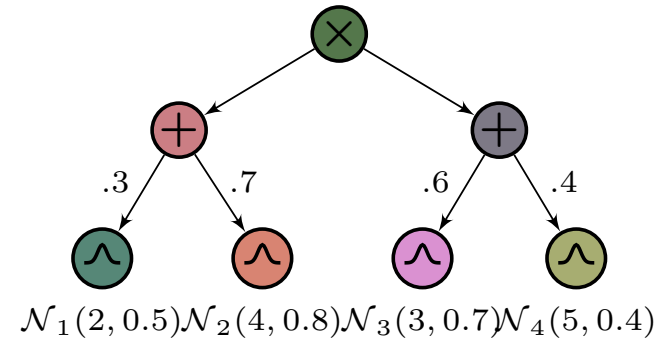
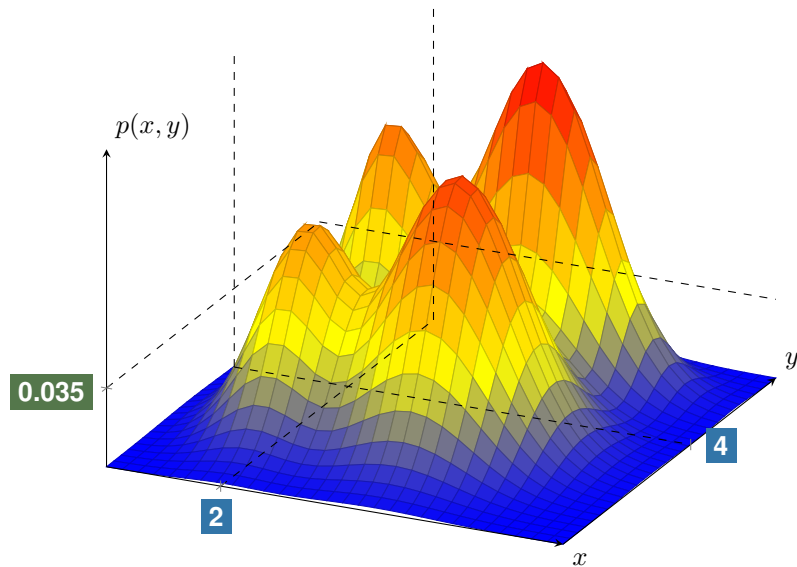
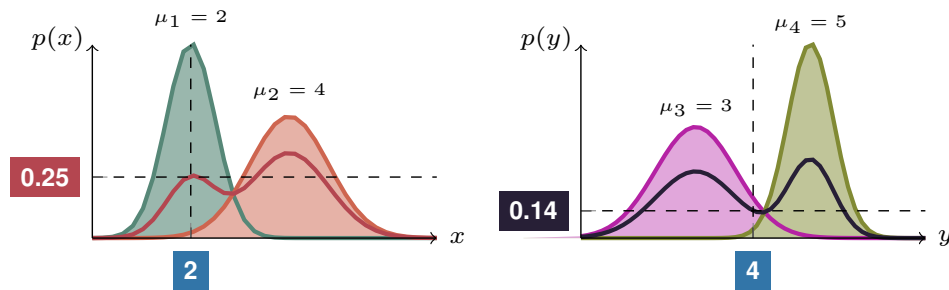


Definition 2 (Determinism).

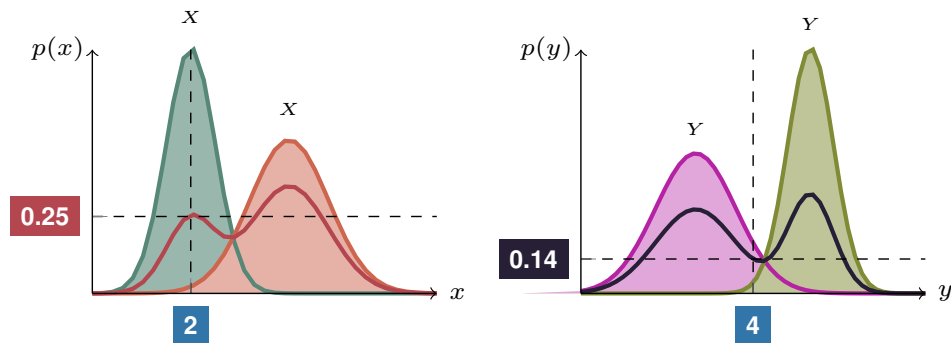
At most one sum node child has a positive value.



