

# hw4

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```
library(faraway)
```

## 3.2

```
data("cheddar")
summary(cheddar)
```

```
##      taste      Acetic      H2S      Lactic
## Min.   : 0.70   Min.   :4.477   Min.   : 2.996   Min.   :0.860
## 1st Qu.:13.55   1st Qu.:5.237   1st Qu.: 3.978   1st Qu.:1.250
## Median :20.95   Median :5.425   Median : 5.329   Median :1.450
## Mean   :24.53   Mean   :5.498   Mean   : 5.942   Mean   :1.442
## 3rd Qu.:36.70   3rd Qu.:5.883   3rd Qu.: 7.575   3rd Qu.:1.667
## Max.   :57.20   Max.   :6.458   Max.   :10.199   Max.   :2.010
```

a.

```
model_a <- lm(taste ~ ., cheddar)
summary(model_a)
```

```
##
## Call:
## lm(formula = taste ~ ., data = cheddar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.390  -6.612  -1.009   4.908  25.449
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -28.8768    19.7354  -1.463  0.15540
## Acetic       0.3277     4.4598   0.073  0.94198
## H2S          3.9118     1.2484   3.133  0.00425 **
## Lactic      19.6705     8.6291   2.280  0.03108 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 10.13 on 26 degrees of freedom
## Multiple R-squared:  0.6518, Adjusted R-squared:  0.6116
## F-statistic: 16.22 on 3 and 26 DF,  p-value: 3.81e-06
```

At 5% level, H2S and Lactic are statistically significant predictors.

b.

```
model_b <- lm(taste ~ exp(Acetic) + exp(H2S) + Lactic, data = cheddar)
summary(model_b)

##
## Call:
## lm(formula = taste ~ exp(Acetic) + exp(H2S) + Lactic, data = cheddar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.209  -7.266  -1.651   7.385  26.335
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.897e+01  1.127e+01  -1.684  0.1042
## exp(Acetic)  1.891e-02  1.562e-02   1.210  0.2371
## exp(H2S)     7.668e-04  4.188e-04   1.831  0.0786 .
## Lactic       2.501e+01  9.062e+00   2.760  0.0105 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 11.19 on 26 degrees of freedom
## Multiple R-squared:  0.5754, Adjusted R-squared:  0.5264
## F-statistic: 11.75 on 3 and 26 DF,  p-value: 4.746e-05
```

At 5% level, Lactic and exp(H2S) are statistically significant predictors.

c.

We can use F-test to compare these model because we want to compare between a base model (model\_a) and a modified new model (model\_b).

```
anova_test <- anova(model_a, model_b)
anova_test

## Analysis of Variance Table
##
## Model 1: taste ~ Acetic + H2S + Lactic
## Model 2: taste ~ exp(Acetic) + exp(H2S) + Lactic
##   Res.Df    RSS Df Sum of Sq F Pr(>F)
## 1      26 2668.4
## 2      26 3253.6  0    -585.2
```

The RSS of model\_a is much lower (585.2) than that of model\_b, hence model\_a is better fit for the data.

d.

```
delta_H2S <- 0.01
predicted_change <- coef(model_a)["H2S"] * log(delta_H2S)
predicted_change
```

```
##      H2S
## -18.01469
```

Since H2S in model\_a is in logarithmic form, when H2S increases by 0.01, the change in taste will be the previous coefficient times the log of the delta increase. The final result is -18.01469.

e.

```
delta_log_H2S <- 0.01
percentage_change <- (exp(delta_log_H2S) - 1) * 100
percentage_change
```

```
## [1] 1.005017
```

If H2S increases by 0.01, The percentage change in H2S is 1%.

4.

```
data("sat")
summary(sat)
```

```
##      expend      ratio      salary      takers
##  Min.   :3.656   Min.   :13.80   Min.   :25.99   Min.    : 4.00
##  1st Qu.:4.882   1st Qu.:15.22   1st Qu.:30.98   1st Qu.: 9.00
##  Median :5.768   Median :16.60   Median :33.29   Median :28.00
##  Mean   :5.905   Mean   :16.86   Mean   :34.83   Mean   :35.24
##  3rd Qu.:6.434   3rd Qu.:17.57   3rd Qu.:38.55   3rd Qu.:63.00
##  Max.   :9.774   Max.   :24.30   Max.   :50.05   Max.   :81.00
##      verbal      math      total
##  Min.   :401.0   Min.   :443.0   Min.   : 844.0
##  1st Qu.:427.2   1st Qu.:474.8   1st Qu.: 897.2
##  Median :448.0   Median :497.5   Median : 945.5
##  Mean   :457.1   Mean   :508.8   Mean   : 965.9
##  3rd Qu.:490.2   3rd Qu.:539.5   3rd Qu.:1032.0
##  Max.   :516.0   Max.   :592.0   Max.   :1107.0
```

