Homework 4

Hoang Chu

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Problem 1.

Solution: From Equation 4.69:

1. $\mathbf{p(x_1)} = \mathbf{N}(\mu_1, \Sigma_{11}) = \mathbf{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}6 & 8\\8 & 13\end{bmatrix}\right)$
2. $\mathbf{p(x_2)} = \mathbf{N}(\mu_2, \Sigma_{22}) = (N)(5, 14)$
3. $\mathbf{p(x_1|x_2)} = \mathbf{N}(\mu_{1|2}, \Sigma_{1|2}) = \mathbf{N}(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{x_2} - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma 21)$
   Computing each element, we have:
   $$\mathbf{N}(\mu_{1|2}, \Sigma_{1|2}) = \mathbf{N}\left(\frac{1}{14}\begin{bmatrix}0\\0\end{bmatrix}(\mathbf{x_2} - 5), \begin{bmatrix}\frac{59}{14} & \frac{57}{14}\\\frac{57}{14} & \frac{61}{14}\end{bmatrix}\right)$$
4. $\mathbf{p(x_1|x_2)} = \mathbf{N}(\mu_{1|2}, \Sigma_{1|2}) = \mathbf{N}(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{x_2} - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma 21)$
   Computing each element, we have:
   $$\mathbf{N}(\mu_{1|2}, \Sigma_{1|2}) = \mathbf{N}\left(\frac{1}{14}\begin{bmatrix}0\\0\end{bmatrix}(\mathbf{x_2} - 5), \begin{bmatrix}\frac{59}{14} & \frac{57}{14}\\\frac{57}{14} & \frac{61}{14}\end{bmatrix}\right)$$

Problem 2.

Solution:

1. Optimal regularization parameter is 5.0000. See the repo for plots.
2. Accuracy is 0.9221 for regularization parameter = 0.01. See repo for plots.