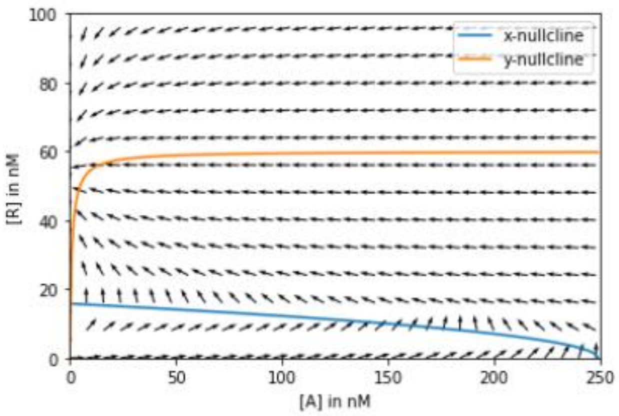
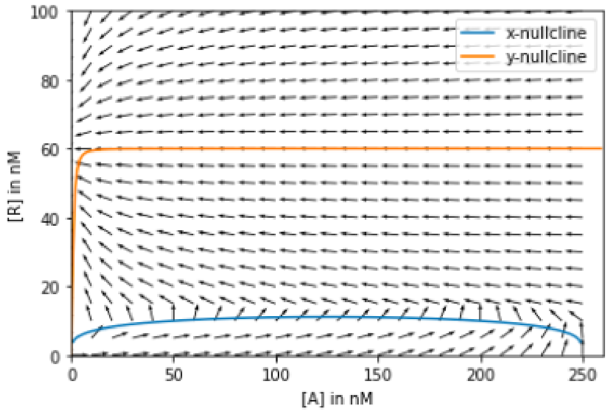
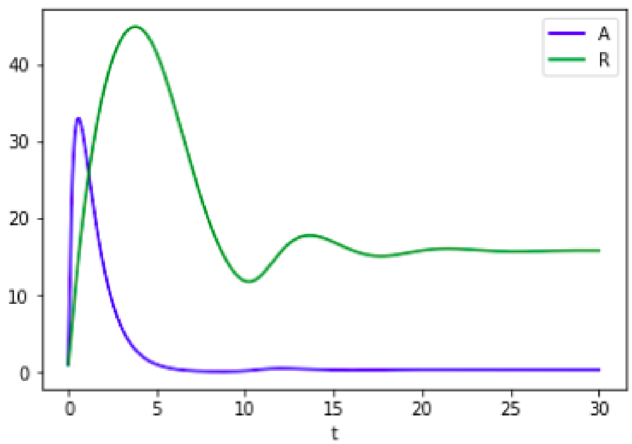
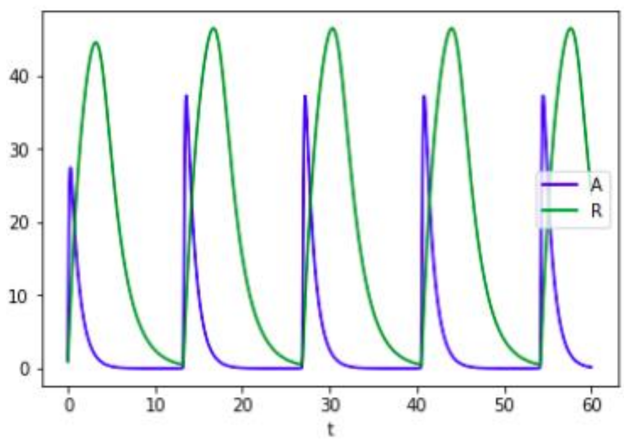
1.

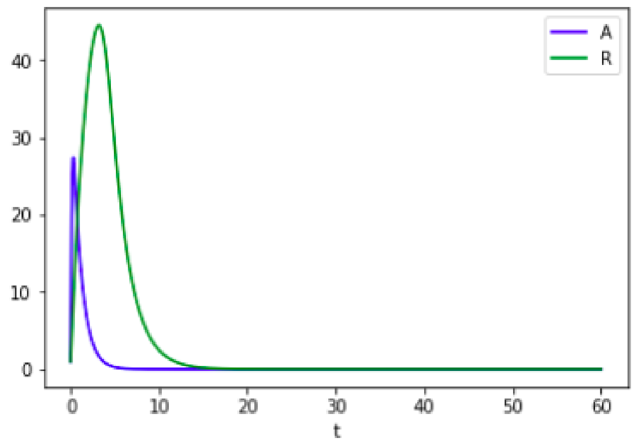
a. When the reporter is expressed on the repressilator plasmid but not on a separate high-copy plasmid, oscillations are more regular.

b. The competition for shared proteases for the degradation tagged reporter proteins, result in ‘retroactivity’ effects on oscillations. Removing the interference created very regular oscillations.

c. The variance in the three interpeak distances showed that the noisiest phase was when TetR levels were low, so they use titration sponge, which is plasmid with PLtet-binding sites, to soak up TetR molecules away from the TetR-controlled promoter, which raising the threshold effectively and reduced noise.

