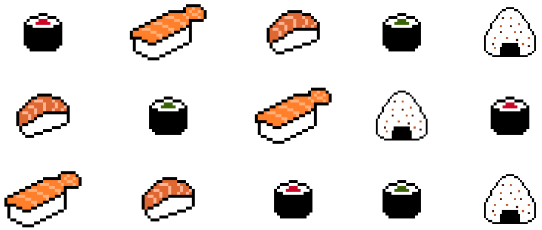
Two Perspectives to Learning with Circuits

Motivation

Given a selection of sushi...

...and people's preferences...

Alice:
Bob:
Carol:



...how can we model this as a probability distribution...

$p(1^{\text{st}} =$  $, 3^{\text{rd}} =$  $)$

$p(2^{\text{nd}} =$  $| 1^{\text{st}} =$  $)$

$\arg\max p(1^{\text{st}} = ?, 2^{\text{nd}} = ?, 3^{\text{rd}} = ?, 4^{\text{th}} =$  $, 5^{\text{th}} =$  $)$

$p((3^{\text{rd}} =$  $\rightarrow 1^{\text{st}} =$  $) \vee 2^{\text{nd}} =$  $)$

...and extract meaningful queries from it?

Kamishima [2003]

Motivation

Given a selection of sushi...

...and people's preferences...

Alice:

Bob:

Carol:

...how can we model this as a probability distribution...

$$p(1^{\text{st}} = \quad , 3^{\text{rd}} = \quad)$$

Marginals

$$p(2^{\text{nd}} = \quad | 1^{\text{st}} = \quad)$$

Conditionals

$$\arg\max p(1^{\text{st}} = ?, 2^{\text{nd}} = ?, 3^{\text{rd}} = ?, 4^{\text{th}} = \quad , 5^{\text{th}} = \quad)$$

MPE

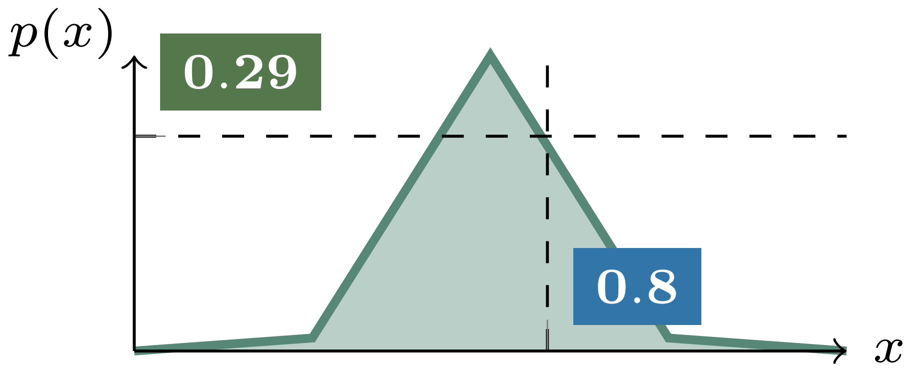
$$p((3^{\text{rd}} = \quad \rightarrow 1^{\text{st}} = \quad) \vee 2^{\text{nd}} = \quad)$$

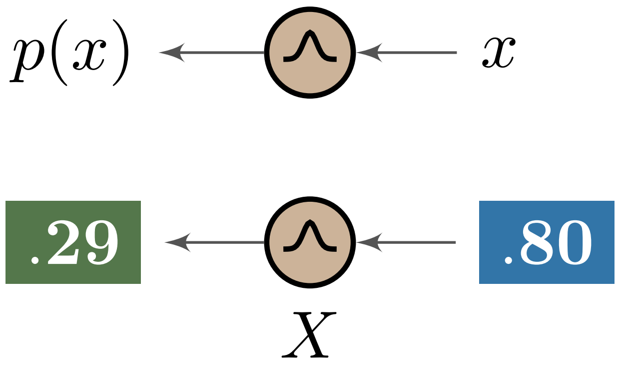
Logical events

...and extract meaningful queries from it?

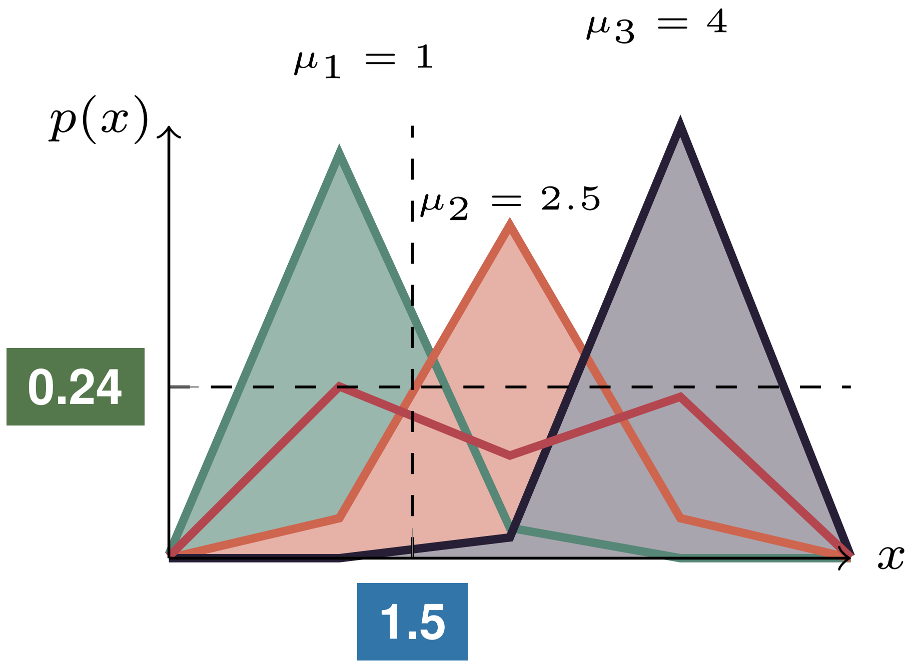
Kamishima [2003]

