

Outputs inspection half CIFAR10

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 4.0.5
```

```
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 4.0.5
```

```
##
```

```
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
##      filter, lag
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##      intersect, setdiff, setequal, union
```

```
library(tidyr)
```

```
## Warning: package 'tidyr' was built under R version 4.0.5
```

```
library("ggpubr")
```

```
## Warning: package 'ggpubr' was built under R version 4.0.5
```

```
library(LDATS)
```

```
## Warning: package 'LDATS' was built under R version 4.0.5
```

```
library(stringr)
```

```
library(reshape2)
```

```
## Warning: package 'reshape2' was built under R version 4.0.3
```

```
##
```

```
## Attaching package: 'reshape2'
```

```
## The following object is masked from 'package:tidyr':
```

```
##
```

```
##      smiths
```

```
library(ggVennDiagram)
```

```
## Warning: package 'ggVennDiagram' was built under R version 4.0.5
```

```
library(reticulate)
```

```
## Warning: package 'reticulate' was built under R version 4.0.5
```

```
library(abind)
```

```
## Warning: package 'abind' was built under R version 4.0.3
```

```
np <- import("numpy")
```

```
source("utils.R")
```

```
## Warning: package 'hash' was built under R version 4.0.5
```

```
## hash-2.2.6.1 provided by Decision Patterns
```

```
## Warning: package 'berryFunctions' was built under R version 4.0.5
```

```
##
```

```
## Attaching package: 'berryFunctions'
```

```
## The following object is masked from 'package:ggVennDiagram':
```

```
##
```

```
## circle
```

```
## The following object is masked from 'package:dplyr':
```

```
##
```

```
## between
```

```
## Warning: package 'purrr' was built under R version 4.0.3
```

Visualization on CIFAR10. We are using data of three neural networks trained on reduced CIFAR10 training set. Half of the CIFAR10 training set was extracted as a validation set. We then divided both the reduced training set and validation set into 50 disjoint subsets and trained an ensemble on each of them. In this visualization, we are trying to inspect the outputs deeper, mainly to make sense of strange behavior of nll metric for ensemble outputs.

```
base_dir <- "../data/data_train_val_half_c10"
```

```
repls <- 0:0
```

```
folds <- 0:49
```

```
classes <- 10
```

```
nets_outputs <- load_network_outputs(base_dir, repls)
```

```
ens_outputs <- load_ensemble_outputs(base_dir, repls, folds)
```

```
net_results <- read.csv(file.path(base_dir, "net_accuracies.csv"))
```

```
ens_results <- read.csv(file.path(base_dir, "ensemble_accuracies.csv"))
```

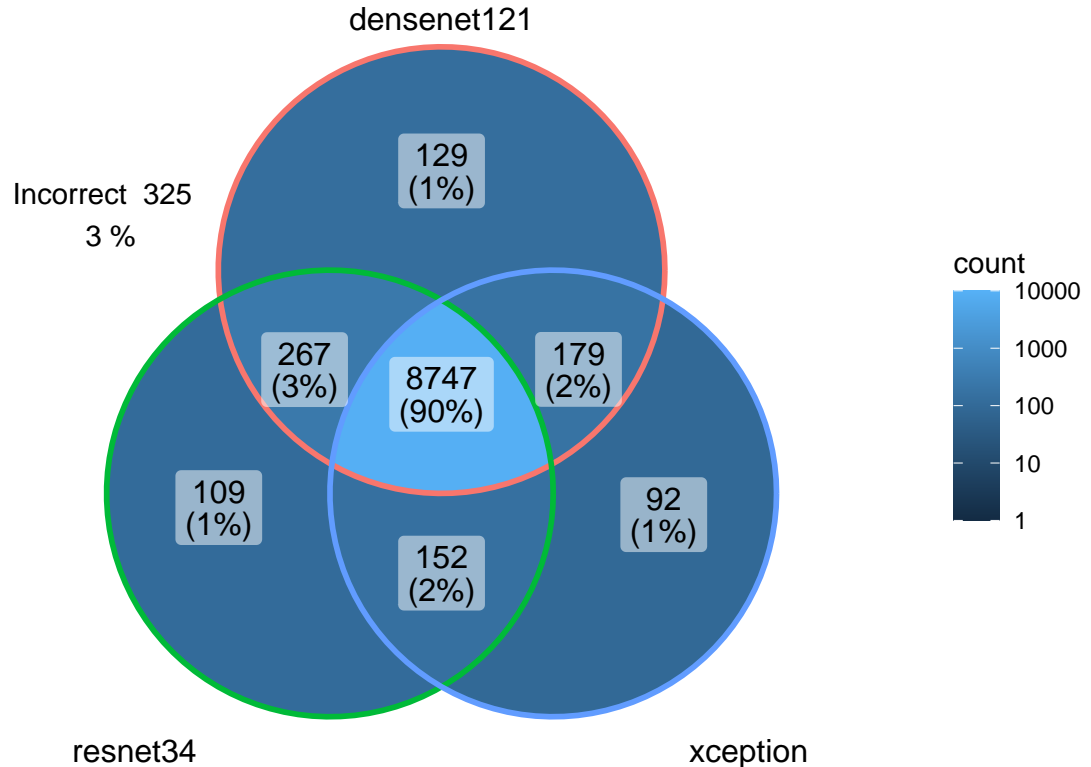
```

sort_ind <- function(lst)
{
  return(sort(lst, index.return=TRUE, decreasing=TRUE)$ix)
}
nets_test_top_indices <- apply(X=nets_outputs$test_outputs, MARGIN=c(1, 2, 3), FUN=sort_ind)[1, , , ]
r_n <- length(repls)
samples_n <- dim(nets_outputs$test_labels)[2]
nets_n <- length(nets_outputs$networks)
test_labs <- nets_outputs$test_labels + 1
dim(test_labs) <- c(r_n, 1, samples_n)
test_labs <- aperm(abind(array(rep(aperm(test_labs, perm=c(2, 1, 3))), nets_n), c(r_n, samples_n, nets_n,
if (r_n == 1)
{ dim(test_labs) <- dim(test_labs)[-1] }
nets_test_cor_preds <- test_labs == nets_test_top_indices

nets_cor_list <- list()
incor <- 1:samples_n
for (ni in 1:nets_n)
{
  cor_list <- which(nets_test_cor_preds[ni, ])
  nets_cor_list[[nets_outputs$networks[ni]]] = cor_list
  incor <- setdiff(incor, cor_list)
}
incor_n <- length(incor)
venn_diag <- ggVennDiagram(nets_cor_list) + scale_fill_gradient(trans="log10", name="count", limits=c(1
  annotate(geom="text", x=-4, y=5, label=paste("Incorrect ", incor_n, "\n", round(incor_n / samples_n *
    ggtitle("Correct predictions by network") +
    scale_x_continuous(limits=c(-8, 10))
print(venn_diag)

```

Correct predictions by network

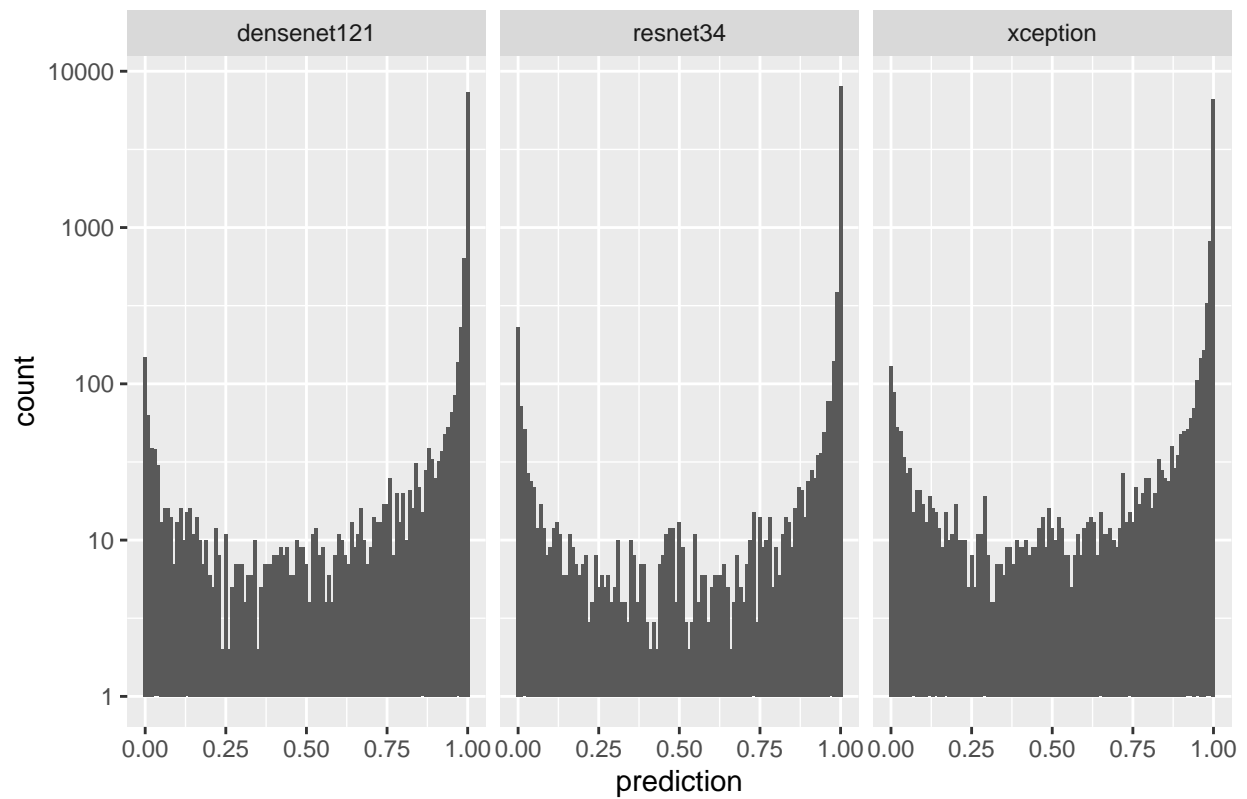


Compared to training the networks on almost complete CIFAR 10 training set, in this case all networks are correct in fewer samples. Otherwise the observations hold.

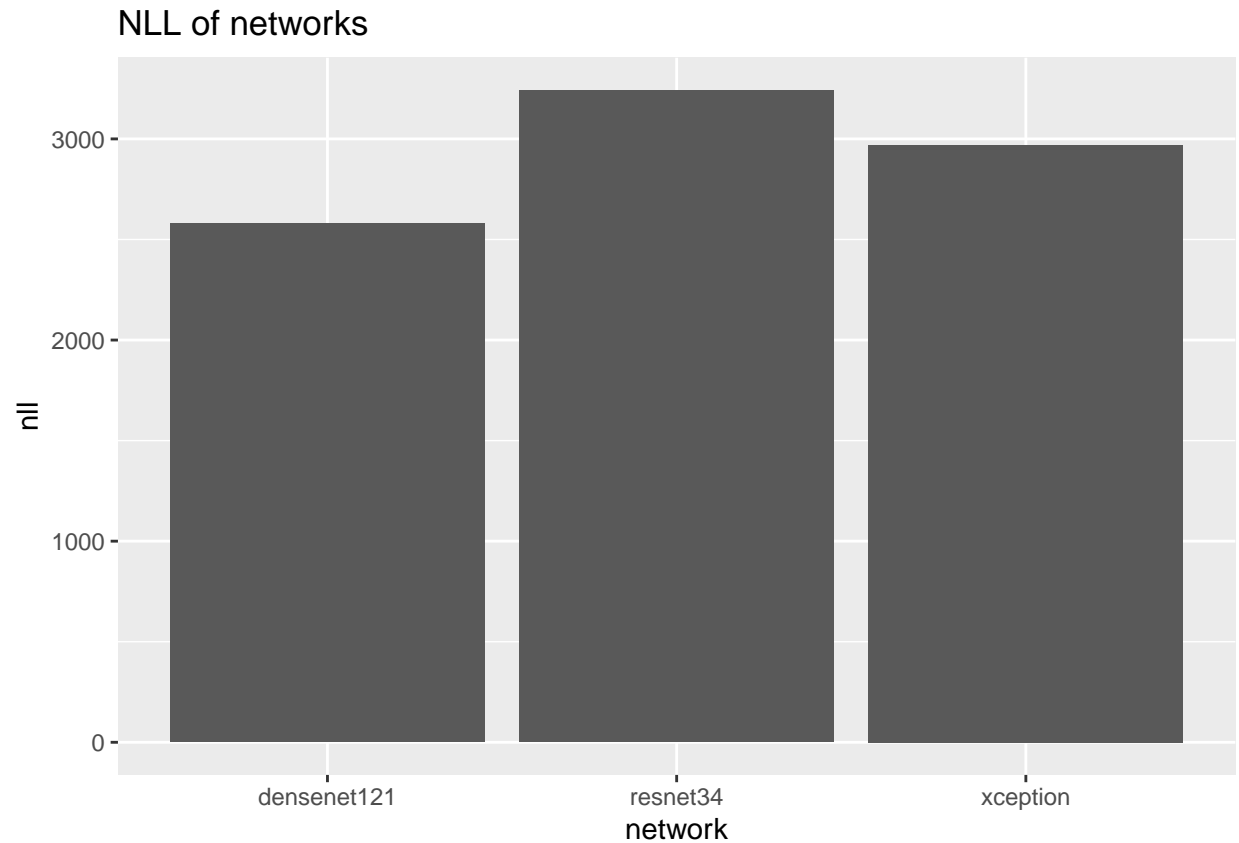
```
preds <- nets_outputs$test_outputs
for (ri in repls + 1)
{
  for (net_i in seq_along(nets_outputs[["networks"]]))
  {
    preds[ri, net_i, ,] <- softmax(preds[ri, net_i, ,])
  }
}
nets_test_cor_probs <- gather(preds, 1 + nets_outputs$test_labels[1, ], 3, 4)
nets_test_cor_probs <- melt(nets_test_cor_probs)
nets_test_cor_probs <- nets_test_cor_probs[, c(-3, -4)]
names(nets_test_cor_probs) <- c("replication", "network", "prediction")
nets_test_cor_probs$network <- as.factor(nets_test_cor_probs$network)
levels(nets_test_cor_probs$network) <- nets_outputs$networks
```

```
nets_cor_preds_histo <- ggplot(data=nets_test_cor_probs) + geom_histogram(mapping=aes(x=prediction), binwidth=0.05)
ggtitle("Histograms of predicted probability for the correct class") + facet_wrap(~network) + scale_y_continuous()
nets_cor_preds_histo
```

Histograms of predicted probability for the correct class



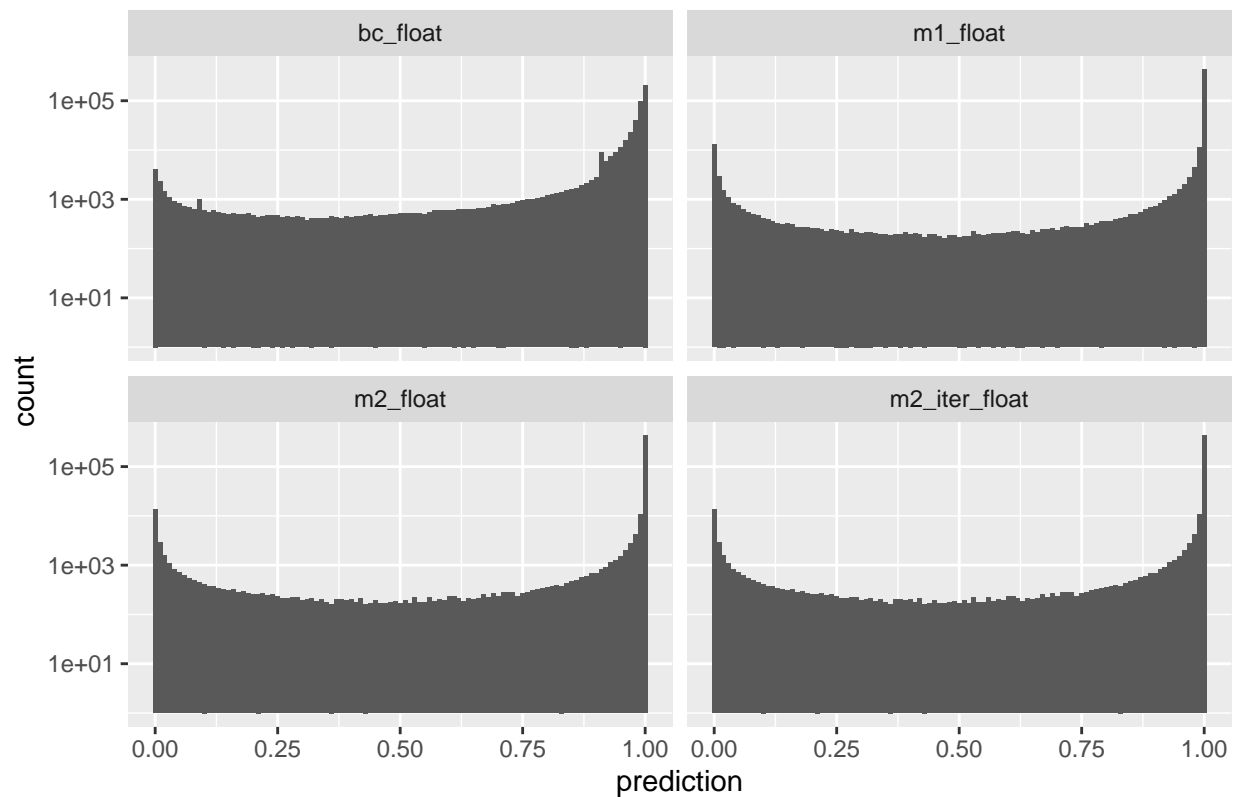
```
networks_nll <- ggplot(data=net_results) + geom_bar(mapping=aes(x=network, y=nll), stat="identity") + g  
networks_nll
```



```
val_ens_cor_probs <- gather(ens_outputs$val_training, 1 + nets_outputs$test_labels[1, ], 4, 5)
val_ens_cor_probs <- melt(val_ens_cor_probs)
val_ens_cor_probs <- val_ens_cor_probs[, c(-4, -5)]
names(val_ens_cor_probs) <- c("replication", "method", "fold", "prediction")
val_ens_cor_probs$method <- as.factor(val_ens_cor_probs$method)
levels(val_ens_cor_probs$method) <- ens_outputs$methods
```

```
val_ens_cor_preds_histo <- ggplot(data=val_ens_cor_probs) + geom_histogram(mapping=aes(x=prediction), b
val_ens_cor_preds_histo
```

Probabilities predicted for the correct class – ens trained on val



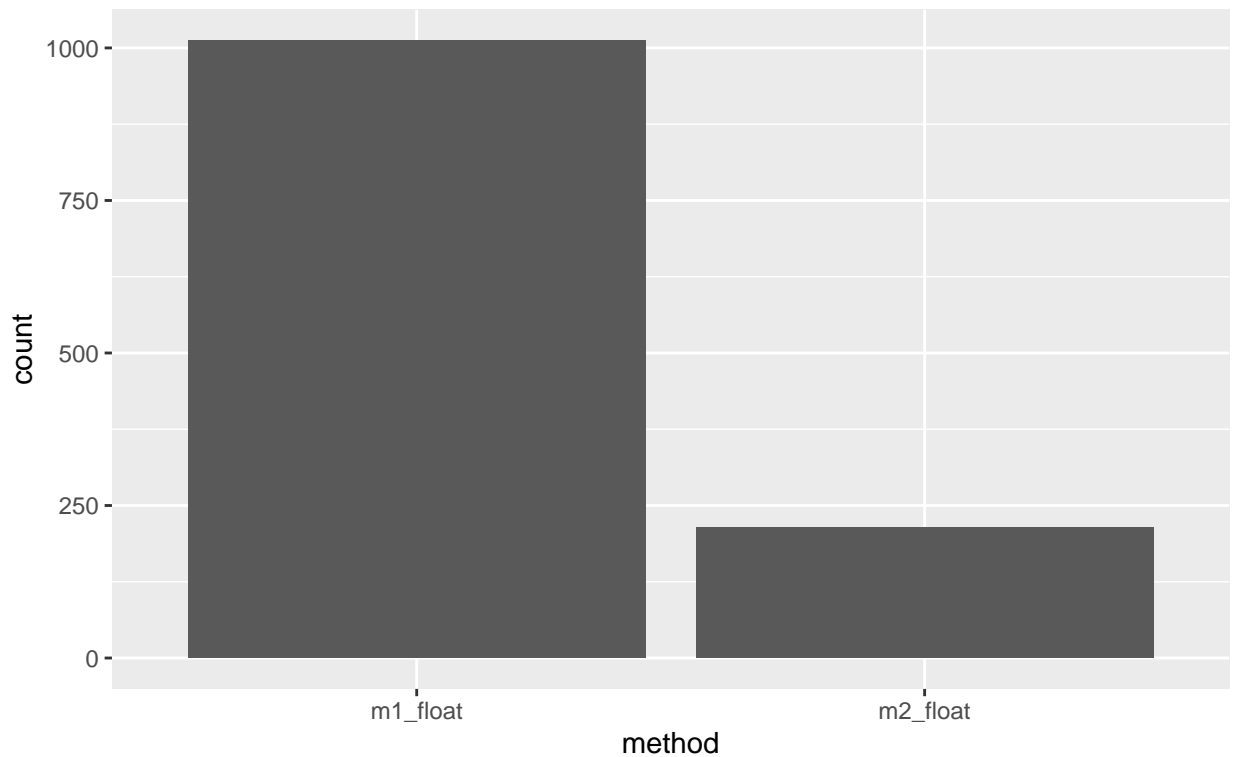
Coupling method bc produces fewer probabilities falling into the lowest bin for the correct class than m1 and m2.

```
val_ens_zero_counts <- ggplot(data=val_ens_cor_probs[val_ens_cor_probs$prediction <= 0, ]) + geom_histogram
```

```
## Warning: Ignoring unknown parameters: binwidth, bins, pad
```

```
val_ens_zero_counts
```

Counts of subzero probabilities predicted for the correct class by coup m
Validation training

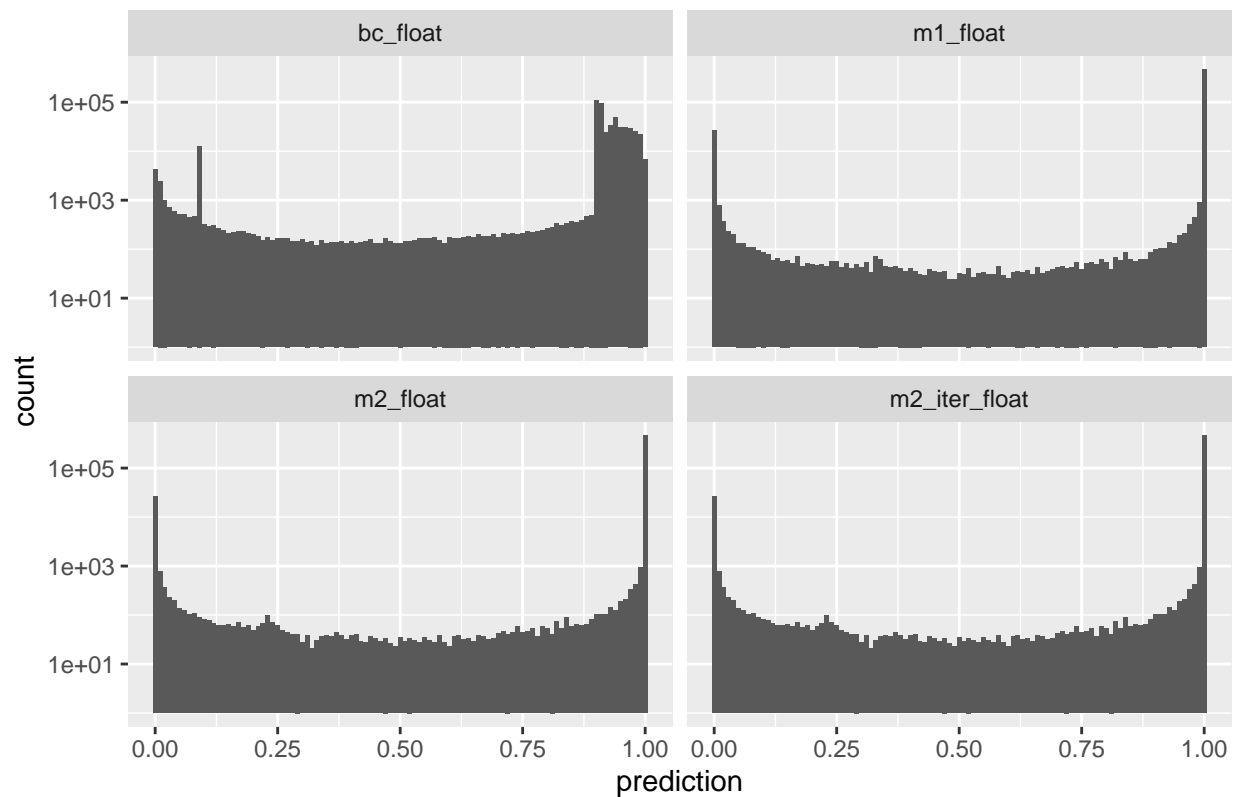


m2_iter and bc didn't produce any zero probability outputs.

```
train_ens_cor_probs <- gather(ens_outputs$train_training, 1 + nets_outputs$test_labels[1, ], 4, 5)
train_ens_cor_probs <- melt(train_ens_cor_probs)
train_ens_cor_probs <- train_ens_cor_probs[, c(-4, -5)]
names(train_ens_cor_probs) <- c("replication", "method", "fold", "prediction")
train_ens_cor_probs$method <- as.factor(train_ens_cor_probs$method)
levels(train_ens_cor_probs$method) <- ens_outputs$methods
```

```
train_ens_cor_preds_histo <- ggplot(data=train_ens_cor_probs) + geom_histogram(mapping=aes(x=prediction))
train_ens_cor_preds_histo
```


Probabilities predicted for the correct class – ens trained on train



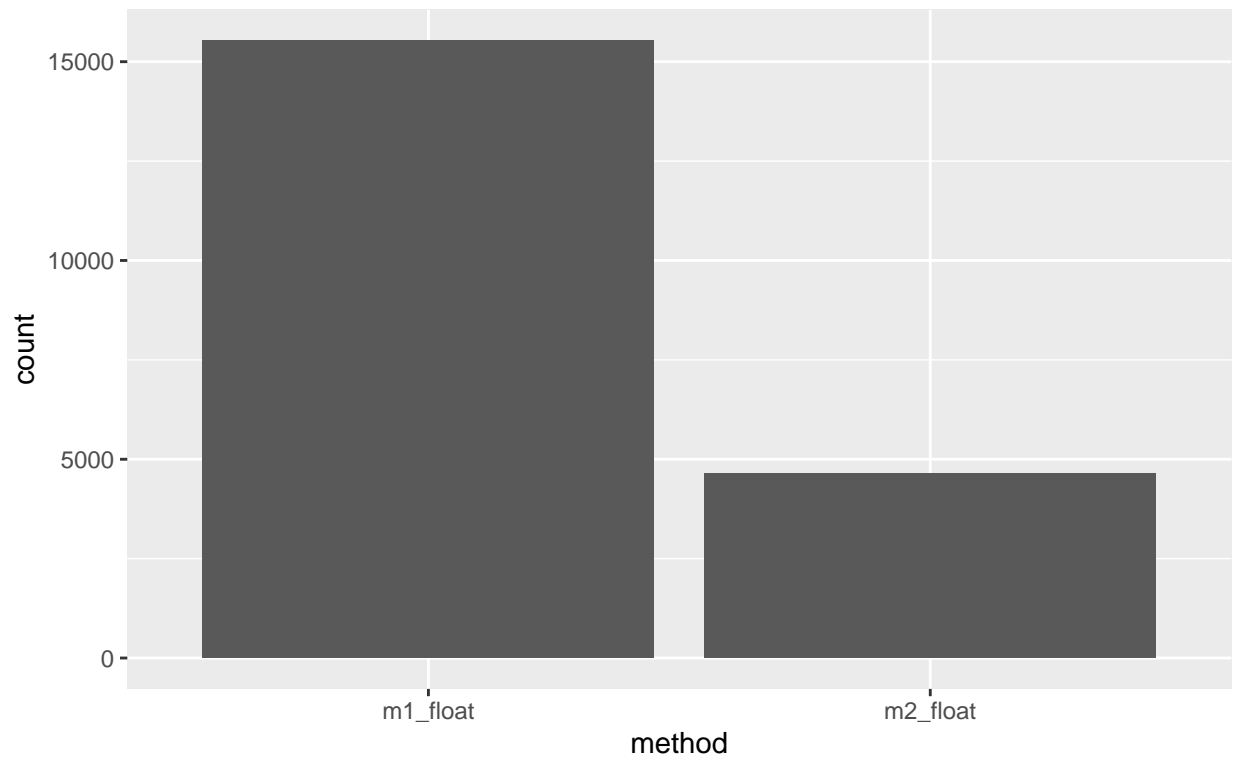
Again, coupling method bc produces fewer probabilities falling into the lowest bin for the correct class than m1 and m2.