2.

<table>
<tr><td colspan="2">a. Both are present the leak of transcription rate of A and R, respectively. The unit is as AlphaA, $nM*hrs^{-1}$. (What we derive from beginning goff to gon, the Alpha0 was usually separate with the probability. It would be a little confusing when is divided by denominator.)</td></tr>
<tr><td>b_i.

</td><td>b_ii.

</td></tr>
<tr><td>c.(n=1, m=2)

</td><td>c.(n=2, m=4) sustained

</td></tr>
</table>

<table>
<tr>
<td>d.(n=2, m=4, alphaA0=0)

</td>
<td>d.
When the alphaA0 = 0, the reboost or restart of the signal A is harder or can't be. The oscillation begins again when the signal hit 0, but when there's no leak of transcription of A and A still be degraded, the system absents of A, then there's no next round, in the other words, the whole system shut down.</td>
</tr>
<tr>
<td colspan="2">e. When gammaA=gammaR, or gammaA bit smaller than gammaR, result in oscillations (ex: gammaA=0.5, gammaR=0.5), but the response drops down through t, finally the signal approaching 0. When the gammaA is bit higher than gammaR (ex: gammaA=0.51, gammaR=0.5, it results in sustained oscillation. However, when gammaA is too big (ex: gammaA=3.1622, gammaR=0.5), there's no oscillation. The rate setting is pretty critical, it still sustained oscillation when (ex: gammaA=3.1621, gammaR=0.5). When the gammaA bit higher than gammaR, the frequency of oscillation is highest, the higher the gammaA, the lower the frequency.</td>
</tr>
</table>

3.

a. $[m]_{ss} = \frac{\alpha_m}{\gamma_m + \frac{k\times C\times \alpha_m}{\gamma_s}}$

b. $\gamma_m \ll \frac{k\times C\times \alpha_m}{\gamma_s}$

$[m]_{ss} = \frac{\gamma_s}{k\times C}$

The $[m]_{ss}$ is independent with its own production rate, α_m.

c. $S\left(\frac{\frac{\Delta[m]_{ss}}{[m]_{ss}}}{\frac{\Delta\alpha_m}{\alpha_m}}\right) = \frac{\alpha_m}{[m]_{ss}} \times \frac{d[m]_{ss}}{d\alpha_m} = \frac{\gamma_m}{1+\gamma_m}$

When γm = 0, sensitivity = 0.

When γm = 1, sensitivity = 1/2.

The calculation procedure and plot are showed in picture below: (next page)

$$S([M]_{ss}, \alpha_m)$$

$$c) = \frac{\Delta [M]_{ss}/[M]_{ss}}{\Delta \alpha_m / \alpha_m} = \frac{\alpha_m}{[M]_{ss}} \cdot \frac{\Delta [M]_{ss}}{\Delta \alpha_m} = \frac{\alpha_m}{[M]_{ss}} \cdot \frac{d[M]_{ss}}{d\alpha_m}$$

$$= \Big(\gamma_m + \underbrace{\frac{K \cdot c \cdot \alpha_m}{\gamma_s}}_{=1}\Big) \cdot \frac{d[M]_{ss}}{d\alpha_m} = (\gamma_m + 1) \cdot \frac{d \dfrac{\alpha_m}{\gamma_m + \frac{K \cdot c \alpha_m}{\gamma_s}}}{d\alpha_m}$$

$$\frac{d}{d\alpha_m} \frac{\alpha_m}{\gamma_m + \frac{K \cdot c \alpha_m}{\gamma_s}} = \frac{d}{d\alpha_m} \frac{\alpha_m / \gamma_m}{\underbrace{\frac{K \cdot c \alpha_m}{\gamma_s \gamma_m}}_{\mu} + 1}$$

$$= \frac{1}{\gamma_m} \frac{d}{d\alpha_m} \frac{\alpha_m}{\mu \alpha_m + 1}$$

$$= \frac{1}{\gamma_m} \times \frac{1 + \cancel{\mu \alpha_m} - \cancel{\alpha_m \mu}}{(1 + \mu \alpha_m)^2}$$

$$= \frac{1}{\gamma_m} \times \frac{1}{\Big[1 + \Big(\underbrace{\frac{K \cdot c}{\gamma_s \gamma_m} \alpha_m}_{\text{Assume} = 1}\Big)\Big]^2} = \frac{1}{\gamma_m} \times \frac{1}{\left(1 + \frac{1}{\gamma_m}\right)^2}$$

$$= \frac{\gamma_m}{(1 + \gamma_m)^2}$$

$$S = \cancel{(\gamma_m + 1)} \cdot \frac{\gamma_m}{(1 + \gamma_m)^{\cancel{2}}} = \frac{\gamma_m}{1 + \gamma_m}$$