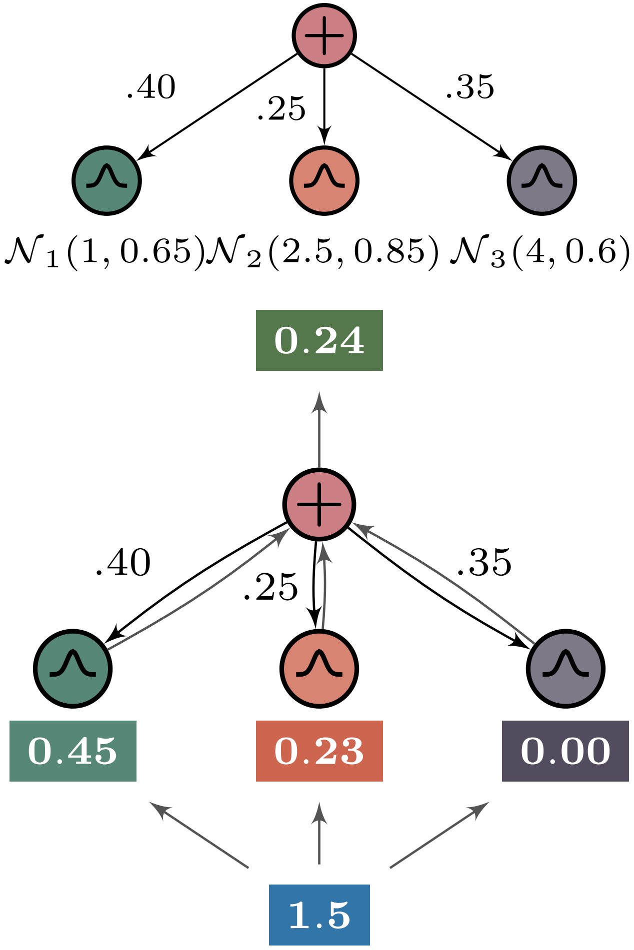
Probabilistic Circuits – Inputs



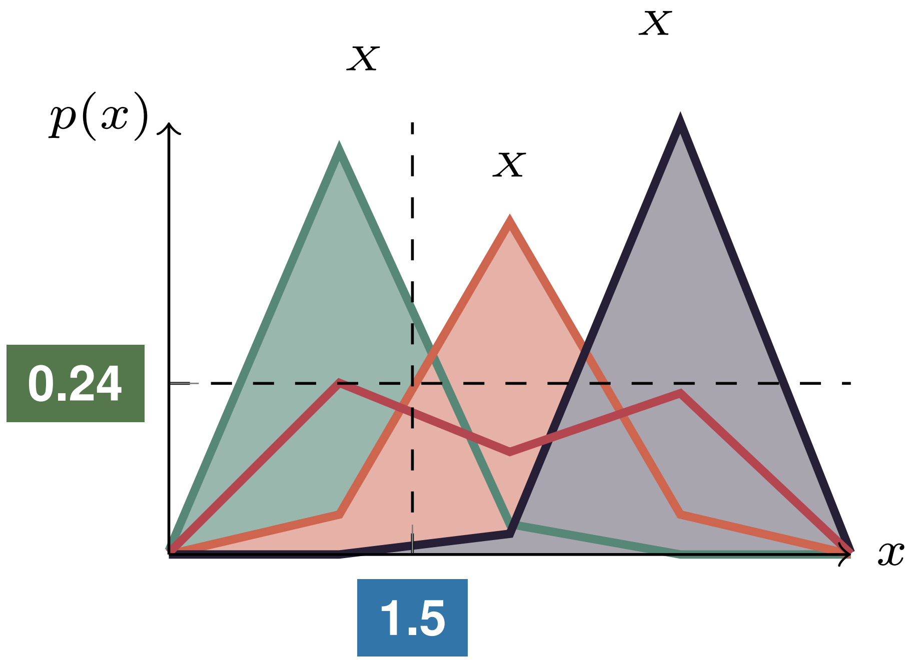


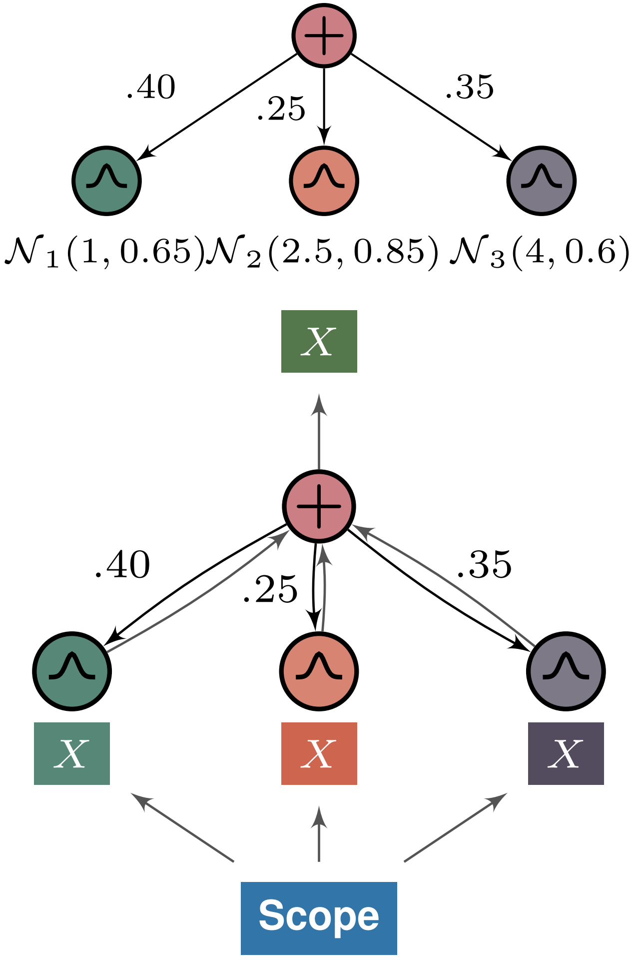
Probabilistic Circuits – Sums





Probabilistic Circuits – Smoothness

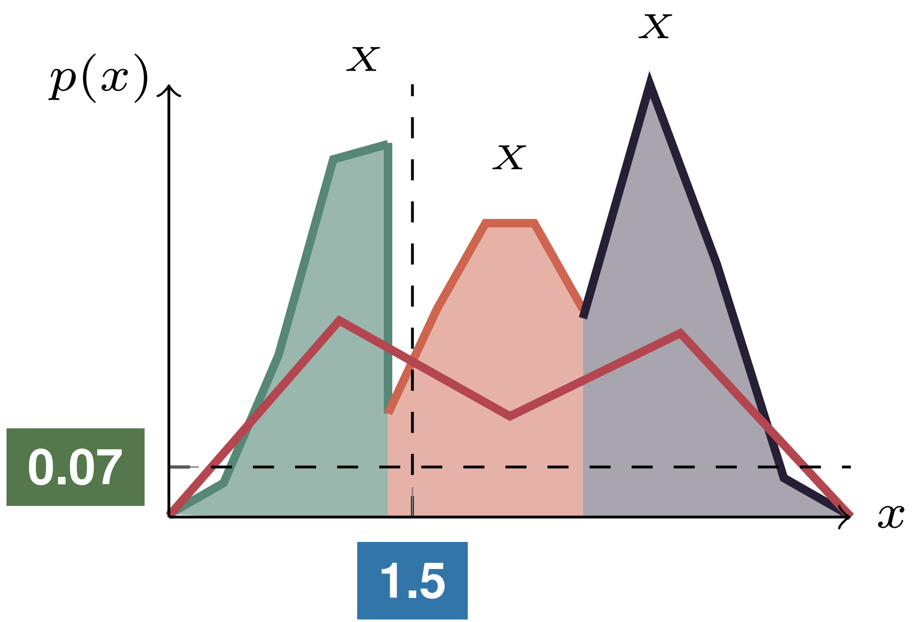


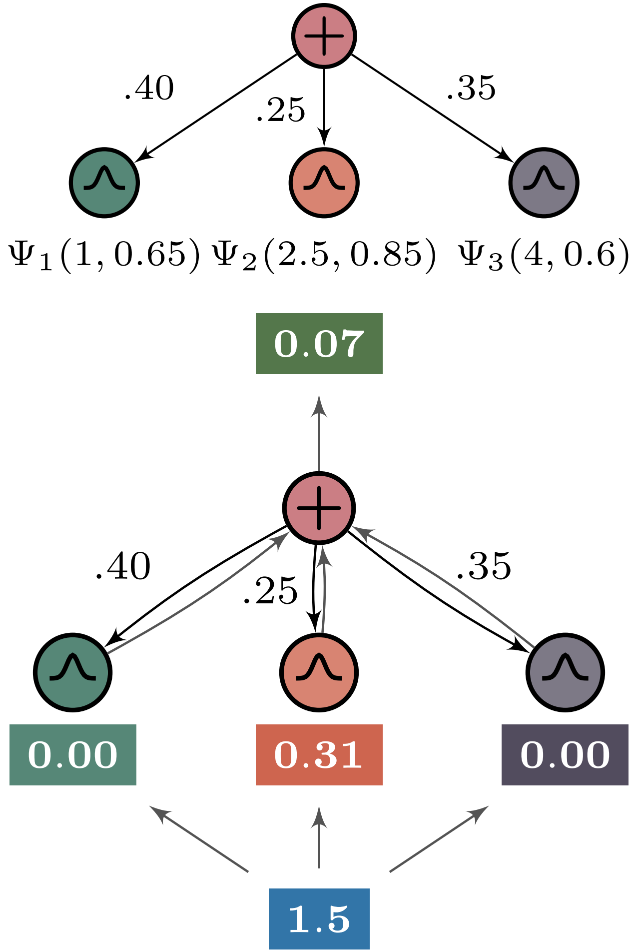


Definition 1 (Smoothness)**.**
Every sum node child mentions the <u>same</u> variables.

Darwiche [2001a]

Probabilistic Circuits – Determinism

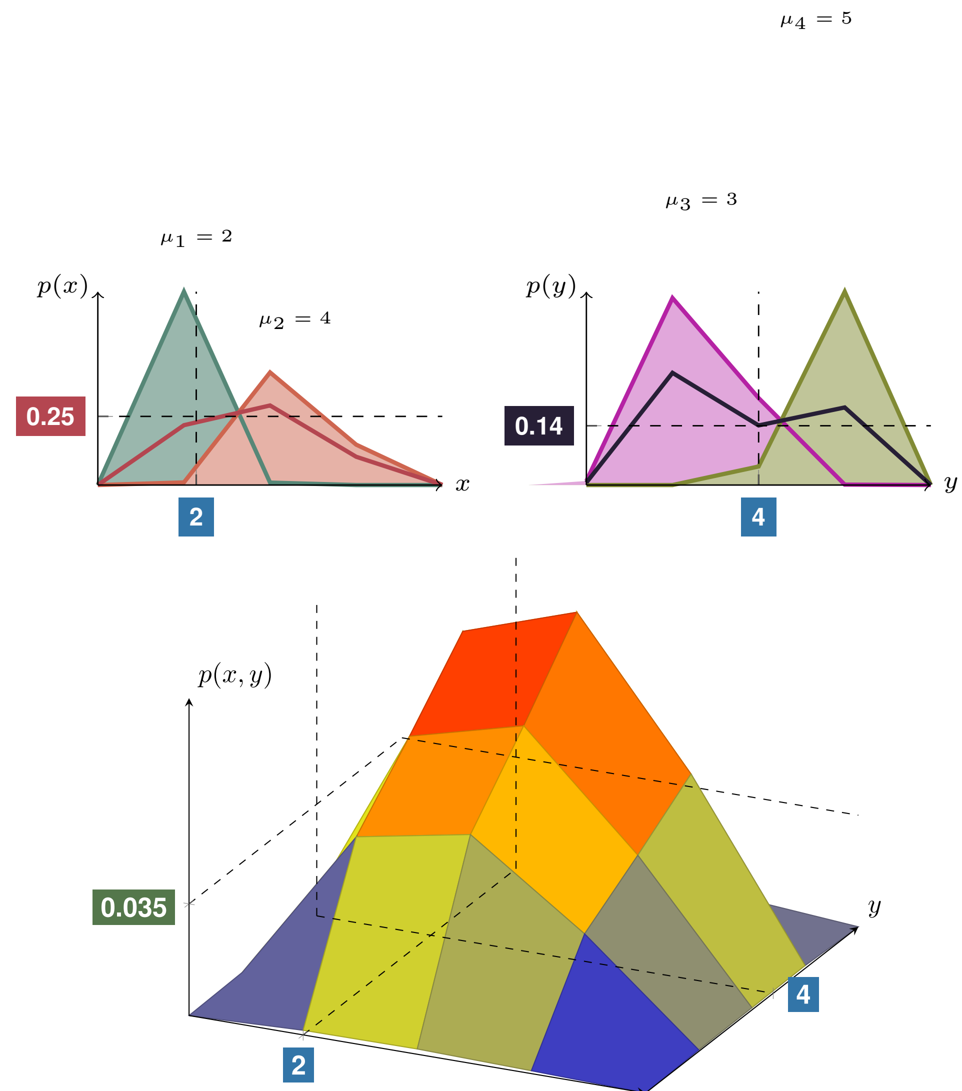


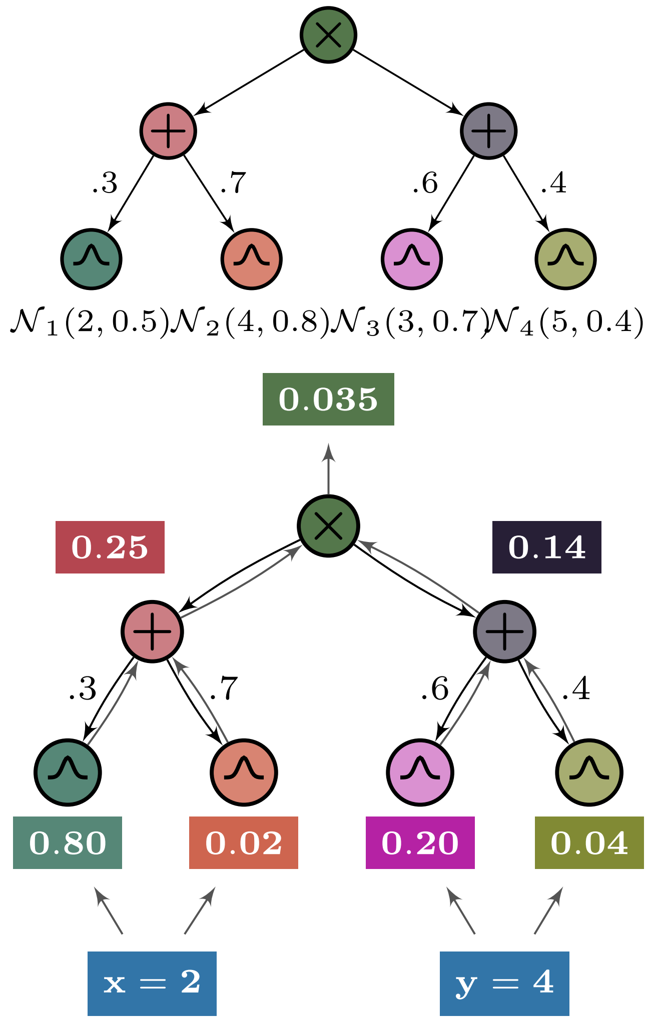


Definition 2 (Determinism)**.**
<u>At most one</u> sum node child has a positive value.