```
model_a <- lm(total ~ expend + ratio + salary, data = sat)
summary(model_a)
```

```
##
## Call:
## lm(formula = total ~ expend + ratio + salary, data = sat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -140.911  -46.740   -7.535   47.966  123.329
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1069.234    110.925   9.639 1.29e-12 ***
## expend       16.469     22.050   0.747  0.4589
## ratio        6.330      6.542   0.968  0.3383
## salary      -8.823      4.697  -1.878  0.0667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 68.65 on 46 degrees of freedom
## Multiple R-squared:  0.2096, Adjusted R-squared:  0.1581
## F-statistic: 4.066 on 3 and 46 DF,  p-value: 0.01209
```

a.

Test of beta of salary is 0:

```
t_test_salary <- coef(summary(model_a))["salary", "t value"]
p_value_salary <- coef(summary(model_a))["salary", "Pr(>|t|)"]

cat("T-Test for Salary Coefficient:\n")
```

```
## T-Test for Salary Coefficient:
```

```
cat("t-value:", t_test_salary, "\n")
```

```
## t-value: -1.878437
```

```
cat("p-value:", p_value_salary, "\n")
```

```
## p-value: 0.06666771
```

We don't reject the null, hence salary does not have an effect on the response variable.

Test of beta of salary, ratio, and expend all 0.

```
f_test_all <- anova(model_a)
cat("\nF-Test for All Coefficients:\n")
```

```
##
```

```
## F-Test for All Coefficients:
```

```
f_test_all
```

```
## Analysis of Variance Table
##
## Response: total
##           Df Sum Sq Mean Sq F value    Pr(>F)
## expend     1  39722   39722   8.4276 0.005658 **
## ratio      1   1143    1143   0.2424 0.624795
## salary     1  16631   16631   3.5285 0.066668 .
## Residuals 46 216812    4713
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the ANOVA table, only extend is a statistically significant predictor to the responses.

b.

```
model_b <- lm(total ~ expend + ratio + salary + takers, data = sat)
```

```
t_test_takers <- coef(summary(model_b))["takers", "t value"]
p_value_takers <- coef(summary(model_b))["takers", "Pr(>|t|)"]

cat("\nT-Test for Takers Coefficient:\n")
```

```
##
## T-Test for Takers Coefficient:
```

```
cat("t-value:", t_test_takers, "\n")
```

```
## t-value: -12.55937
```

```
cat("p-value:", p_value_takers, "\n")
```

```
## p-value: 2.606559e-16
```

Based on the p-value, we reject the null. Hence, takers is a statistically significant predictor.

```
anova_test_ab <- anova(model_a, model_b)
anova_test_ab
```

```
## Analysis of Variance Table
##
## Model 1: total ~ expend + ratio + salary
## Model 2: total ~ expend + ratio + salary + takers
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1      46 216812
## 2      45  48124  1    168688 157.74 2.607e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the ANOVA table, we can see that, as the difference between model\_a and model\_b is just the new predictor takers, and model\_b shows a significant improvement in RSS from model\_a, takers definitely contribute to the response, which is the same conclusion as the t-test above.

6.

```
data("happy")
summary(happy)
```

```
##      happy      money      sex      love
## Min.   : 2.000   Min.    : 0.00   Min.    :0.0000   Min.    :1.000
## 1st Qu.: 5.000   1st Qu.: 42.50   1st Qu.:0.0000   1st Qu.:2.000
## Median : 7.000   Median : 50.00   Median :1.0000   Median :3.000
## Mean   : 6.744   Mean    : 62.15   Mean    :0.6923   Mean    :2.462
## 3rd Qu.: 8.000   3rd Qu.: 78.00   3rd Qu.:1.0000   3rd Qu.:3.000
## Max.   :10.000   Max.    :175.00   Max.    :1.0000   Max.    :3.000
##      work
## Min.    :1.000
## 1st Qu.:3.000
## Median :4.000
## Mean    :3.359
## 3rd Qu.:4.000
## Max.    :5.000
```

a.