```
train_ens_zero_counts <- ggplot(data=train_ens_cor_probs[train_ens_cor_probs$prediction <= 0, ]) + geom
```

```
## Warning: Ignoring unknown parameters: binwidth, bins, pad
```

```
train_ens_zero_counts
```

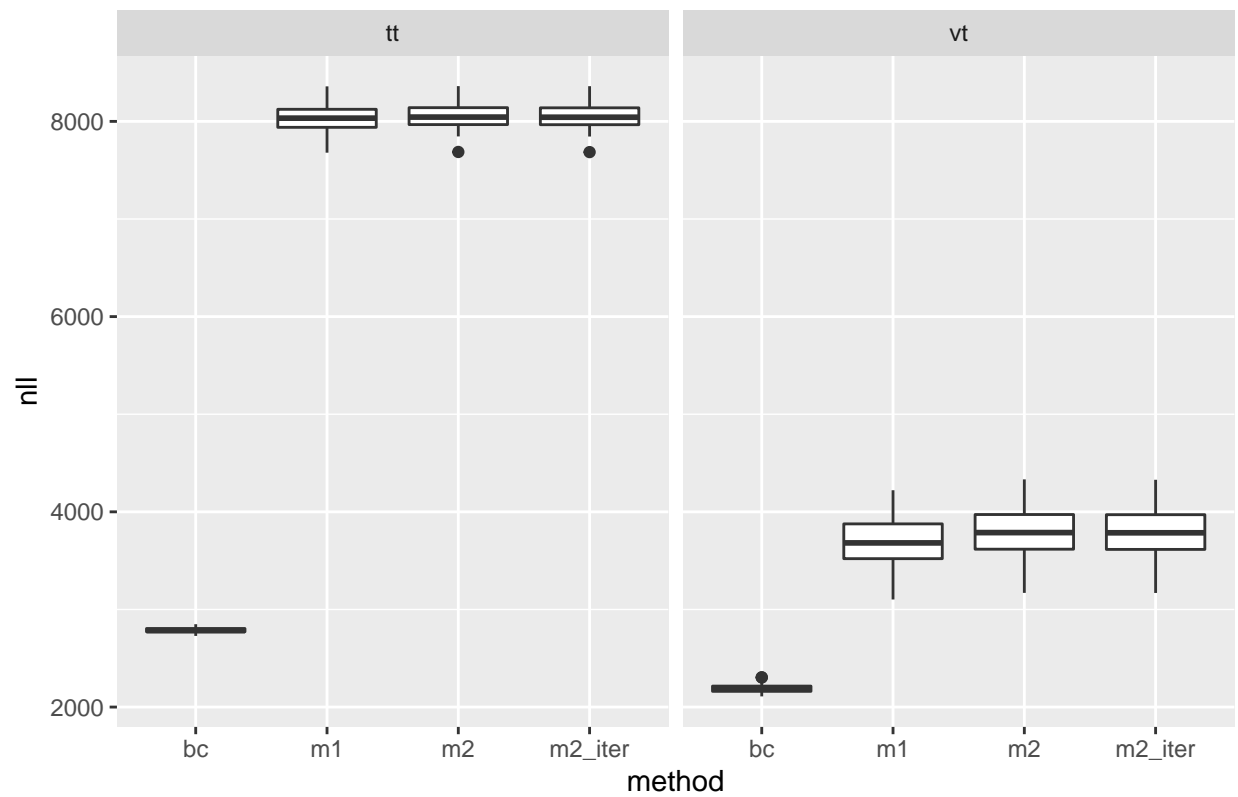
Counts of zero or lower probabilities predicted for the correct class by coupling method
Train training



m2_iter and bc didn't produce any zero probability outputs.

```
val_ens_nll <- ggplot(data=ens_results) + geom_boxplot(mapping=aes(x=method, y=nll)) + facet_wrap(~train)
  ggtitle("Comparison of nll for coupling methods for different LDA train methodologies")
val_ens_nll
```

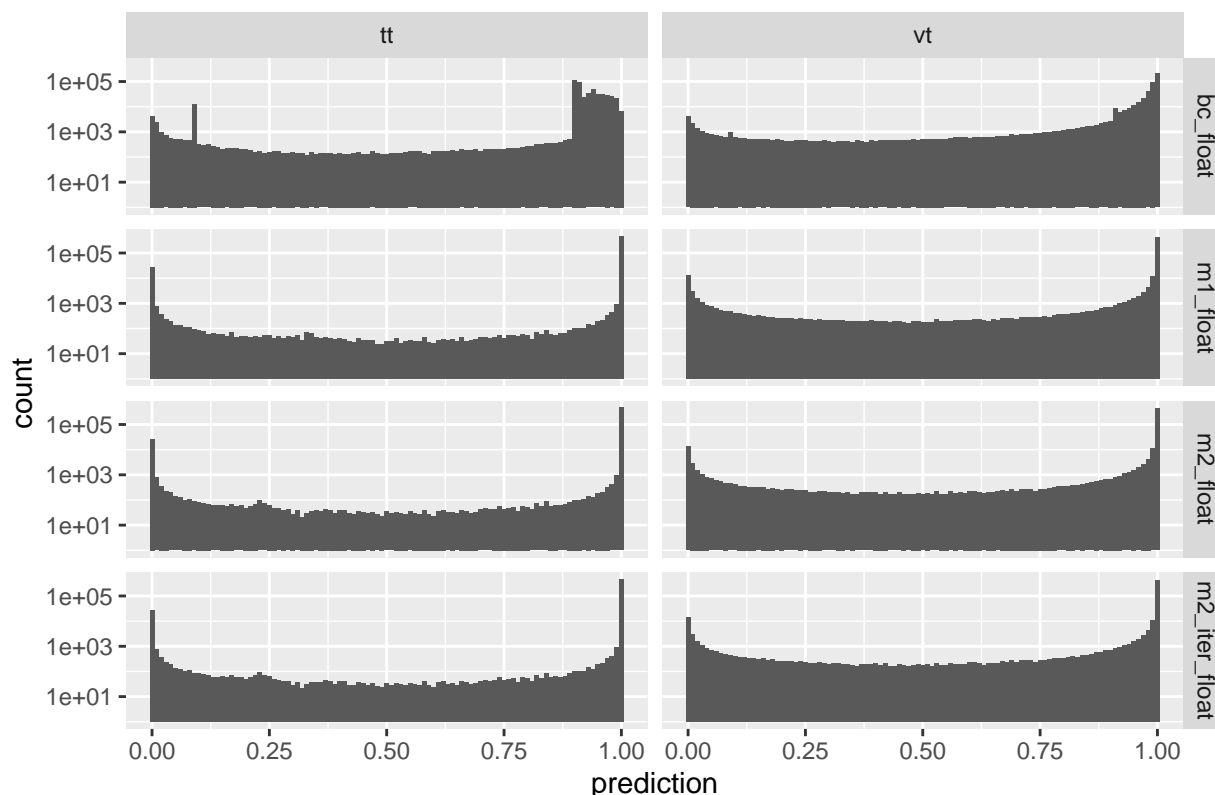
Comparison of nll for coupling methods for different LDA train methodologies



```
val_ens_cor_probs$train_type <- "vt"
train_ens_cor_probs$train_type <- "tt"
ens_cor_probs <- rbind(val_ens_cor_probs, train_ens_cor_probs)
```

```
ens_cor_preds_histo <- ggplot(data=ens_cor_probs) + geom_histogram(mapping=aes(x=prediction), binwidth=
ens_cor_preds_histo
```

Probabilities predicted for the correct class



Bayes covariant coupling method produces more uniformly distributed predictions than methods m1 and m2. Also, there is a big difference in each method between ensemble trained on validation and ensemble trained on train set. Ensembles trained on validation set produce generally more uniformly distributed predictions. However, ensembles trained on training set attain statistically significantly higher accuracy. Similar results to those in `visualizations_ensemble_outputs_CIF10`.

```
aggreg_Rs <- load_class_averaged_R_matrices(base_dir, nets_outputs$test_labels[1, ], repls, folds=folds)
```

```
## 'summarise()' has grouped output by 'class1', 'class2'. You can override using the '.groups' argument
## 'summarise()' has grouped output by 'class1', 'class2'. You can override using the '.groups' argument
## 'summarise()' has grouped output by 'class1', 'class2'. You can override using the '.groups' argument
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## 'summarise()' has grouped output by 'class1', 'class2'. You can override using the '.groups' argument
## 'summarise()' has grouped output by 'class1', 'class2'. You can override using the '.groups' argument
```

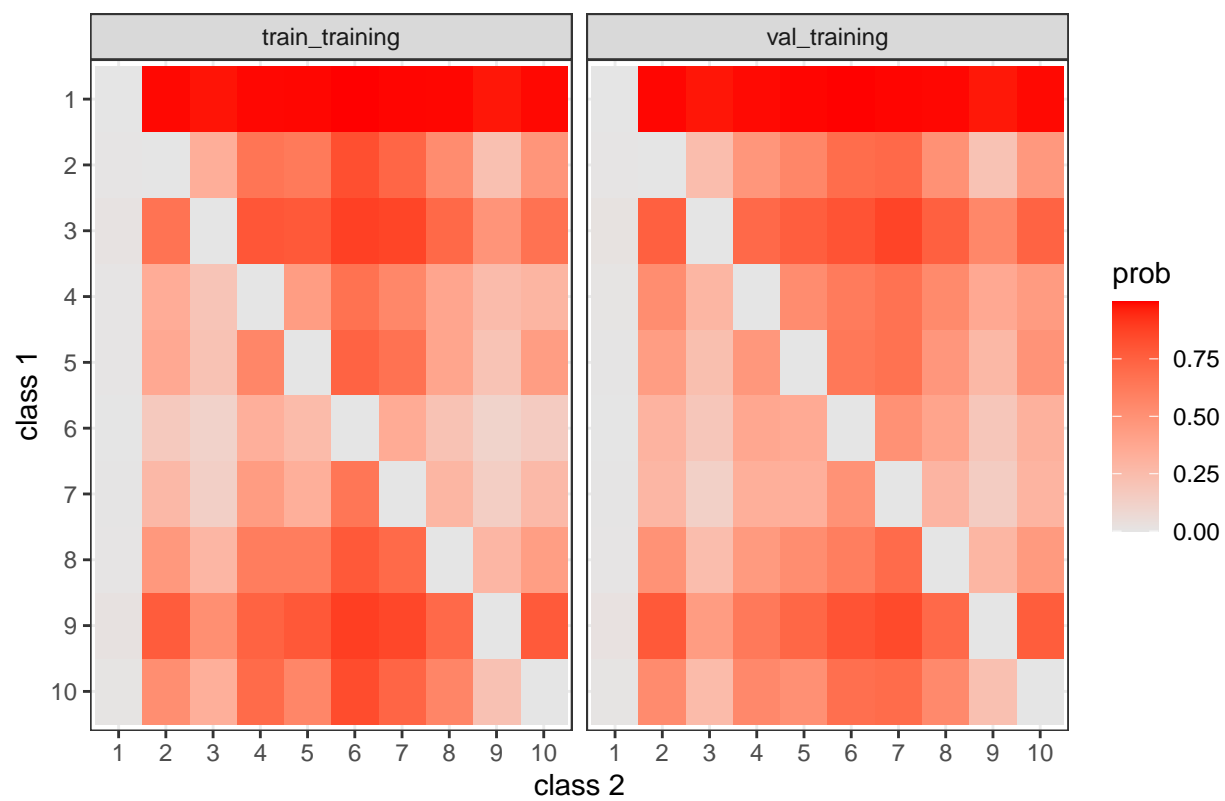
[illegible]

```
## 'summarise()' has grouped output by 'precision', 'train_type', 'class1', 'class2'. You can override
```

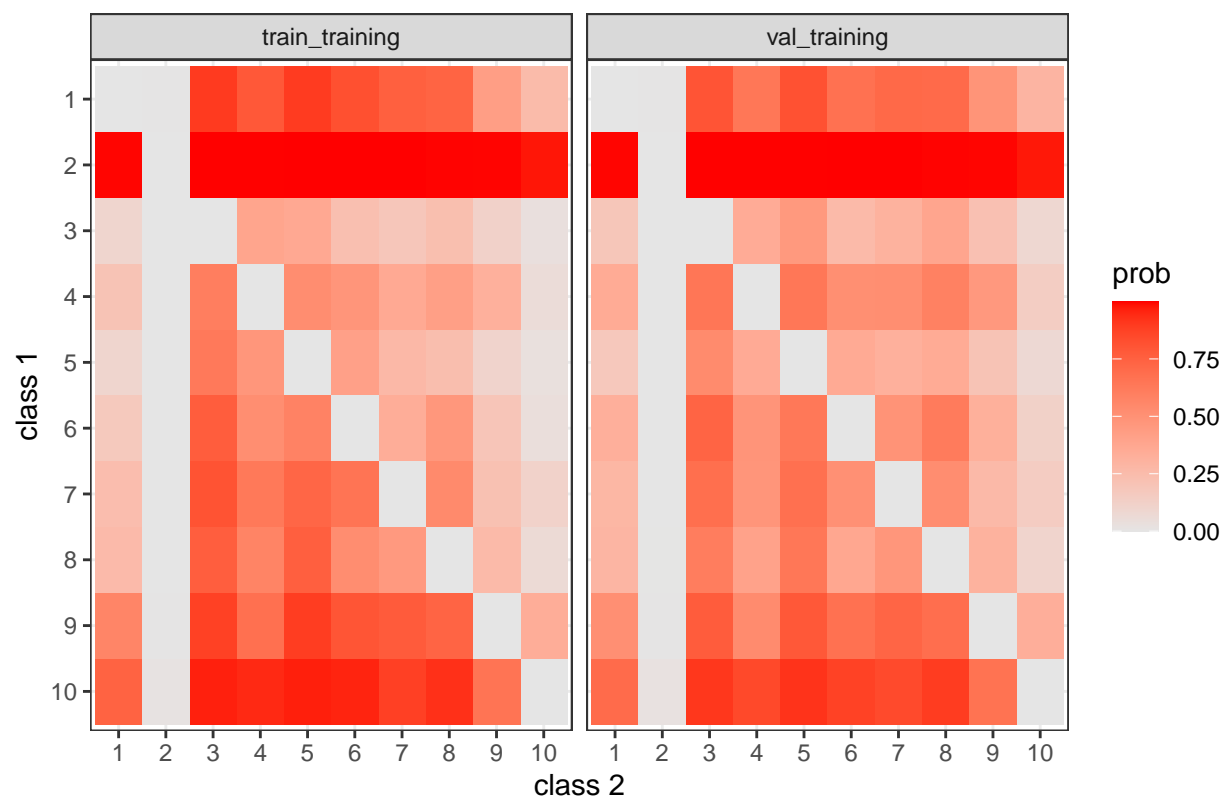
```
for (cls in 1:classes)
{
  cur_class_Rs <- aggreg_Rs %>% filter(class == cls)
  plot_cls <- ggplot(cur_class_Rs, aes(x = class2, y = class1)) +
    geom_raster(aes(fill=prob)) +
    facet_wrap(~train_type) +
    scale_fill_gradient(low="grey90", high="red") +
    scale_y_discrete(limits=rev) +
    labs(x="class 2", y="class 1", title=paste("Average pairwise probabilities - class ", cls)) +
    theme_bw()