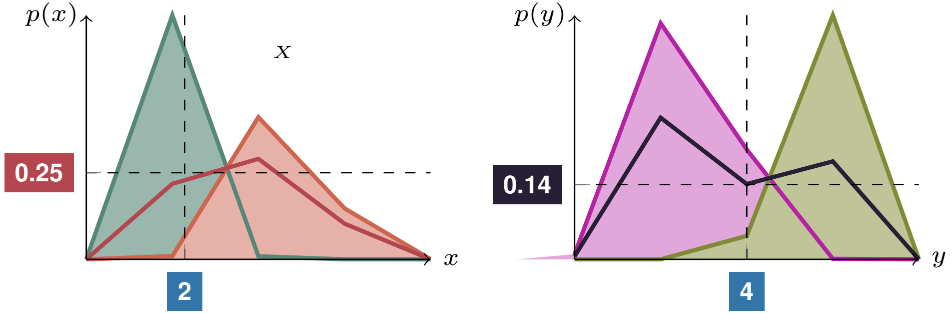
Darwiche [2001a]

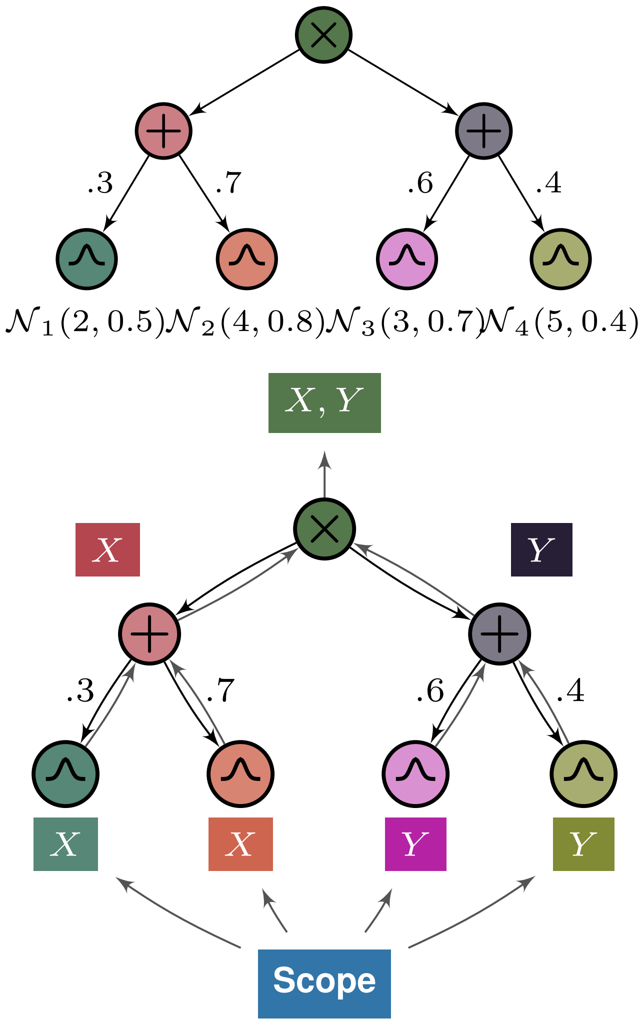
Probabilistic Circuits – Products





Probabilistic Circuits – Decomposability

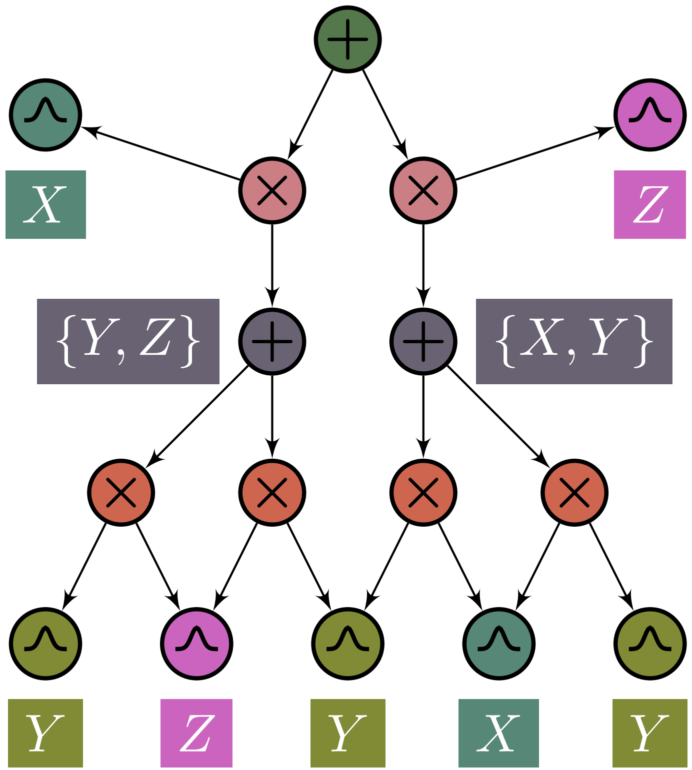


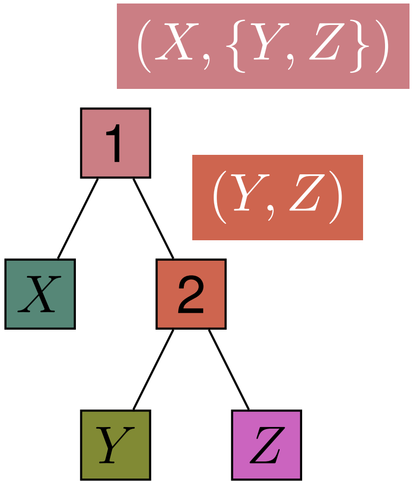


Definition 3 (Decomposability).
Every product node child mentions different variables.

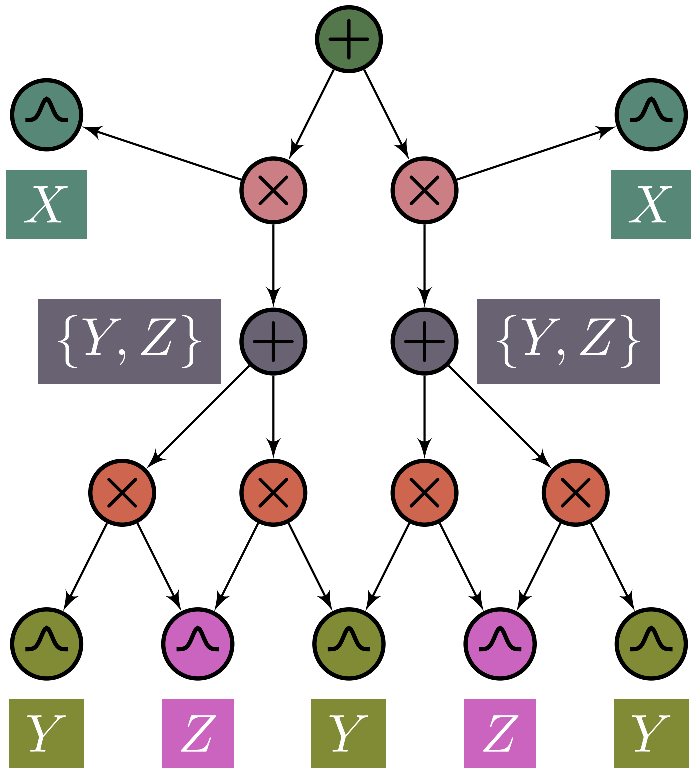
Darwiche [1999, 2001b]

Probabilistic Circuits – Structured Decomposability





vtree



Definition 4 (Structured decomposability)**.** *Every product node follows a vtree decomposition.*

Pipatsrisawat and Darwiche [2008]

Probabilistic Circuits – Tractability

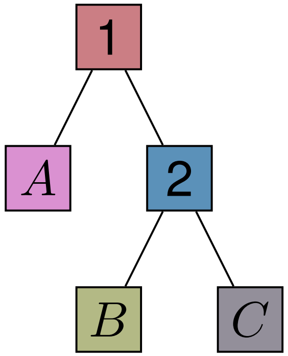
Query	+Sm?	+Dec?	+Det?	+Str Dec?
Evidence	✓	✓	✓	✓
Marginals	✗	✓	✓	✓
Conditionals	✗	✓	✓	✓
MPE	✗	✗	✓	✓
Shannon Entropy	✗	✗	✓	✓
Rényi Entropy	✗	✗	✓	✓
Cross Entropy	✗	✗	✗	✓
Kullback-Leibler Div	✗	✗	✗	✓
Rényi's Alpha Div	✗	✗	✗	✓
Cauchy-Schwarz Div	✗	✗	✗	✓
Logical Events	✗	✗	✗	✓
Mutual Information	✗	✗	✗	✓

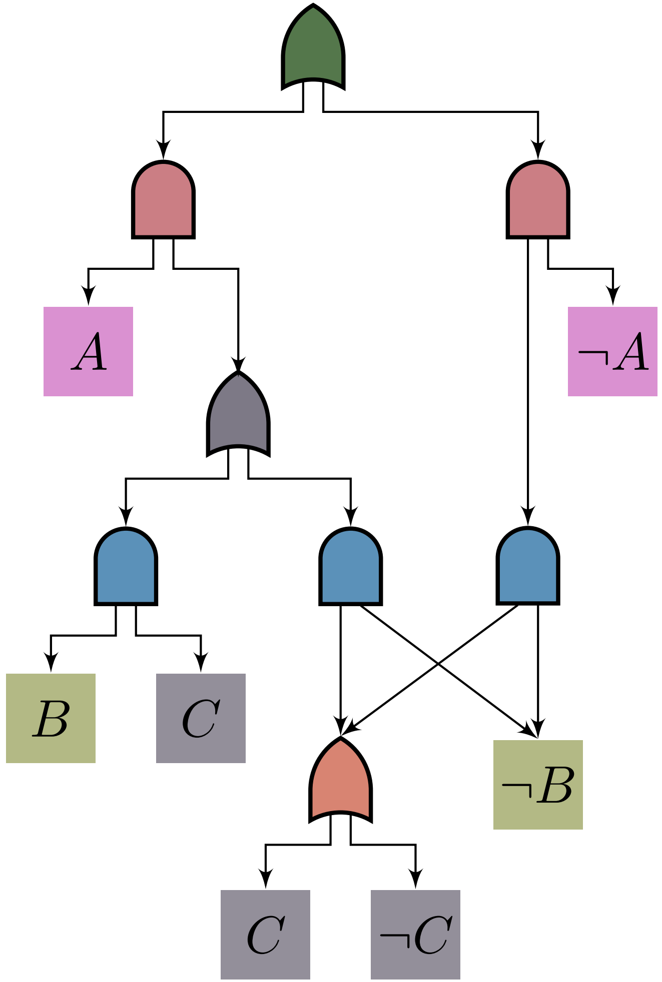
Vergari et al. [2021], Poon and Domingos [2011], Peharz et al. [2016]