```
model <- lm(happy ~ money + sex + love + work, data = happy)
summary(model)
```

```
##
## Call:
## lm(formula = happy ~ money + sex + love + work, data = happy)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.7186 -0.5779 -0.1172  0.6340  2.0651
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.072081   0.852543  -0.085   0.9331
## money        0.009578   0.005213   1.837   0.0749 .
## sex         -0.149008   0.418525  -0.356   0.7240
## love         1.919279   0.295451   6.496 1.97e-07 ***
## work         0.476079   0.199389   2.388   0.0227 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.058 on 34 degrees of freedom
## Multiple R-squared:  0.7102, Adjusted R-squared:  0.6761
## F-statistic: 20.83 on 4 and 34 DF, p-value: 9.364e-09
```

At 1% level, love and work are statistically significant predictors.

b.

```
summary_table <- table(happy)
summary_table
```

```
## , , sex = 0, love = 1, work = 1
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 1, work = 1
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
```

```

##      10      0
##
## , , sex = 0, love = 2, work = 1
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##      10     0
##
## , , sex = 1, love = 2, work = 1
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##      10     0
##
## , , sex = 0, love = 3, work = 1
##

```



```

##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 3, work = 1
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 1, work = 2
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 1, work = 2
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 2, work = 2
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 2, work = 2
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 3, work = 2
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money

```

```

## happy 175
## 2 0
## 3 0
## 4 0
## 5 0
## 6 0
## 7 0
## 8 0
## 9 0
## 10 0
##
## , , sex = 1, love = 3, work = 2
##
## money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
## 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
## 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## money
## happy 175
## 2 0
## 3 0
## 4 0
## 5 0
## 6 0
## 7 0
## 8 0
## 9 0
## 10 0
##
## , , sex = 0, love = 1, work = 3
##
## money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
## 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## money
## happy 175
## 2 0
## 3 0
## 4 0

```

```

##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 1, love = 1, work = 3
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 0, love = 2, work = 3
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0

```

```

##      9      0
##     10      0
##
## , , sex = 1, love = 2, work = 3
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 0, love = 3, work = 3
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 1, love = 3, work = 3

```

```

##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 1, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 1, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 2, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 2, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```



```

##      7 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 0, love = 3, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2 0
##      3 0
##      4 0
##      5 0
##      6 0
##      7 0
##      8 0
##      9 0
##     10 0
##
## , , sex = 1, love = 3, work = 4
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 1 0 1 0 0
##      9 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

##      money
## happy 175
##    2    0
##    3    0
##    4    0
##    5    0
##    6    0
##    7    0
##    8    1
##    9    1
##   10    0
##
## , , sex = 0, love = 1, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##    2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##   10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##    2    0
##    3    0
##    4    0
##    5    0
##    6    0
##    7    0
##    8    0
##    9    0
##   10    0
##
## , , sex = 1, love = 1, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##    2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##    9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##   10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##    2    0
##    3    0

```

```

##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 0, love = 2, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 1, love = 2, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0

```

```

##      8      0
##      9      0
##     10      0
##
## , , sex = 0, love = 3, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0
##
## , , sex = 1, love = 3, work = 5
##
##      money
## happy 0 31 35 36 40 41 44 45 47 50 53 55 56 60 62 65 70 75 81 85 88 90 112 115
##      2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      5 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      6 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      9 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##     10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##      money
## happy 175
##      2      0
##      3      0
##      4      0
##      5      0
##      6      0
##      7      0
##      8      0
##      9      0
##     10      0

```

The questionable assumption is that some predictors should not have numerical values in the real number domain e.g 'sex', but they instead should have had categorical values.

c.

Permutation Test

```
set.seed(123) # Set a random seed for reproducibility
n_permutations <- 1000
permutation_results <- numeric(n_permutations)

for (i in 1:n_permutations) {
  happy_permuted <- happy
  happy_permuted$money <- sample(happy$money)
  model_permuted <- lm(happy ~ money + work + sex + love, data = happy_permuted)
  permutation_results[i] <- coef(model_permuted)["money"]
}

observed_coef <- coef(model)["money"]
p_value_permutation <- sum(abs(permutation_results) >= abs(observed_coef)) / n_permutations
cat("\nP-value from Permutation Test (money predictor):\n")

##
## P-value from Permutation Test (money predictor):

cat("p-value:", p_value_permutation, "\n")
```