  print(plot_cls)
}
```

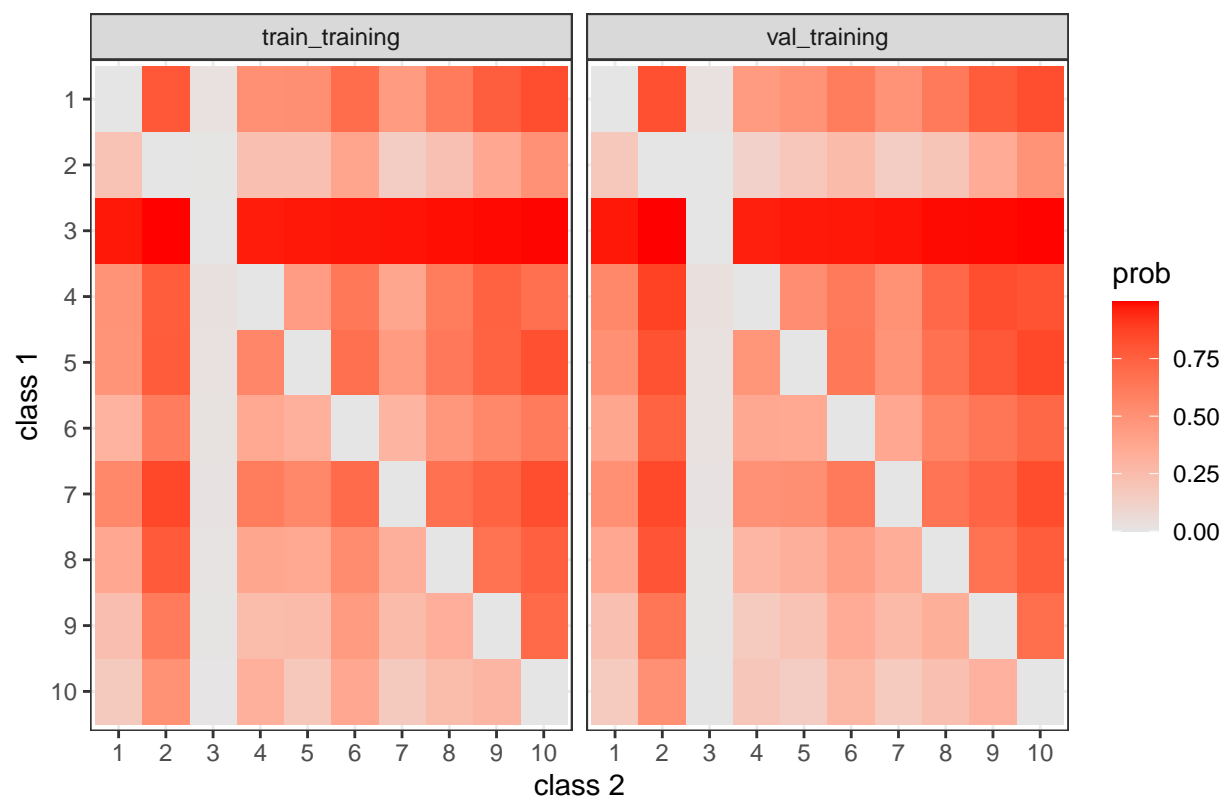
Average pairwise probabilities – class 1



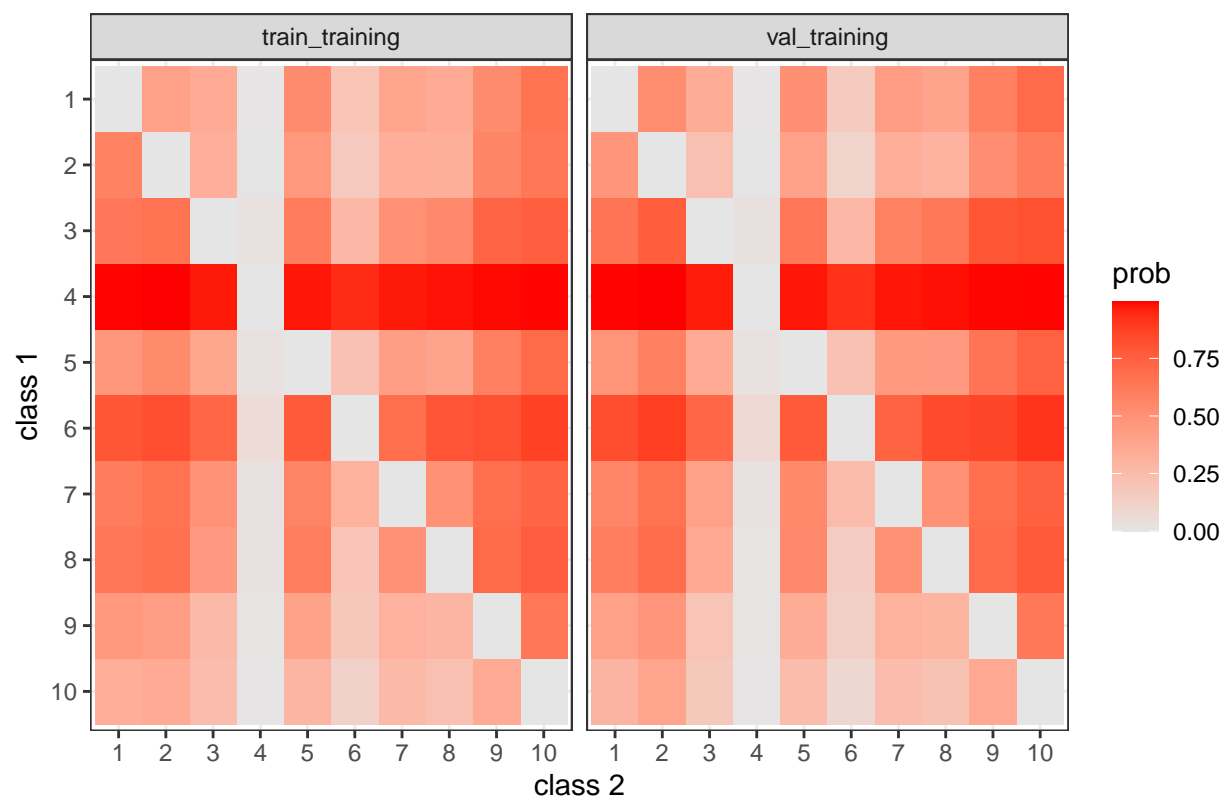
Average pairwise probabilities – class 2



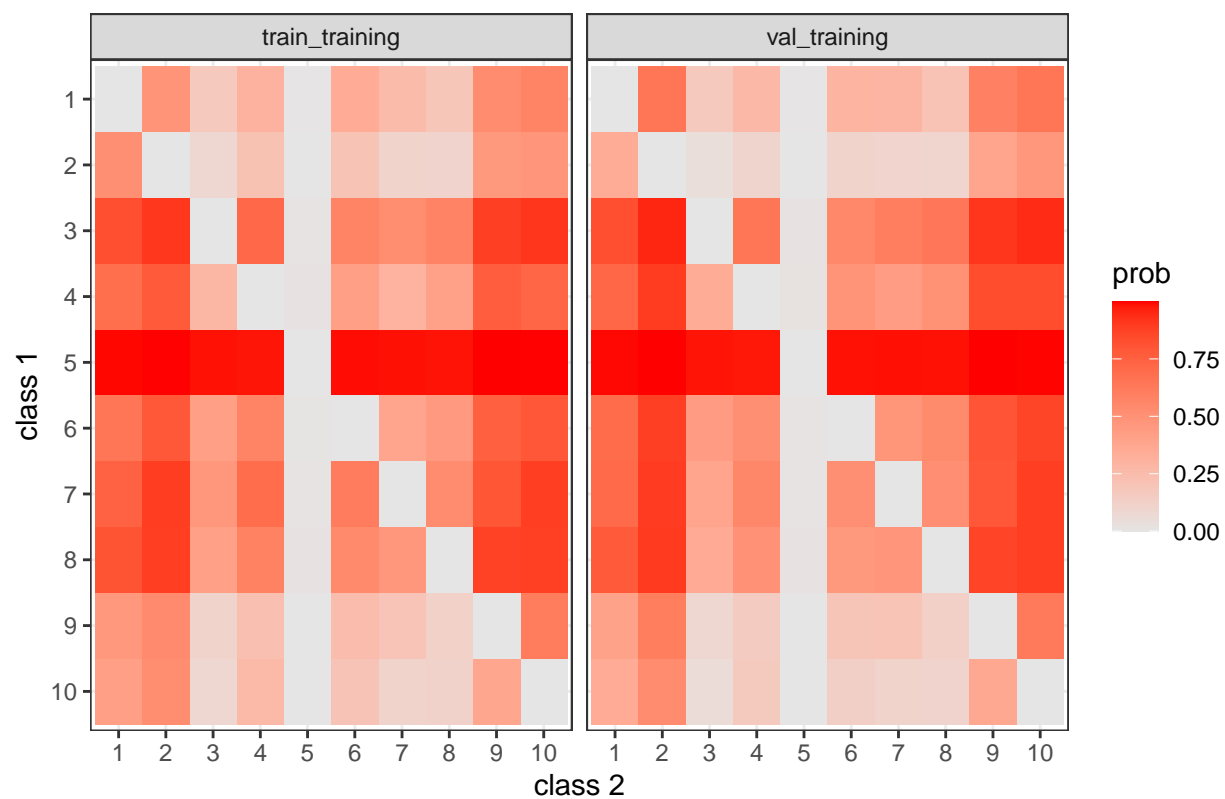
Average pairwise probabilities – class 3



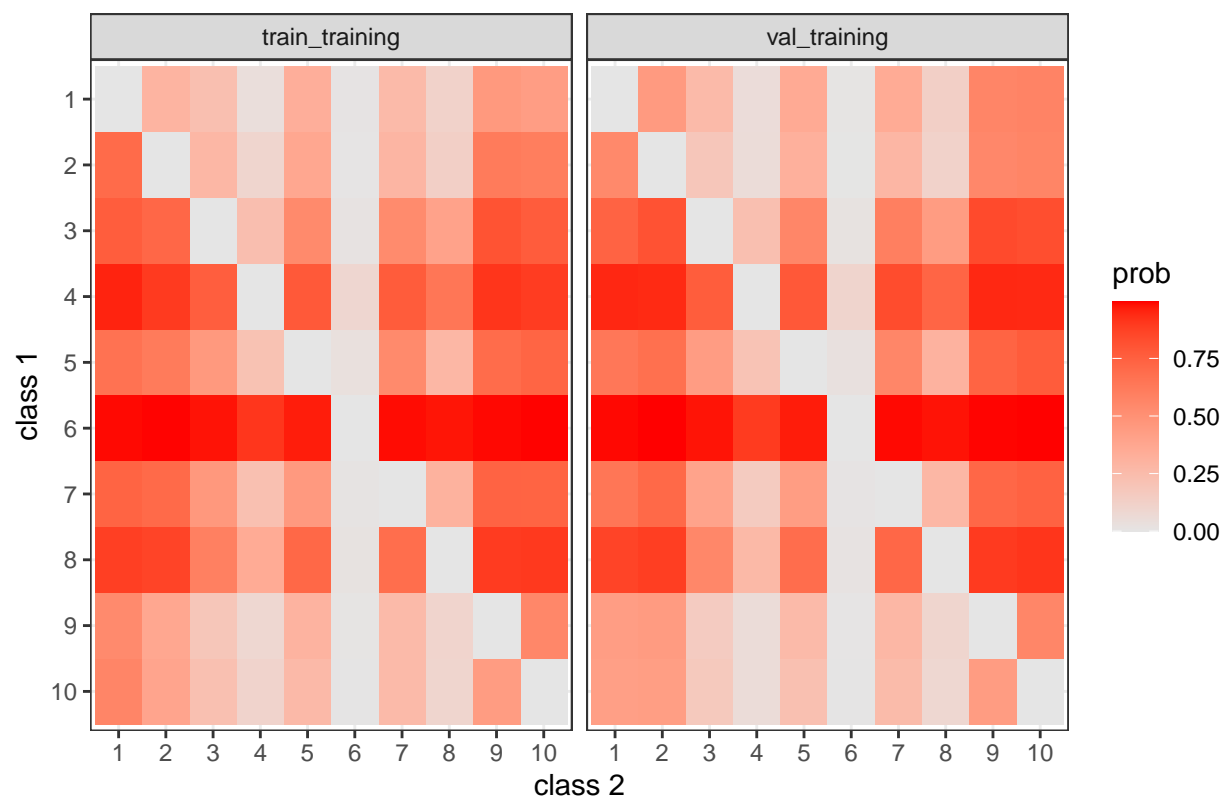
Average pairwise probabilities – class 4



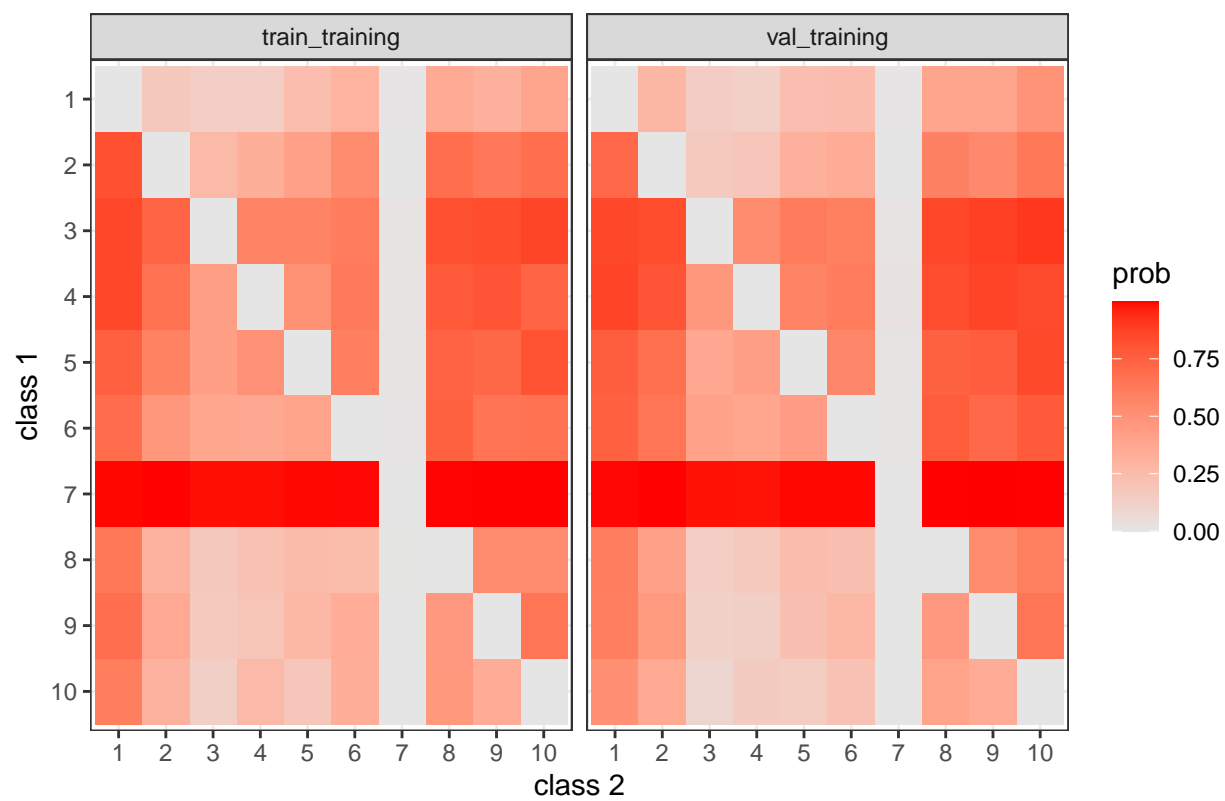
Average pairwise probabilities – class 5



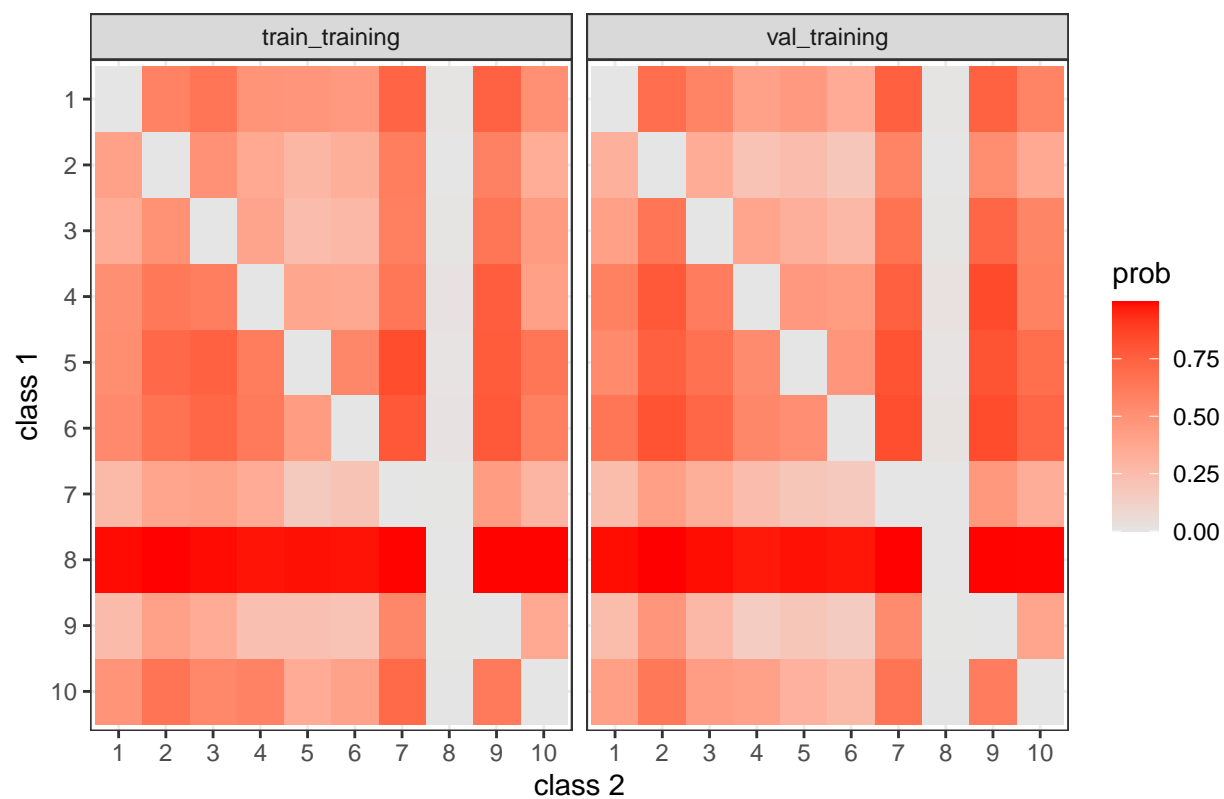
Average pairwise probabilities – class 6



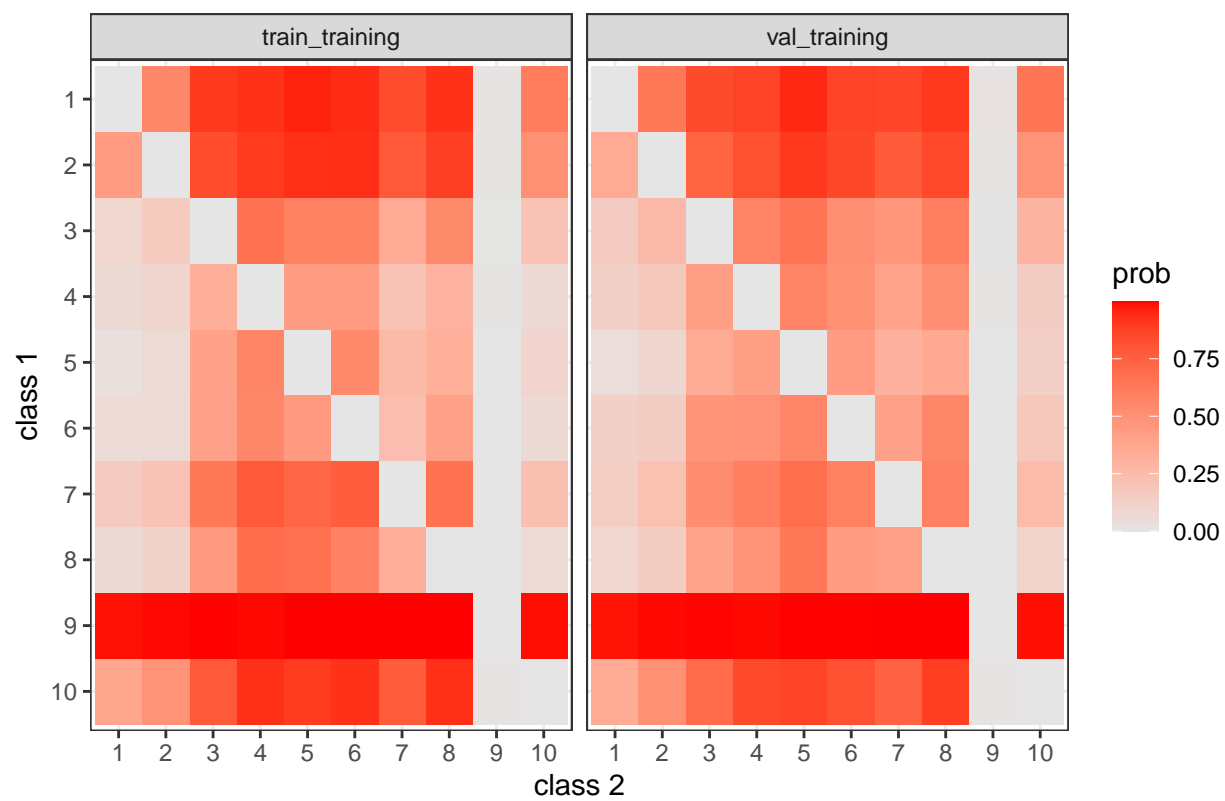
Average pairwise probabilities – class 7



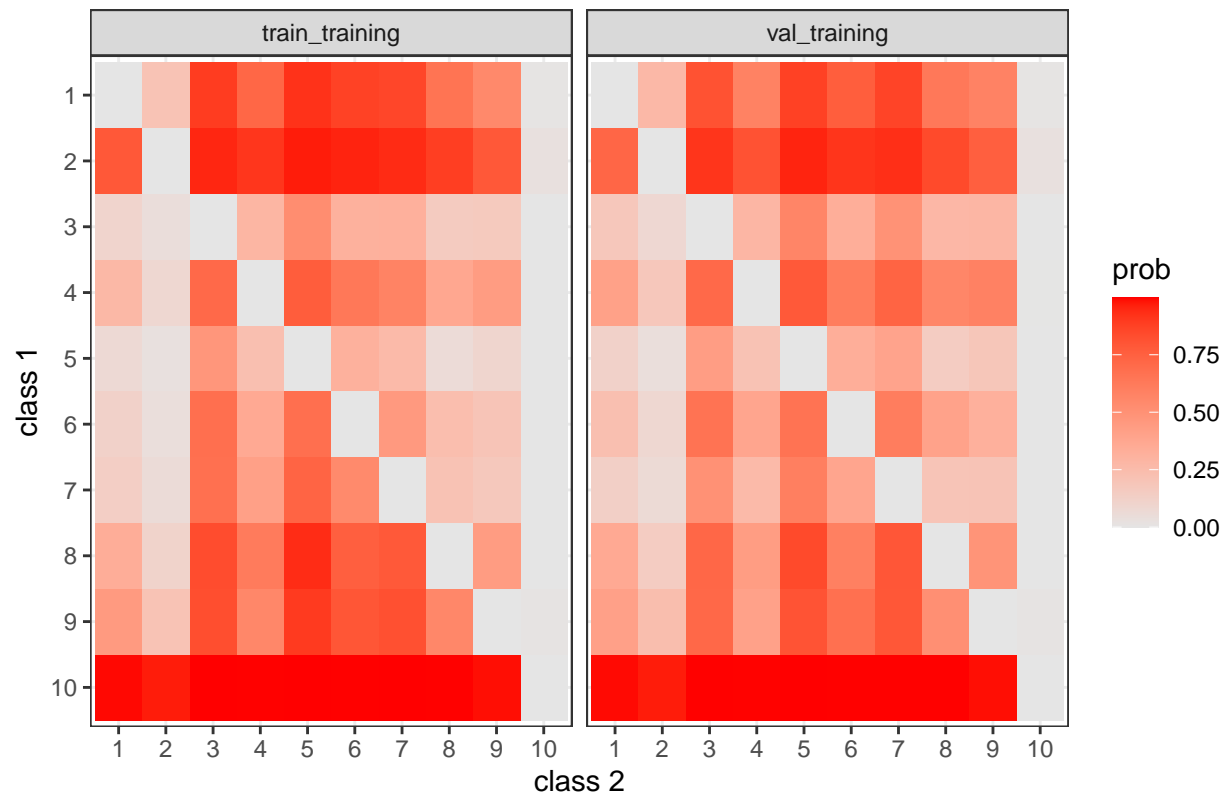
Average pairwise probabilities – class 8



Average pairwise probabilities – class 9



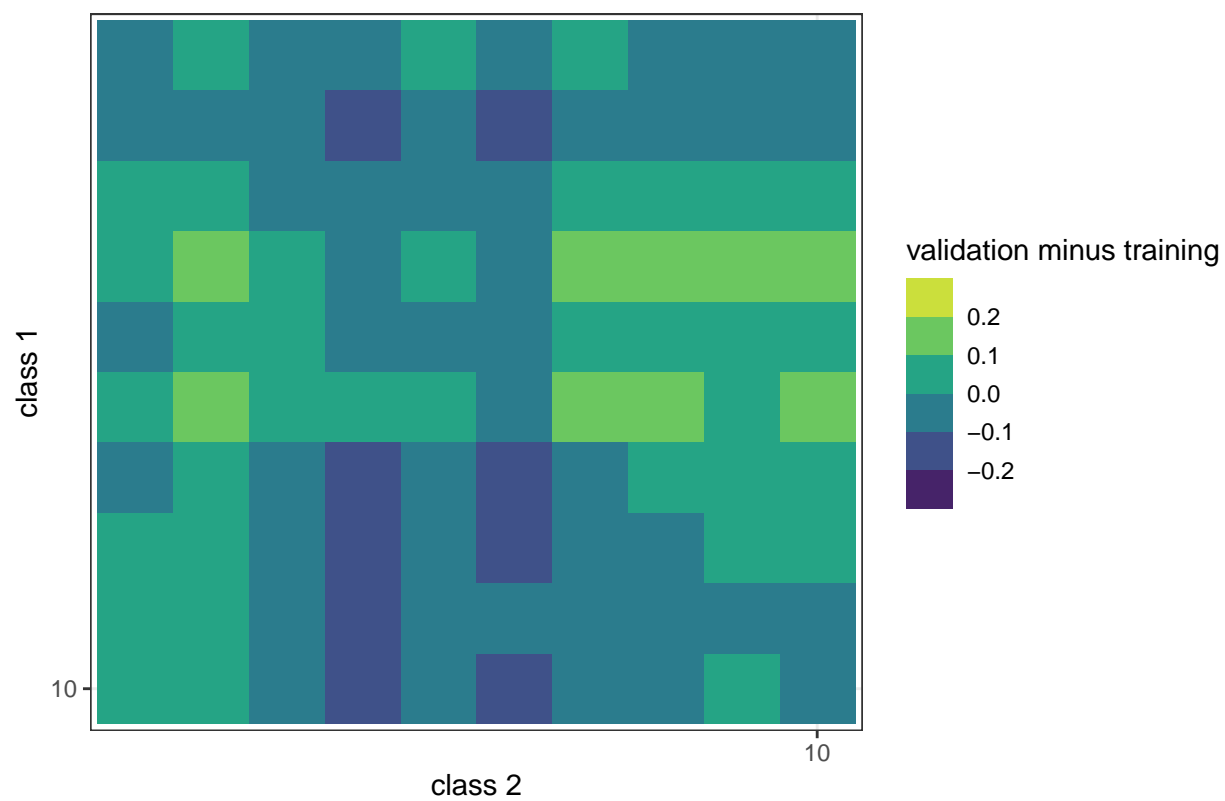
Average pairwise probabilities – class 10



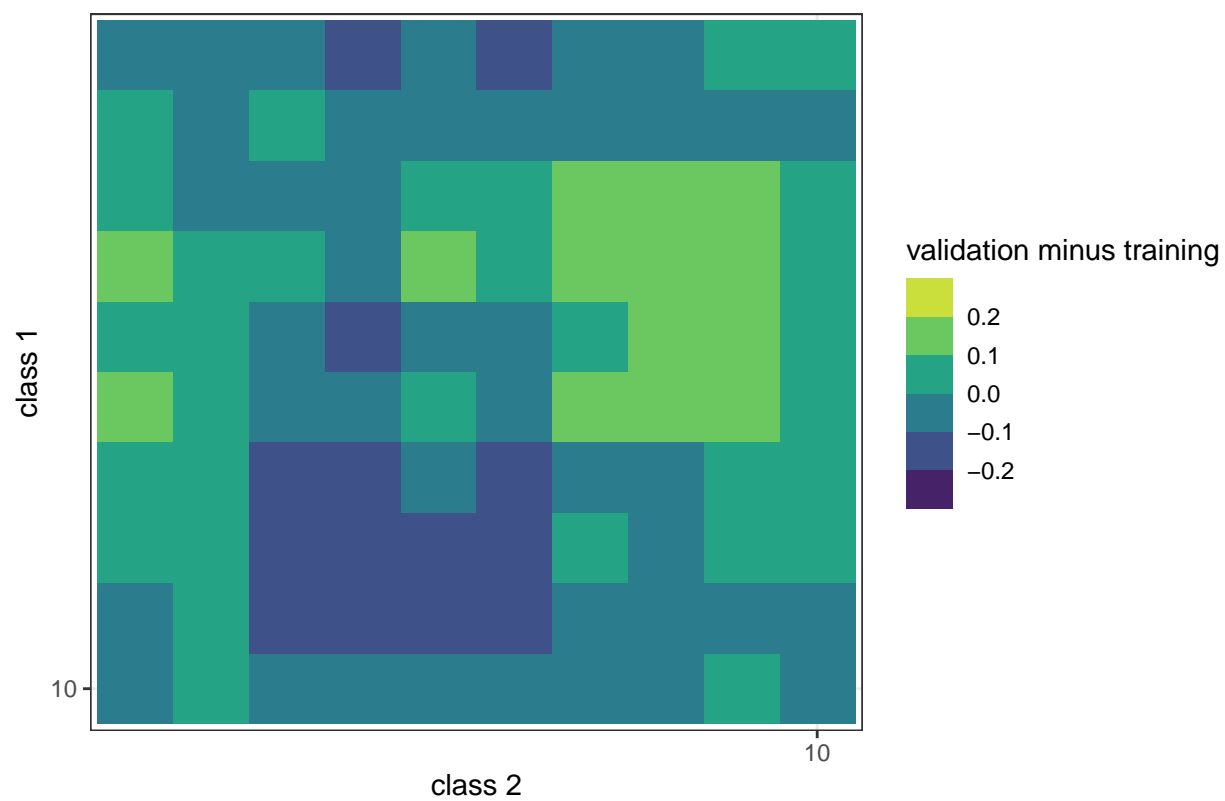
```
for (cls in 1:classes)
{
  cur_class_Rs <- df_aggr_Rs_diff %>% filter(class == cls)
  plot_cls <- ggplot(cur_class_Rs, aes(x = class2, y = class1)) +
    geom_raster(aes(fill=val_min_train)) +
    scale_fill_binned(type="viridis", limits=c(-0.3, 0.3), name="validation minus training") +
    scale_y_discrete(limits=rev, breaks=seq(0, classes, 10)) +
    scale_x_discrete(breaks=seq(0, classes, 10)) +
    labs(x="class 2", y="class 1", title=paste("Differences between average pairwise probabilities - class", cls))
    theme_bw()