Probabilistic Circuits – Logic Circuits

A	B	C	$\phi(\mathbf{x})$
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	0
0	0	1	1
1	0	1	1
0	1	1	0
1	1	1	1

$$\phi(A, B, C) = (A \vee B) \wedge (\neg B \vee C)$$



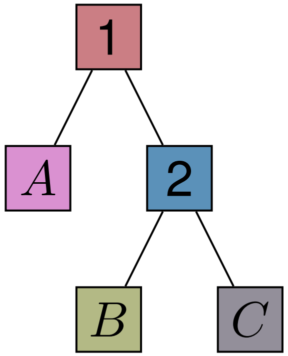


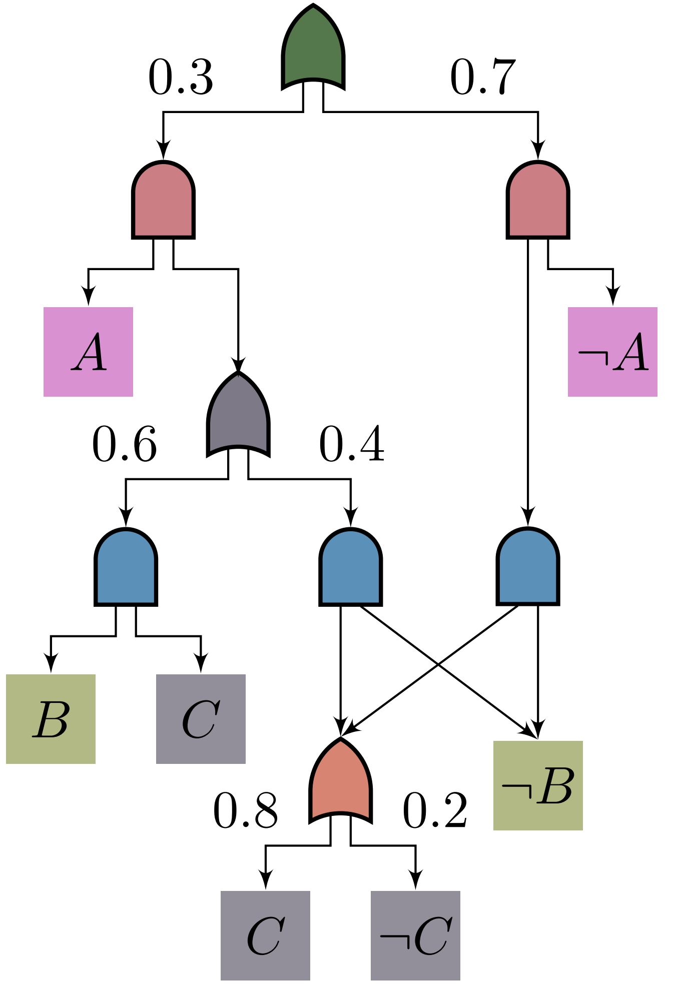
Darwiche [2011]

Probabilistic Circuits – Support

A	B	C	$\phi(\mathbf{x})$	$p(\mathbf{x})$
0	0	0	1	0.140
1	0	0	1	0.024
0	1	0	0	0.000
1	1	0	0	0.000
0	0	1	1	0.560
1	0	1	1	0.096
0	1	1	0	0.000
1	1	1	1	0.180

$$\phi(A, B, C) = (A \lor B) \land (\neg B \lor C)$$





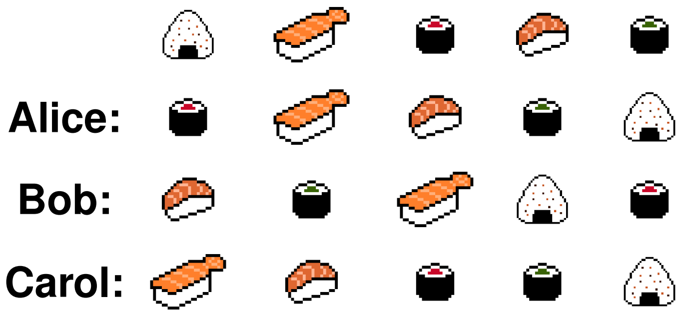
Kisa et al. [2014]

Learning Probabilistic Circuits – Where are we right now?

Name	Class	Time Complexity	# hyperparams	Accepts logic?	Sm?	Dec?	Det?	Str Dec?	$\{0,1\}$?	$\mathbb{N}$?	$\mathbb{R}$?	Reference
LearnSPN	DIV	$\begin{cases}\mathcal{O}(nkmc) & \text{, if sum}\\ \mathcal{O}(nm^3) & \text{, if product}\end{cases}$	≥ 2	✗	✓	✓	✗	✗	✓	✓	✓	Gens and Domingos [2013]
ID-SPN	DIV	$\begin{cases}\mathcal{O}(nkmc) & \text{, if sum}\\ \mathcal{O}(nm^3) & \text{, if product}\\ \mathcal{O}(ic(rn+m)) & \text{, if input}\end{cases}$	$\geq 2+3$	✗	✓	✓	✗	✗	✓	✓	✗	Rooshenas and Lowd [2014]
Prometheus	DIV	$\begin{cases}\mathcal{O}(nkmc) & \text{, if sum}\\ \mathcal{O}(m(\log m)^2) & \text{, if product}\end{cases}$	≥ 1	✗	✓	✓	✗	✗	✓	✓	✓	Jaini et al. [2018]
LearnPSDD	INCR	$\begin{cases}\mathcal{O}(m^2) & \text{, top-down vtree}\\ \mathcal{O}(m^4) & \text{, bottom-up vtree}\\ \mathcal{O}(i\lvert\mathcal{C}\rvert^2) & \text{, circuit structure}\end{cases}$	1	✓	✓	✓	✓	✓	✓	✗	✗	Liang et al. [2017]
Strudel	INCR	$\begin{cases}\mathcal{O}(m^2 n) & \text{, CLT + vtree}\\ \mathcal{O}(i(\lvert\mathcal{C}\rvert n+m^2)) & \text{, circuit structure}\end{cases}$	1	✓	✓	✓	✓	✓	✓	✗	✗	Dang et al. [2020]
RAT-SPN	RAND	$\mathcal{O}(rd(s+l))$	4	✗	✓	✓	✗	✗	✓	✓	✓	Peharz et al. [2020]
XPC	RAND	$\mathcal{O}(i(t+kn)+ikm^2n)$	3	✗	✓	✓	✓	✓	✓	✗	✗	Mauro et al. [2021]
SamplePSDD	RAND	$\begin{cases}\mathcal{O}(m) & \text{, random vtree}\\ \mathcal{O}(kc\log c+\log_2^2 k) & \text{, per call}\end{cases}$	1	✓	✓	✓	✓	✓	✓	✗	✗	Geh and Mauá [2021]
LearnRP	RAND	$\begin{cases}\mathcal{O}(m^2) & \text{, top-down vtree}\\ \mathcal{O}(m^4) & \text{, bottom-up vtree}\\ \mathcal{O}(knm) & \text{, per call}\end{cases}$	0	✗	✓	✓	✗	✓	✓	✓	✓	To appear

A Logical Perspective

Motivation



If we assume

- n sushi types,
- k sized rankings with $k \leq n$,
- X_{ij} binary variables; i is sushi type, j is position in ranking;

then the total number of possible assignments of the $n \cdot k$ variables is 2^{nk}...

...but many of these are zero probability assignments!

If we can embed total ranking constraints...

...we go down to k! total assignments!

Takeaway: models which exploit domain knowledge are much more efficient!

Example:

$n = 3,\ k = 3$

X_{11}	X_{12}	X_{13}	X_{21}	$\cdots$	X_{33}	$p(\mathbf{x}) > 0$
0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
1	1	0	0	0	0	0
0	0	1	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
0	1	1	1	1	1	0
1	1	1	1	1	1	0

Assignments: $2^{3 \cdot 3} = 512$

Positive assignments: $3! = 6$

Choi et al. [2015]

Motivation

Existing approaches:

LearnPSDD (Liang et al. [2017]):

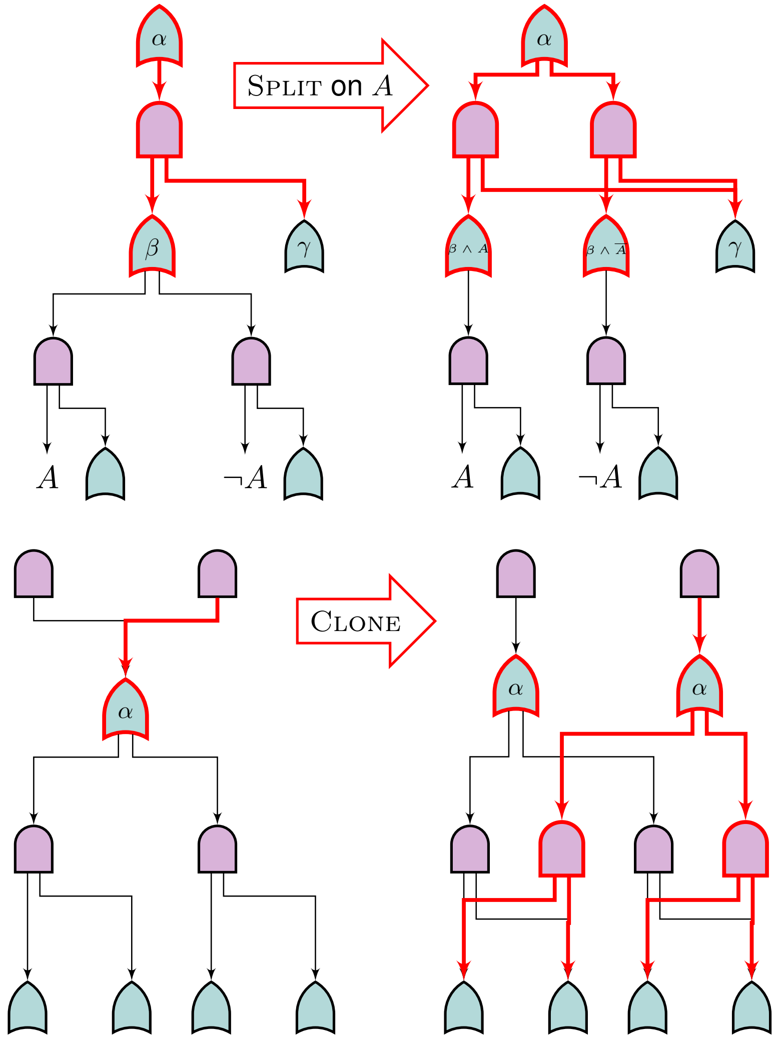
- ✗ Requires initial logic circuit encoding the support...
- ✗ Scales poorly to complex formulae and/or high dimension...
- ✗ Costly whole circuit evaluation at every iteration...
- ✓ Very good performance!