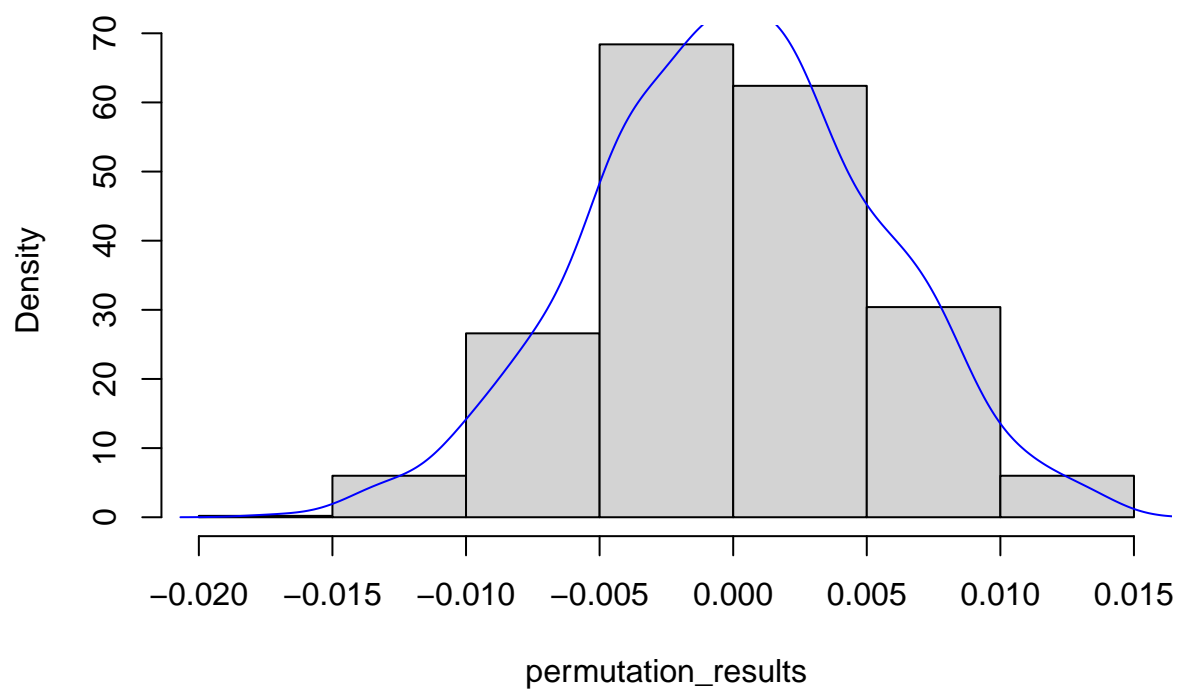
## p-value: 0.073

Based on the p-value, 'money' is not a statistically significant predictor at 5% level.

d.

```
hist(permutation_results, probability = TRUE, main = "Permutation Test for money Predictor")
lines(density(permutation_results), col = "blue")
```

