THERE ARE 3 HANDOUTS BY THE DOOR.

PLEASE FILL OUT THE BACKGROUND SURVEY WHILE YOU WAIT.

Lecture 1: Welcome!

August 28, 2023

CS 81: Computability and Logic

> Contrariwise, if it was so, it might be; and if it were so, it would be;
> but as it isn't, it ain't. That's logic.
>
> —Lewis Carroll

> Science may be described as the art of systematic oversimplification — the art of discerning what we may with advantage omit.
>
> —Karl Popper

Read the Syllabus! But for now...

Faculty:

- Two professors for the price of one! Chris Stone and Calden Wloka

Homework:

- Due Tuesdays before midnight, submit via Gradescope (PDF and/or code)
- Don't handwrite (except pictures/diagrams)

Homework Collaborations:

- Try the problems on your own first!
- Then OK to discuss with classmates (but focus on joint learning)
- Hand in only what you understand, in *your own* words.

Grade: 55% homework, 20% midterm, 25% final

Logic as a Topic of Study

We'd like to reason *about* correct logical arguments.

Specific questions:

- Is this step in our argument valid?
- Is this entire argument logically correct?

General questions:

- How many *kinds* of proof steps do we need to ensure that we can prove all true statements?
- What reasoning principles are safe to use in our proofs?

Inferences and Valid Inferences

An *inference* has assumptions and a conclusion.

Assumption 1	Everyone loves a lover.
Assumption 2	Romeo loves Juliet.
Conclusion	Everyone loves Juliet.

How can we recognize a *valid* (correct) inference?

Maybe we should check the reasoning?

1. Even if the reasoning is wrong, the conclusion might be "right"
2. Each step is an inference; how do we know *those* are valid ?!

We need a way to define "valid inference" that is not circular.

The *Entailment* Relation

Assumptions $\mathcal{A}_1, \ldots, \mathcal{A}_n$ *entail* a conclusion $\mathcal{B}$

when

in <u>every</u> situation where all the assumptions are true,
the conclusion is true.

Equivalently:

there's <u>no</u> counterexample situation where
the assumptions are true, but the conclusion is false.

In this case we write

$$\mathcal{A}_1, \ldots, \mathcal{A}_n \models \mathcal{B}$$

and say the inference from $\mathcal{A}_1, \ldots, \mathcal{A}_n$ to $\mathcal{B}$ is *valid*.

Examples

This dish is a pudding, All nice things are unwholesome, All puddings are nice	$\models$	This dish is unwholesome.
This dish is a pudding, All unwholesome things are nice, All puddings are nice	$\not\models$	This dish is unwholesome.
Everyone loves a lover, Romeo loves Juliet	$\models$	Everyone loves Juliet.

Problem: Language can be Ambiguous

This cat is a calico,
This cat is mine $\models$ This cat is my calico

This dog is a father,
This dog is mine $\overset{?}{\models}$ This dog is my father

Problem: Language can be Ambiguous

These steelworkers are unionized $\overset{?}{\models}$ These steelworkers are electrically neutral

He saw a girl with a telescope $\overset{?}{\models}$ The girl had a telescope

Do We Even Agree What a Proposition Is?

Are these propositions?

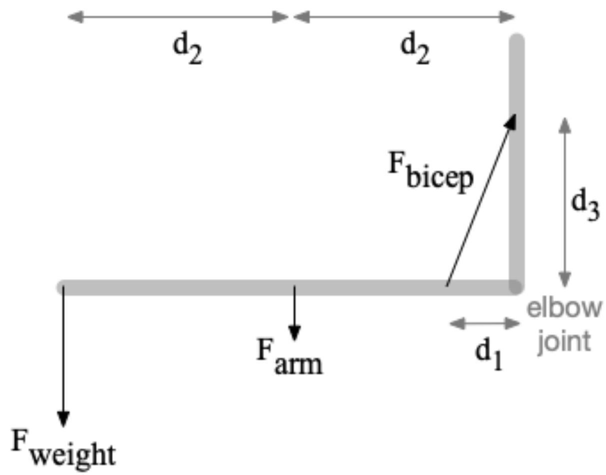
Živim v Claremontu.

Polish sausage bowls.

Beige.

This sentence is false.

Formal Models (Mathematically Precise)



$$NaHCO_3 + CH_3COOH \longrightarrow CO_2 + H_2O + CH_3COONa$$

- Propositional Logic, Predicate Logic: formal models of reasoning
- Finite Automata, Turing Machines: formal models of computation

Formal Logic

Formal logic is logic done using an *artificial* language designed with mathematical precision.