  print(plot_cls)
}
```

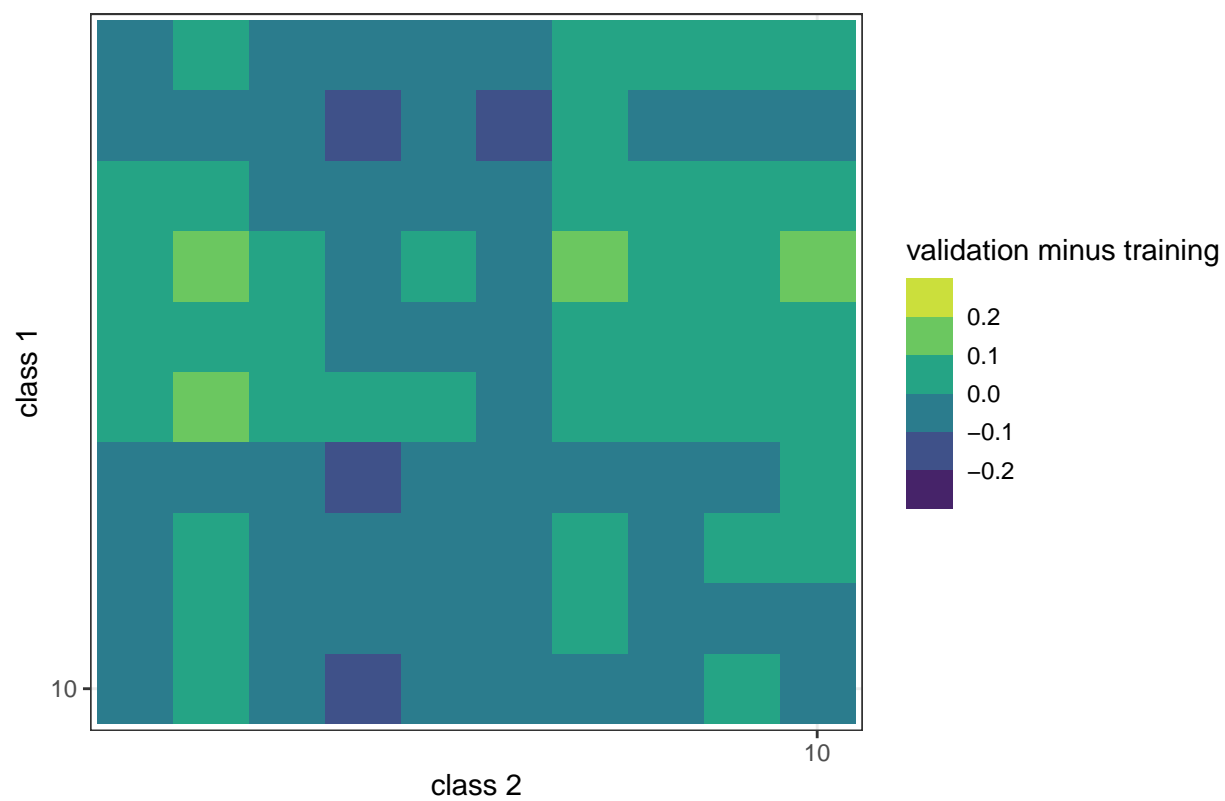

Differences between average pairwise probabilities – class 1



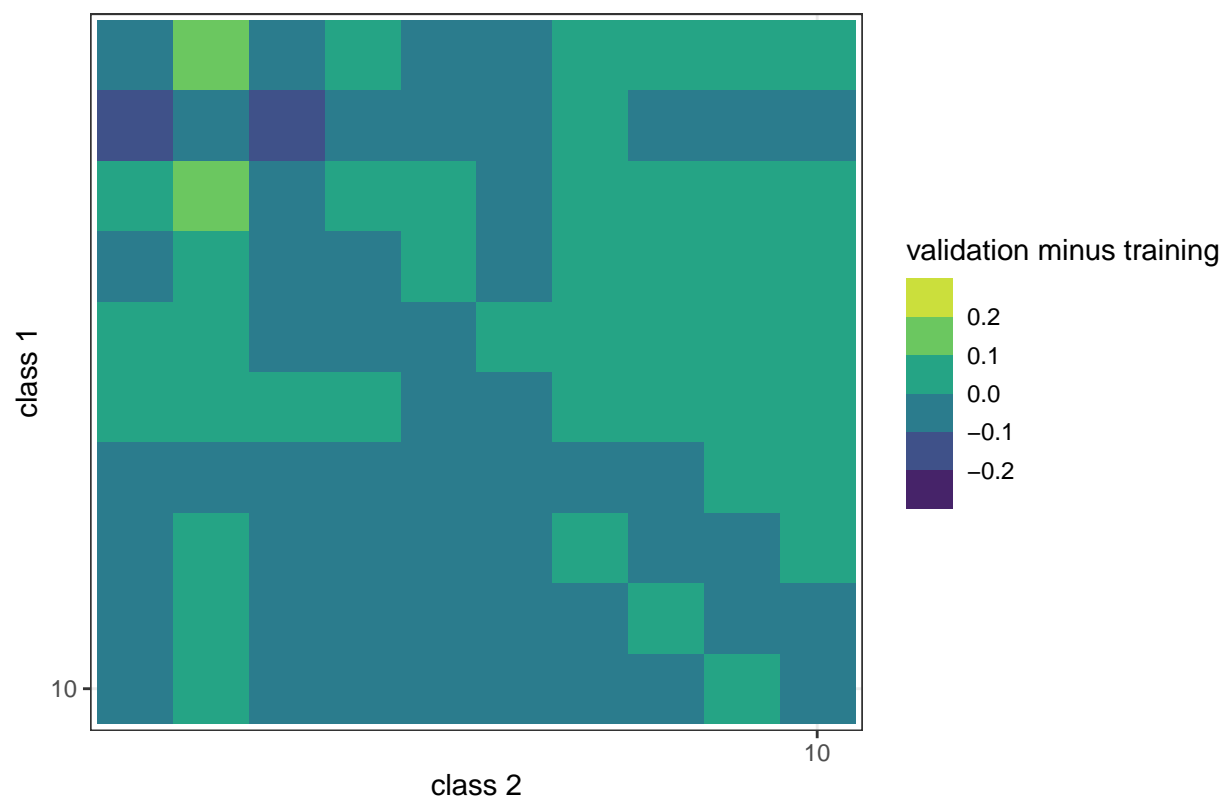
Differences between average pairwise probabilities – class 2



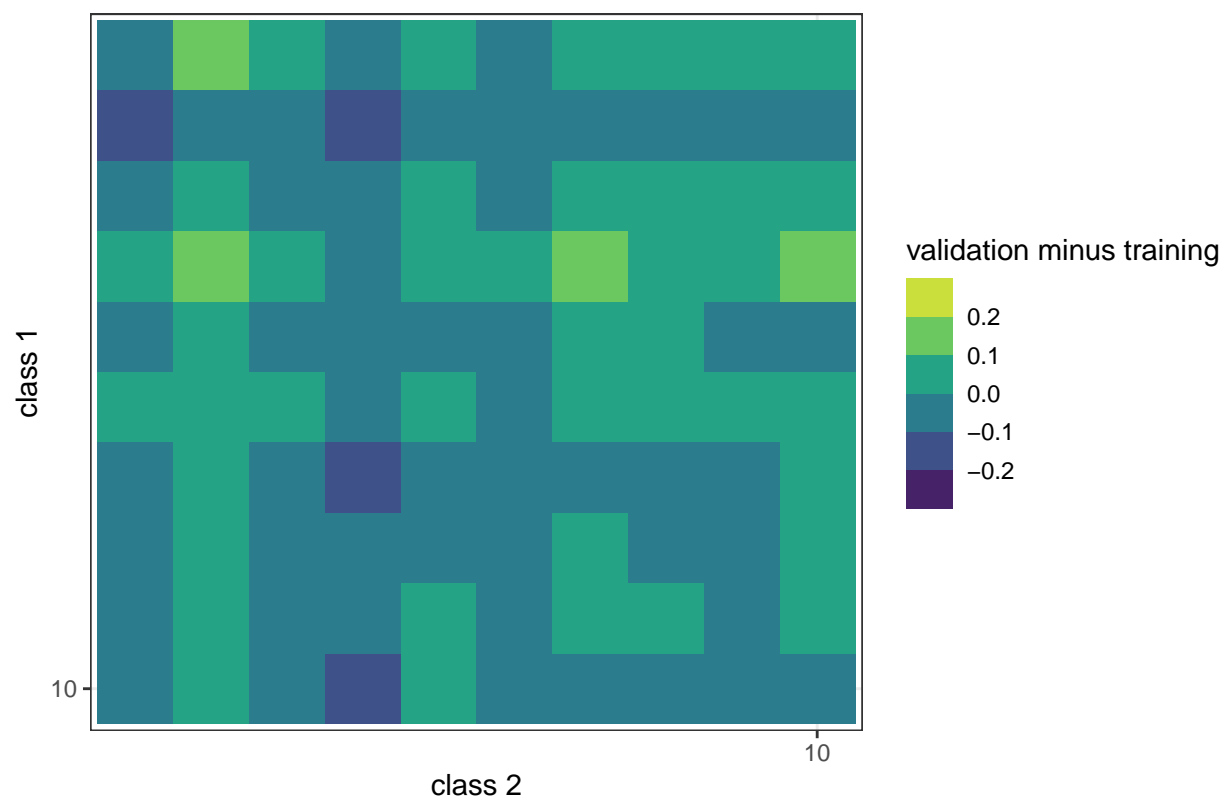
Differences between average pairwise probabilities – class 3



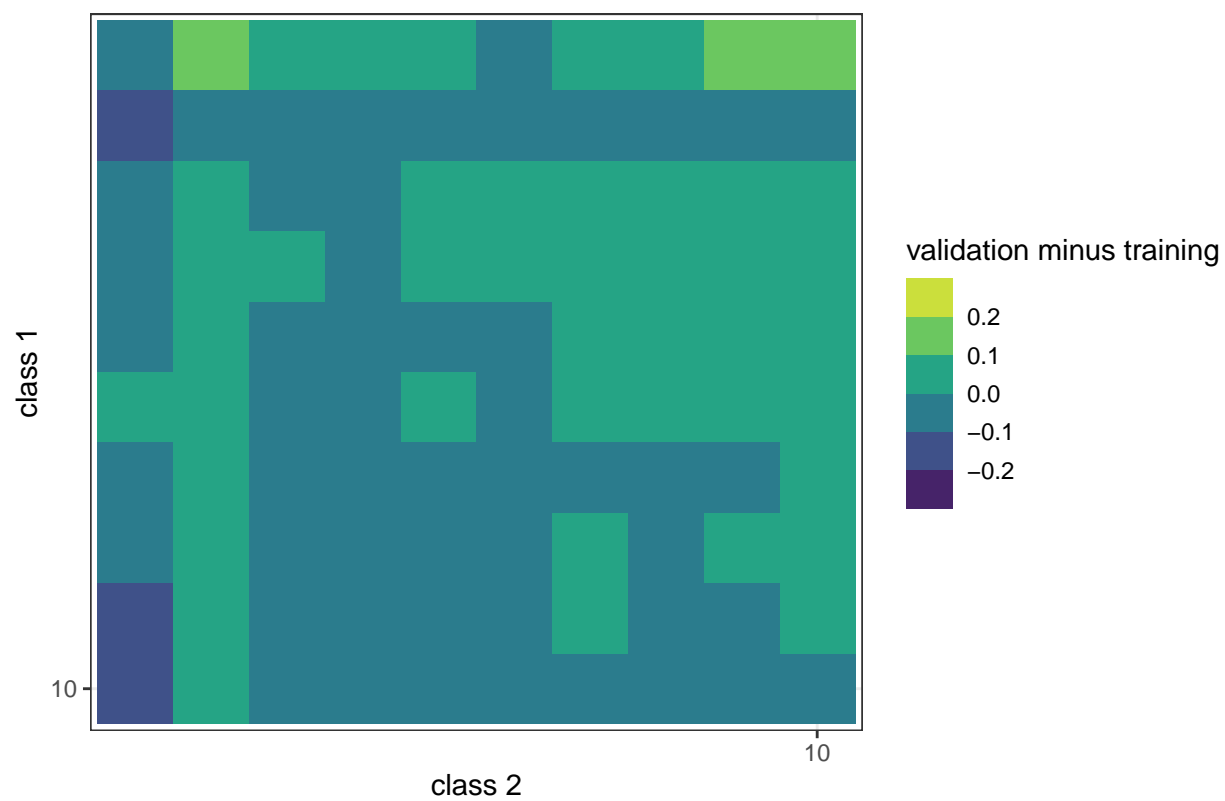
Differences between average pairwise probabilities – class 4



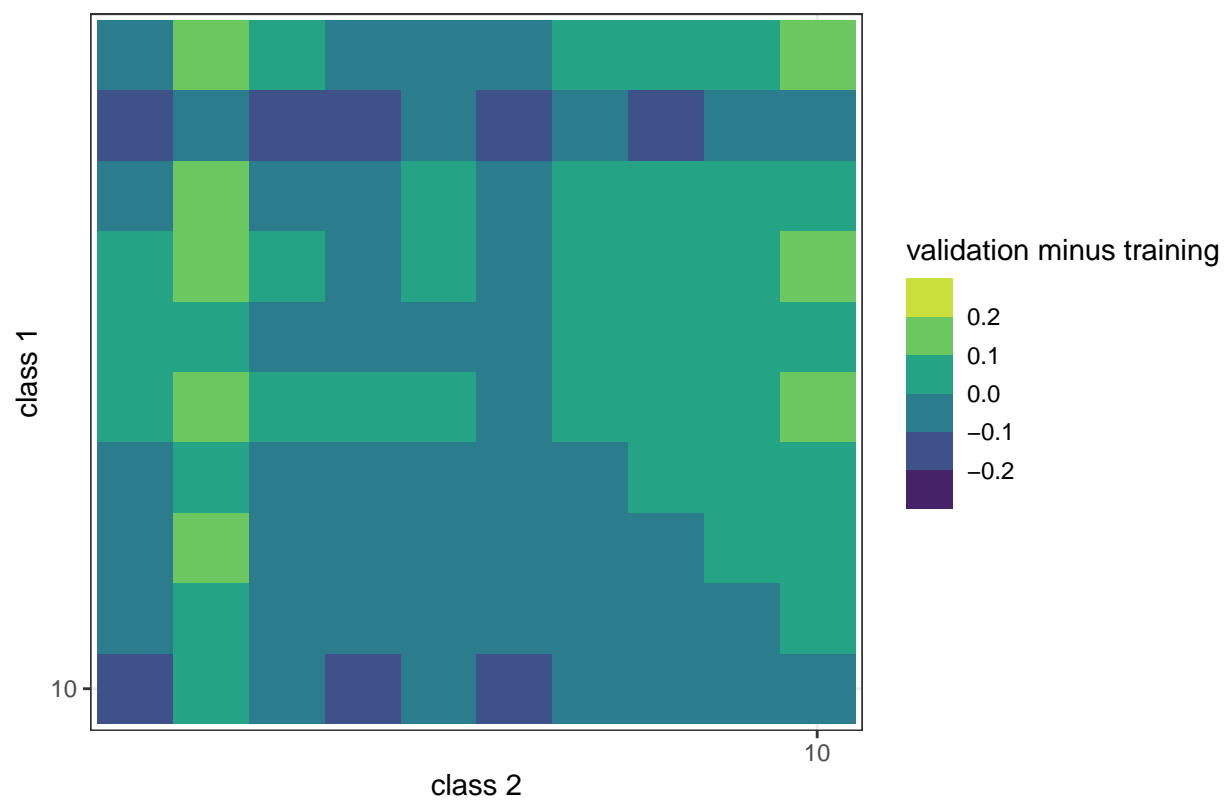
Differences between average pairwise probabilities – class 5



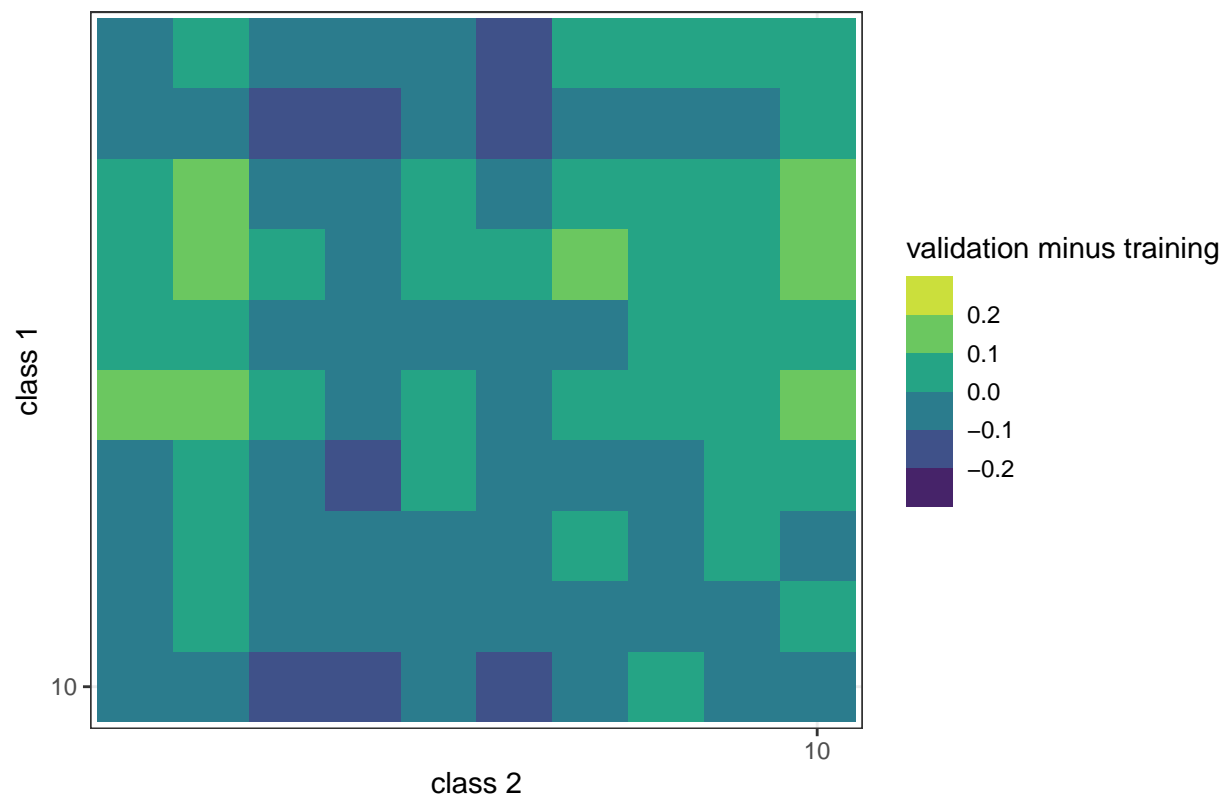
Differences between average pairwise probabilities – class 6



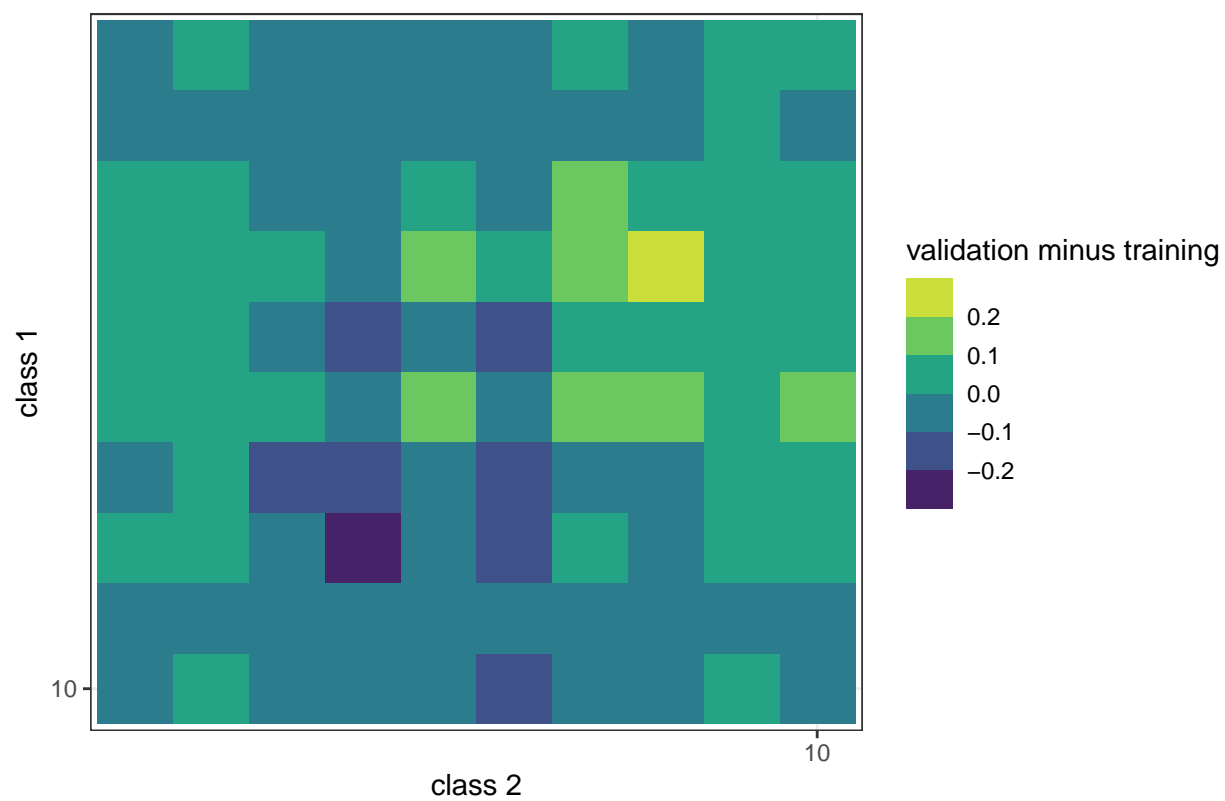
Differences between average pairwise probabilities – class 7



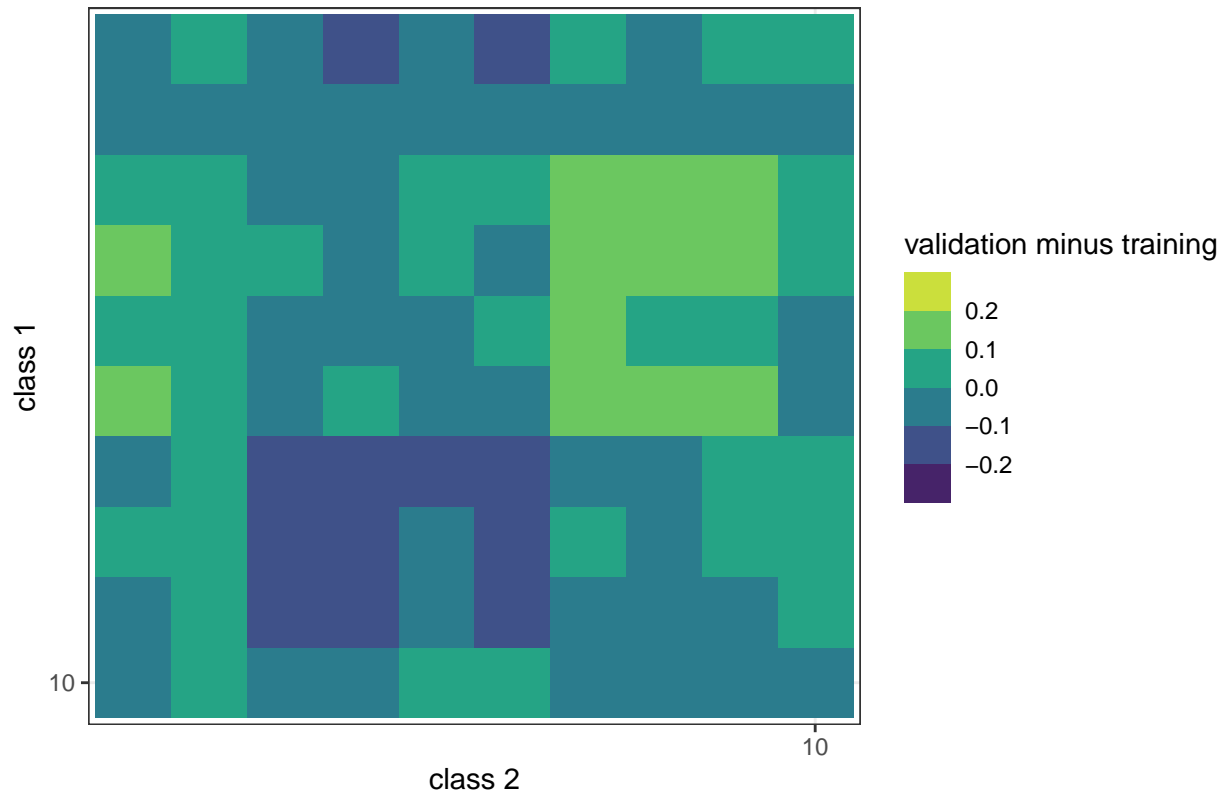
Differences between average pairwise probabilities – class 8



Differences between average pairwise probabilities – class 9



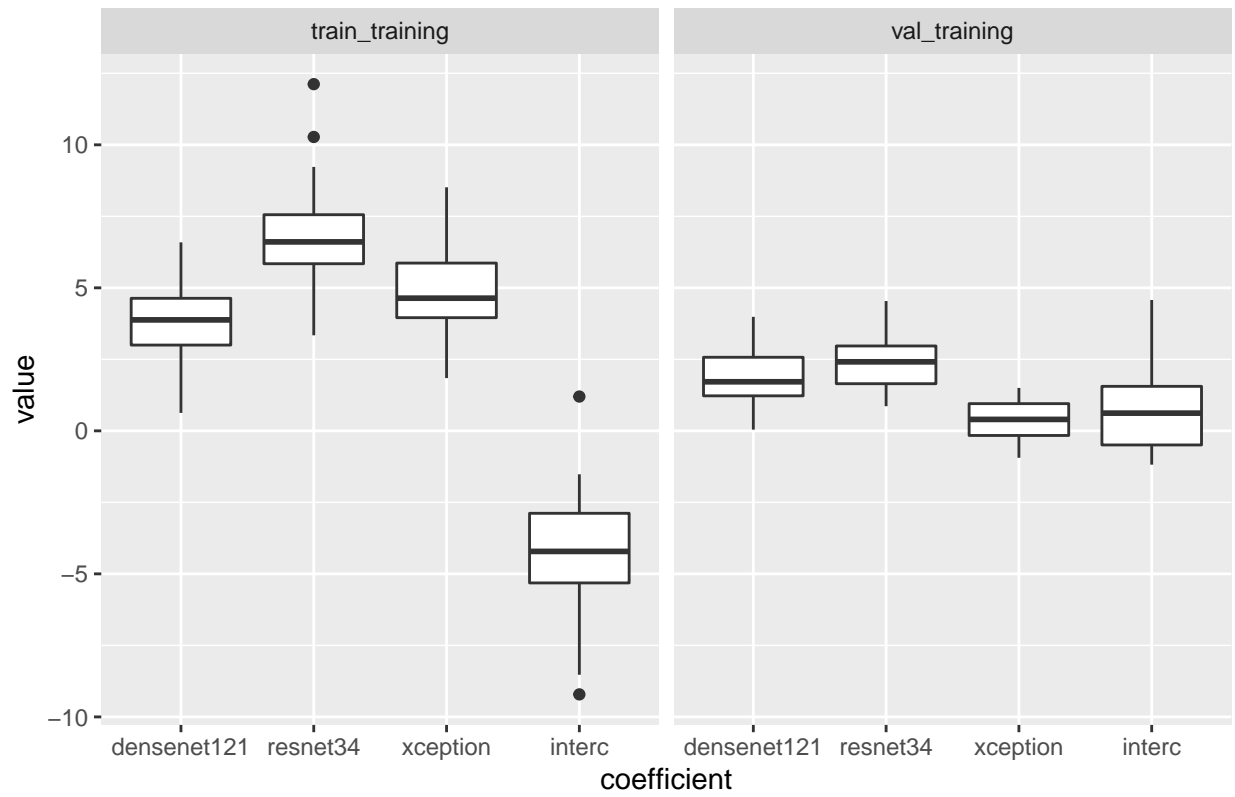
Differences between average pairwise probabilities – class 10



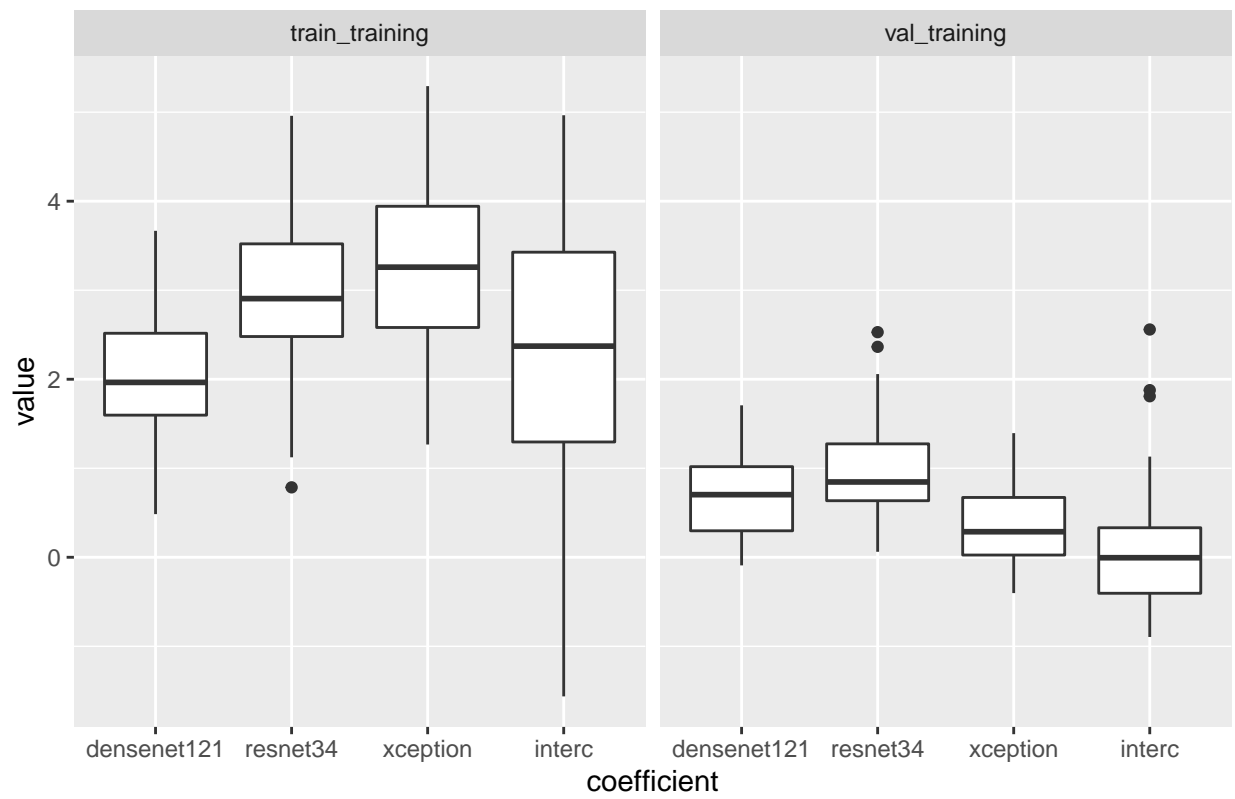
```
lda_coefs <- load_lda_coefs(base_dir, repls, folds)
```