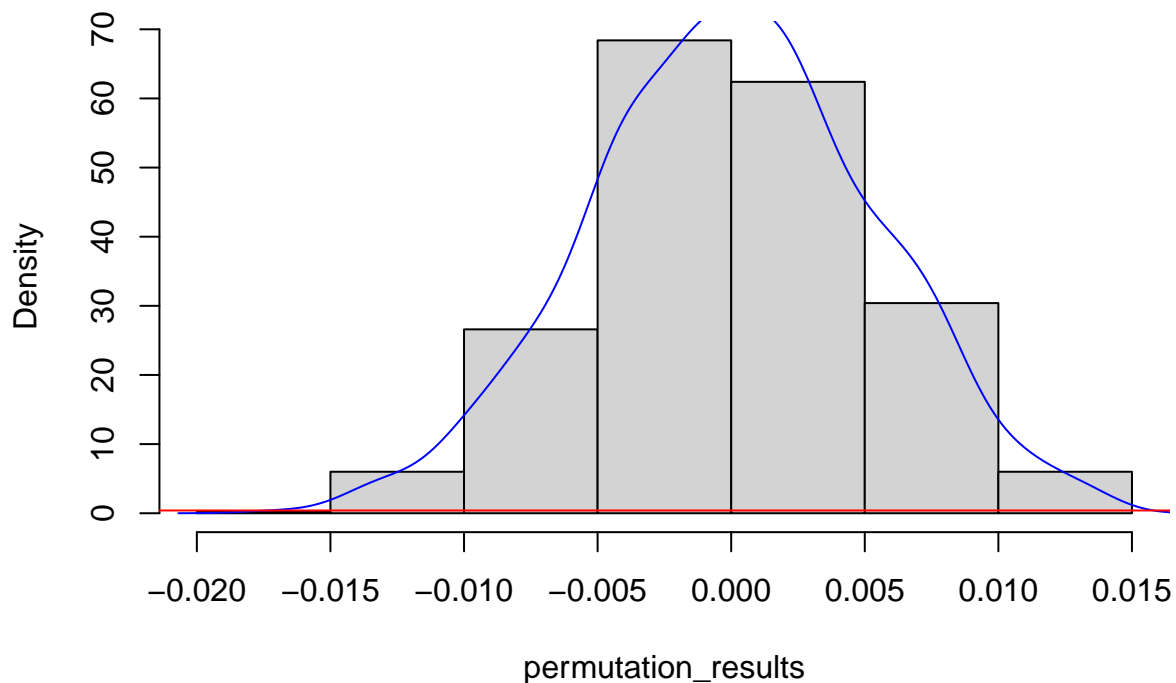
## Permutation Test for money Predictor



### e.

```
hist(permutation_results, probability = TRUE, main = "Permutation Test for money Predictor")
lines(density(permutation_results), col = "blue")
grid <- seq(-3, 3, length = 300)
t_density <- dt(grid, df = length(permutation_results) - 1)
lines(grid, t_density, col = "red")
```

## Permutation Test for money Predictor



f.

```
set.seed(123)
n_bootstrap <- 1000
bootstrap_results <- numeric(n_bootstrap)

for (i in 1:n_bootstrap) {
  sample_indices <- sample(1:length(happy$money), replace = TRUE)
  bootstrap_sample <- happy[sample_indices, ]
  model_bootstrap <- lm(happy ~ money + love + sex + work, data = bootstrap_sample)
  bootstrap_results[i] <- coef(model_bootstrap)["money"]
}

# Calculate bootstrap confidence intervals
conf_interval_90 <- quantile(bootstrap_results, c(0.05, 0.95))
conf_interval_95 <- quantile(bootstrap_results, c(0.025, 0.975))

# Check if zero falls within these confidence intervals
zero_in_interval_90 <- conf_interval_90[1] <= 0 && conf_interval_90[2] >= 0
zero_in_interval_95 <- conf_interval_95[1] <= 0 && conf_interval_95[2] >= 0

# Output the results
cat("\nBootstrap Confidence Intervals for money:\n")
```

```
##
## Bootstrap Confidence Intervals for money:

cat("90% Confidence Interval:", conf_interval_90, "\n")

## 90% Confidence Interval: 0.001322696 0.02432515

cat("95% Confidence Interval:", conf_interval_95, "\n")

## 95% Confidence Interval: 0.0001812671 0.02748471

cat("Zero within 90% Confidence Interval:", zero_in_interval_90, "\n")

## Zero within 90% Confidence Interval: FALSE

cat("Zero within 95% Confidence Interval:", zero_in_interval_95, "\n")

## Zero within 95% Confidence Interval: FALSE
```

ZERO does NOT fall within these confidence intervals. This is consistent with the result from the permutation test above as we will reject the null that money is a statistically significant predictor at 5% level.

## 4.1

a.

```
data("prostate")
summary(prostate)
```

```
##      lcavol      lweight      age      lbph
## Min.   :-1.3471  Min.   :2.375  Min.   :41.00  Min.   :-1.3863
## 1st Qu.: 0.5128  1st Qu.:3.376  1st Qu.:60.00  1st Qu.: -1.3863
## Median : 1.4469  Median :3.623  Median :65.00  Median : 0.3001
## Mean   : 1.3500  Mean   :3.653  Mean   :63.87  Mean   : 0.1004
## 3rd Qu.: 2.1270  3rd Qu.:3.878  3rd Qu.:68.00  3rd Qu.: 1.5581
## Max.   : 3.8210  Max.   :6.108  Max.   :79.00  Max.   : 2.3263
##      svi      lcp      gleason      pgg45
## Min.   :0.0000  Min.   :-1.3863  Min.   :6.000  Min.   : 0.00
## 1st Qu.:0.0000  1st Qu.: -1.3863  1st Qu.:6.000  1st Qu.: 0.00
## Median :0.0000  Median :-0.7985  Median :7.000  Median : 15.00
## Mean   :0.2165  Mean   :-0.1794  Mean   :6.753  Mean   : 24.38
## 3rd Qu.:0.0000  3rd Qu.: 1.1786  3rd Qu.:7.000  3rd Qu.: 40.00
## Max.   :1.0000  Max.   : 2.9042  Max.   :9.000  Max.   :100.00
##      lpsa
## Min.   :-0.4308
## 1st Qu.: 1.7317
## Median : 2.5915
## Mean   : 2.4784
## 3rd Qu.: 3.0564
## Max.   : 5.5829
```



```
model <- lm(lpsa ~ ., data = prostate)
summary(model)
```

```
##
## Call:
## lm(formula = lpsa ~ ., data = prostate)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.7331 -0.3713 -0.0170  0.4141  1.6381
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.669337   1.296387   0.516  0.60693
## lcavol       0.587022   0.087920   6.677 2.11e-09 ***
## lweight      0.454467   0.170012   2.673  0.00896 **
## age         -0.019637   0.011173  -1.758  0.08229 .
## lbph         0.107054   0.058449   1.832  0.07040 .
## svi          0.766157   0.244309   3.136  0.00233 **
## lcp          -0.105474   0.091013  -1.159  0.24964
## gleason      0.045142   0.157465   0.287  0.77503
## pgg45        0.004525   0.004421   1.024  0.30886
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7084 on 88 degrees of freedom
## Multiple R-squared:  0.6548, Adjusted R-squared:  0.6234
## F-statistic: 20.86 on 8 and 88 DF,  p-value: < 2.2e-16
```