Strudel (Dang et al. [2020]):

- ✓ Constructs an initial structure (from a CLT)!
- ✗ But does not encode constraints...
- ✓ Scales to high dimension!
- ✗ As long as the circuit doesn't get too big...

SamplePSDD (Geh and Mauá [2021]):

- ✓ Scales to high dimension and complex formulae!
- ✓ Constructs a structure consistent with constraints!
- ✗ But does so by relaxing the formula...
- ✗ Performance varies on set bounds and vtree structure...

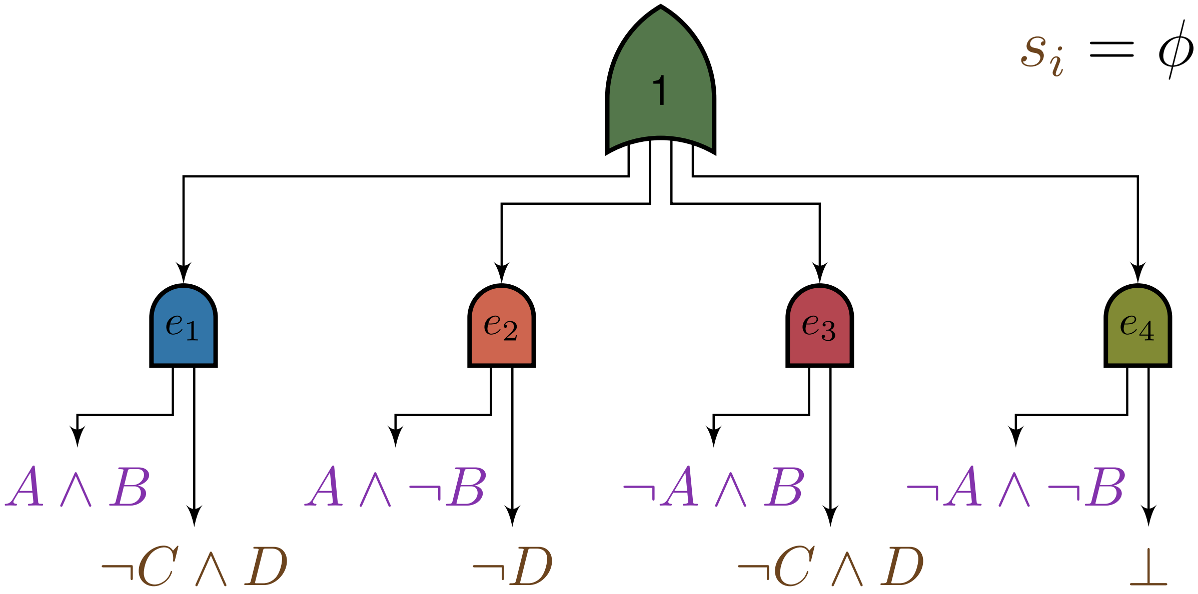


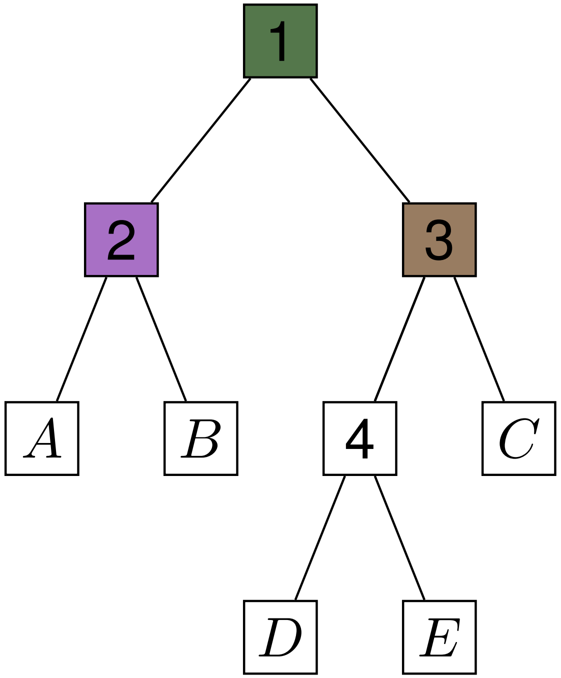
SamplePSDD

Common assumption: p_i are conjunctions of literals.

$$\phi(A, B, C, D) = (A \wedge \neg B \wedge \neg D) \vee (B \wedge \neg C \wedge D)$$

$$s_i = \phi|_{p_i}$$



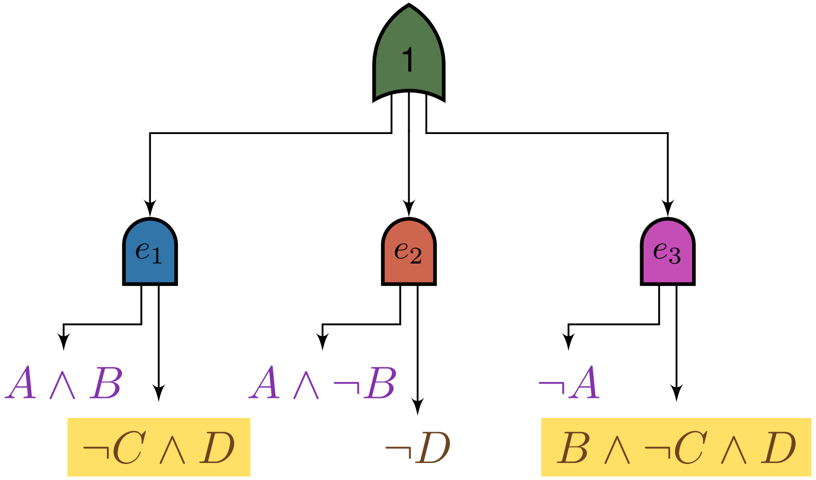


Problem: size of circuit is exponential in the size of p_i's scope.

SamplePSDD

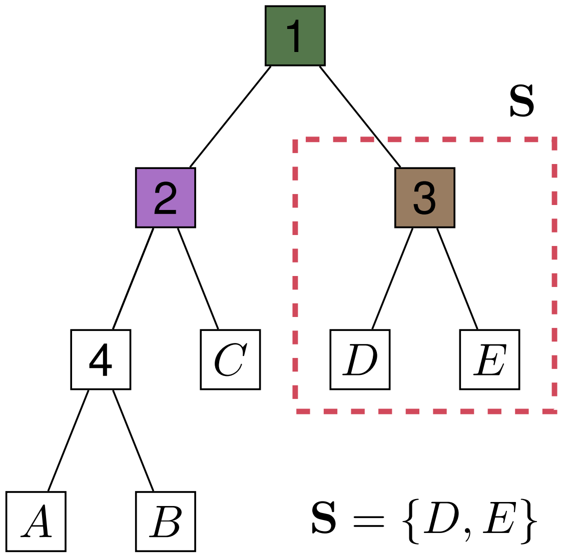
Solution: randomly sample a bounded number (k) of p_i

$$\phi(A, B, C, D) = (A \wedge \neg B \wedge \neg D) \vee (B \wedge \neg C \wedge D)$$



$$s_i = \phi|_{p_i}$$

$\mathrm{Sc}(s_3) \not\subseteq \mathbf{S}$



$\mathbf{S} = \{D, E\}$

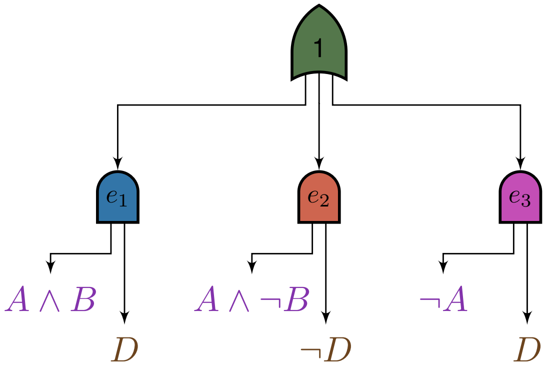
But: this violates structured decomposability:

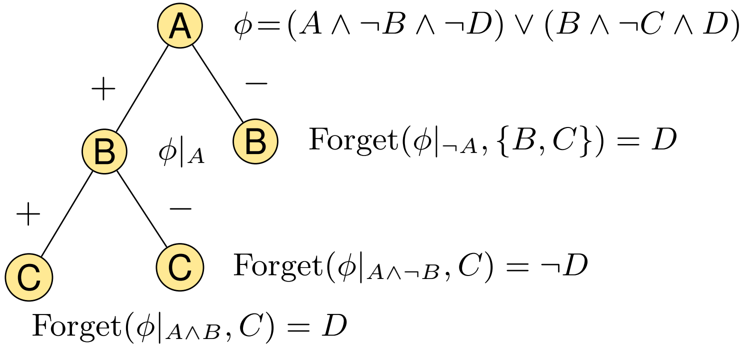
$\neg C \wedge D$ contains C, and $C \notin \mathbf{S}$