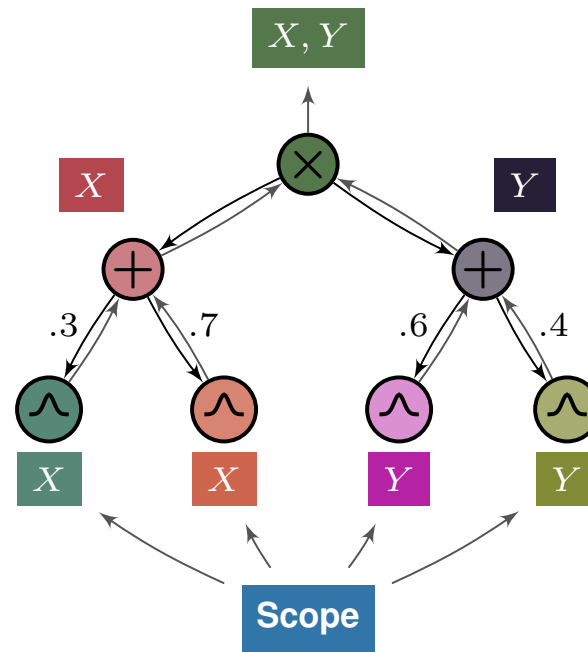
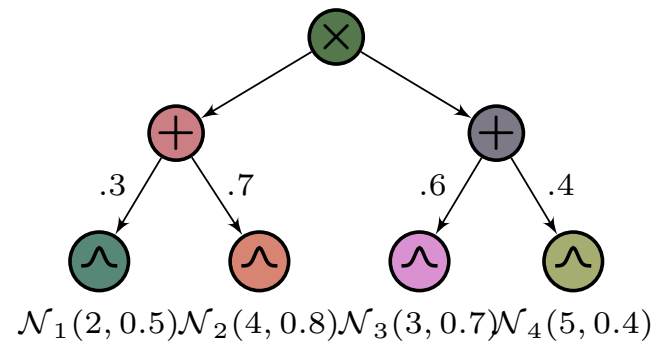
Probabilistic Circuits – Products