```
for (cl1 in 1:(classes - 1))
{
  for (cl2 in (cl1 + 1):classes)
  {
    cur_plt <- lda_coefs %>% filter(class1 == cl1 & class2 == cl2) %>% ggplot() + geom_boxplot(aes(x=co
      facet_wrap(~train_type) + ggtitle(paste("Coefficients for class", cl1, "vs", cl2))
    print(cur_plt)
  }
}
```

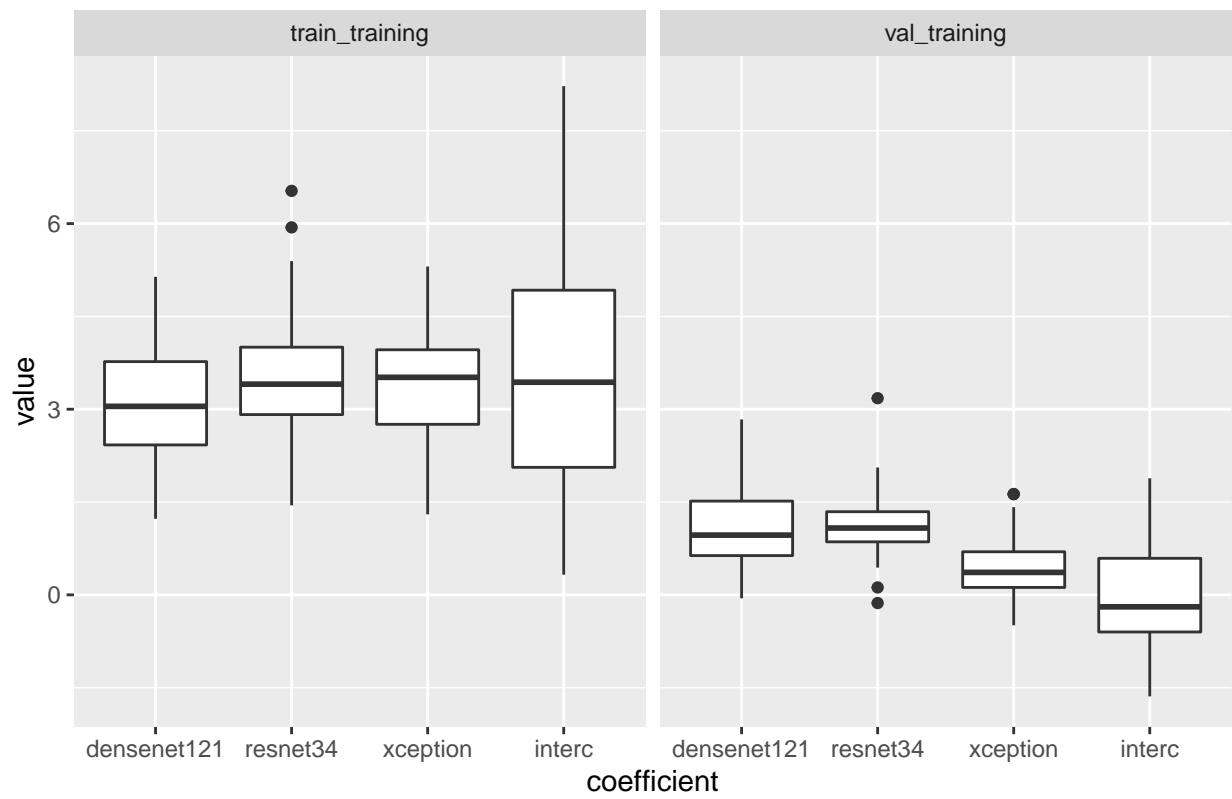
Coefficients for class 1 vs 2



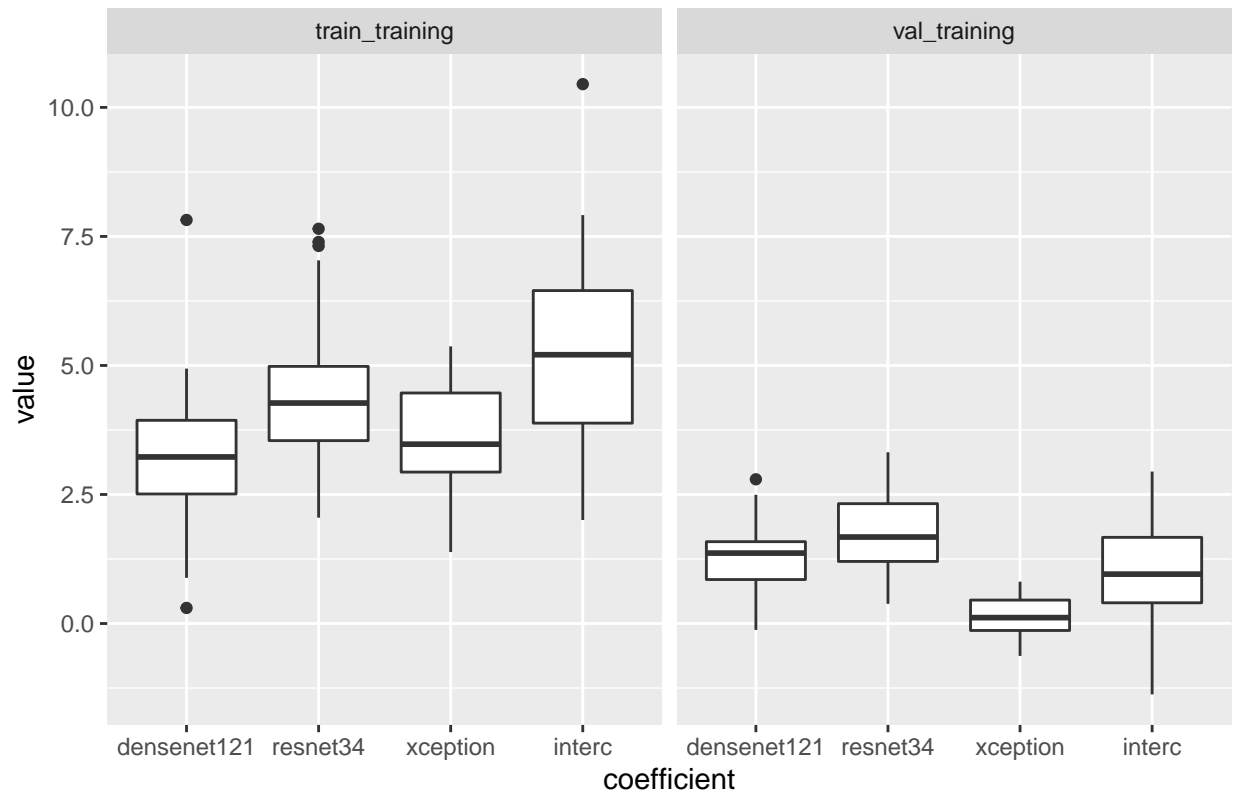
Coefficients for class 1 vs 3



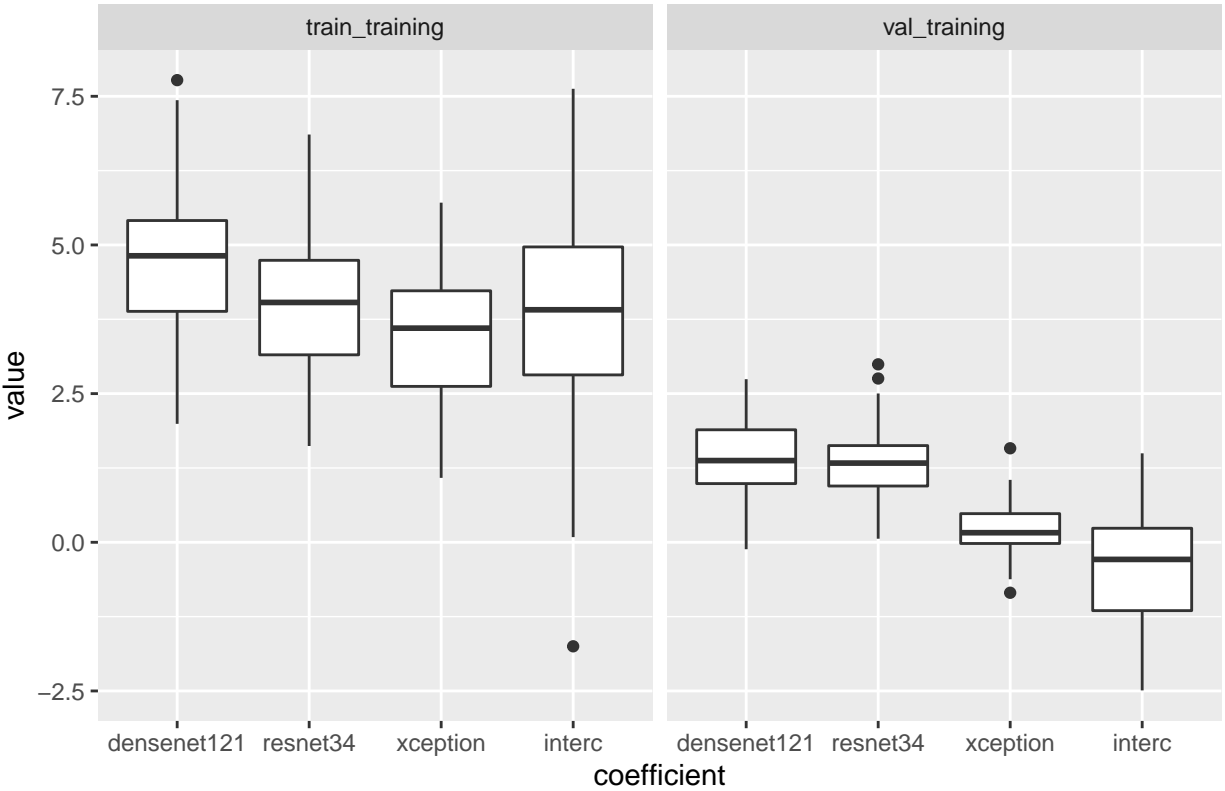
Coefficients for class 1 vs 4



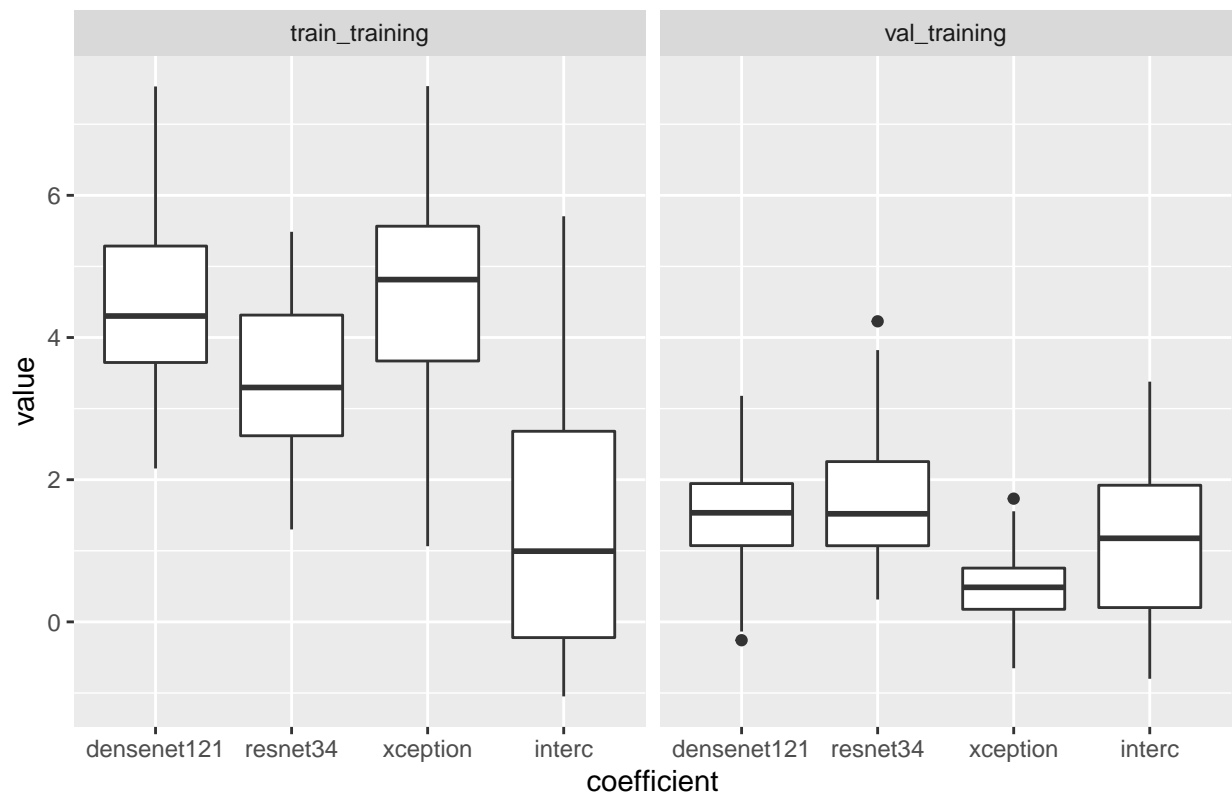
Coefficients for class 1 vs 5



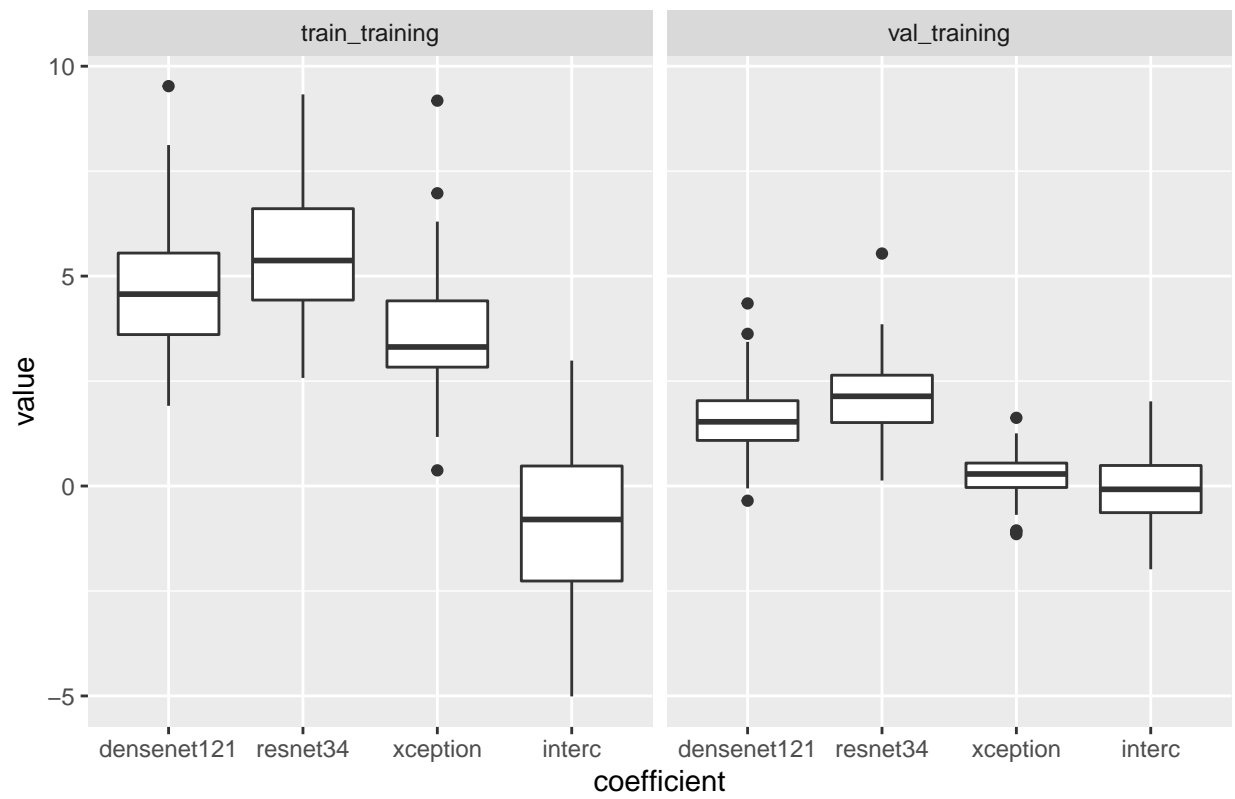
Coefficients for class 1 vs 6



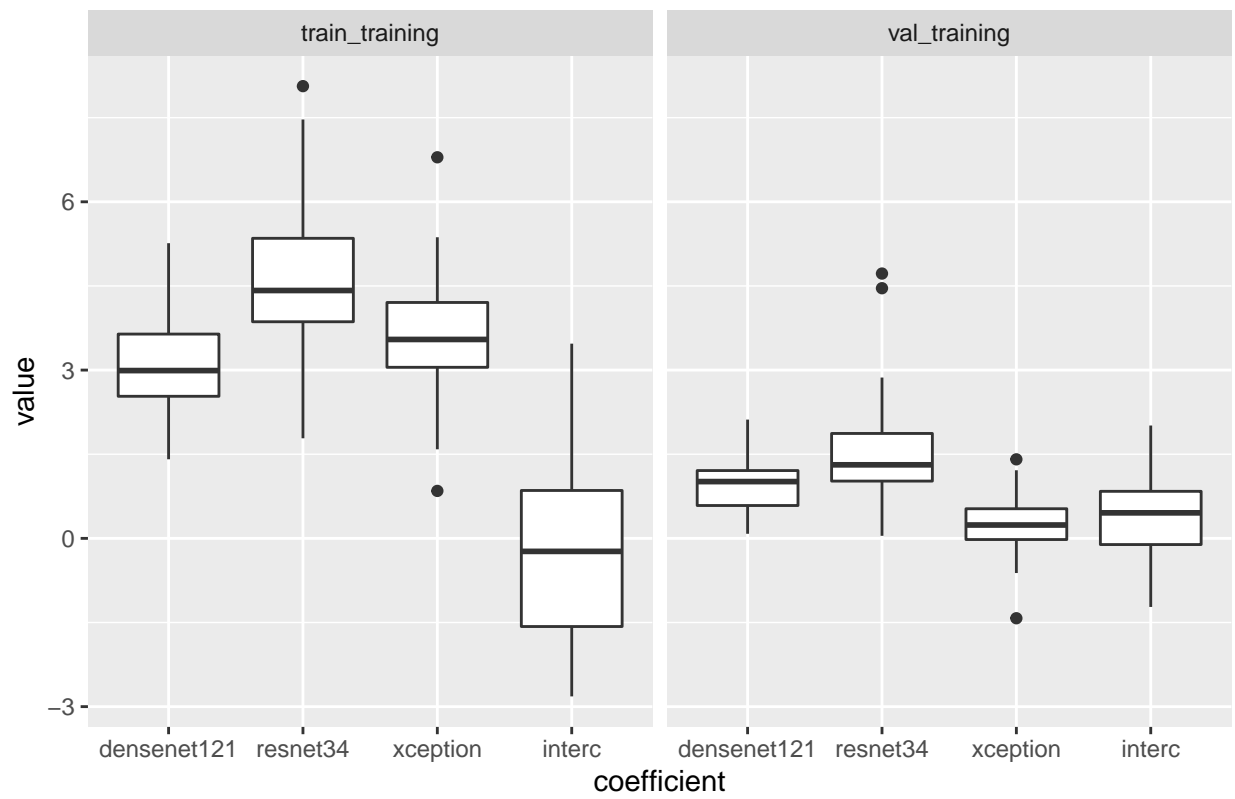
Coefficients for class 1 vs 7