- Unambiguous whether a sequence of symbols is a proposition
- Unambiguous how to interpret each proposition
- Unambiguous whether a proposition is true or false in a situation

Classical Propositional Logic

Classical Propositional Logic

1. We define a set of strings (*Well-Formed Formulas*) that count as propositions.

2. We define *entailment* (valid inferences) for WFFs. Specifically,
   - We define what counts as a situation (a *model*).
   - We show how to decide if a WFF is true or false in a model.

3. (Next lecture) We define what counts as a correct *proof*.
   - More than one option here!
   - If we get our definition right, all true entailments are provable, and all provable entailments are valid!

Well-Formed Formulas of Propositional Logic

The WFFs of Propositional Logic are **strings of symbols**, as follows:

- Any capital regular letter (possibly with primes and subscripts) is a WFF.

  e.g., $P, Q, R, P', Q_5, \ldots$

- $\top$ and $\bot$ are WFFs.
- If $\mathcal{A}$ stands for some WFF, then $\neg\mathcal{A}$ is a WFF.

$P \quad Q' \quad \bot$

$\neg P \quad \neg Q' \quad \neg\bot$

$\neg\neg P \quad \neg\neg Q' \quad \neg\neg\bot$

Well-Formed Formulas of Propositional Logic

The WFFs of Propositional Logic are **strings of symbols**, as follows:

- Any capital regular letter (possibly with primes and subscripts) is a WFF.
  e.g., $\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{P}', \mathrm{Q}_5, \ldots$
- $\top$ and $\bot$ are WFFs.
- If $\mathcal{A}$ stands for some WFF, then $\neg\mathcal{A}$ is a WFF.
- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \wedge \mathcal{B})$ is a WFF.
- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \vee \mathcal{B})$ is a WFF.
- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \to \mathcal{B})$ is a WFF.

Nothing else is a WFF.

Which are WFFs (according to our definition)?

1. $(P \wedge Q)$ ✓
2. $((P \to P) \to (P \to P))$ ✓
3. $p \wedge q$ X   P∧Q X
4. $(P \wedge (Q \vee R)$ X
5. $(\mathcal{P} \wedge \mathcal{Q})$
6. $P \wedge Q \vee R$

S = "hello"

- Any capital letter (possibly with primes and subscripts) is a WFF.
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- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \wedge \mathcal{B})$ is a WFF.
- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \vee \mathcal{B})$ is a WFF.
- If $\mathcal{A}$ and $\mathcal{B}$ stand for WFFs, then $(\mathcal{A} \to \mathcal{B})$ is a WFF.

The Problem with Parentheses

$P \land Q \lor R$ is not a WFF *because* it *doesn't* have enough parentheses.

Requiring parentheses makes WFFs unambiguous, which is useful.

$((P \land Q) \lor R)$ is a WFF.

$(P \land (Q \lor R))$ is a *different* WFF.

But for humans, it's a pain to write all the parentheses all the time.

$(((((P \land Q) \land S) \land T) \land U) \to (X \land Y))$

A Compromise: Parenthesis Conventions

$\neg P \wedge Q \vee R$

$\approx$

$-x * y + z$

WFFs are officially fully parenthesized.

We allow ourselves to write *abbreviations* for WFFs.

- Precedence: $\neg$ binds most tightly, then $\wedge$, then $\vee$, then $\rightarrow$.
- Associativity: $\wedge$ and $\vee$ group to the left, and $\rightarrow$ to the right.

$A \wedge B \wedge C \equiv (A \wedge B) \wedge C$

$A \rightarrow B \rightarrow C \equiv A \rightarrow (B \rightarrow C)$

For example, when we write

- $P \wedge Q \vee R$ what we really mean is $((P \wedge Q) \vee R)$ not $P \wedge (Q \vee R)$
- $\neg P \wedge Q$ what we really mean is $(\neg P \wedge Q)$ not $\neg(P \wedge Q)$
- $((P \wedge Q) \rightarrow (((R \vee S) \vee T) \rightarrow U))$

Classical Propositional Logic

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   - We define what counts as a situation (a *model*).
   - We show how to decide if a WFF is true or false in a model.

3. We define what counts as a correct *proof*.
   - More than one option here!
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Classical Truth and Falsehood (for Propositional WFFs)

Truth and Falsehood

Is $(P \rightarrow (Q \wedge \neg R))$ true or false?

It depends on the situation!

Models in Classical Propositional Logic

The technical term for a "situation" is *model*, *assignment*, or *interpretation*.