$\neg B \wedge \neg C \wedge D$ contains B and C, and $B, C \notin \mathbf{S}$

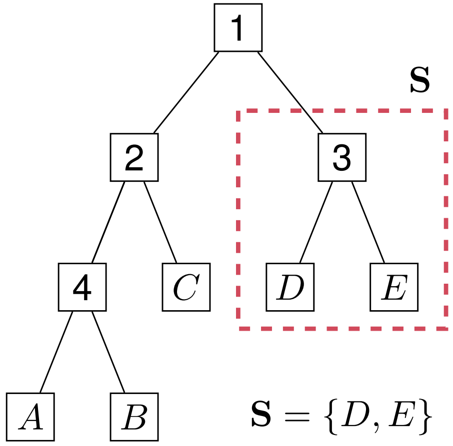
SAMPLEPSDD

New solution: relax logical constraints ϕ



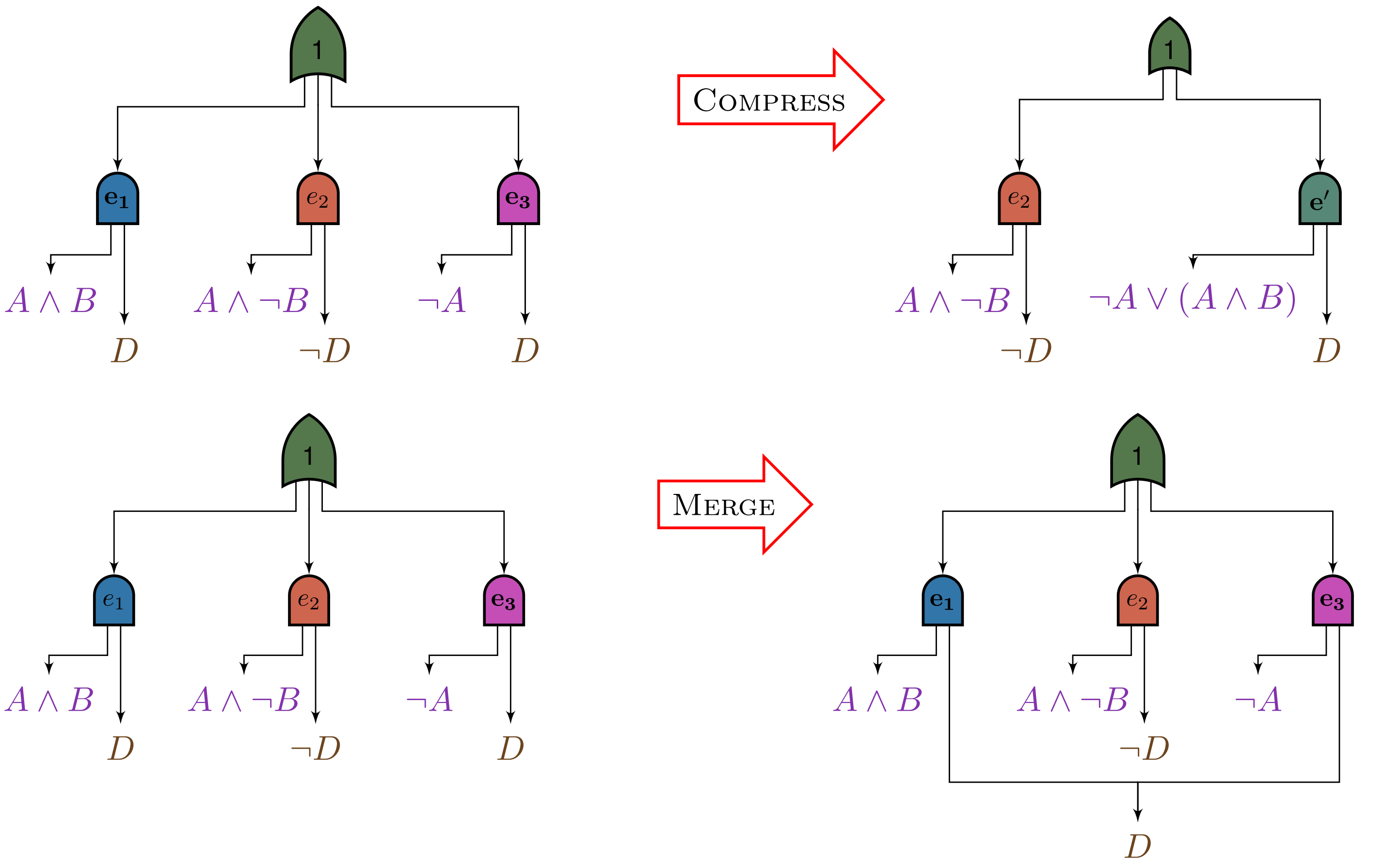


Now all s_i respect $\mathbf{S}$



SamplePSDD

Apply **local transformations** for variety and size reduction



Darwiche [2011]

Experiments

Evaluation: we sample 30 PSDDs and use 5 ensemble strategies:

- ● **Likelihood weighting (LLW)**,
- ■ **Uniform weights**,
- ◆ **Expectation Maximization (EM)**,
- ▲ **Stacking**,
- ▼ **Bayesian Model Combination (BMC)**;

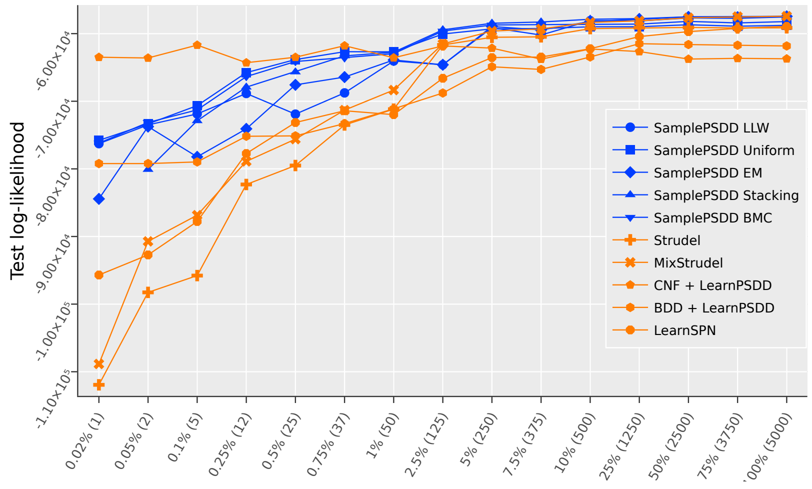
comparing against STRUDEL, LEARNPSDD and LEARNSPN.

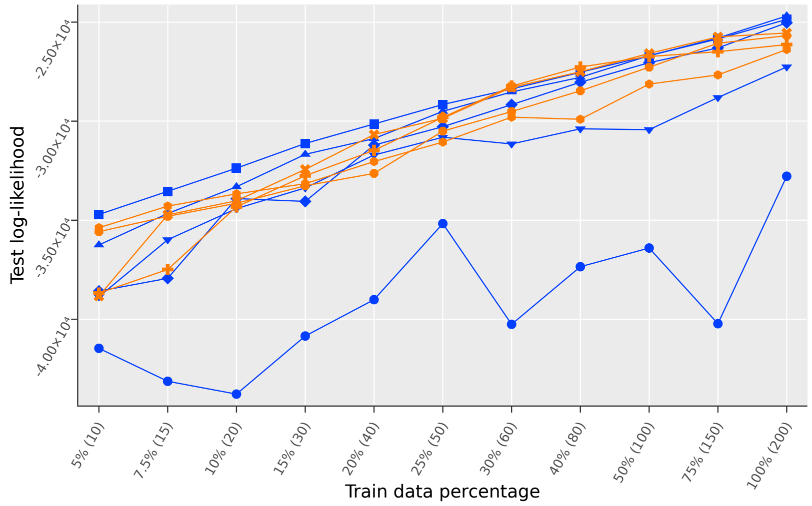
Datasets: we evaluate with 5 data + knowledge as logic constraints:

	Dataset	#vars	#train	ϕ's size
⇒	LED	14	5000	23
⇒	LED + IMAGES	157	700	39899
	SUSHI RANKING	100	3500	17413
	SUSHI TOP 5	10	3500	37
	DOTA 2 GAMES	227	92650	1308

Our approach fares **better** with **fewer** data, yet remains **competitive** under **lots of data**.

Mattei et al. [2020], Kamishima [2003], Shen et al. [2017], Choi et al. [2015], Gens and Domingos [2013], Dang et al. [2020]





Experiments

Evaluation: we sample 30 PSDDs and use 5 ensemble strategies:

- ● **Likelihood weighting (LLW)**,
- ■ **Uniform weights**,
- ◆ **Expectation Maximization (EM)**,
- ▲ **Stacking**,
- ▼ **Bayesian Model Combination (BMC)**;

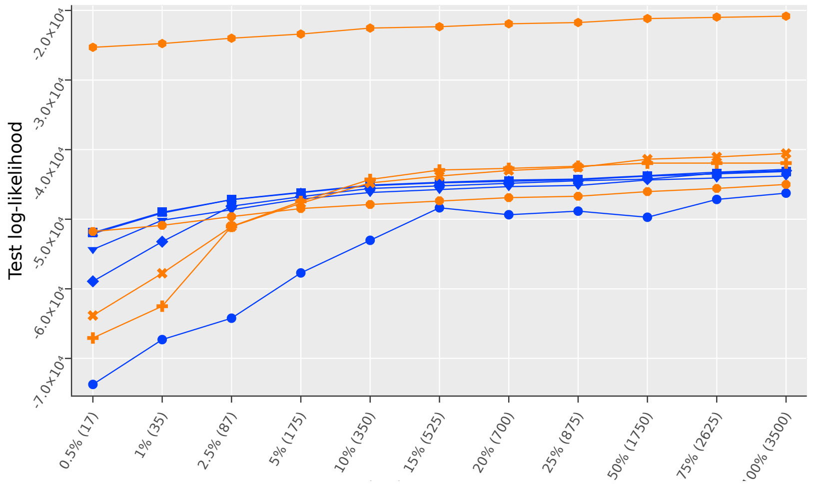
comparing against **Strudel**, **LearnPSDD** and **LearnSPN**.

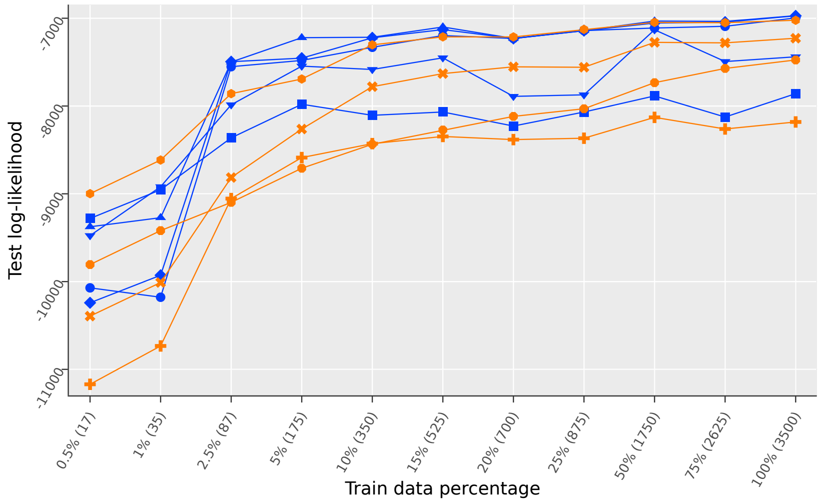
Datasets: we evaluate with 5 data + knowledge as logic constraints:

	Dataset	#vars	#train	ϕ's size
	LED	14	5000	23
	LED + Images	157	700	39899
$\Rightarrow$	Sushi Ranking	100	3500	17413
$\Rightarrow$	Sushi Top 5	10	3500	37
	Dota 2 Games	227	92650	1308

Our approach fares **better** with **fewer** data, yet remains **competitive** under **lots of data**.