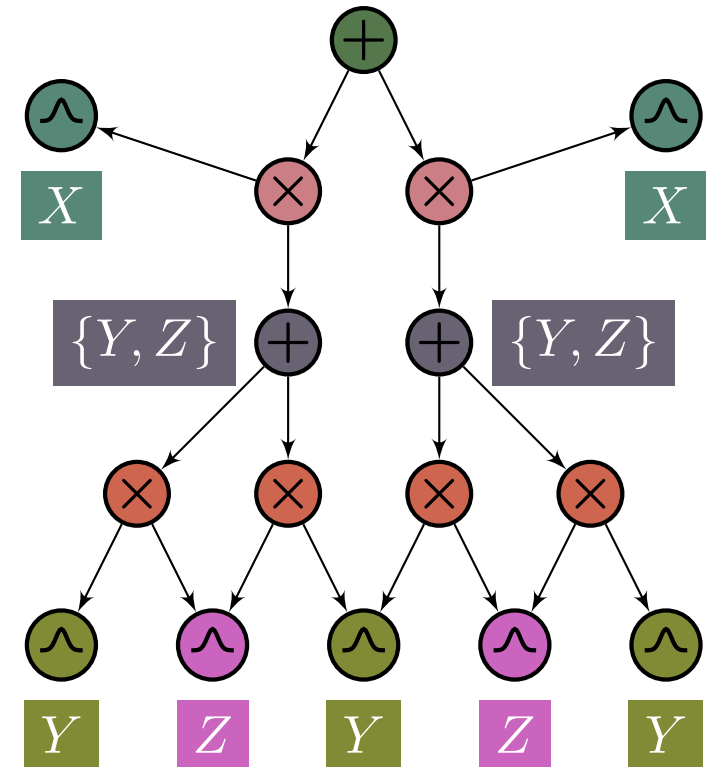
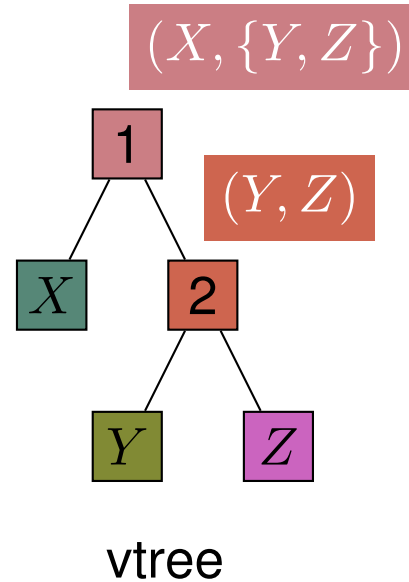
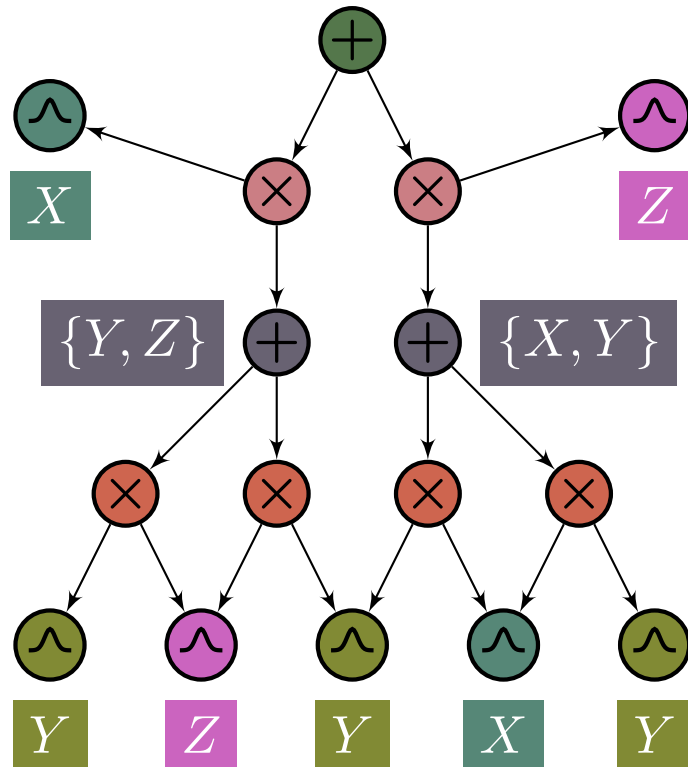
Probabilistic Circuits – Decomposability



Definition 3 (Decomposability).
Every product node child mentions different variables.



Probabilistic Circuits – Structured Decomposability

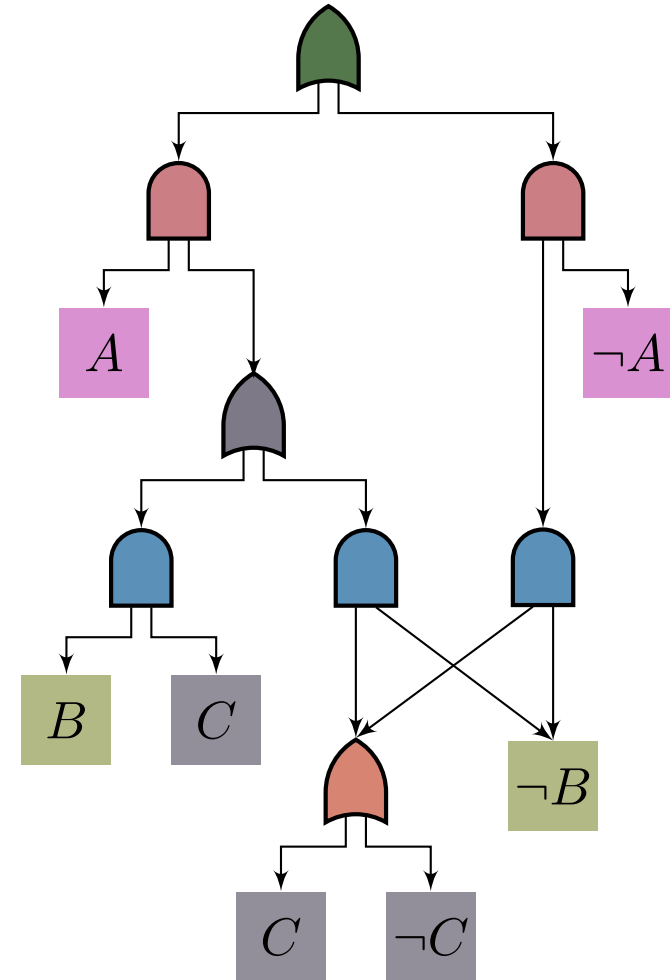
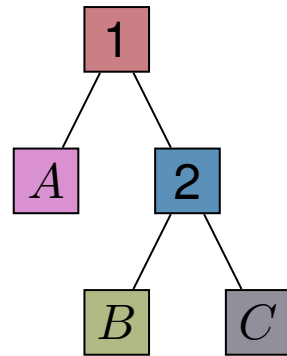


