

## 1. Warmup

1.
  - a. or
  - b. and
2.
  - a.  $P \rightarrow Q$  means  $\neg(P \wedge (\neg Q))$  which means  $((\neg P) \vee Q)$
  - b.  $\neg(P \wedge Q)$  means  $((\neg P) \vee (\neg Q))$
  - c.  $\neg(P \vee Q)$  means  $((\neg P) \wedge (\neg Q))$
  - d.  $\neg(P \rightarrow Q)$  means  $(\neg(\neg P \vee Q))$  which means  $(P \wedge (\neg Q))$

## 2. English to Logic

1.  $B \wedge (\neg R)$
2.  $(\neg R) \wedge W \wedge B$
3.  $B \rightarrow (W \leftrightarrow (\neg R))$
4.  $(\neg W) \wedge (\neg R) \wedge B$
5.  $W \rightarrow ((\neg B) \wedge (\neg R))$
6.  $(R \wedge B) \rightarrow (\neg W)$

## 3. Legal English to Logic

Define the terminologies:

1F = Search the 1st floor

1A = Occupant on the 1st floor accepts delivery

2F = Search the 2nd floor

2A = Occupant on the 2nd floor accepts delivery

O = Open the package

We have:  $((1F \leftrightarrow 1A) \vee O) \vee ((2F \leftrightarrow 2A) \vee O)$  which abbreviates to

$$(1F \leftrightarrow 1A) \vee (2F \leftrightarrow 2A) \vee O$$

## 4. Boolean Expressions in Code

1.  $(1 \leq n) \ \&\& \ (n \leq 10)$
2.
  - a)  $!(a < b)$  means  **$a \geq b$**
  - b)  $!(count < 99) \ || \ (len \neq 0) \ || \ !atEndFlag$  )  
means  $!(count < 99) \ \&\& \ !(len \neq 0) \ \&\& \ !(atEndFlag)$   
means  **$((count \geq 99) \ \&\& \ (len == 0) \ \&\& \ atEndFlag)$**
  - c)  $!(x > 3 \ \&\& \ (y == 4 \ || \ z \leq 5))$   
means  $(!(x > 3) \ || \ !(y == 4 \ || \ z \leq 5))$   
means  $(x \leq 3) \ || \ ( !(y == 4) \ \&\& \ !(z \leq 5) )$   
means  **$(x \leq 3) \ || \ (y \neq 4 \ \&\& \ z > 5)$**

3.

a)  $((b < c) \parallel (d < a))$

b)  $!((b < c) \parallel (d < a))$

means  $!(b < c) \&\& !(d < a)$

means  $(b \geq c) \&\& (d \geq a)$

c) If 2 rectangles don't overlap, then either their horizontal or vertical intervals should also not overlap, assuming we meant up = positive y-axis and right = positive x-axis.

Hence, for the 2 rectangles to not overlap:

$((u_1 < x_2) \parallel (u_2 < x_1)) \parallel ((v_1 < y_2) \parallel (v_2 < y_1))$

Therefore, to have 2 rectangles overlapping at least 1 point:

$!((u_1 < x_2) \parallel (u_2 < x_1) \parallel (v_1 < y_2) \parallel (v_2 < y_1))$

means  $((u_1 \geq x_2) \&\& (u_2 \geq x_1) \&\& (v_1 \geq y_2) \&\& (v_2 \geq y_1))$

## 5. Models and Entailment

1.

P	Q	$\neg P$	$(\neg P \rightarrow Q)$	$\neg Q$	$(\neg Q \rightarrow P)$
1	0	0	1	1	1
1	1	0	1	0	1
0	1	1	1	0	1
0	0	1	0	1	0

As highlighted in red, whenever the assumption is True, the conclusion is also True.

Hence, we have an entailment.

2.

F	G	H	$F \rightarrow G$	$F \rightarrow H$	$G \wedge H$	$(F \rightarrow G) \wedge (F \rightarrow H)$	$F \rightarrow (G \wedge H)$
0	0	0	1	1	0	1	1
0	0	1	1	1	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	0	0	0

1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

As highlighted in red, whenever the assumption is True, the conclusion is also True.  
Hence, we have an entailment.