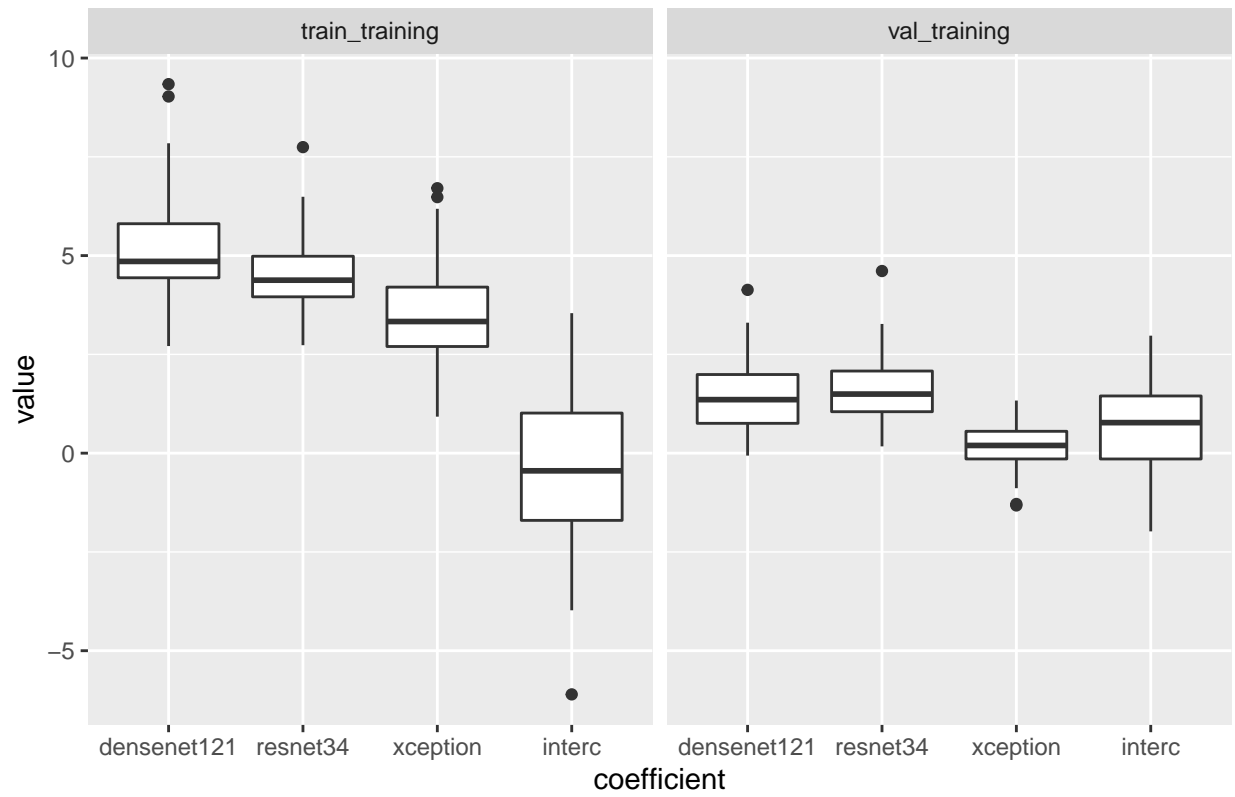
Coefficients for class 1 vs 8



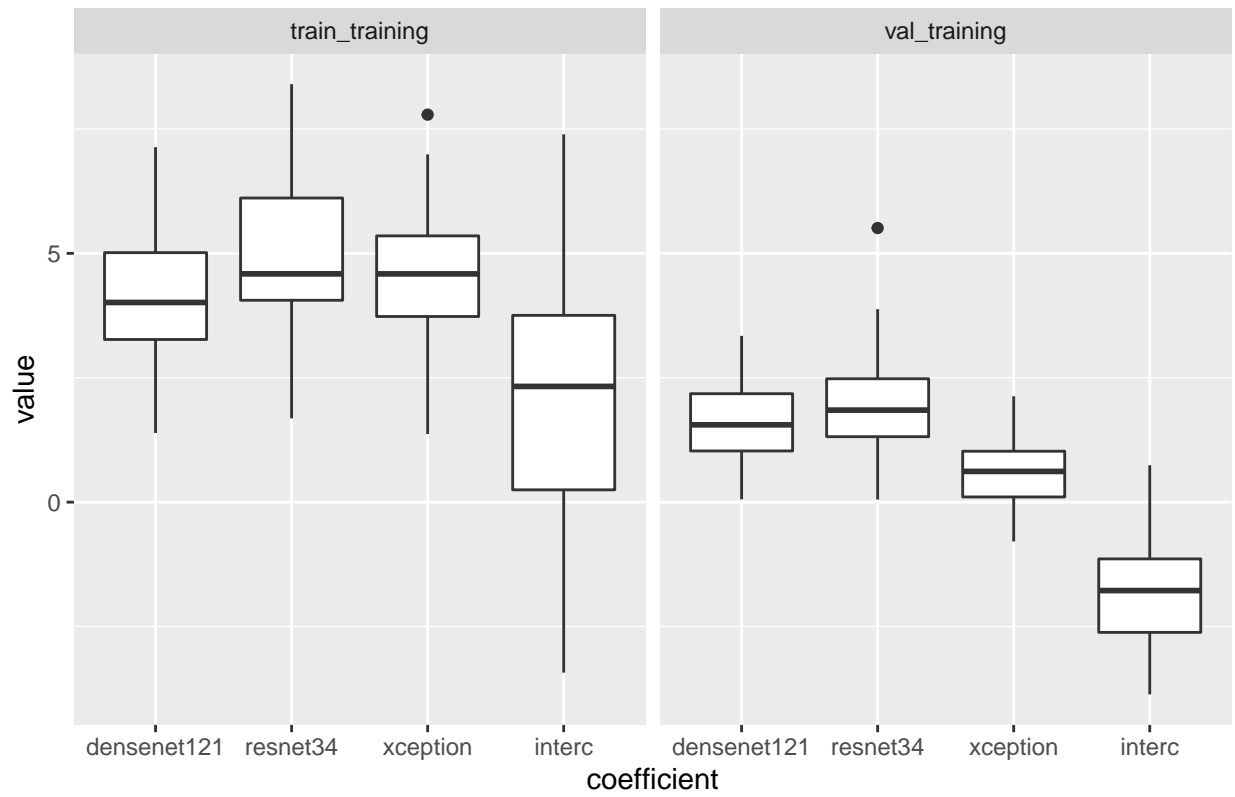
Coefficients for class 1 vs 9



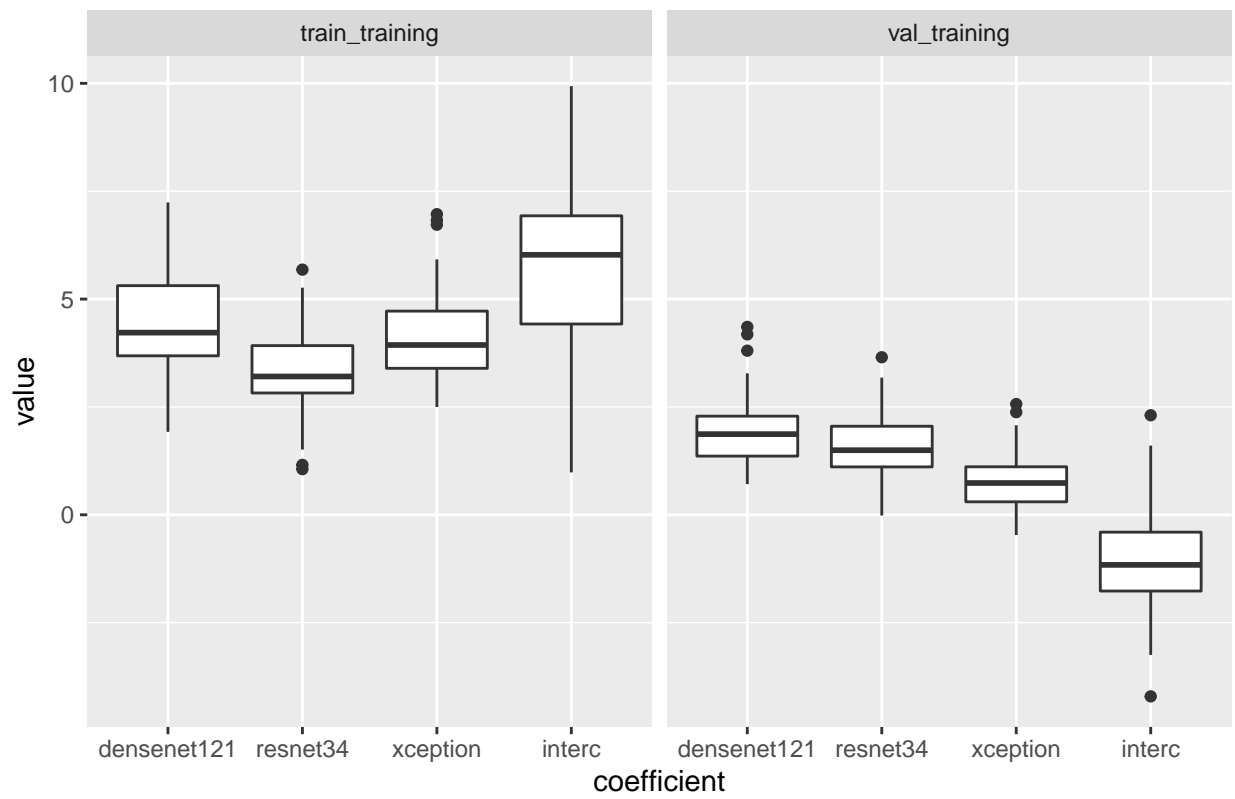
Coefficients for class 1 vs 10



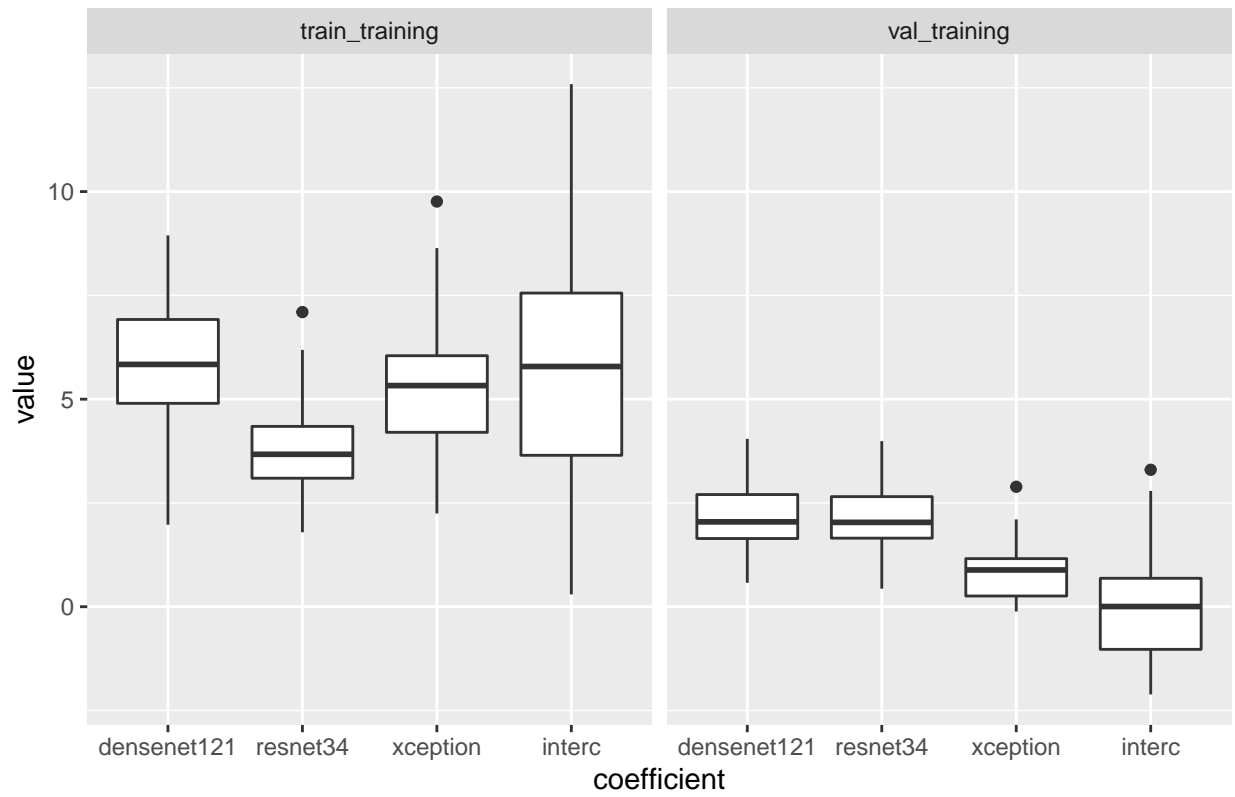
Coefficients for class 2 vs 3



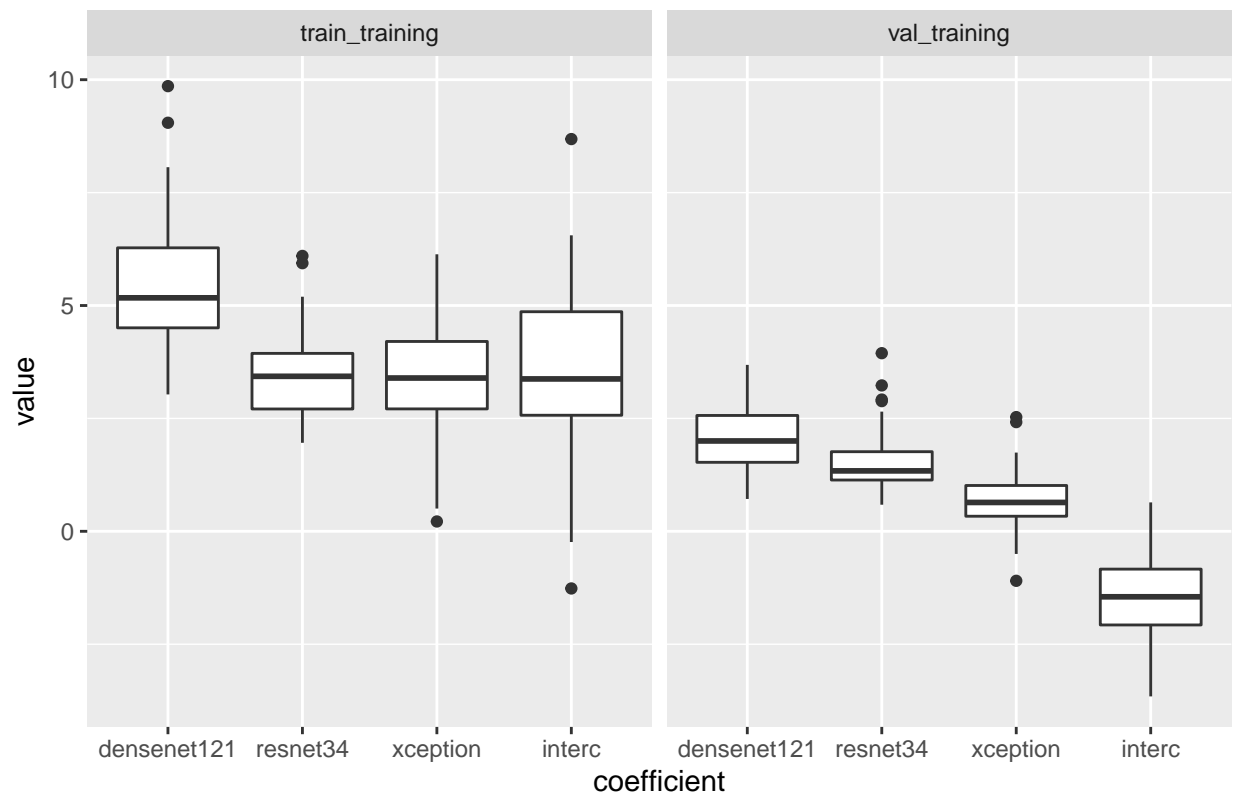
Coefficients for class 2 vs 4



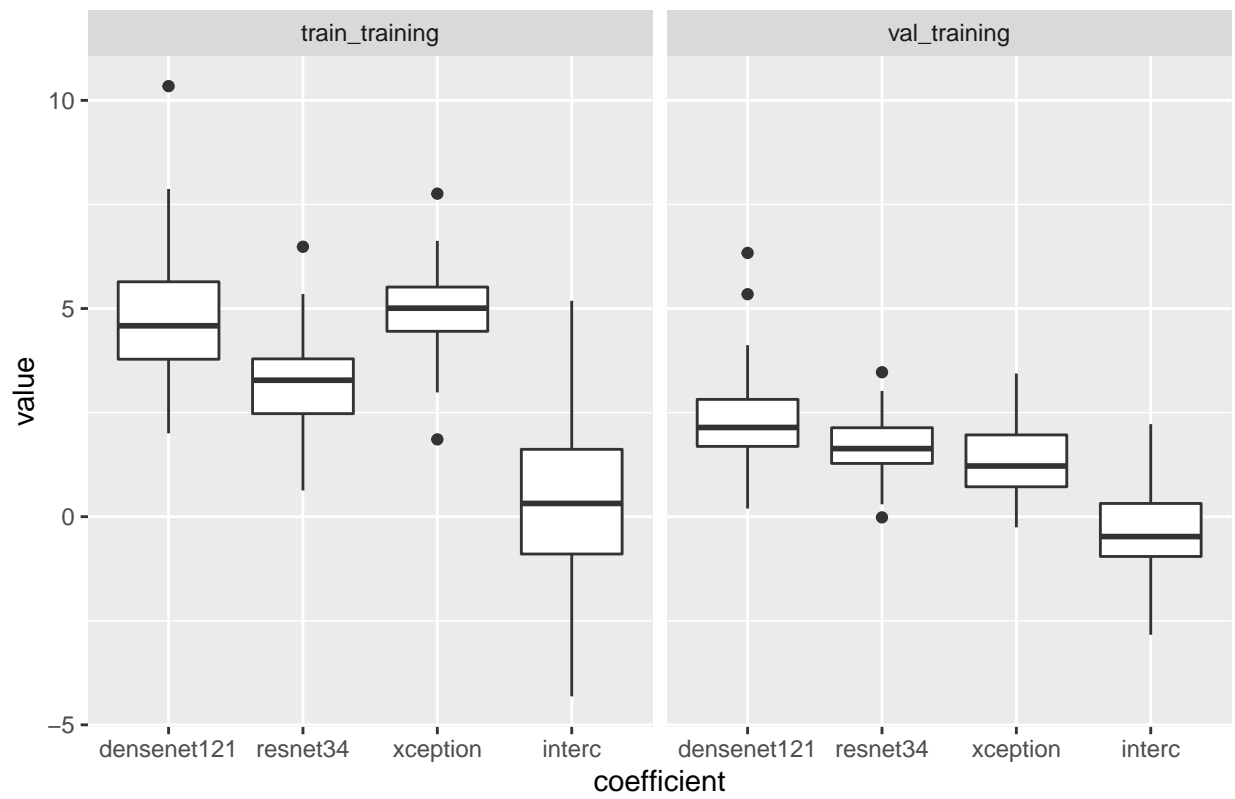
Coefficients for class 2 vs 5



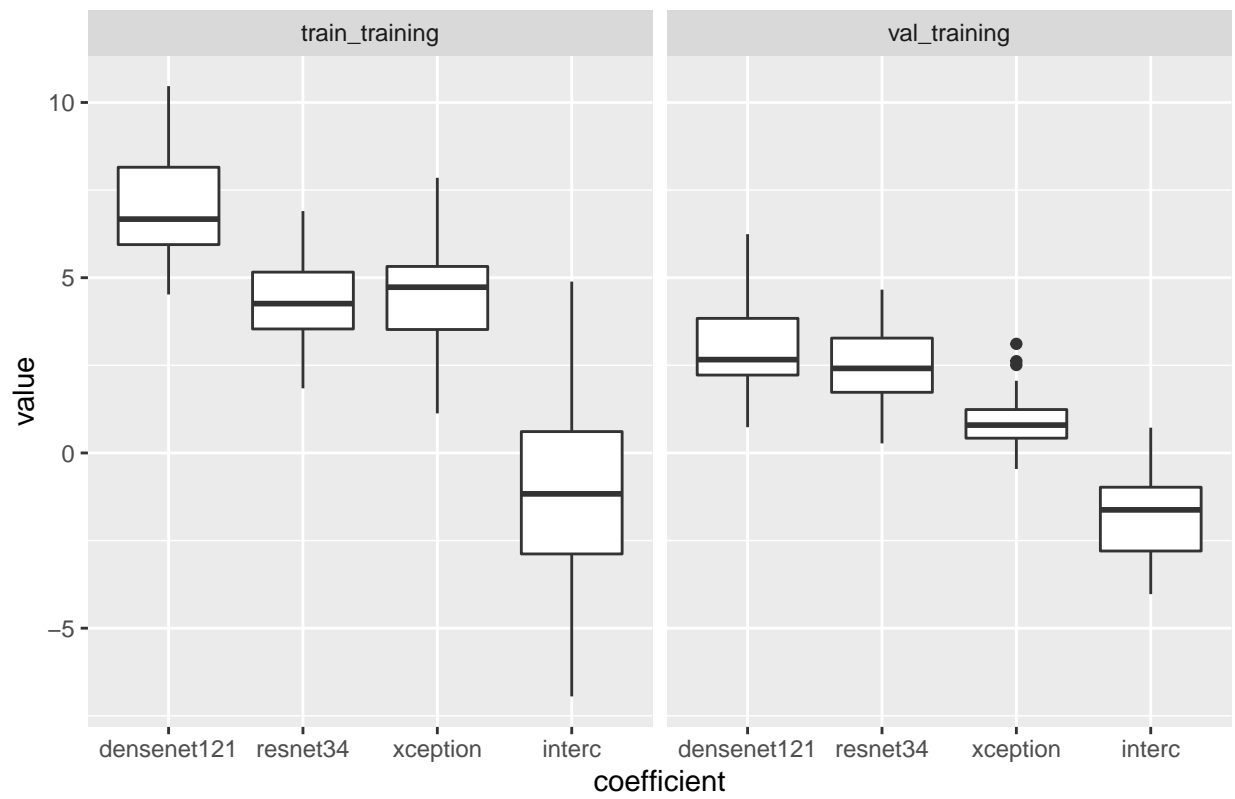
Coefficients for class 2 vs 6



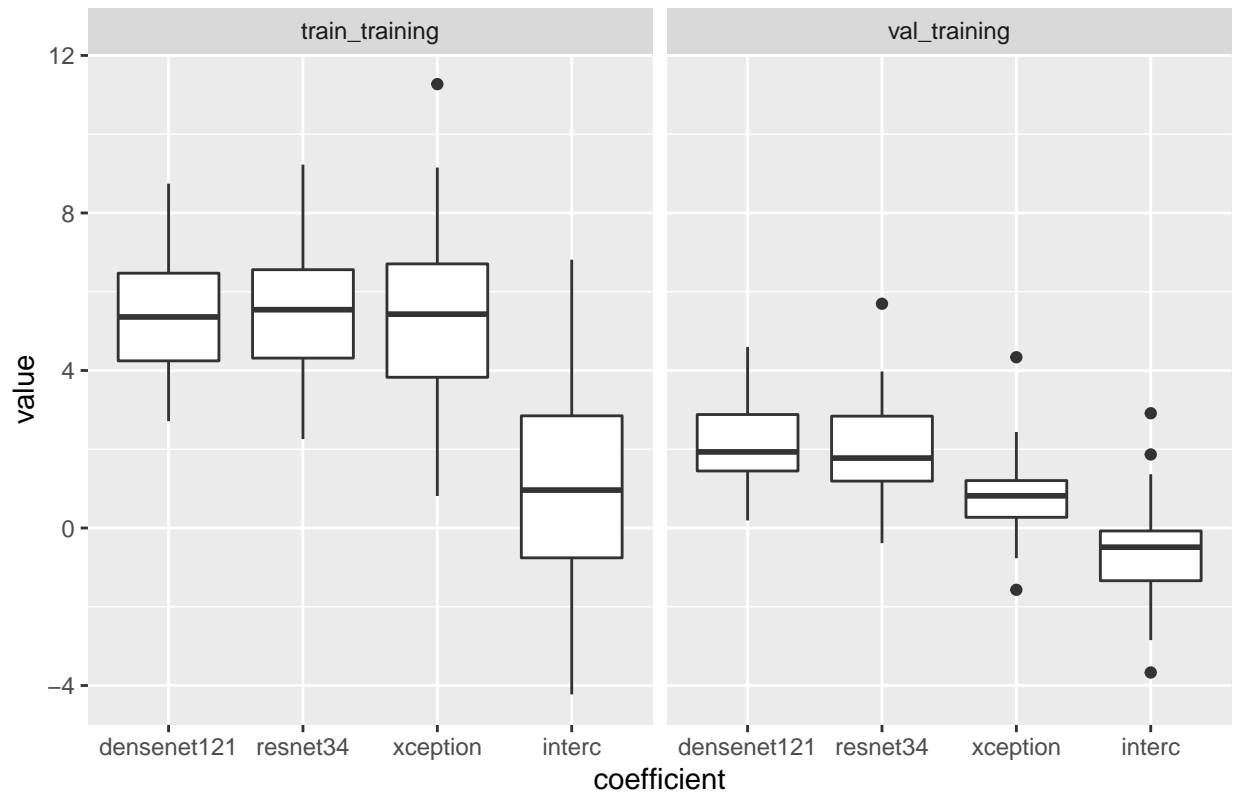
Coefficients for class 2 vs 7



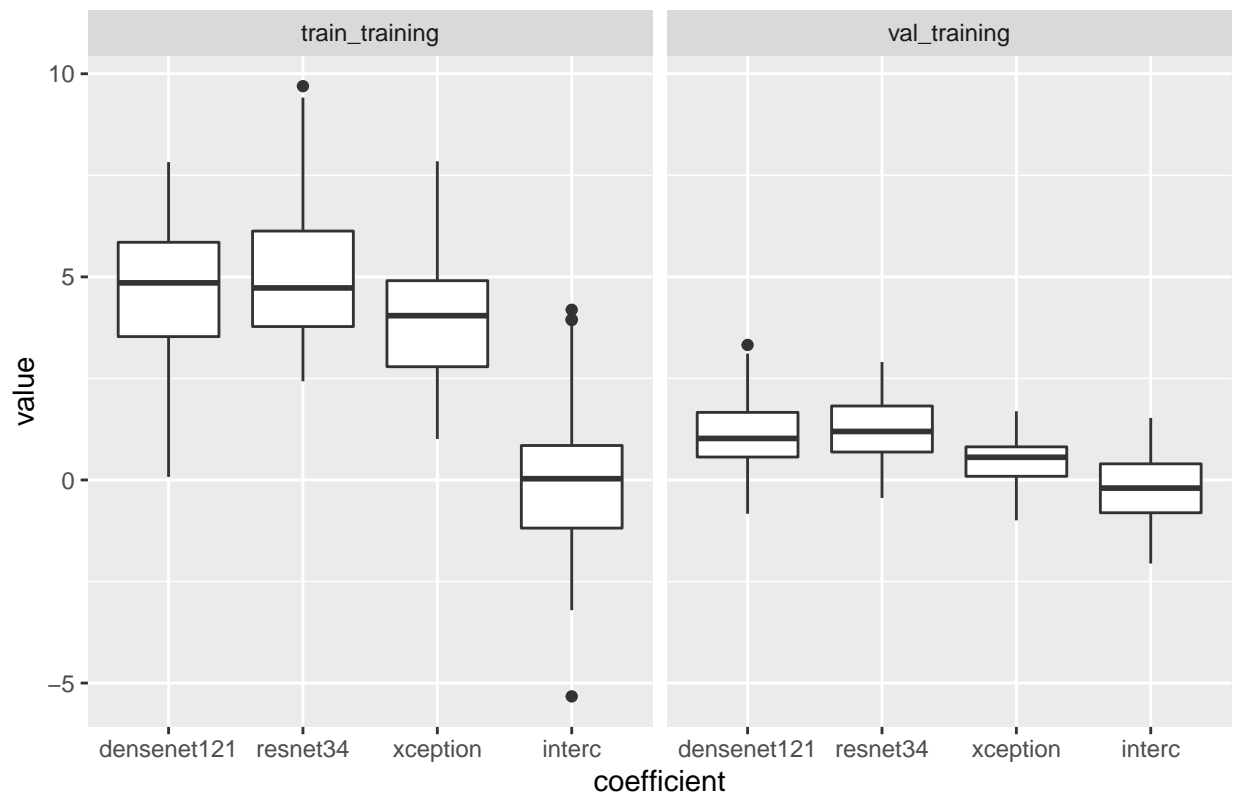
Coefficients for class 2 vs 8



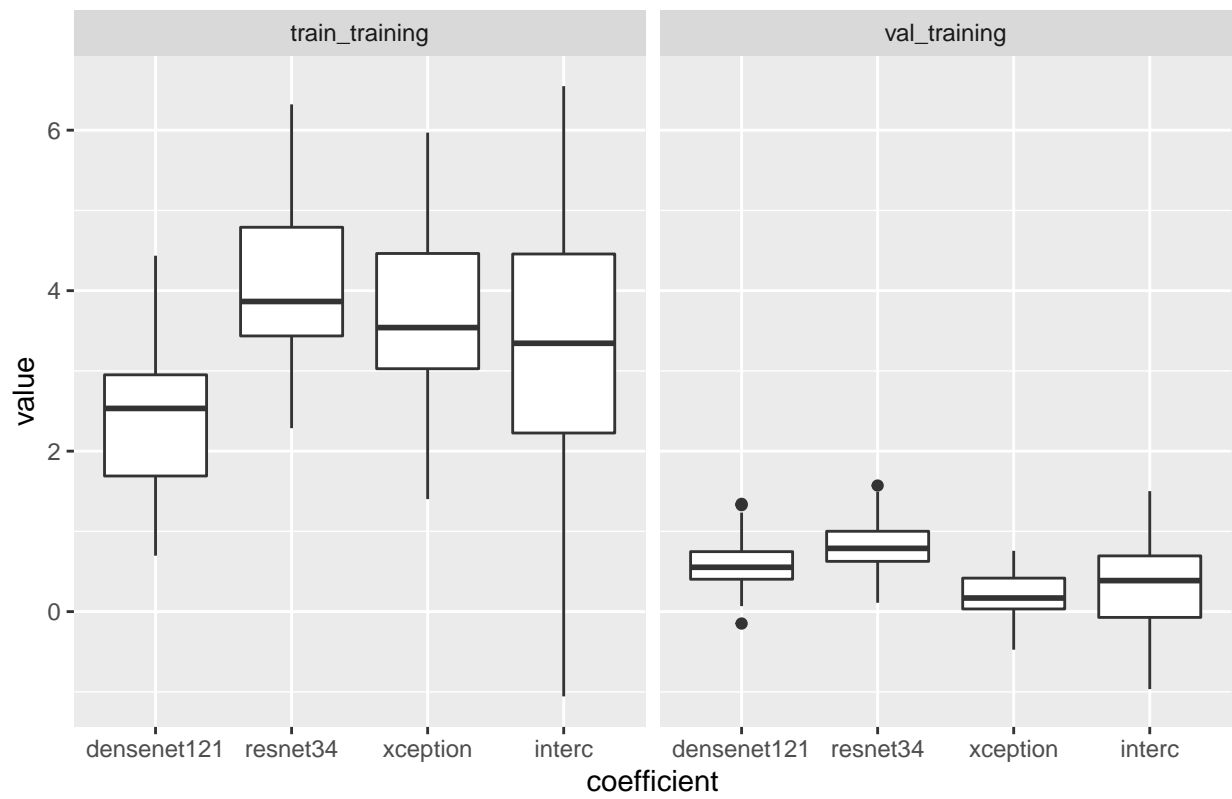
Coefficients for class 2 vs 9



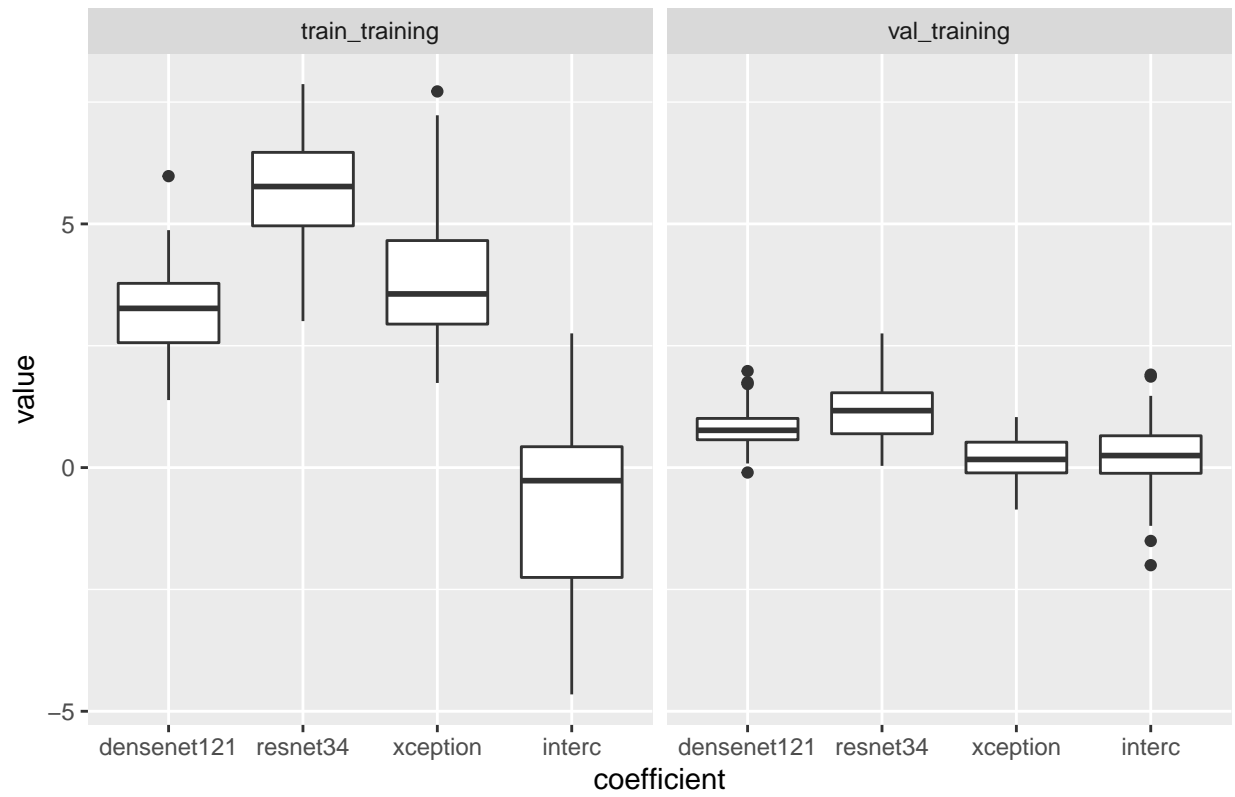