Definition 4 (Structured decomposability). *Every product node follows a vtree decomposition.*

Probabilistic Circuits – Logic Circuits

A	B	C	$\phi(\mathbf{x})$
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	0
0	0	1	1
1	0	1	1
0	1	1	0
1	1	1	1

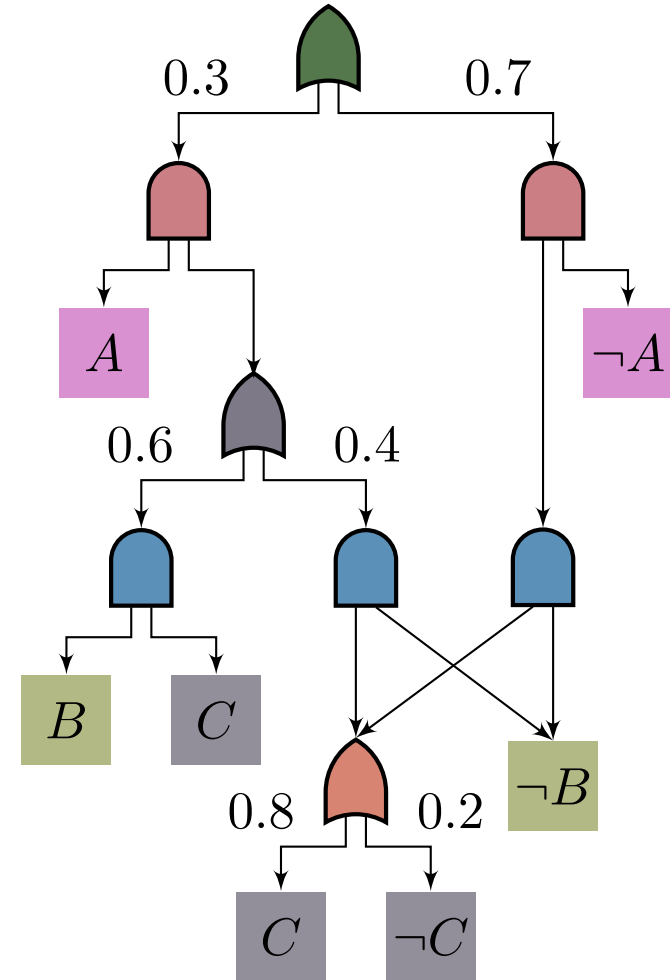
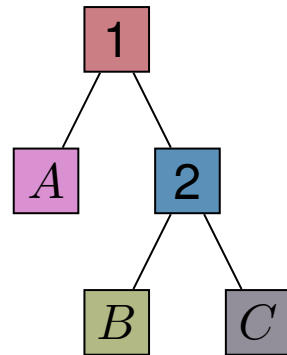
$$\phi(A, B, C) = (A \vee B) \wedge (\neg B \vee C)$$