Mattei et al. [2020], Kamishima [2003], Shen et al. [2017], Choi et al. [2015], Gens and Domingos [2013], Dang et al. [2020]





Experiments

Evaluation: we sample 30 PSDDs and use 5 ensemble strategies:

- ● **Likelihood weighting (LLW)**,
- ■ **Uniform weights**,
- ◆ **Expectation Maximization (EM)**,
- ▲ **Stacking**,
- ▼ **Bayesian Model Combination (BMC)**;

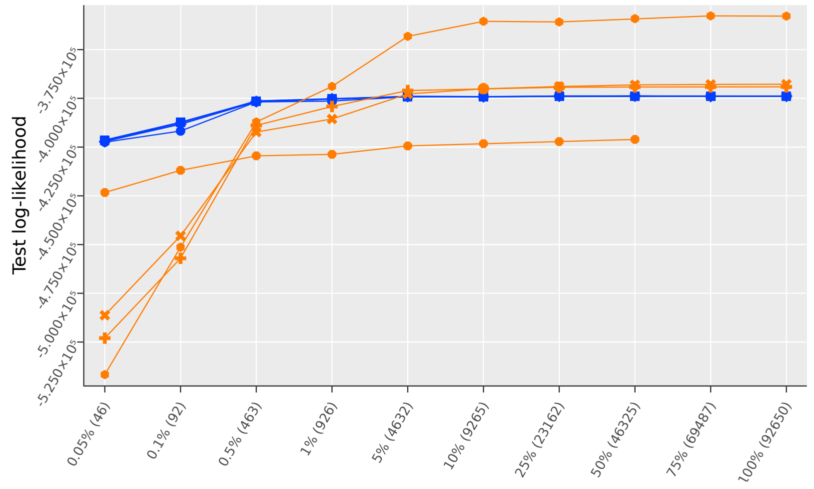
comparing against **Strudel**, **LearnPSDD** and **LearnSPN**.

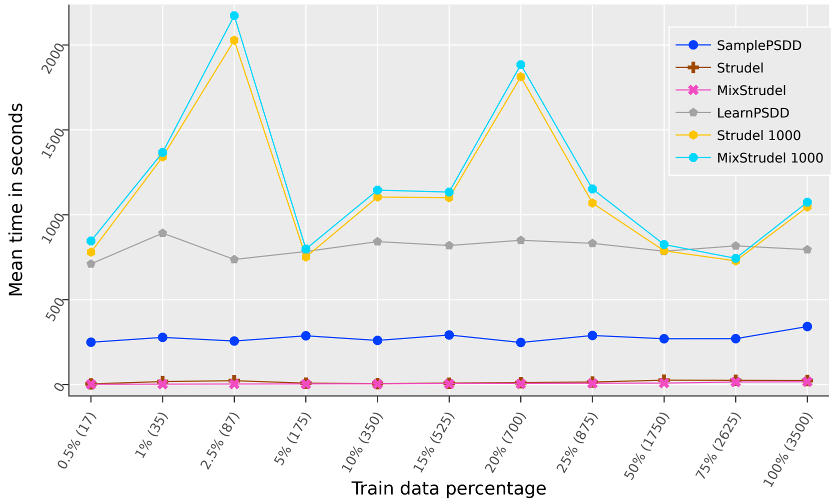
Datasets: we evaluate with 5 data + knowledge as logic constraints:

Dataset	#vars	#train	ϕ's size
LED	14	5000	23
LED + Images	157	700	39899
Sushi Ranking	100	3500	17413
Sushi Top 5	10	3500	37
$\Rightarrow$ Dota 2 Games	227	92650	1308

Our approach fares **better** with **fewer** data, yet remains **competitive** under **lots of data**.

Mattei et al. [2020], Kamishima [2003], Shen et al. [2017], Choi et al. [2015], Gens and Domingos [2013], Dang et al. [2020]

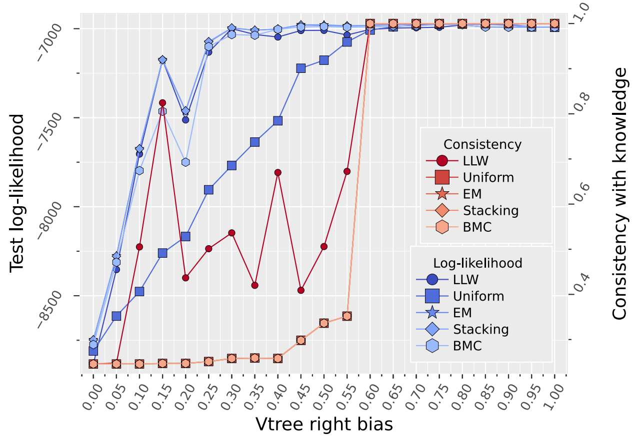


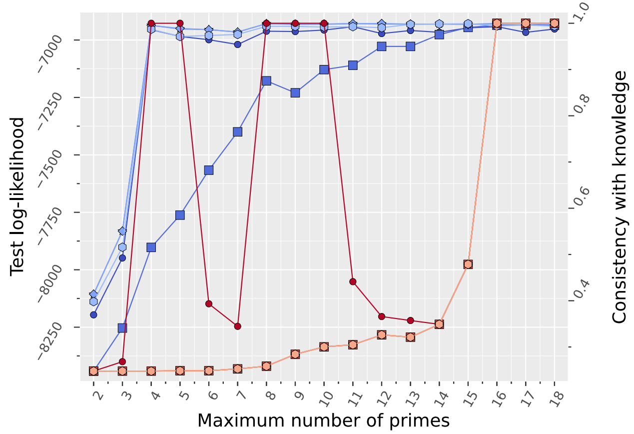


Experiments

What is the impact of higher k's and right-leaning vtrees

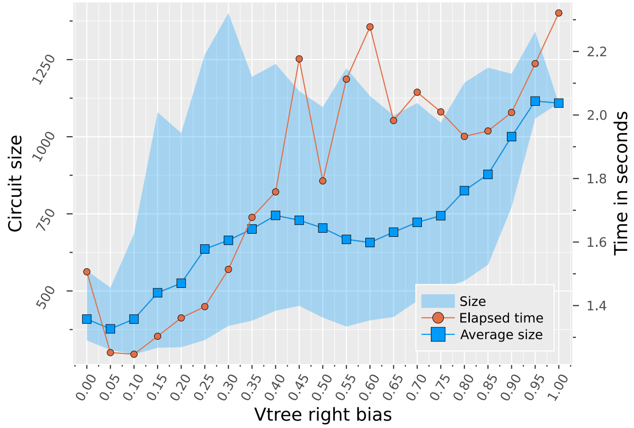
in log-likelihood and consistency?

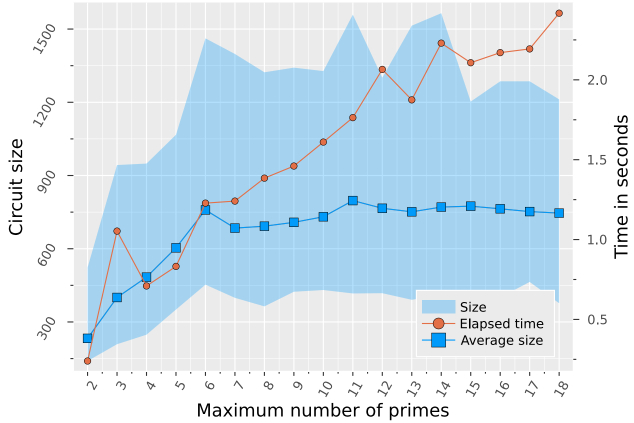




Samples perform **better** with higher k's and right-leaning vtrees...

...but at a **cost** to complexity.





What do we gain from this?

Available queries:

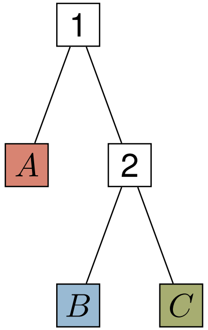
- ☑ Probability of Evidence;
- ☑ Marginal Probability;
- ☑ Conditional Probability;
- ☑ Most Probable Explanation;
- ☑ Shannon Entropy;
- ☑ Cross Entropy;
- ☑ Kullback-Leibler Divergence;
- ☑ Rényi's Alpha Divergence;
- ☑ Cauchy-Schwarz Divergence;
- ☑ Probability of Logical Events;
- ☑ Mutual Information.

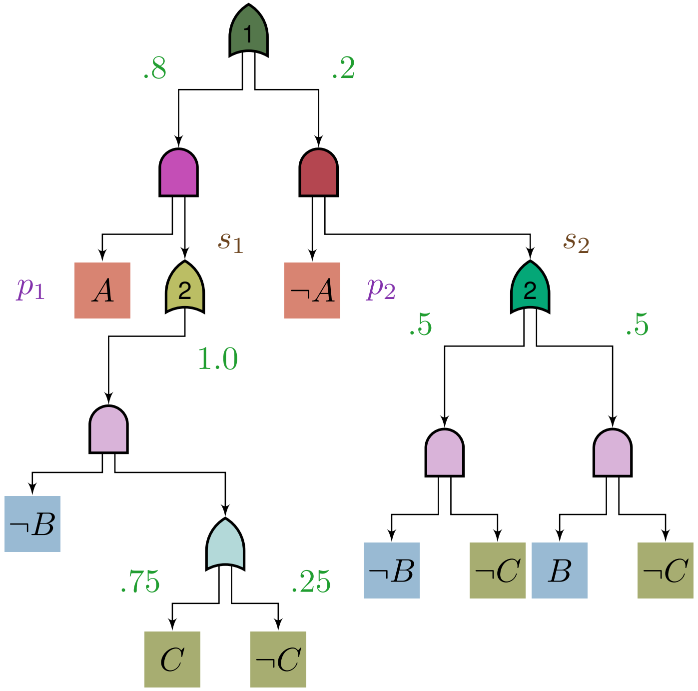
Support:

- ☑ Defineable as a logic formula;
- ✗ Consistent with a relaxation;
- ☑ Ensembles mitigate relaxation.

A	B	C	$p(\mathbf{x})$
0	0	0	0.1
0	1	0	0.1
1	0	0	0.2
1	0	1	0.6

$$\phi(A,B,C) = (A \to \neg B) \land (C \to A)$$





A Data Perspective

Motivation

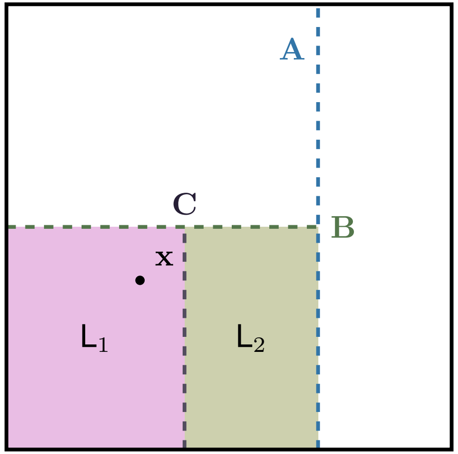
Density Estimation Trees...