## 6. Valid, Satisfiable, and Unsatisfiable

1. A is valid  $\rightarrow$  A is True in every model  $\rightarrow (\neg A)$  is False in every model  $\rightarrow (\neg A)$  is **unsatisfiable**.
2. A is satisfiable  $\rightarrow$  A is True in at least 1 model  $\rightarrow$  A is False in some, but not all, models  $\rightarrow (\neg A)$  is True in some, but not all, models  $\rightarrow (\neg A)$  is either False in all models or True in some, but not all, models  $\rightarrow (\neg A)$  is **either satisfiable or unsatisfiable**. In other words,  $(\neg A)$  is **not valid**.
3. A is unsatisfiable  $\rightarrow$  A is False in every model  $\rightarrow (\neg A)$  is True in every mode  $\rightarrow (\neg A)$  is **valid**.

## 7. Writing Natural Deduction Proofs

Construct a proof for the argument:  $(A \wedge B) \rightarrow C, A \rightarrow B \therefore A \rightarrow C$

1		$(A \wedge B) \rightarrow C$	
2		$A \rightarrow B$	
3			$A$
4			$B$
			$\rightarrow E$ 2, 3
5		$A \wedge B$	$\wedge I$ 3, 4
6		$C$	$\rightarrow E$ 1, 5
7		$A \rightarrow C$	$\rightarrow I$ 3-6

NEW LINE

NEW SUBPROOF

1. 🎉 Congratulations! This proof is correct.

Construct a proof for the argument:  $F \rightarrow [G \vee (H \wedge I)], \neg G, \neg I \therefore \neg F$

1	$F \rightarrow [G \vee (H \wedge I)]$	
2	$\neg G$	
3	$\neg I$	
4	$F$	
5	$G \vee (H \wedge I)$	$\rightarrow E 1, 4$
6	$G$	
7	$\neg G$	R 2
8	$\perp$	$\perp I 6, 7$
9	$H \wedge I$	
10	$I$	$\wedge E 9$
11	$\neg I$	R 3
12	$\perp$	$\perp I 10, 11$
13	$\perp$	$\vee E 5, 6-8, 9-12$
14	$\neg F$	$\neg I 4-13$

NEW LINE

NEW SUBPROOF

2. 😊 Congratulations! This proof is correct.

1	$(F \rightarrow G) \wedge (F \rightarrow H)$	
2	$F \rightarrow G$	$\wedge E 1$
3	$F \rightarrow H$	$\wedge E 1$
4	$F$	
5	$G$	$\rightarrow E 2, 4$
6	$H$	$\rightarrow E 3, 4$
7	$G \wedge H$	$\wedge I 5, 6$
8	$F \rightarrow (G \wedge H)$	$\rightarrow I 4-7$

NEW LINE

NEW SUBPROOF

3. 😊 Congratulations! This proof is correct.

Construct a proof for the argument:  $F \rightarrow G \therefore [(F \wedge G) \rightarrow F] \wedge [F \rightarrow (F \wedge G)]$

1	$F \rightarrow G$	
2	$F$	
3	$G$	$\rightarrow E 1, 2$
4	$F \wedge G$	$\wedge I 2, 3$
5	$F \rightarrow (F \wedge G)$	$\rightarrow I 2-4$
6	$F \wedge G$	
7	$F$	$\wedge E 6$
8	$(F \wedge G) \rightarrow F$	$\rightarrow I 6-7$
9	$((F \wedge G) \rightarrow F) \wedge (F \rightarrow (F \wedge G))$	$\wedge I 5, 8$

NEW LINE

NEW SUBPROOF

4. 😊 Congratulations! This proof is correct.

Construct a proof for the argument:  $\neg(F \vee G) \therefore \neg F \wedge \neg G$

1	$\neg(F \vee G)$	
2	$F$	
3	$F \vee G$	$\vee I 2$
4	$\perp$	$\neg E 1, 3$
5	$\neg F$	$\neg I 2-4$
6	$G$	
7	$F \vee G$	$\vee I 6$
8	$\perp$	$\neg E 1, 7$
9	$\neg G$	$\neg I 6-8$
10	$\neg F \wedge \neg G$	$\wedge I 5, 9$

NEW LINE

NEW SUBPROOF

5. 😊 Congratulations! This proof is correct.

## 8. Survey

- 4 hours
- I learned a bit on the algorithm of constructing natural deduction proof: at least the first step should be deciding if you want to start from the top, the bottom, or both, and if you see what sign what action is recommended.