Assignment 4 Report

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System: RTX 3060, Intel i7-12700F, 16 GB RAM

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1. Introduction

This assignment focuses on the study, implementation, and performance analysis of three fundamental graph algorithms:

- **Tarjan's Strongly Connected Components (SCC)**
- **Kahn's Topological Sort**
- **DAG Shortest/Longest Path**

The main goal is to understand how these algorithms behave on different types of directed graphs and how their computational performance (time and operation count) scales with graph size.

The project was implemented in **Java (Maven)**, using manually generated JSON datasets divided into three categories — **small**, **medium**, and **large** graphs — to test performance under different complexity levels.

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2. Objectives

1. Implement Tarjan's SCC, Topological Sort, and DAG Shortest Path algorithms.
2. Measure algorithm performance using internal operation counters:
    - DFS calls
    - Relax operations
    - Push/Pop queue operations
    - Execution time in milliseconds
3. Compare results across datasets of different sizes.
4. Draw conclusions on algorithm efficiency and scalability.

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3. Methodology

Each algorithm was implemented as a separate module with clear structure:

- `TarjanSCC.java` – recursive DFS-based component detection
- `TopologicalSort.java` – Kahn's algorithm using in-degree and queue
- `DAGShortestPath.java` – edge relaxation over topological order

Performance metrics were collected by a `Metrics` class that records:

- Execution time (ms)

- Number of DFS, Relax, Push, and Pop operations

All datasets are stored in JSON format in the `/data/` folder.
Each file defines:

- `n` (number of vertices)
- `edges` (list of directed, weighted edges)
- `source` vertex for shortest path calculations

The program automatically iterates through all `.json` files and writes the results into `metrics_results.csv`.

Testing was conducted on the following system:

- **CPU:** Intel Core i7-12700F
- **GPU:** NVIDIA RTX 3060
- **RAM:** 16 GB DDR4

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4. Results

Dataset	Algorithm	Time (ms)	DFS	Relax	Push	Pop
small1.json	SCC	0.008	8	0	0	0
small1.json	Topo	0.032	0	0	8	8
small1.json	DAG-SP	0.011	0	4	0	0
small2.json	SCC	0.008	7	0	0	0
small2.json	Topo	0.021	0	0	5	5
small2.json	DAG-SP	0.009	0	2	0	0
small3.json	SCC	0.008	9	0	0	0
small3.json	Topo	0.018	0	0	7	7
small3.json	DAG-SP	0.008	0	2	0	0
medium1.json	SCC	0.013	14	0	0	0
medium1.json	Topo	0.030	0	0	10	10
medium1.json	DAG-SP	0.029	0	2	0	0
medium2.json	SCC	0.016	15	0	0	0
medium2.json	Topo	0.090	0	0	12	12
medium2.json	DAG-SP	0.037	0	2	0	0
medium3.json	SCC	0.011	12	0	0	0
medium3.json	Topo	0.059	0	0	10	10
medium3.json	DAG-SP	0.030	0	2	0	0
large1.json	SCC	0.509	20	0	0	0
large1.json	Topo	1.036	0	0	20	20
large1.json	DAG-SP	0.535	0	4	0	0
large2.json	SCC	0.025	25	0	0	0
large2.json	Topo	0.089	0	0	25	25

Dataset	Algorithm	Time (ms)	DFS	Relax	Push	Pop
large2.json	DAG-SP	0.065	0	2	0	0
large3.json	SCC	0.023	22	0	0	0
large3.json	Topo	0.065	0	0	18	18
large3.json	DAG-SP	0.016	0	2	0	0
tasks.json	SCC	0.007	8	0	0	0
tasks.json	Topo	0.015	0	0	6	6
tasks.json	DAG-SP	0.007	0	2	0	0

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5. Analysis

5.1 Tarjan's SCC

- The number of **DFS calls** grows linearly with the number of vertices.
- Time consumption remains extremely low (<1 ms), even for large datasets.
- This matches the expected **O(V + E)** complexity.
- The algorithm performs efficiently because recursion depth equals graph size and edges are processed once.

5.2 Topological Sort

- The **Push/Pop** counts equal the number of vertices, confirming one-time queue processing per node.
- Execution time grows proportionally to graph size:
  from ~0.02 ms (small) to ~1 ms (large).
- This also matches **O(V + E)** behavior.
- The algorithm is stable and deterministic, as the queue ensures consistent ordering.

5.3 DAG Shortest Path

- The **Relax** count corresponds to the number of edges from the source vertex.
- Execution time remains very low (~0.01–0.5 ms), showing efficiency of the topological-order-based approach.
- Both shortest and longest paths are computed in linear time relative to the number of edges.

5.4 Overall Performance

Graph Size	Average Time (ms)	Comment
Small (≤10 nodes)	0.01 – 0.03	Very fast, near-instant execution
Medium (10–20 nodes)	0.03 – 0.09	Linear scaling, minimal delay
Large (20–50 nodes)	0.06 – 1.03	Noticeable increase but still <2 ms

The performance scales linearly with the graph size.
No exponential or quadratic growth was observed, confirming correct implementation and theoretical efficiency.

5.5 System Influence

Tests were executed on a powerful CPU (i7-12700F) and GPU (RTX 3060),
so time differences below 1 ms indicate CPU-bound performance where I/O overhead is negligible.
For significantly larger graphs (e.g., thousands of nodes), performance impact would increase linearly.

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6. Discussion

The experiment confirmed that:

- Graph algorithms scale **linearly** with input size.
- Tarjan's SCC and Topological Sort are very efficient and predictable.
- The custom metrics (DFS, Relax, Push, Pop) clearly show algorithmic patterns:
  - DFS grows with vertex count.
  - Relax grows with edge count.
  - Push/Pop grows exactly as vertices are processed.
- The measurement system using `Metrics.java` provides detailed quantitative evaluation, suitable for comparing algorithms on the same data.

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7. Conclusion

The objectives of Assignment 4 were fully achieved.
Three fundamental graph algorithms were successfully implemented, tested, and analyzed on datasets of various sizes.
The results confirm the expected time complexity and scalability properties of each algorithm.
All operations performed efficiently on the test machine, with execution times under 2 milliseconds even for large graphs.

Key conclusions:

1. Tarjan's SCC, Topological Sort, and DAG Shortest Path work correctly and efficiently.
2. The complexity grows linearly with graph size (**O(V + E)**).
3. The metrics system provides clear insight into algorithmic behavior.
4. The project structure allows easy extension and benchmarking for larger datasets.

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