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1. Warmup
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- 1.
- a. or
- b. and
- 2.
- a. P -> Q means \neg (P ^ (\neg Q)) which means ((\neg P) v Q)
- b. ¬(P ^ Q) means ((¬P) v (¬Q))
- c. ¬(P v Q) means ((¬P) ^ (¬Q))
- d. $\neg(P \rightarrow Q)$ means $(\neg(\neg P \lor Q))$ which means $(P \land (\neg Q))$

English to Logic

- 1. B ^ (¬R)
- 2. (¬R) ^ W ^ B
- 3. B -> (W <-> (¬R))
- 4. (¬W) ^ (¬R) ^ B
- 5. W -> ((¬B) ^ (¬R))
- 6. (R ^ B) -> (¬W)

3. Legal English to Logic

Define the terminologies:

1F = Search the 1st floor

1A = Occupant on the 1st floor accepts delivery

2F = Search the 2nd floor

2A = Occupant on the 2nd floor accepts delivery

O = Open the package

We have: ((1F <-> 1A) v O) v ((2F <-> 2A) v O) which abbreviates to

4. Boolean Expressions in Code

- 1. (1 <= n) && (n <= 10)
- 2.
 - a) !(a < b) means **a >= b**
 - b) !((count < 99) || (len != 0) || !atEndFlag)
 means (!(count < 99) && !(len != 0) && !(!atEndFlag)

c)
$$!(x > 3 && (y == 4 || z <= 5))$$

means (
$$!(x > 3) || !((y == 4 || z <= 5))$$

means (
$$(x \le 3) || (!(y == 4) \&\& !(z \le 5)))$$

means (
$$(x \le 3) || (y != 4 \&\& z > 5))$$

3.

a)
$$((b < c) || (d < a))$$

c) If 2 rectangles don't overlap, then either their horizontal or vertical intervals should also not overlap, assuming we meant up = positive y-axis and right = positive x-axis.

Hence, for the 2 rectangles to not overlap: $((1, 1 \le x + 2) \parallel (1, 2 \le x + 1)) \parallel ((y + 1 \le y + 2) \parallel (y + 2 \le y + 1)) \parallel ((y + 1 \le y + 2) \parallel (y + 2 \le y + 1)) \parallel ((y + 1 \le y + 2) \parallel (y + 2 \le y + 1)) \parallel ((y + 1 \le y + 2) \parallel (y + 2 \le y + 1)) \parallel ((y + 1 \le y + 2) \parallel ((y + 1 \le y + 2)) \parallel ((y + 1 \le y +$

$$((u_1 < x_2) \mid\mid (u_2 < x_1)) \mid\mid ((v_1 < y_2) \mid\mid (v_2 < y_1))$$

Therefore, to have 2 rectangles overlapping at least 1 point:

$$!((u_1 < x_2) || (u_2 < x_1) || (v_1 < y_2) || (v_2 < y_1))$$
 means $((u_1 >= x_2) && (u_2 >= x_1) && (v_1 >= y_2) && (v_2 >= y_1))$

5. Models and Entailment

1.

Р	Q	¬P	(¬P -> Q)	¬Q	(¬Q -> P)
1	0	0	1	1	1
1	1	0	1	0	1
0	1	1	1	0	1
0	0	1	0	1	0

As highlighted in red, whenever the assumption is True, the conclusion is also True. Hence, we have an entailment.

2.

F	G	Н	F -> G	F -> H	G^H	(F->G) ^ (F -> H)	F->(G ^ H)
0	0	0	1	1	0	1	1
0	0	1	1	1	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	0	0	0

1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

As highlighted in red, whenever the assumption is True, the conclusion is also True. Hence, we have an entailment.

6. Valid, Satisfiable, and Unsatisfiable

- 1. A is valid -> A is True in every model -> $(\neg A)$ is False in every model -> $(\neg A)$ is **unsatisfiable**.
- 2. A is satisfiable -> A is True in at least 1 model -> A is False in some, but not all, models -> $(\neg A)$ is True in some, but not all, models -> $(\neg A)$ is either False in all models or True in some, but not all, models -> $(\neg A)$ is **either satisfiable** or **unsatisfiable**. In other words, $(\neg A)$ is **not valid**.
- 3. A is unsatisfiable -> A is False in every model -> $(\neg A)$ is True in every mode -> $(\neg A)$ is **valid**.

7. Writing Natural Deduction Proofs

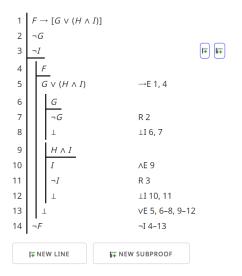
Construct a proof for the argument: $(A \land B) \rightarrow C, A \rightarrow B :: A \rightarrow C$

1
$$(A \land B) \rightarrow C$$

2 $A \rightarrow B$
3 $A \land B$
6 $C \rightarrow E 1, 5$
7 $A \rightarrow C \rightarrow I 3-6$

1. ② Congratulations! This proof is correct.

Construct a proof for the argument: $F \to [G \lor (H \land I)], \neg G, \neg I :: \neg F$



2. © Congratulations! This proof is correct.

1
$$(F \rightarrow G) \land (F \rightarrow H)$$

2 $F \rightarrow G$ $\land E 1$
3 $F \rightarrow H$ $\land E 1$
4 F
5 G $\rightarrow E 2, 4$
6 H $\rightarrow E 3, 4$
7 $G \land H$ $\land I 5, 6$
8 $F \rightarrow (G \land H)$ $\rightarrow I 4-7$

 $_{3.}$ $^{ ext{\textcircled{o}}}$ Congratulations! This proof is correct.

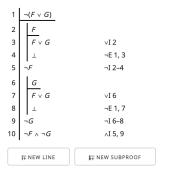
Construct a proof for the argument: $F \to G$:. $[(F \land G) \to F] \land [F \to (F \land G)]$

1
$$F \rightarrow G$$

2 F
3 G \rightarrow E 1, 2
4 $F \land G$ \land I 2, 3
5 $F \rightarrow (F \land G)$ \rightarrow I 2-4
6 $F \land G$
7 F \land AE 6
8 $(F \land G) \rightarrow F$ \rightarrow I 6-7
9 $((F \land G) \rightarrow F) \land (F \rightarrow (F \land G)) \land$ I 5, 8

4. © Congratulations! This proof is correct.

Construct a proof for the argument: $\neg (F \lor G) :. \neg F \land \neg G$



5. © Congratulations! This proof is correct.

8. Survey

- a. 4 hours
- b. I learned a bit on the algorithm of constructing natural deduction proof: at least the first step should be deciding if you want to start from the top, the bottom, or both, and if you see what sign what action is recommended.