Introduction to Numerical Optimization – Project 2024

Investment Portfolio Management

In this project, you will work on an investment portfolio management project. Assume you are a financial analyst advising a client, an individual investor, who wants to compose a portfolio of diverse stocks with a total capital of €500,000 to invest. The weekly closing values for a set of 462 stocks collected over 568 weeks are provided in the file `data.csv`. Stocks can be grouped by sectors, which are provided in the file `sector_mapping.csv`.

Linear Model

In this section, your task is to compose the portfolio of your client in order to maximize the expected return of the invested amount of money. In addition, the portfolio should respect a constraint limiting the exposure to the sectors, meaning that stocks should be diversified across all sectors.

The problem will be solved using a simplified model. First, we call a portfolio a set of stocks and the value of the portfolio is the sum of the values of the stocks it is composed of. The return of a stock is the relative difference of the stock value over a defined time period. Similarly, the return of a portfolio is the relative difference of the portfolio value over a defined time period and can be computed as the weighted sum of the return of the stocks composing the portfolio. The return of each stock can be computed using historical time series of the stock prices. The weekly return $r_{i,t}$ for a stock i at week t is the variation, taken as a percentage, in their weekly closing values

$$r_{i,t} = \frac{p_{i,t} - p_{i,t-1}}{p_{i,t-1}} * 100, \tag{1}$$

where $p_{i,t}$ is the closing price of stock i at week t.

Your task is to build a portfolio of stocks with maximum historical average weekly return using the time series you are provided. Moreover, to limit the exposure to sectors, no more than 20% of the total capital should be allocated to a given sector.

1. Formulate mathematically the problem as a linear program. Define explicitly the variables, objective function, and constraints.
2. Solve numerically the problem. Give the composition of the portfolio and the means of historical return. Discuss the results.
3. Formulate the dual optimization problem. Interpret theoretically and physically the dual variables.
4. Based on the primal solution of the problem, explain how to compute the optimal dual variables. Using an attribute of the solver, give these values and interpret the results.
5. Compare the primal and the dual simplex algorithms for solving the problem.
6. Assume the limit l_6 on the capital allocated to the sector number 6 may be changed by a value Δl_6. Give the interval outside of which the optimal basis would change. In this interval, give the new optimal solution for any value of Δl_6. Interpret the results.

Non-Linear Model: Risk Management

In this section, we want to maximize the utility, representing the trade-off between the average historical return and the risk of the portfolio. The trade-off is implemented as an affine combination of both quantities, where the weight of the risk of the portfolio γ is the risk-aversion coefficient. It controls how much weight is placed on risk versus return. The risk of the portfolio is measured by the variance of the portfolio's average historical return

$$\sigma^2 = x^T \Sigma x, \tag{2}$$

where x is the vector of the fractions of the capital invested in each stock, and Σ is the covariance matrix of the average historical return of each stock.

7. Formulate the problem accounting for the risk mathematically.

8. Solve the problem for the values of γ given in the file `gamma_vals.csv`. Plot the expected return as a function of the risk for the different values of γ, which is called the efficient frontier. Give a physical interpretation of this figure and discuss the results.

Deliverables

Students will work in pairs or individually on this project. For organisational reasons, each group is expected to send an email to Laurie Boveroux (`laurie.boveroux@uliege.be`) before 17 October with the names of the group members.

Each group is expected to submit a short report in PDF format (maximum 4 pages) describing the problem formulations used and discussing the results. The code supporting the report should be implemented in Julia. The report and the Julia file must be submitted to Gradescope before 20:00 on 23 November.

Presentations of methods and results will take place on 27 November. However, groups will only present if the evaluators consider the content submitted sufficient. The exact format of the presentation will be communicated in due time.

If the instructions are not followed, the group will be penalised. **No second session will be organised for the project**, the grade will be final.