Coefficients for class 2 vs 10



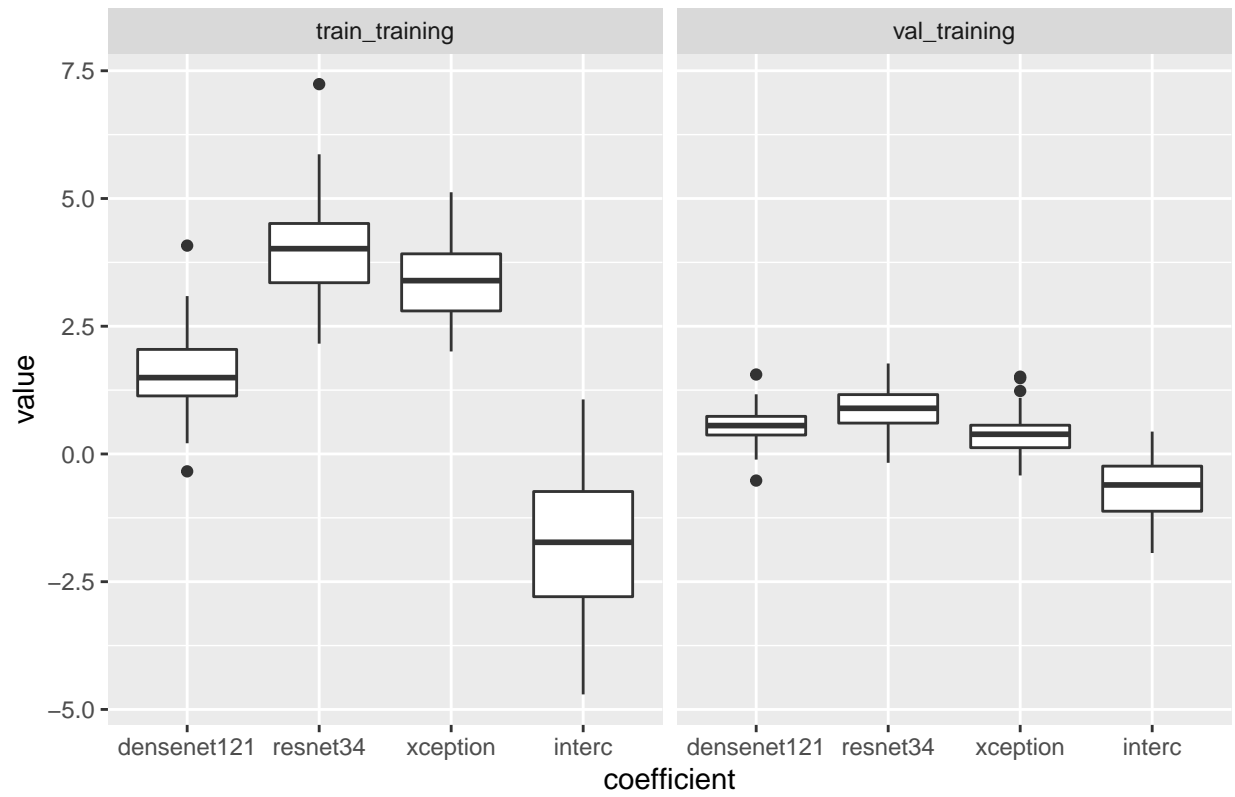
Coefficients for class 3 vs 4



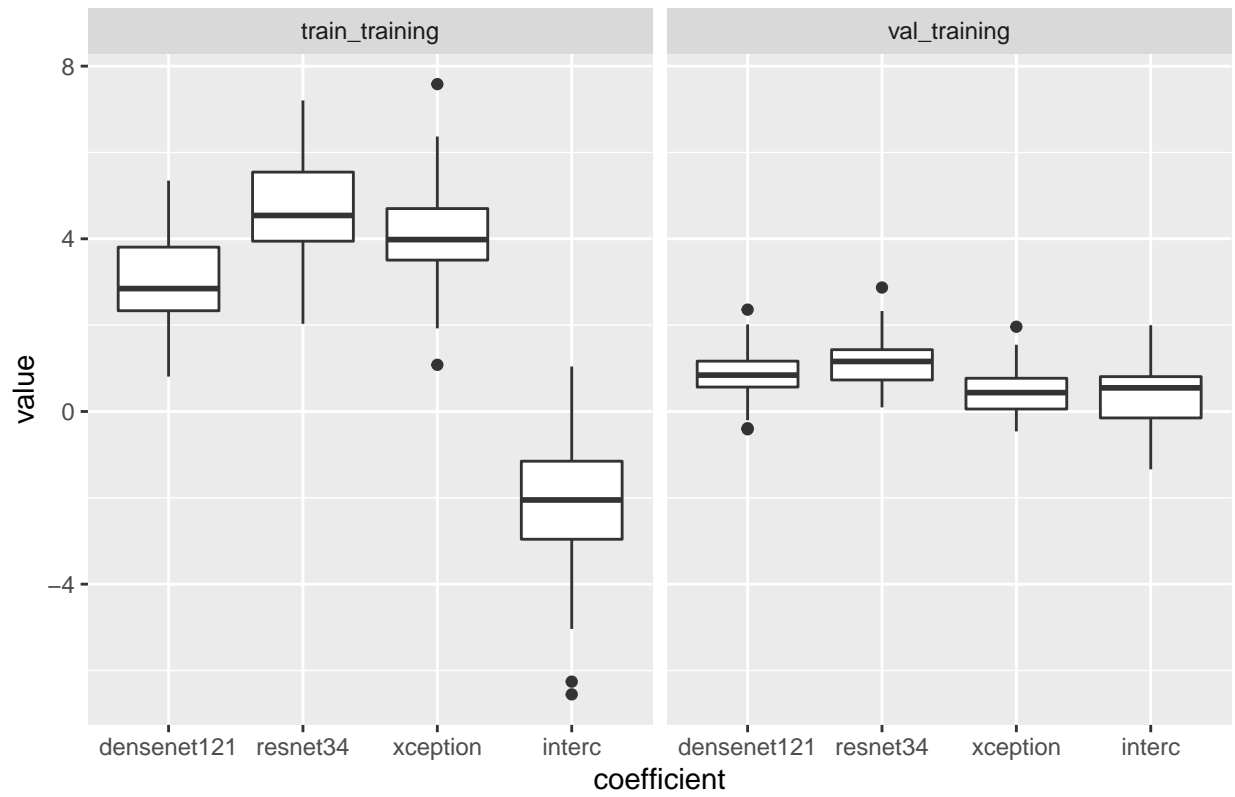
Coefficients for class 3 vs 5



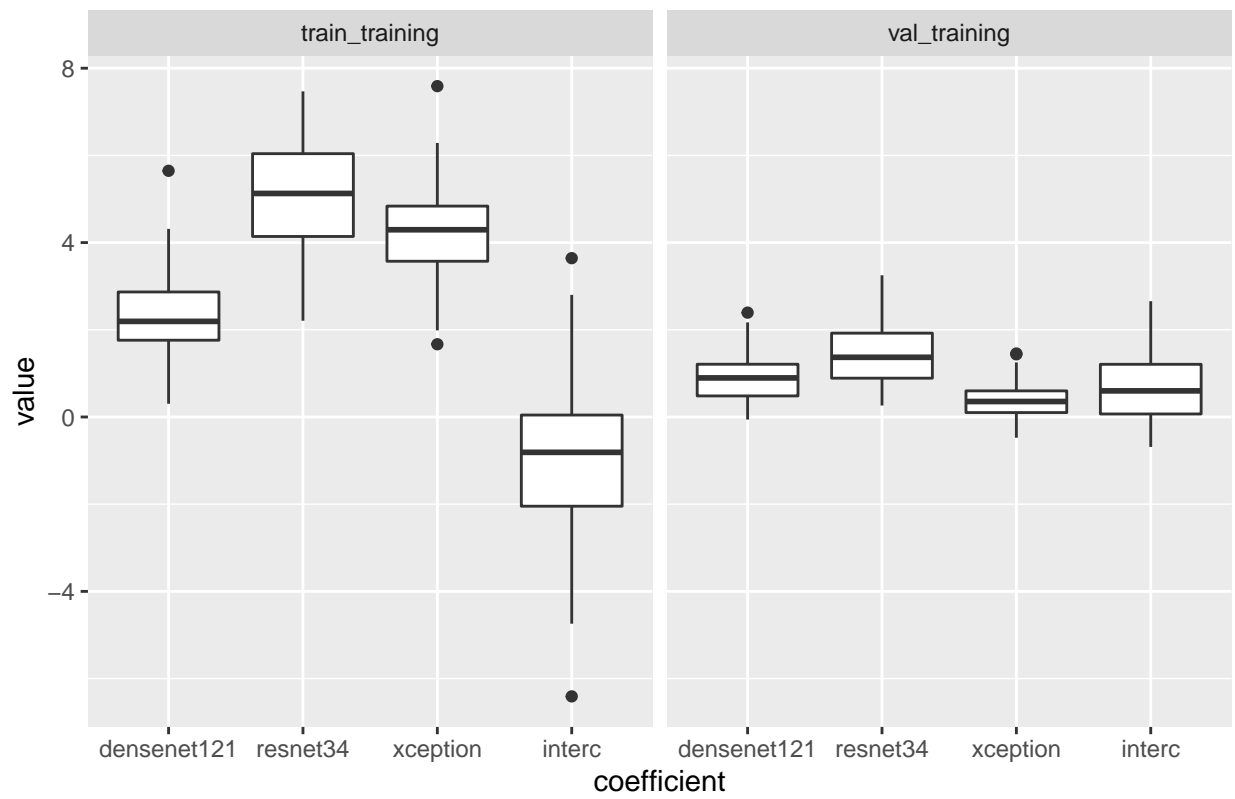
Coefficients for class 3 vs 6



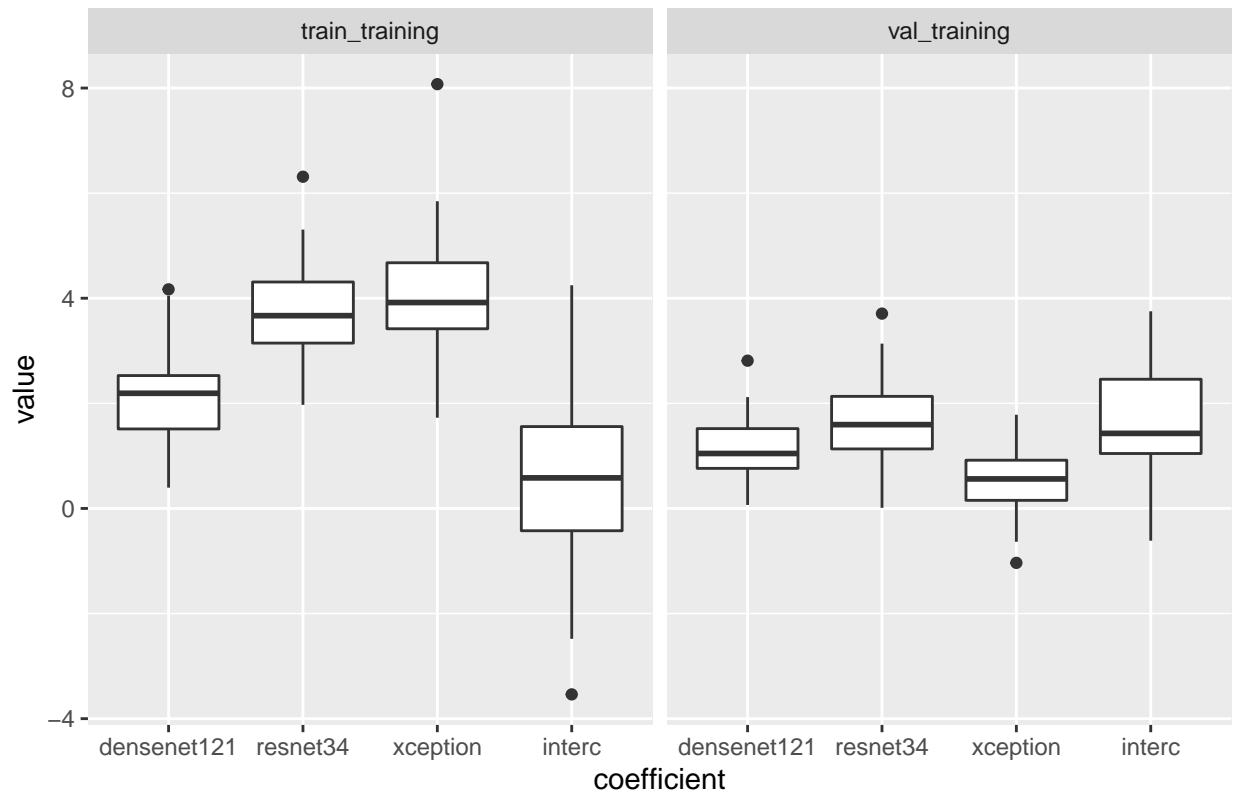
Coefficients for class 3 vs 7



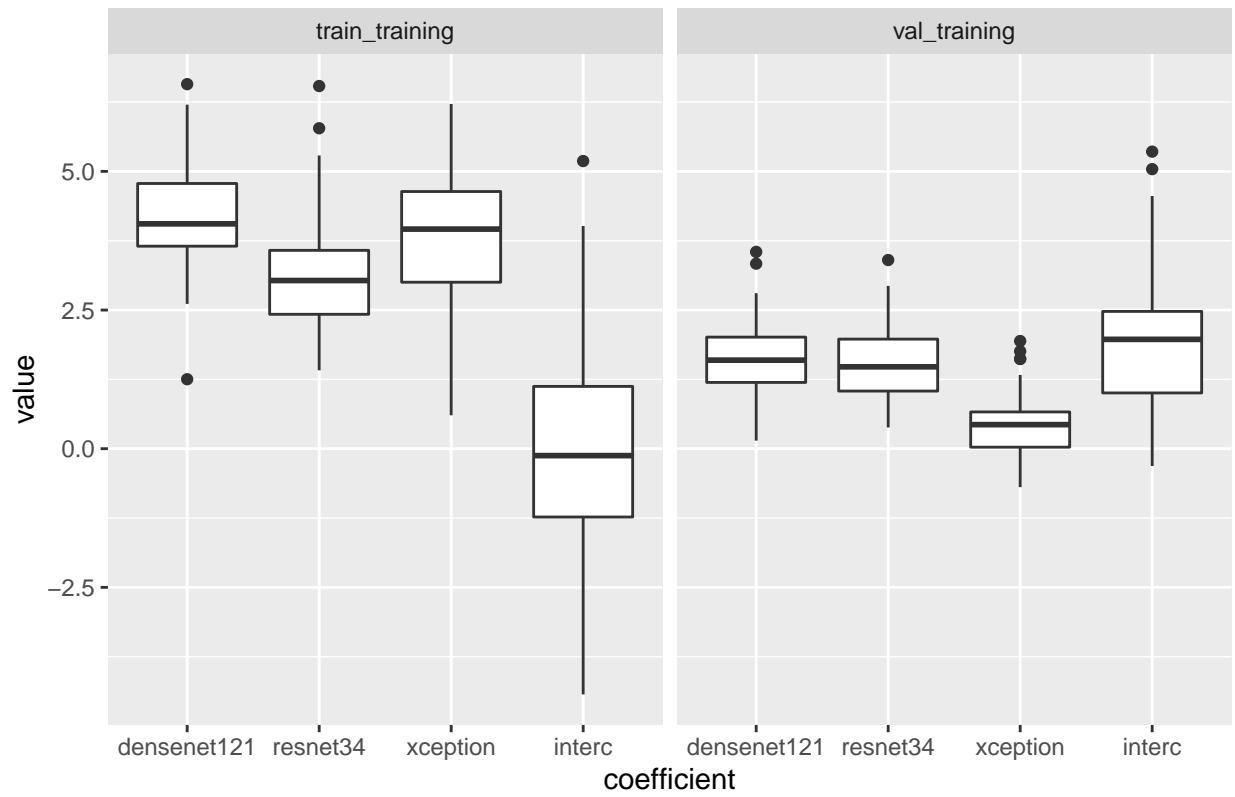
Coefficients for class 3 vs 8



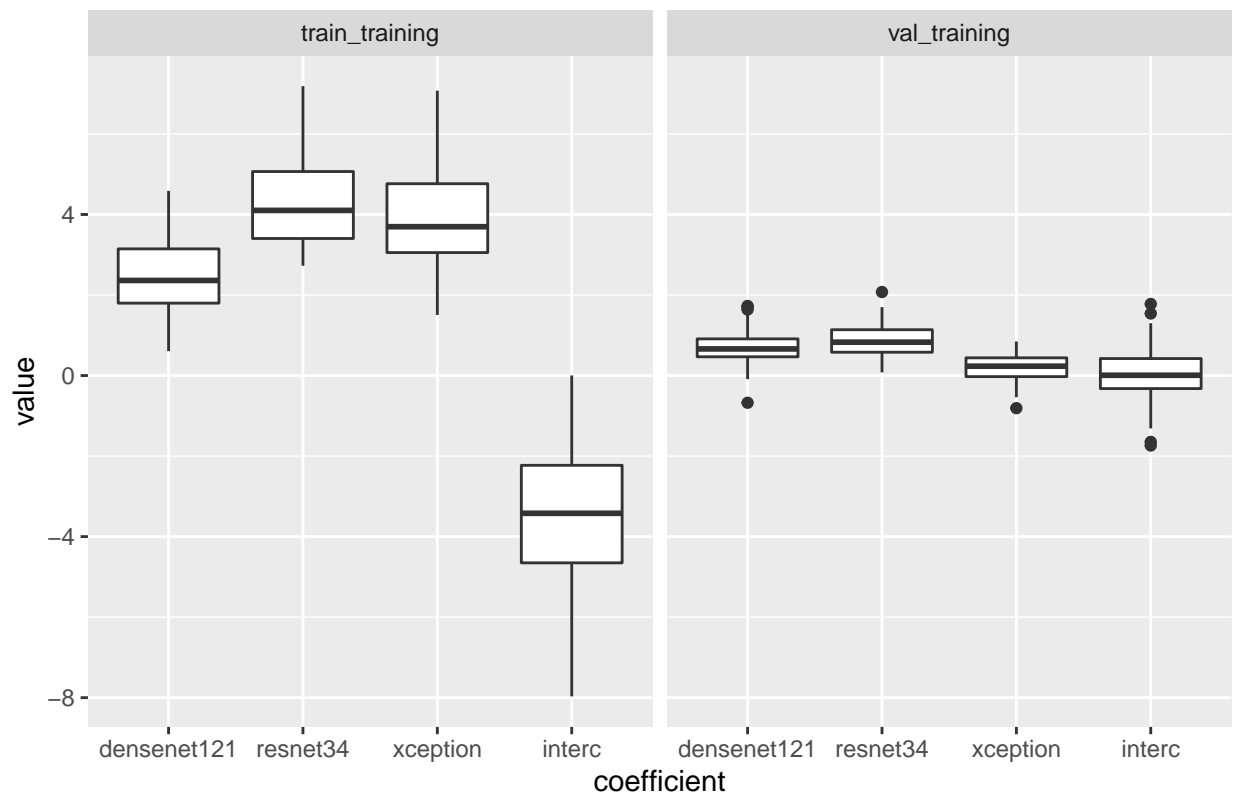
Coefficients for class 3 vs 9



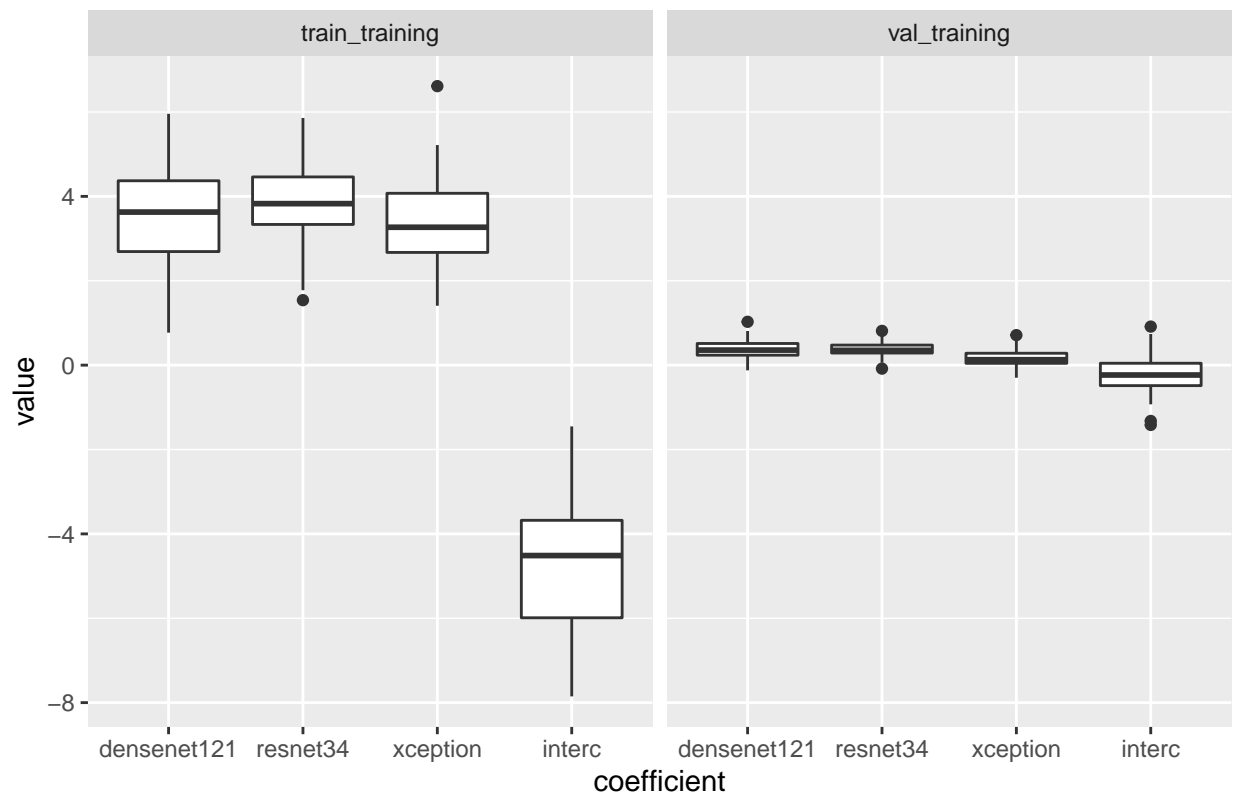
Coefficients for class 3 vs 10



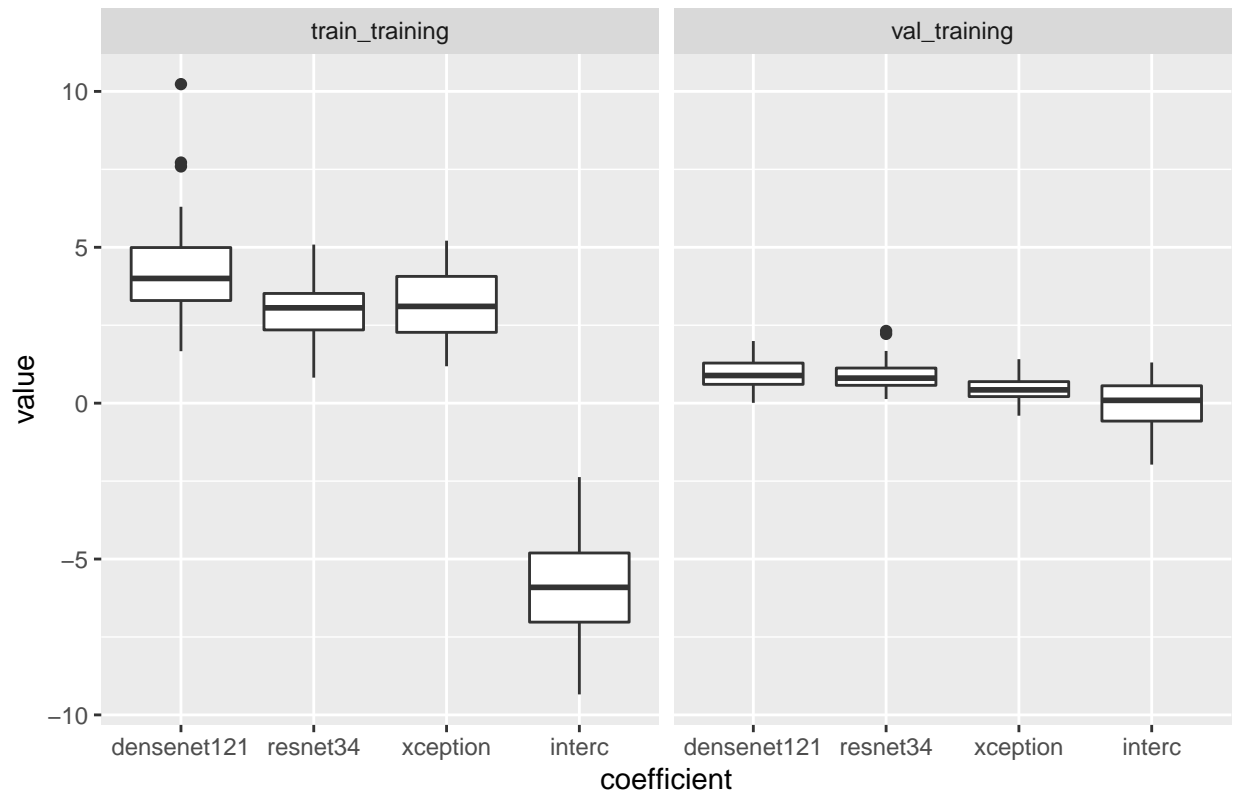
Coefficients for class 4 vs 5



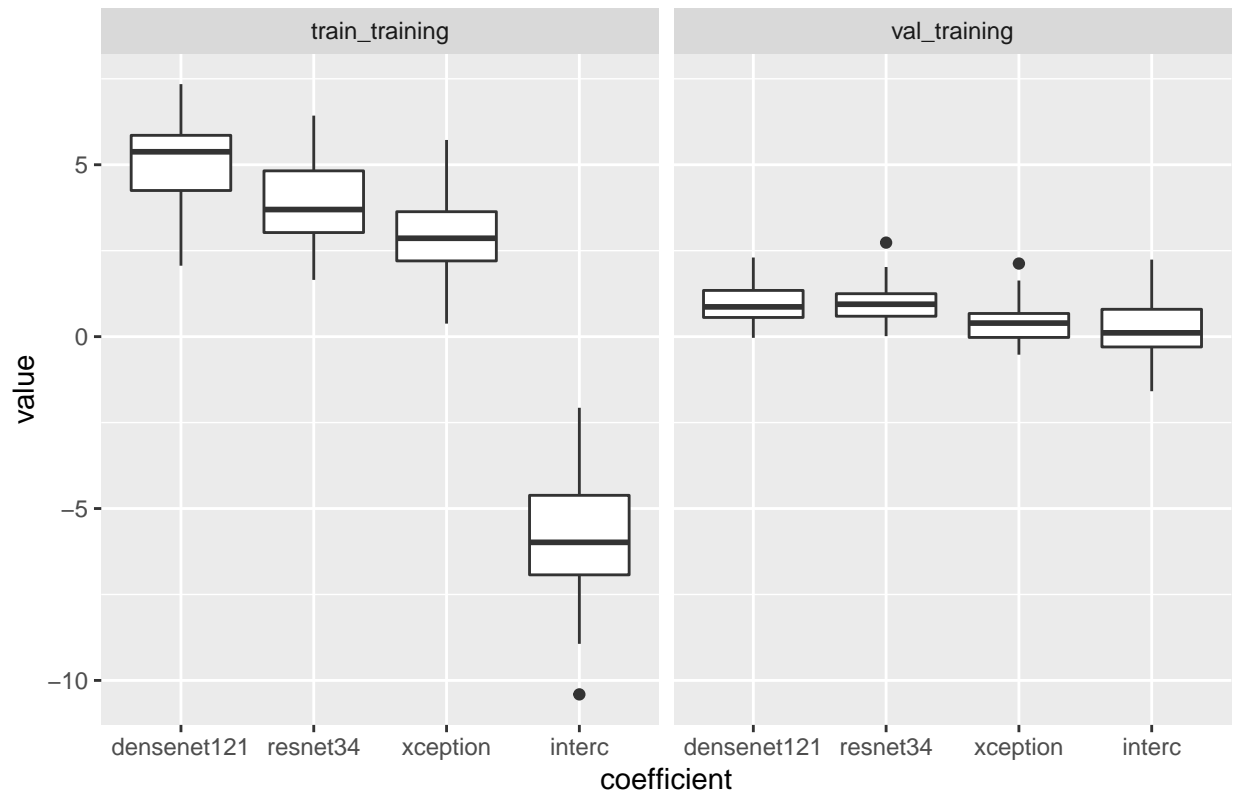
Coefficients for class 4 vs 6



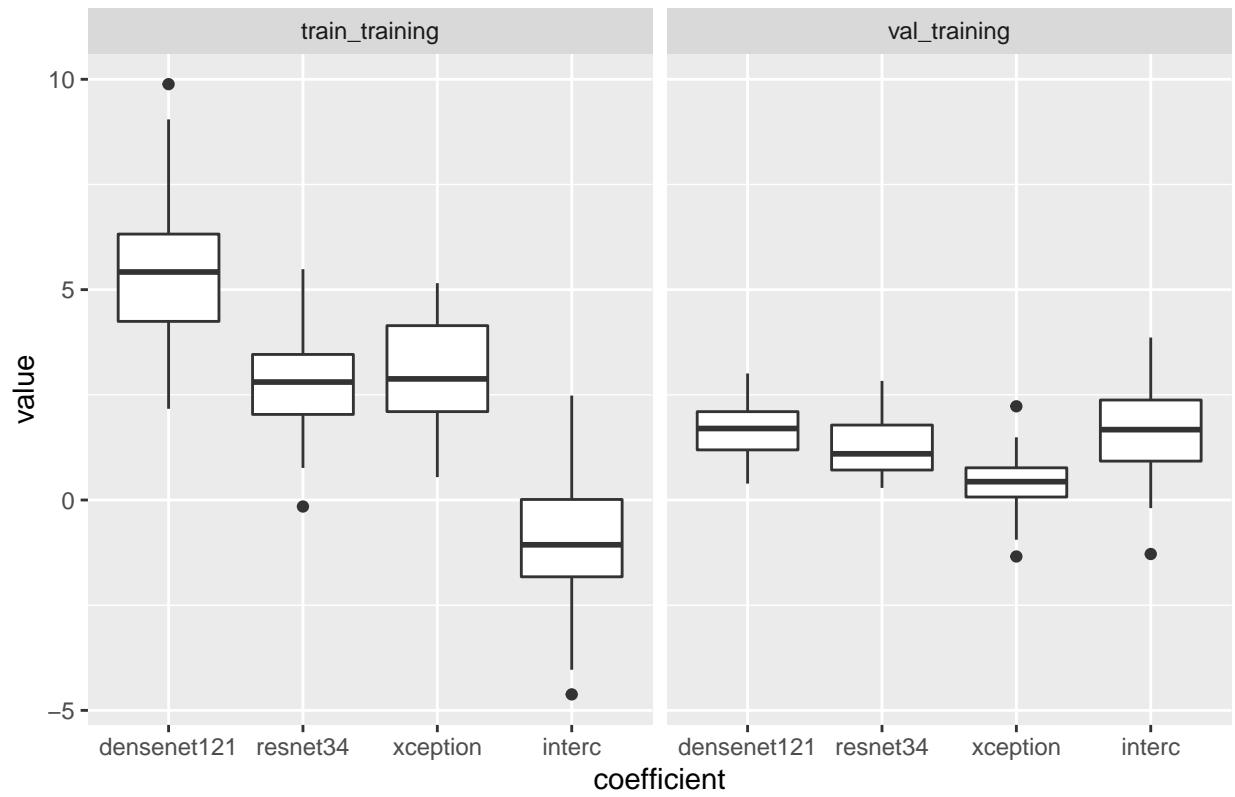