a.

```
new_patient <- data.frame(
  lcavol = 1.44692,
  lweight = 3.62301,
  age = 65.00000,
  lbph = 0.30010,
  svi = 0.00000,
  lcp = -0.79851,
  gleason = 7.00000,
  pgg45 = 15.00000
)

predicted_lpsa <- predict(model, newdata = new_patient, interval = "prediction", level = 0.95)

cat("Predicted lpsa:", predicted_lpsa[1], "\n")

## Predicted lpsa: 2.389053

cat("95% Confidence Interval for lpsa:", predicted_lpsa[2], "to", predicted_lpsa[3], "\n")

## 95% Confidence Interval for lpsa: 0.9646584 to 3.813447
```

b.

```
new_patient_age_20 <- data.frame(
  lcavol = 1.44692,
  lweight = 3.62301,
  age = 20.00000, # Updated age
  lbph = 0.30010,
  svi = 0.00000,
  lcp = -0.79851,
  gleason = 7.00000,
  pgg45 = 15.00000
)

predicted_lpsa_age_20 <- predict(model, newdata = new_patient_age_20, interval = "prediction", level = 0.95)

cat("Predicted lpsa (age = 20):", predicted_lpsa_age_20[1], "\n")

## Predicted lpsa (age = 20): 3.272726

cat("95% Confidence Interval for lpsa (age = 20):", predicted_lpsa_age_20[2], "to", predicted_lpsa_age_20[3], "\n")

## 95% Confidence Interval for lpsa (age = 20): 1.538744 to 5.006707
```

The CI is wider for the patient with age = 20 compared to the patient with age = 65 because at 5% level, age is one of the predictors in the regression model. When age is set to 20, it results in a prediction that is further away from the mean response value of the model compared to when age is 65. This increased deviation from the mean response leads to greater uncertainty in the prediction, resulting in a wider CI.

c.

```
significant_predictors_model <- lm(lpsa ~ lcavol + lweight + age + lbph + svi, data = prostate)
summary(significant_predictors_model)

##
## Call:
## lm(formula = lpsa ~ lcavol + lweight + age + lbph + svi, data = prostate)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.83505 -0.39396  0.00414  0.46336  1.57888
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.95100    0.83175   1.143 0.255882
## lcavol         0.56561    0.07459   7.583 2.77e-11 ***
## lweight        0.42369    0.16687   2.539 0.012814 *
## age          -0.01489    0.01075  -1.385 0.169528
## lbph          0.11184    0.05805   1.927 0.057160 .
## svi           0.72095    0.20902   3.449 0.000854 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7073 on 91 degrees of freedom
## Multiple R-squared:  0.6441, Adjusted R-squared:  0.6245
## F-statistic: 32.94 on 5 and 91 DF,  p-value: < 2.2e-16

new_patient_age_20 <- data.frame(
  lcavol = 1.44692,
  lweight = 3.62301,
  age = 20.00000, # Updated age
  lbph = 0.30010,
  svi = 0.00000,
  lcp = -0.79851,
  gleason = 7.00000,
  pgg45 = 15.00000
)

# Predict lpsa for the new patient with age = 20 using the model with significant predictors
predicted_lpsa_age_20_significant <- predict(significant_predictors_model, newdata = new_patient_age_20)

# Output the prediction and 95% confidence interval for lpsa with age = 20 using the model with significant predictors
cat("Predicted lpsa (age = 20, significant predictors only):", predicted_lpsa_age_20_significant[1], "\n")

## Predicted lpsa (age = 20, significant predictors only): 3.040145

cat("95% Confidence Interval for lpsa (age = 20, significant predictors only):", predicted_lpsa_age_20_significant[2:3], "\n")

## 95% Confidence Interval for lpsa (age = 20, significant predictors only): 1.340103 to 4.740187
```

