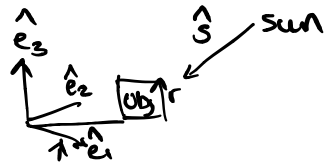
L4 CH3 Using Non-Square Matrices to do Projections Quiz

Matematika 2 (Universitas Katolik Widya Mandala Surabaya)

PROBLEM #1.



shadow $= r' = r + \lambda \hat{s}$

Get r' in terms of r.

Eq 1. $r' = r + \lambda \hat{s}$

Eq 2. $r'_3 = 0$, since $r' \cdot \hat{e}_3 = 0$

Eq 3. $0 = r \cdot \hat{e}_3 + \lambda(\hat{s} \cdot \hat{e}_3)$

① Get lambda in terms of r → that is

$$r \cdot \hat{e}_3 = (-1) \operatorname{proj}_{r} \hat{e}_3$$

$$0 = r \cdot \hat{e}_3 + \lambda(\hat{s} \cdot \hat{e}_3)$$

$$\lambda = \frac{-r \cdot \hat{e}_3}{\hat{s} \cdot \hat{e}_3}$$

, only makes sense because $\hat{e}_3$ is unit orthogonal vector.
↳ basically how many vertical comps of $\hat{e}_3$ needed to match vert of r.

② $$r' = r - \hat{s}\left(\frac{r \cdot \hat{e}_3}{\hat{s} \cdot \hat{e}_3}\right)$$

$\square$.

PROBLEM #2.

Write $r' = r - \hat{s}\left(\frac{r \cdot \hat{e}_3}{\hat{s} \cdot \hat{e}_3}\right)$ as a linear transformation of r. (e.g $Ar = r'$)

Also write einstein summation convention of $Ar = r'$.

As vectors:

$$r' = r - \hat{s} * \frac{r_3}{s_3}$$

Einstein:

$$r' = r_i - \frac{s_i r_3}{s_3} \rightarrow r'_i = r_i - \frac{s_i r_3}{s_3}$$

$$r'_i = (I_{ij} - s_i I_{3j}/s_3) r_j$$

As matrix:

$$\left[r_1 - \frac{s_1 r_3}{s_3} \quad r_2 - \frac{s_2 r_3}{s_3} \quad r_3 - \frac{s_3 r_3}{s_3} \right]^T = \begin{bmatrix} r_1 - \frac{s_1 r_3}{s_3} \\ r_2 - \frac{s_2 r_3}{s_3} \\ 0 \end{bmatrix}$$

PROBLEM #3.

Given the einstein sum. conv. for the matrix

$$r'_i = \left(I_{ij} - s_i I_{3j}/s_3\right) r_j$$

Write the component form of A.

$$\begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix}$$

PROBLEM #4.

· Write the third row of A from Prob #2 if the matrix went from $\mathbb{R}^3 \rightarrow \mathbb{R}^3$:

$$A_3 = \left[0 \quad 0 \quad (1 - s_i/s_3)\right]$$

$$= [0 \quad 0 \quad (1-1)]$$

$$= [0 \quad 0 \quad 0]$$

PROBLEM #5

Calculate b from $Ax = b$.

$$A = \begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix} \quad x = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} \quad s = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix}$$

$$b = \begin{bmatrix} (1)(6) + (0)(2) + (3)(-1)\dfrac{(4/13)}{(-12/13)} \\ (0)(6) + (1)(2) + (3)(-1)\dfrac{(-3/13)}{(-12/13)} \end{bmatrix}$$

$$= \begin{matrix} 6 + 1 \\ 2 - 3/4 \end{matrix} = \left[7 \quad 3/4\right]$$

PROBLEM #6

A transformation $r' = Ar$ can be generalized to a matrix equation,

$$R' = AR$$

where R' and R are matrices where each col. corresponds to r' and r vectors.

$$\underbrace{\begin{bmatrix} r'_1 & s'_1 & t'_1 & u'_1 & \\ r'_2 & s'_2 & t'_2 & u'_2 & \cdots \end{bmatrix}}_{R'} = A \begin{bmatrix} r_1 & s_1 & t_1 & u_1 & \\ r_2 & s_2 & t_2 & u_2 & \cdots \\ r_3 & s_3 & t_3 & u_3 & \end{bmatrix}$$

In einstein's notatation

$$r'_i = A_{ij} r_j \quad \text{becomes} \quad R'_{ia} = A_{ij} R_{ja}. \quad \text{cool!}$$

For the same s as previous question, apply A to the matrix,

$$R = \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix} \qquad s = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix}$$

$$R' = \begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix} \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix}, \quad \text{using } s_i = \vec{s}_i$$

Let $c_1 = -s_1/s_3 = (4/13)(-1)(13/-12) = 1/3$, $c_2 = -s_2/s_3 = (-3/13)(-1)(-13/12) = -1/4$,

$$R' = \begin{bmatrix} (1)(5)+(9)c_1^{\,3} & (-1)+3c_1^{\,1} & -3+0c_1^{\,0} & 7+12c_1^{\,4} \\ 4+9c_2^{\,-9/4} & -4+3c_2^{\,-3/4} & 1+0c_2^{\,0} & -2+12c_1^{\,-3} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & -3 & 11 \\ 7/4 & -\frac{9}{4} & 1 & -5 \end{bmatrix}$$