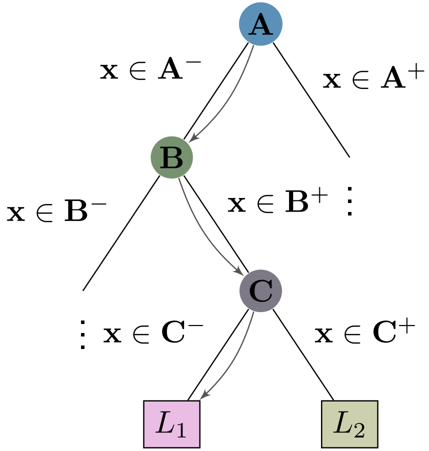
- ☑ ...are fast;
- ☑ ...are interpretable;
- ☑ ...are (somewhat) explainable;
- ☑ ...have extensive literature coverage;
- ☒ ...are not so expressive;
- ☒ ...only accept marginalization queries;
- ☒ ...are not so accurate;

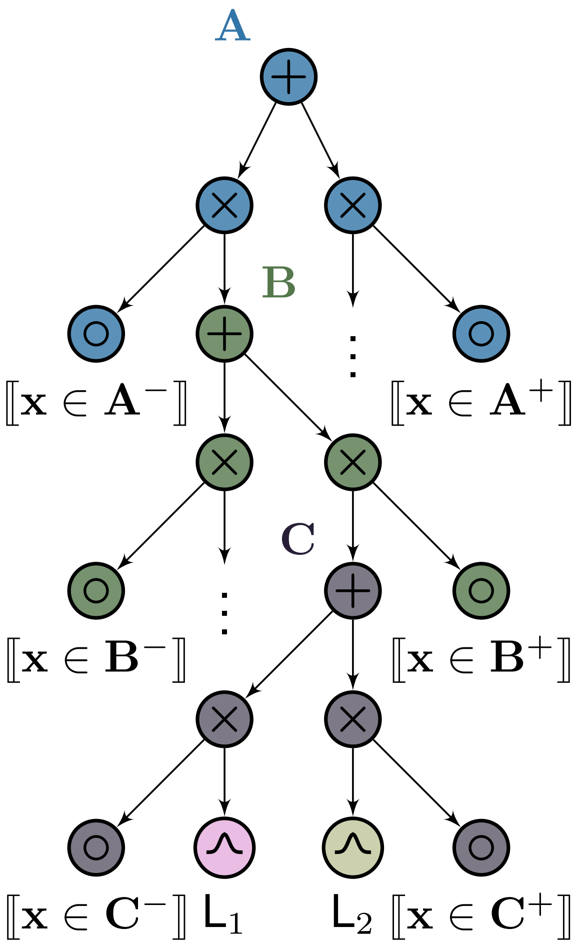
...but are subsumed by circuits!

Learn DETs $\subseteq$ Learn PCs?

Can we take advantage of known learning procedures in DETs and transplant them to more general circuits?

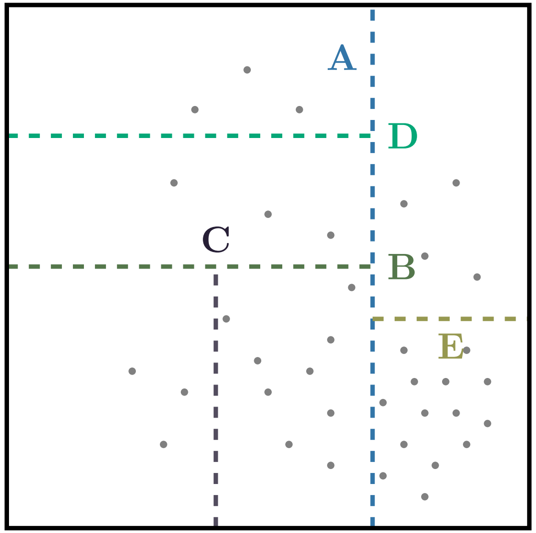




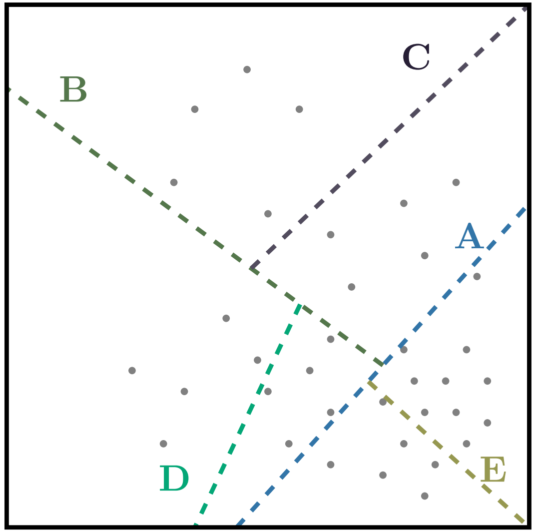


Correia et al. [2020]

Random Projections



Axis-aligned projections

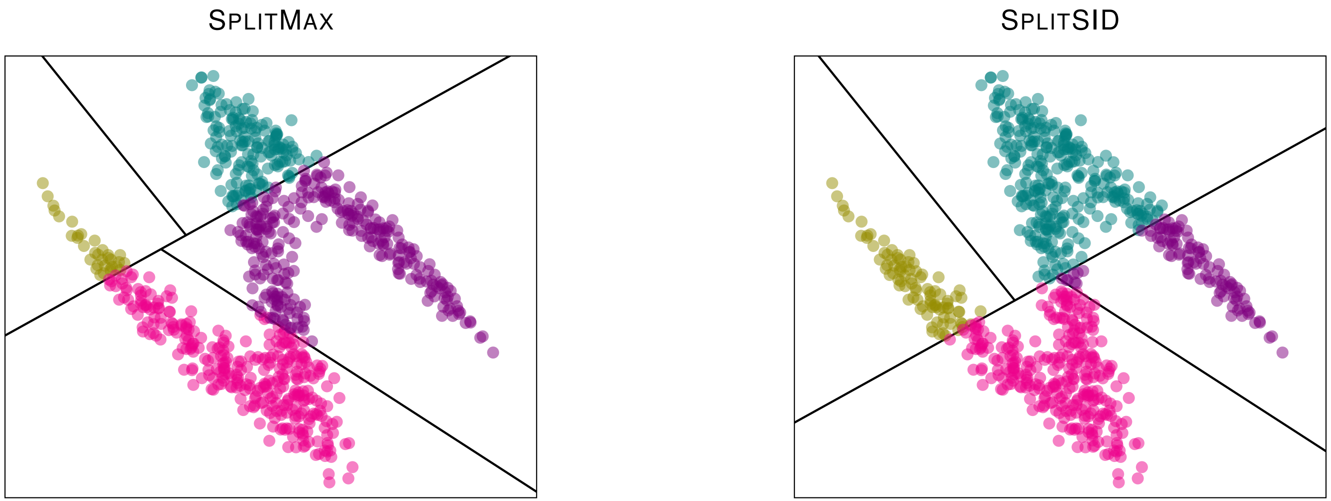


Random projections

If the data has *intrinsic dimension* d, then with constant probability the part of the data at level d or higher of the tree has average diameter less than half of the data.

Freund et al. [2008], Dasgupta and Freund [2008]

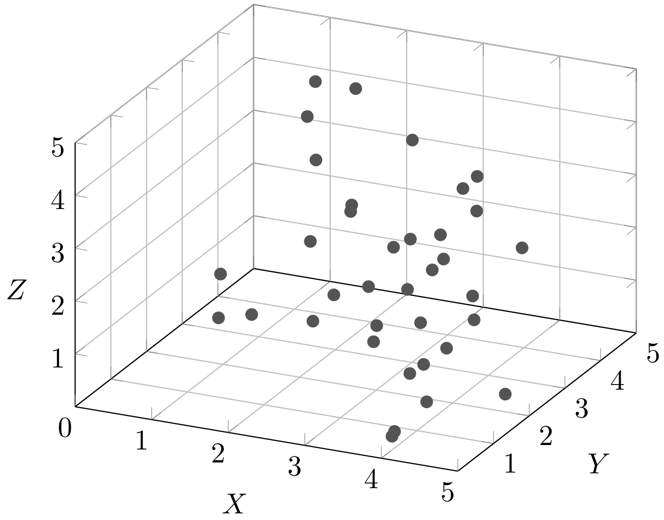
Random Projections

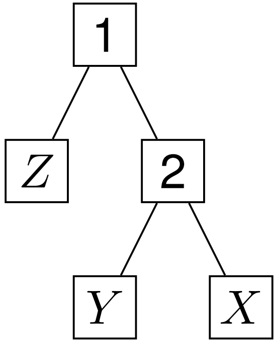


If the data has *intrinsic dimension* d, then with constant probability the part of the data at level d or higher of the tree has average diameter less than half of the data.

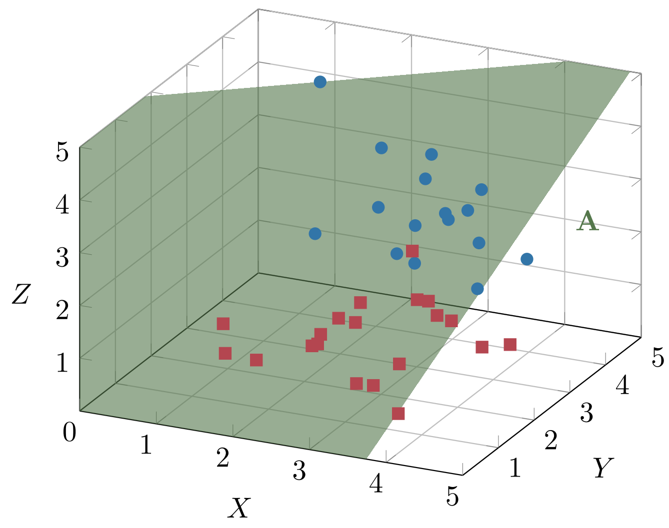
Freund et al. [2008], Dasgupta and Freund [2008]

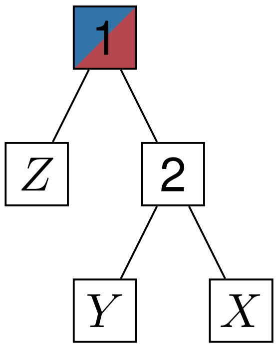
LearnRP



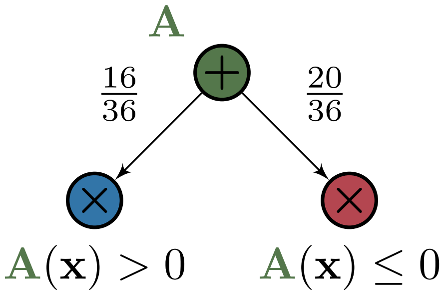


LearnRP





$$\mathbf{A}(x, y, z) = \begin{bmatrix} x & y & z \end{bmatrix} \cdot \begin{bmatrix} -0.31 \\ -0.40 \\ 0.85 \end{bmatrix} + 1$$



LearnRP