The CIs for the predictions made with the model containing only significant predictors are narrower than the CIs from the previous model because the model's predictions are based on fewer variables, resulting in reduced uncertainty.

## 5.

```
data("fat")
summary(fat)
```

```
##      brozek      siri      density      age
##  Min.   : 0.00  Min.   : 0.00  Min.   :0.995  Min.   :22.00
##  1st Qu.:12.80  1st Qu.:12.47  1st Qu.:1.041  1st Qu.:35.75
##  Median :19.00  Median :19.20  Median :1.055  Median :43.00
##  Mean   :18.94  Mean   :19.15  Mean   :1.056  Mean   :44.88
##  3rd Qu.:24.60  3rd Qu.:25.30  3rd Qu.:1.070  3rd Qu.:54.00
##  Max.   :45.10  Max.   :47.50  Max.   :1.109  Max.   :81.00
##      weight      height      adipos      free
##  Min.   :118.5  Min.   :29.50  Min.   :18.10  Min.   :105.9
##  1st Qu.:159.0  1st Qu.:68.25  1st Qu.:23.10  1st Qu.:131.3
##  Median :176.5  Median :70.00  Median :25.05  Median :141.6
```

```
## Mean :178.9 Mean :70.15 Mean :25.44 Mean :143.7
## 3rd Qu.:197.0 3rd Qu.:72.25 3rd Qu.:27.32 3rd Qu.:153.9
## Max. :363.1 Max. :77.75 Max. :48.90 Max. :240.5
## neck chest abdom hip
## Min. :31.10 Min. : 79.30 Min. : 69.40 Min. : 85.0
## 1st Qu.:36.40 1st Qu.: 94.35 1st Qu.: 84.58 1st Qu.: 95.5
## Median :38.00 Median : 99.65 Median : 90.95 Median : 99.3
## Mean :37.99 Mean :100.82 Mean : 92.56 Mean : 99.9
## 3rd Qu.:39.42 3rd Qu.:105.38 3rd Qu.: 99.33 3rd Qu.:103.5
## Max. :51.20 Max. :136.20 Max. :148.10 Max. :147.7
## thigh knee ankle biceps forearm
## Min. :47.20 Min. :33.00 Min. :19.1 Min. :24.80 Min. :21.00
## 1st Qu.:56.00 1st Qu.:36.98 1st Qu.:22.0 1st Qu.:30.20 1st Qu.:27.30
## Median :59.00 Median :38.50 Median :22.8 Median :32.05 Median :28.70
## Mean :59.41 Mean :38.59 Mean :23.1 Mean :32.27 Mean :28.66
## 3rd Qu.:62.35 3rd Qu.:39.92 3rd Qu.:24.0 3rd Qu.:34.33 3rd Qu.:30.00
## Max. :87.30 Max. :49.10 Max. :33.9 Max. :45.00 Max. :34.90
## wrist
## Min. :15.80
## 1st Qu.:17.60
## Median :18.30
## Mean :18.23
## 3rd Qu.:18.80
## Max. :21.40
```

```
full_model <- lm(brozek ~ ., data = fat)
smaller_model <- lm(brozek ~ age + weight + height + abdom, data = fat)
```

```
summary(smaller_model)
```

```
##
## Call:
## lm(formula = brozek ~ age + weight + height + abdom, data = fat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.5105  -2.9346   0.0087   2.8942   9.4179
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -32.769636    6.541902  -5.009 1.04e-06 ***
## age          -0.007051    0.024342  -0.290  0.772
## weight       -0.123722    0.025046  -4.940 1.44e-06 ***
## height       -0.116694    0.082727  -1.411  0.160
## abdom         0.889704    0.067267  13.226 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.126 on 247 degrees of freedom
## Multiple R-squared:  0.7211, Adjusted R-squared:  0.7166
## F-statistic: 159.7 on 4 and 247 DF, p-value: < 2.2e-16
```

```
summary(full_model)
```

```
##
## Call:
## lm(formula = brozek ~ ., data = fat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.11191 -0.04847  0.00277  0.04625  1.47542
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 12.1524013  4.1718589   2.913  0.00393 **
## siri         0.8884085  0.0111341  79.792 < 2e-16 ***
## density     -9.8456305  3.7471770  -2.627  0.00917 **
## age         -0.0005268  0.0012935  -0.407  0.68421
## weight       0.0084855  0.0036200   2.344  0.01991 *
## height     -0.0005459  0.0044439  -0.123  0.90234
## adipos      -0.0153248  0.0124778  -1.228  0.22062
## free        -0.0097388  0.0044270  -2.200  0.02880 *
## neck         0.0005002  0.0094279   0.053  0.95773
## chest        0.0021454  0.0043013   0.499  0.61840
## abdom        0.0014464  0.0044217   0.327  0.74388
## hip         -0.0044514  0.0058941  -0.755  0.45087
## thigh        0.0156926  0.0059507   2.637  0.00892 **
## knee        -0.0252126  0.0098531  -2.559  0.01113 *
## ankle        0.0027790  0.0089580   0.310  0.75667
## biceps      -0.0147134  0.0069201  -2.126  0.03454 *
## forearm      0.0149983  0.0080832   1.855  0.06478 .
## wrist        0.0326518  0.0218000   1.498  0.13554
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1706 on 234 degrees of freedom
## Multiple R-squared:  0.9995, Adjusted R-squared:  0.9995
## F-statistic: 3.046e+04 on 17 and 234 DF, p-value: < 2.2e-16
```

```
anova_test <- anova(full_model, smaller_model)
anova_test
```

```
## Analysis of Variance Table
##
## Model 1: brozek ~ siri + density + age + weight + height + adipos + free +
##      neck + chest + abdom + hip + thigh + knee + ankle + biceps +
##      forearm + wrist
## Model 2: brozek ~ age + weight + height + abdom
##   Res.Df    RSS  Df Sum of Sq    F    Pr(>F)
## 1      234     6.8
## 2      247 4205.0 -13   -4198.2 11096 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

I don't think it's justifiable to use the smaller model instead of the full model because the smaller model has a higher RSS, lower R-Squared, and we can see in the summary of the full model, there are statistically significant predictors we left out e.g 'siri', 'density'.

b.