In Classical Logic, if we want to decide whether this WFF is true

$$(P \rightarrow (Q \wedge \neg R))$$

we don't need to consider all the English propositions that P, Q, and R could stand for.

We just need to consider 8 (= 2^3) models:

P being true or false, Q being true or false, R being true or false.

A *model* in Classical Propositional Logic is an assignment of truth values to each of the propositional variables involved.

Truth in a Model

We cannot say that $(P \wedge Q)$ is true or false until we specify our model.

The model only tells us directly whether P and Q are true.

In Classical Propositional Logic, the truth of a complex formula is determined by standard *truth tables*.

Conjunction (And)

Propositions of the form $(\mathcal{A} \wedge \mathcal{B})$ are called *conjunctions*.

$(\mathcal{A} \wedge \mathcal{B})$ is true in a model when both $\mathcal{A}$ and $\mathcal{B}$ are true.

$\mathcal{A}$	$\mathcal{B}$	$(\mathcal{A} \wedge \mathcal{B})$
t	t	t
t	f	f
f	t	f
f	f	f

Disjunction (Or)

Propositions of the form $(\mathcal{A} \vee \mathcal{B})$ are called *disjunctions*.

$(\mathcal{A} \vee \mathcal{B})$ is true in a model when either $\mathcal{A}$ and $\mathcal{B}$ or both are true.

$\mathcal{A}$	$\mathcal{B}$	$(\mathcal{A} \vee \mathcal{B})$
t	t	t
t	f	t
f	t	t
f	f	f

As in mathematics, “Or” means *inclusive* or.

Negation (Not)

Propositions of the form $\neg\mathcal{A}$ are called *negations*.

$\neg\mathcal{A}$ is true in a model when $\mathcal{A}$ is false in that model.

$\mathcal{A}$	$\neg\mathcal{A}$
t	f
f	t

Implication (If...Then...)

Propositions of the form $(\mathcal{A} \to \mathcal{B})$ are called *implications*.

$(\mathcal{A} \to \mathcal{B})$ is true in a model if $\mathcal{A}$ is false or $\mathcal{B}$ is true.

That is, $(\mathcal{A} \to \mathcal{B})$ has the same truth table as $(\neg\mathcal{A} \vee \mathcal{B})$

$\mathcal{A}$	$\mathcal{B}$	$(\mathcal{A} \to \mathcal{B})$
t	t	t
t	f	f
f	t	t
f	f	t

Warning: This is *Material* Implication

A	B	$(A \to B)$
t	t	t
t	f	f
f	t	t
f	f	t

This is the if-then relationship used in math and logic.

No causal relationship is required or implied.

All the following are true (in the model corresponding to the real world):

- If $2 + 2 = 4$, then the moon is not made of green cheese.
- If the moon is made of green cheese, then $2 + 2 = 4$.
- If the moon is made of green cheese, then the moon is made of spam.
- If you pick a guinea pig up by the tail, then its eyes will fall out.
- If you scare a pregnant guinea pig, its babies will be born without tails.

$\top$ and $\bot$

The WFF $\top$ is true (t) in *every* model.

The WFF $\bot$ is false (f) in *every* model.

Note: We will consistently use

- $\top$ and $\bot$ when writing logical formulas (WFF syntax)
- t and f talking about meanings, truth, and falsity (WFF semantics)

Checking for Entailment

Suppose I want to know whether

Assumption 1 Assumption 2

$$(P \to Q), \neg Q \models \neg P$$

Conclusion.

How many models must be considered?

Which of the models make the assumptions true?

Do all of these make the conclusion true?

	P	Q	P→Q	¬Q	¬P	
Model 1	t	t	t	f	f	✓
Model 2	t	f	f	t	f	✓
" 3	f	t	t	f	t	✓
" 4	f	f	t	t	t	✓

Checking for Entailment

Suppose I want to know whether

$$(P \rightarrow Q),\ \neg P \vDash \neg Q$$

P	Q	$P \to Q$	$\neg P$	$\neg Q$	
t	t	t	f	f	✓
t	f	f	f	t	✓
f	t	t	t	f	oops
f	f	t	t	t	

Checking for Entailment

Suppose I want to know whether

$$\models (P \rightarrow (Q \rightarrow P))$$

Note: if there are no assumptions, then *every* model makes them true! (No model contradicts the assumptions.)

P	Q	$Q \rightarrow P$	$P \rightarrow (Q \rightarrow P)$
t	t	t	t
t	f	t	t
f	t	f	t
f	f	t	t