Integer Programming

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30 August 2022

Abstract

We shall conquer at all costs!

What is it?

Def LP problems with at least one variable that has to be an integer.

Types

- Pure - All decision vars have to be integers. $X \in int$
- Mixed - Some ints and some not (Even int + binary).
- Binary - Decision vars can only take on the values 0 or 1.
  $X \in \{0; 1\}$

Characteristics

- The change from a normal LP to an integer one changes the problem drastically; Making it more complex (Might not even have a solution).

Example 1

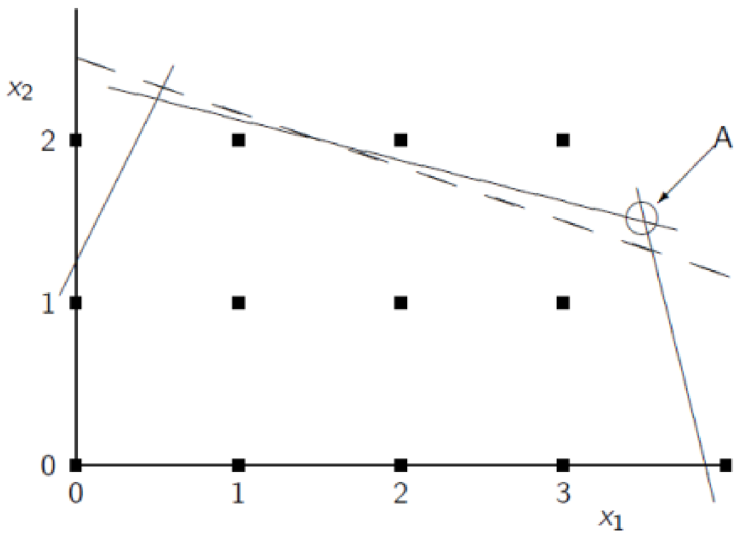

Figure 1: Example 1

Max $X_1 + 3X_2$
$S.t:$
$2X_1 + 8X_2 \leq 9$
$4X_1 + X_2 \leq 15.5$
$8X_1 - 4X_2 \geq -5$
$X_1, X_2 \geq 0$ and Integer

- Point A would be the optimal solution point if this was a normal LP problem.
- Trying to use the same point rounded up would result in one using a point that's actually not in the feasible region.
- If you round down, the solution would be feasible but not optimal.
- Adding an int constraint can only restrict the solution.
- Therefore A is the best solution we could ever hope to achieve having restricted things.
- So shifting the isocost line down from A (Restricting solution), the first int point it touches will be the optimal point.
- Theres an algorithm we'll use to find these solutions.

Example 2

Suppose we have identified four investment opportunities and we have R14000 to invest. Project 1 requires an investment of R5,00 and has a present value (a time-discounted value) of R8,000; Project 2 requires R7,000 and has a value of R11,000; Project 3 requires R4,000 and has a value of R6,000; and Project 4 requires R3,000 and has a value of R4,000. The question is: into which projects should we place our money in order to maximize our total present value?

Figure 2: Example 2

$$Let X_j = \begin{cases} 1 & \text{if project } j \text{ is selected; j = 1,2,3,4} \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

Max $8000X_1 + 11000X_2 + 6000X_3 + 4000X4$ St: $5000X_1 + 7000X_2 + 4000X_3 + 3000X_4 \leq 14000$
$X_1, X_2, X_3, X_4 \in \{0, 1\}$

- Using simplex, Z = R22000 ($X_1 = X_2 = 1, X_3 = 0.5, X_4 = 0$)
- This obviously doesn't satisfy the last constraint.
- If you round down, Z = R19000
- Better solution - $X_1 = 0, X_2 = X_3 = X_4 = 1$
- This is found using the algorithm we'll learn later.