```
median_values <- sapply(fat[, -1], median)
new_data_median <- data.frame(t(median_values))

pred_interval_smaller_median <- predict(smaller_model, newdata = new_data_median, interval = "prediction", level = 0.95)
pred_interval_full_median <- predict(full_model, newdata = new_data_median, interval = "prediction", level = 0.95)

pred_interval_smaller_median

##           fit           lwr           upr
## 1 17.84028  9.696631 25.98392

pred_interval_full_median

##           fit           lwr           upr
## 1 18.99539 18.65831 19.33247
```

The CIs differ by a practically importance amount. The CI in the full model is much smaller than that of in the smaller mode.

c.

```
anomalous_cases_smaller <- residuals(smaller_model)[25:50]

# Identify observations with particularly large residuals
outliers_smaller <- which(abs(anomalous_cases_smaller) > 1.5 * sd(anomalous_cases_smaller))

outliers_smaller

## 32 39
##  8 15
```

Observations 32 and 39 seems to be anomalous as they're 1.5 times standard deviation away from the mean.

d.

```
fat_cleaned <- fat[-outliers_smaller, ]
median_values <- sapply(fat_cleaned[, -1], median)
new_data_median <- data.frame(t(median_values))
```

```
smaller_model_cleaned <- lm(brozek ~ age + weight + height + abdom, data = fat_cleaned)
full_model_cleaned <- lm(brozek ~ ., data = fat_cleaned)
```

```
pred_interval_smaller_cleaned <- predict(smaller_model_cleaned, newdata = new_data_median, interval = "prediction")
pred_interval_full_cleaned <- predict(full_model, newdata = new_data_median, interval = "prediction", level = 0.95)
```

```
pred_interval_smaller_cleaned
```

```
##           fit          lwr          upr
## 1 17.84793  9.678289 26.01757
```

Look like removing the anomalous observations does not make much difference to the CI of the smaller model

```
pred_interval_full_cleaned
```

```
##           fit          lwr          upr
## 1 18.99755 18.66048 19.33463
```

Similarly, look like removing the anomalous observations does not make much difference to the CI of the full model.