Probabilistic Circuits – Support

A	B	C	$\phi(\mathbf{x})$	$p(\mathbf{x})$
0	0	0	1	0.140
1	0	0	1	0.024
0	1	0	0	0.000
1	1	0	0	0.000
0	0	1	1	0.560
1	0	1	1	0.096
0	1	1	0	0.000
1	1	1	1	0.180

$$\phi(A, B, C) = (A \vee B) \wedge (\neg B \vee C)$$



Probabilistic Circuits – Tractability

Query	+Sm?	+Dec?	+Det?	+Str Dec?
Evidence	✓	✓	✓	✓
Marginals	✗	✓	✓	✓
Conditionals	✗	✓	✓	✓
MPE	✗	✗	✓	✓
Shannon Entropy	✗	✗	✓	✓
Rényi Entropy	✗	✗	✓	✓
Cross Entropy	✗	✗	✗	✓
Kullback-Leibler Div	✗	✗	✗	✓
Rényi's Alpha Div	✗	✗	✗	✓
Cauchy-Schwarz Div	✗	✗	✗	✓
Logical Events	✗	✗	✗	✓
Mutual Information	✗	✗	✗	✓

Learning Probabilistic Circuits – Where are we right now?

Name	Class	Time Complexity	# hyperparams	Accepts logic?	Sm?	Dec?	Det?	Str Dec?	$\{0, 1\}$?	\mathbb{N} ?	\mathbb{R} ?	Reference
LEARNSPN	DIV	$\begin{cases} \mathcal{O}(nkmc) & , \text{ if sum} \\ \mathcal{O}(nm^3) & , \text{ if product} \end{cases}$	≥ 2	\times	✓	✓	\times	\times	✓	✓	✓	Gens and Domingos [2013]
ID-SPN	DIV	$\begin{cases} \mathcal{O}(nkmc) & , \text{ if sum} \\ \mathcal{O}(nm^3) & , \text{ if product} \\ \mathcal{O}(ic(rn + m)) & , \text{ if input} \end{cases}$	$\geq 2 + 3$	\times	✓	✓	\times	\times	✓	✓	\times	Rooshenas and Lowd [2014]
PROMETHEUS	DIV	$\begin{cases} \mathcal{O}(nkmc) & , \text{ if sum} \\ \mathcal{O}(m(\log m)^2) & , \text{ if product} \end{cases}$	≥ 1	\times	✓	✓	\times	\times	✓	✓	✓	Jaini et al. [2018]
LEARNSDD	INCR	$\begin{cases} \mathcal{O}(m^2) & , \text{ top-down vtree} \\ \mathcal{O}(m^4) & , \text{ bottom-up vtree} \\ \mathcal{O}(i \mathcal{C} ^2) & , \text{ circuit structure} \end{cases}$	1	✓	✓	✓	✓	✓	✓	\times	\times	Liang et al. [2017]
STRUDEL	INCR	$\begin{cases} \mathcal{O}(m^2n) & , \text{ CLT + vtree} \\ \mathcal{O}(i(\mathcal{C} n + m^2)) & , \text{ circuit structure} \end{cases}$	1	✓	✓	✓	✓	✓	✓	\times	\times	Dang et al. [2020]
RAT-SPN	RAND	$\mathcal{O}(rd(s + l))$	4	\times	✓	✓	\times	\times	✓	✓	✓	Peharz et al. [2020]
XPC	RAND	$\mathcal{O}(i(t + kn) + ikm^2n)$	3	\times	✓	✓	✓	✓	✓	\times	\times	Mauro et al. [2021]
SAMPLESDD	RAND	$\begin{cases} \mathcal{O}(m) & , \text{ random vtree} \\ \mathcal{O}(kc \log c + \log_2^2 k) & , \text{ per call} \end{cases}$	1	✓	✓	✓	✓	✓	✓	\times	\times	Geh and Mauá [2021]
LEARNRP	RAND	$\begin{cases} \mathcal{O}(m^2) & , \text{ top-down vtree} \\ \mathcal{O}(m^4) & , \text{ bottom-up vtree} \\ \mathcal{O}(knm) & , \text{ per call} \end{cases}$	0	\times	✓	✓	\times	✓	✓	✓	✓	To appear