Coefficients for class 4 vs 7



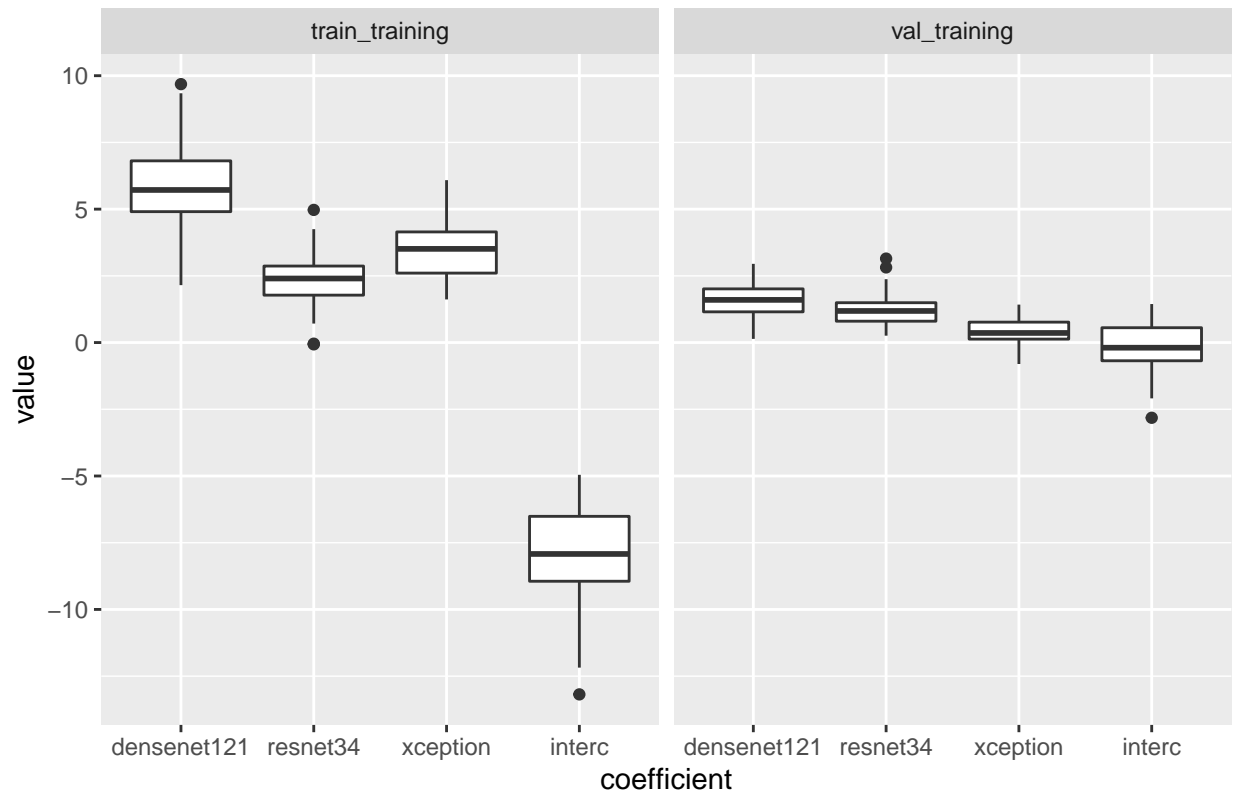
Coefficients for class 4 vs 8



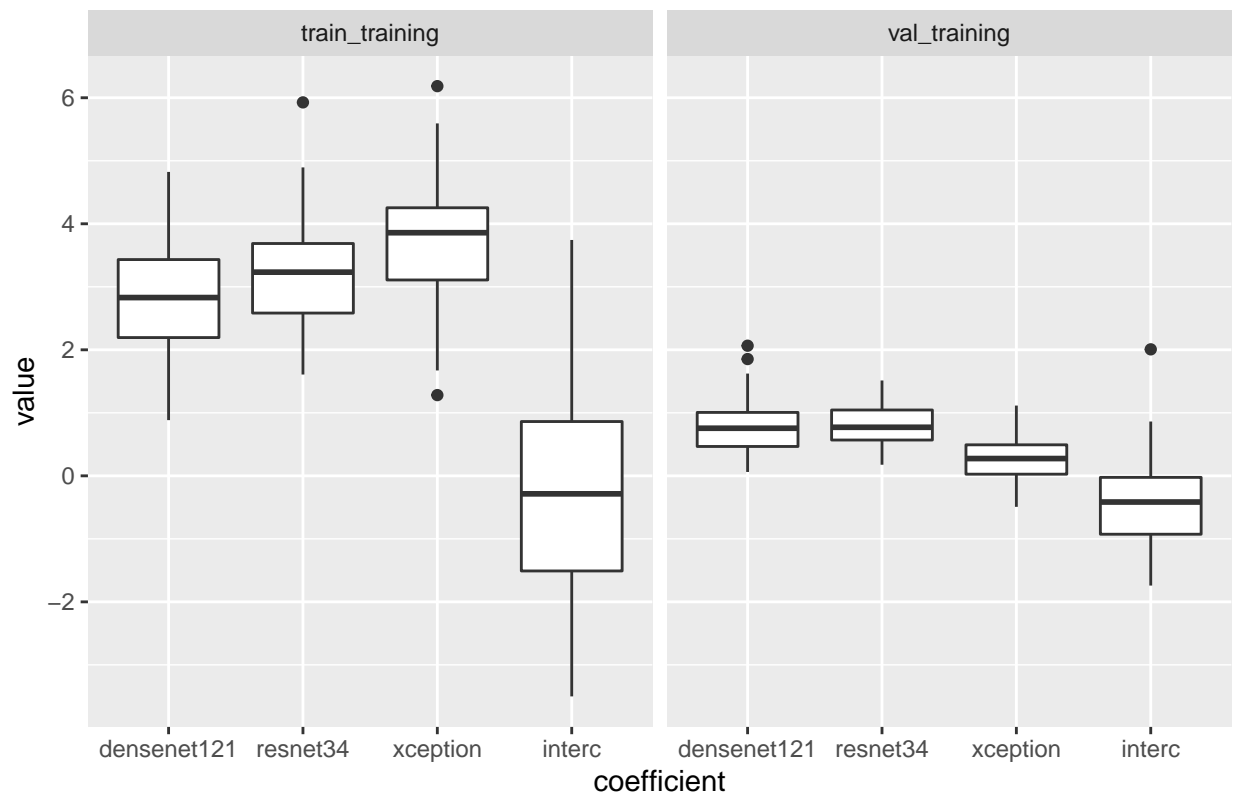
Coefficients for class 4 vs 9



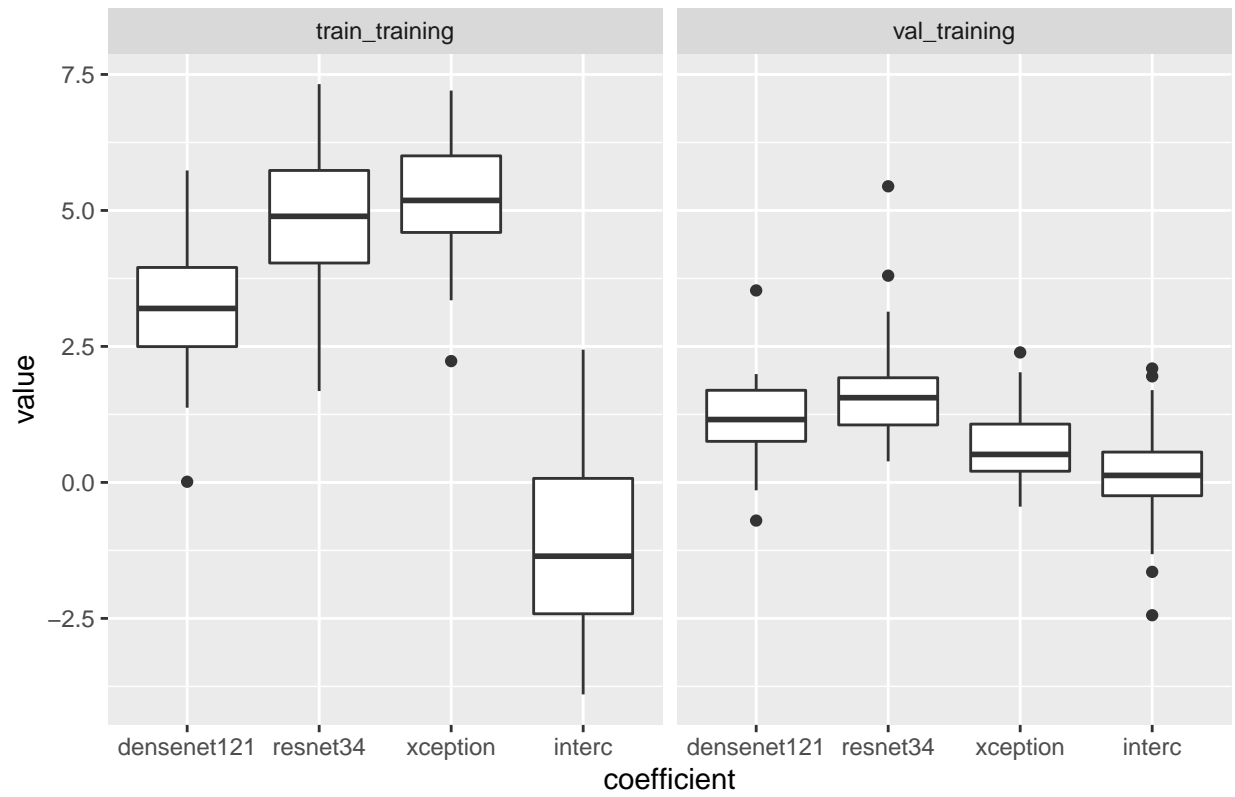
Coefficients for class 4 vs 10



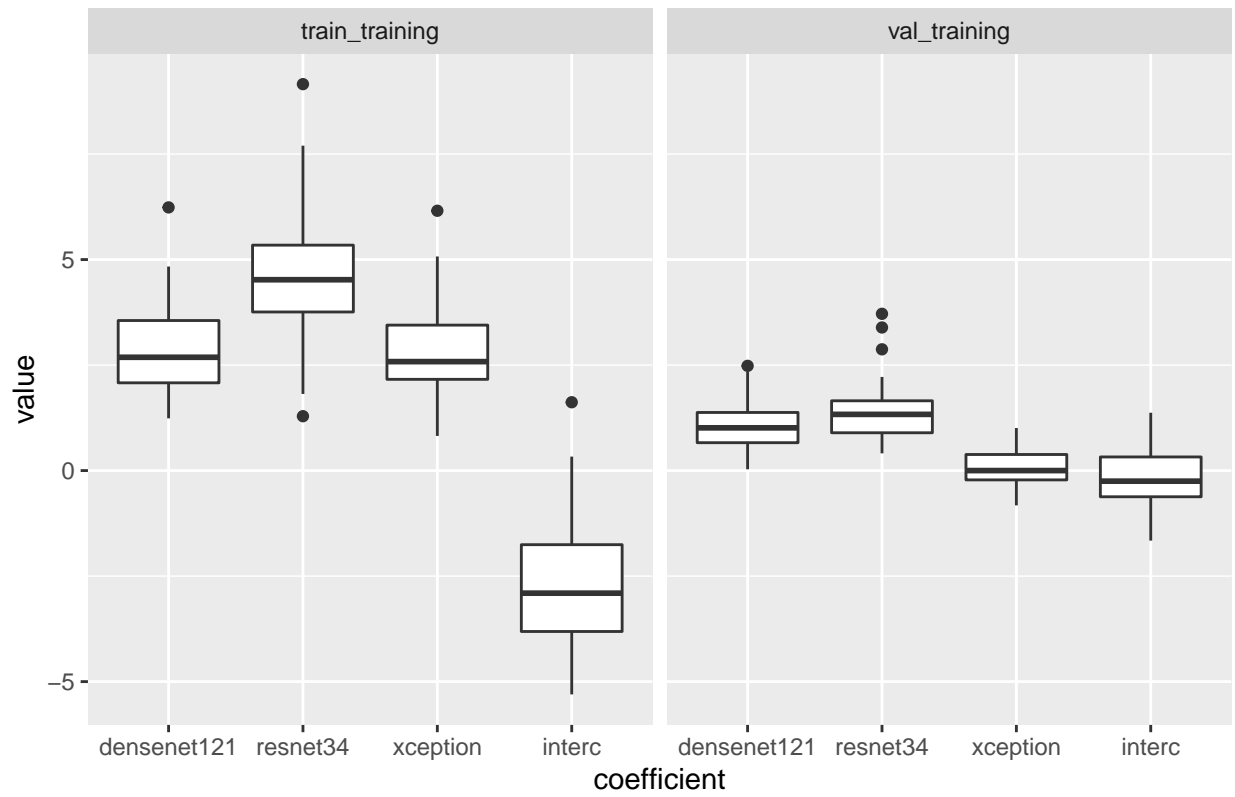
Coefficients for class 5 vs 6



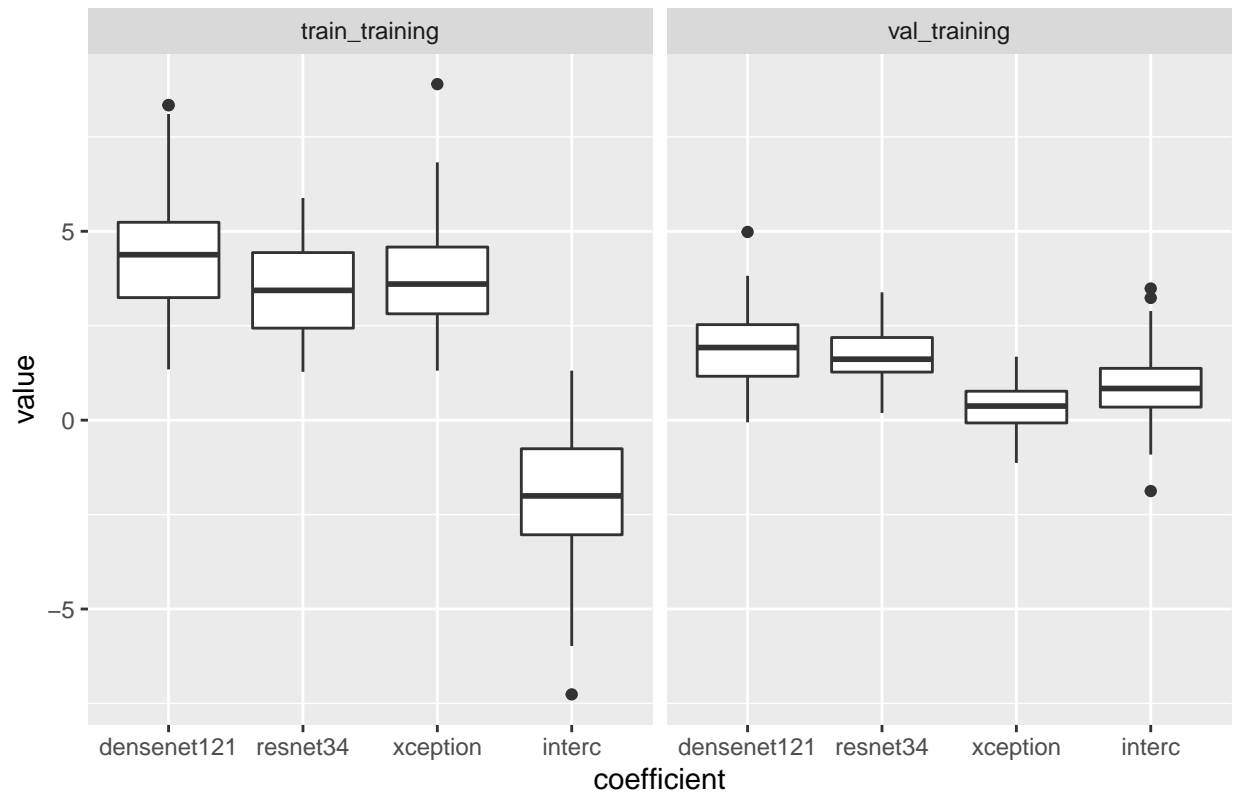
Coefficients for class 5 vs 7



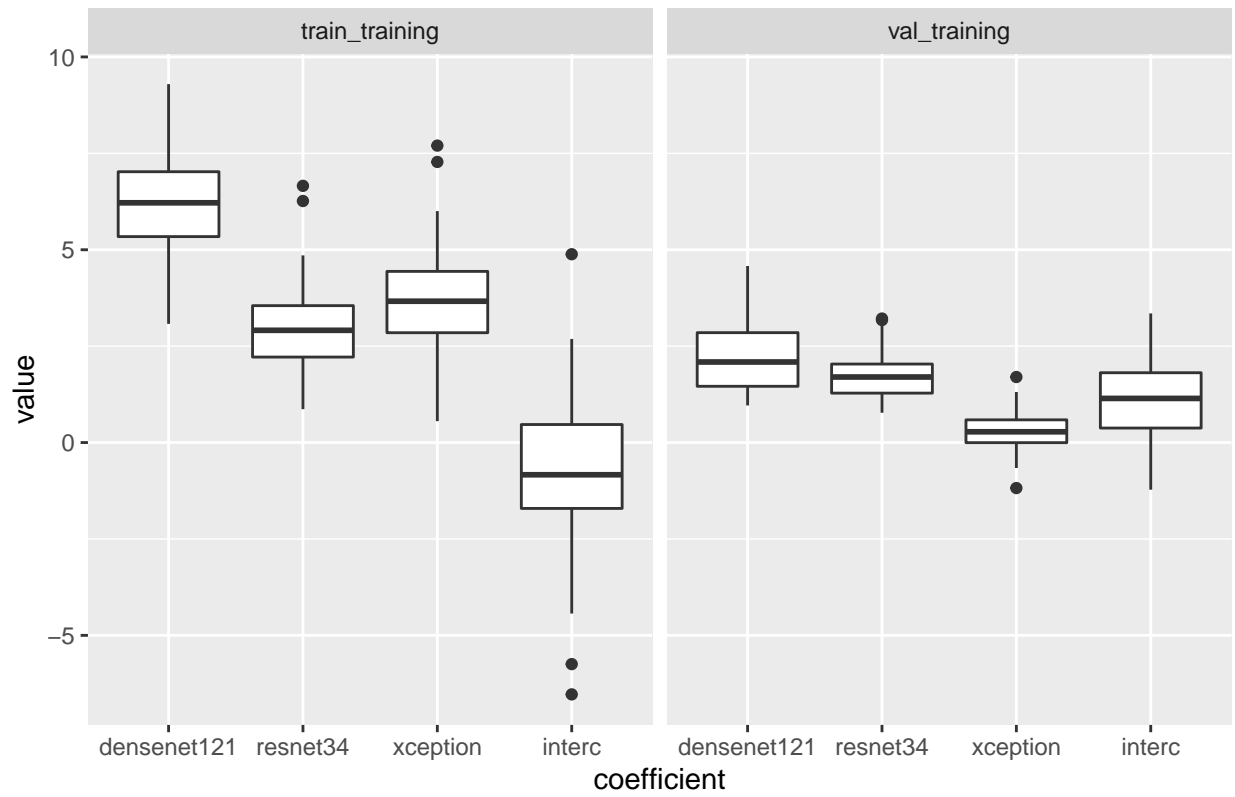
Coefficients for class 5 vs 8



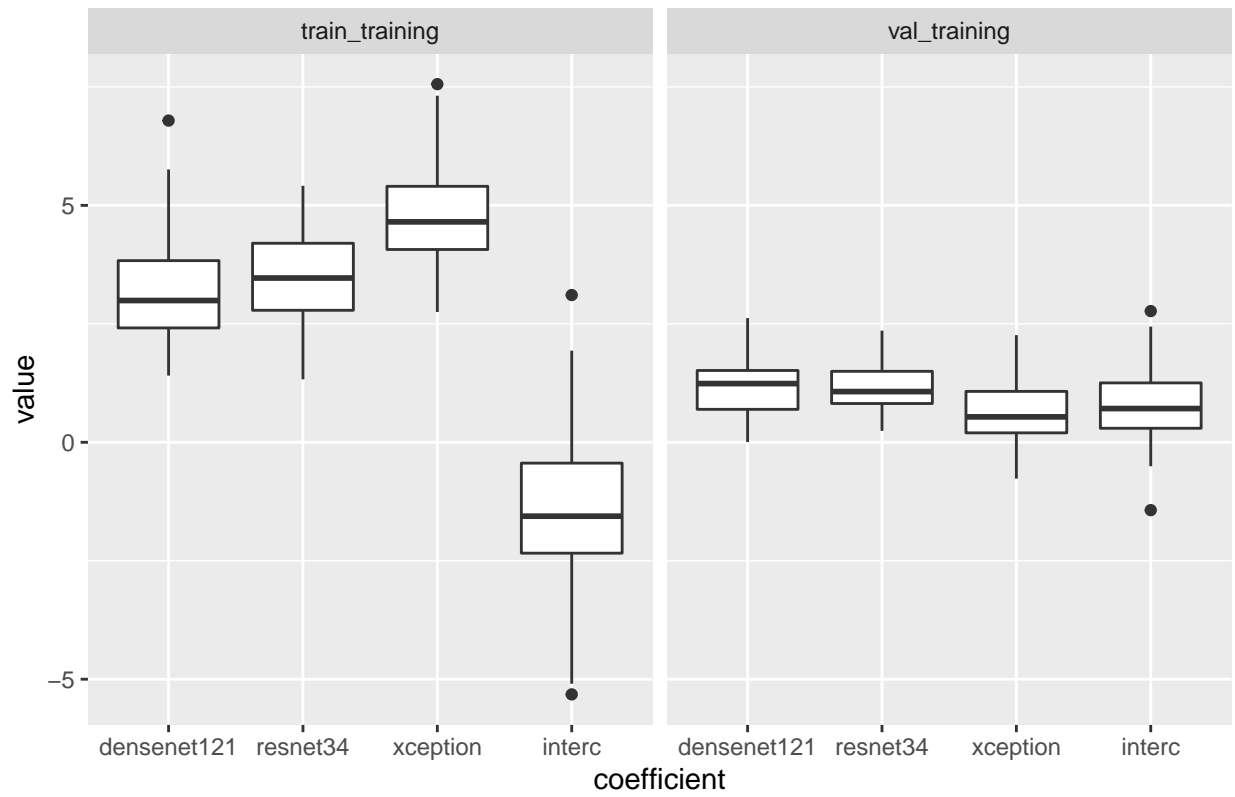
Coefficients for class 5 vs 9



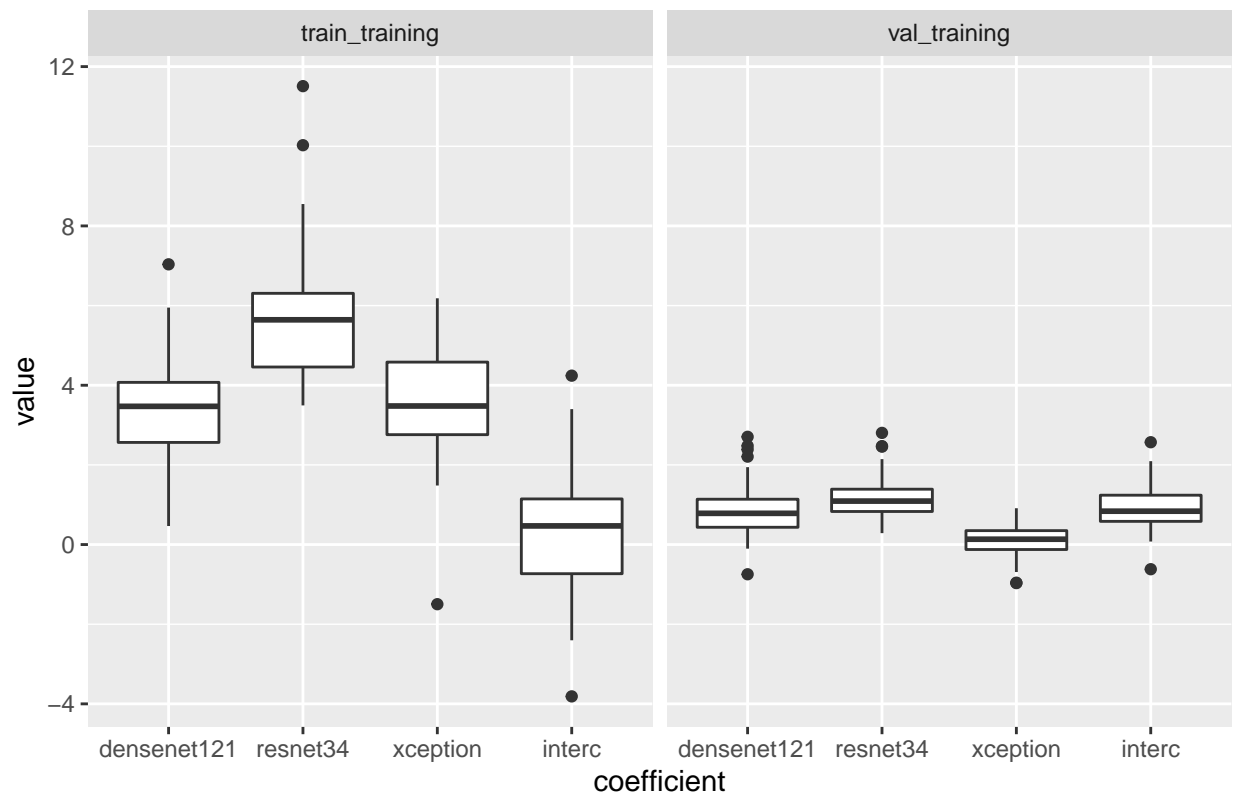
Coefficients for class 5 vs 10



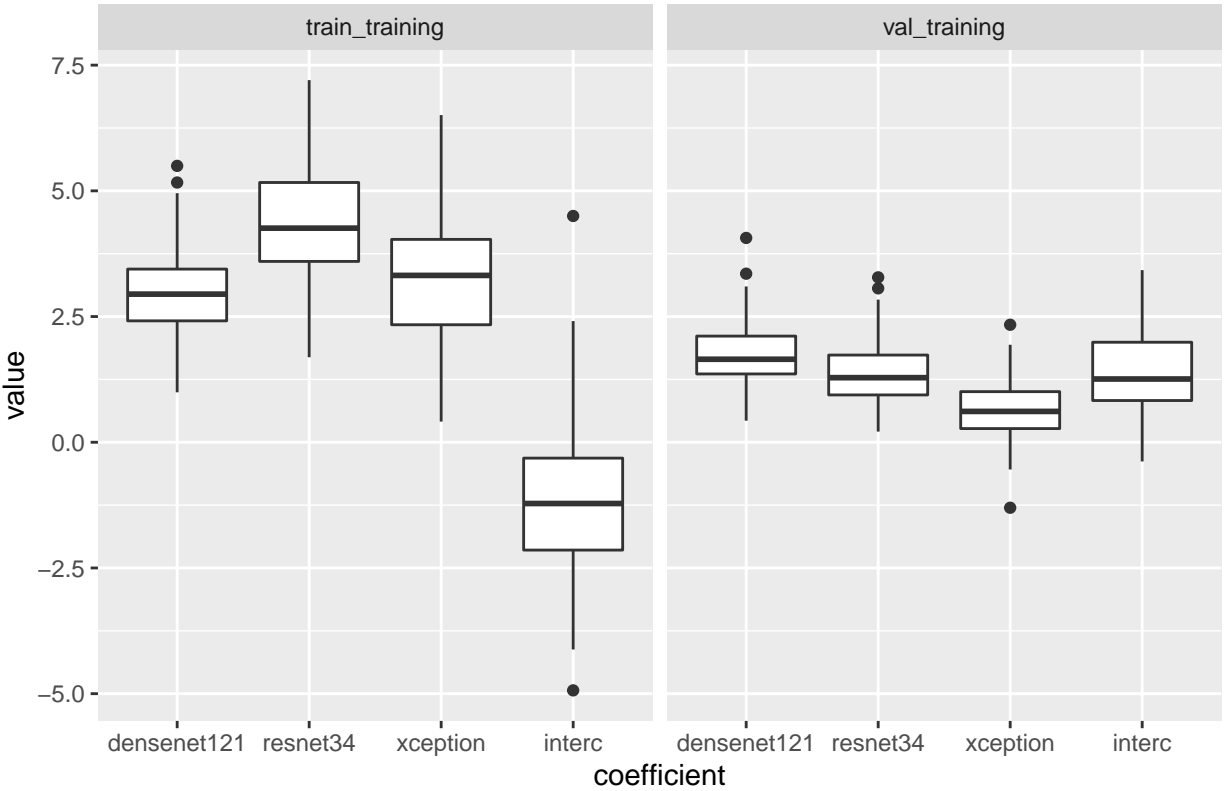
Coefficients for class 6 vs 7



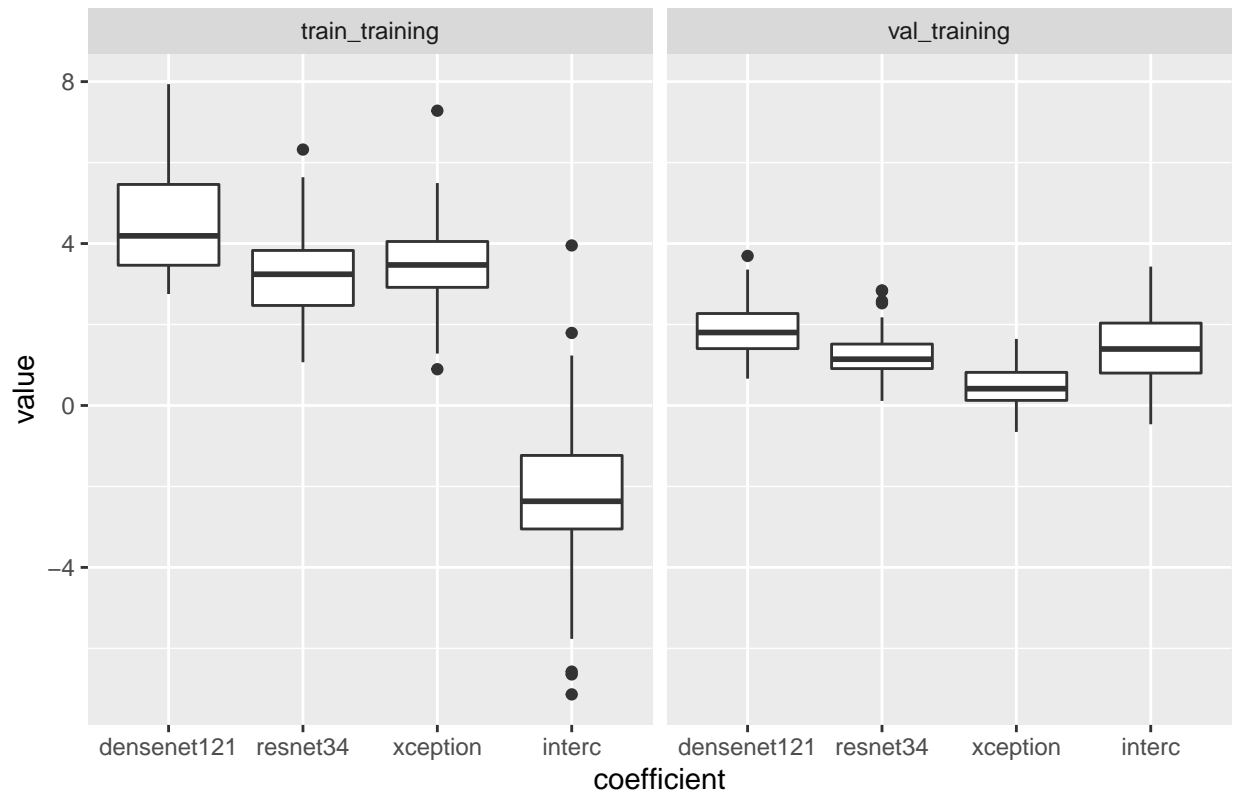
Coefficients for class 6 vs 8



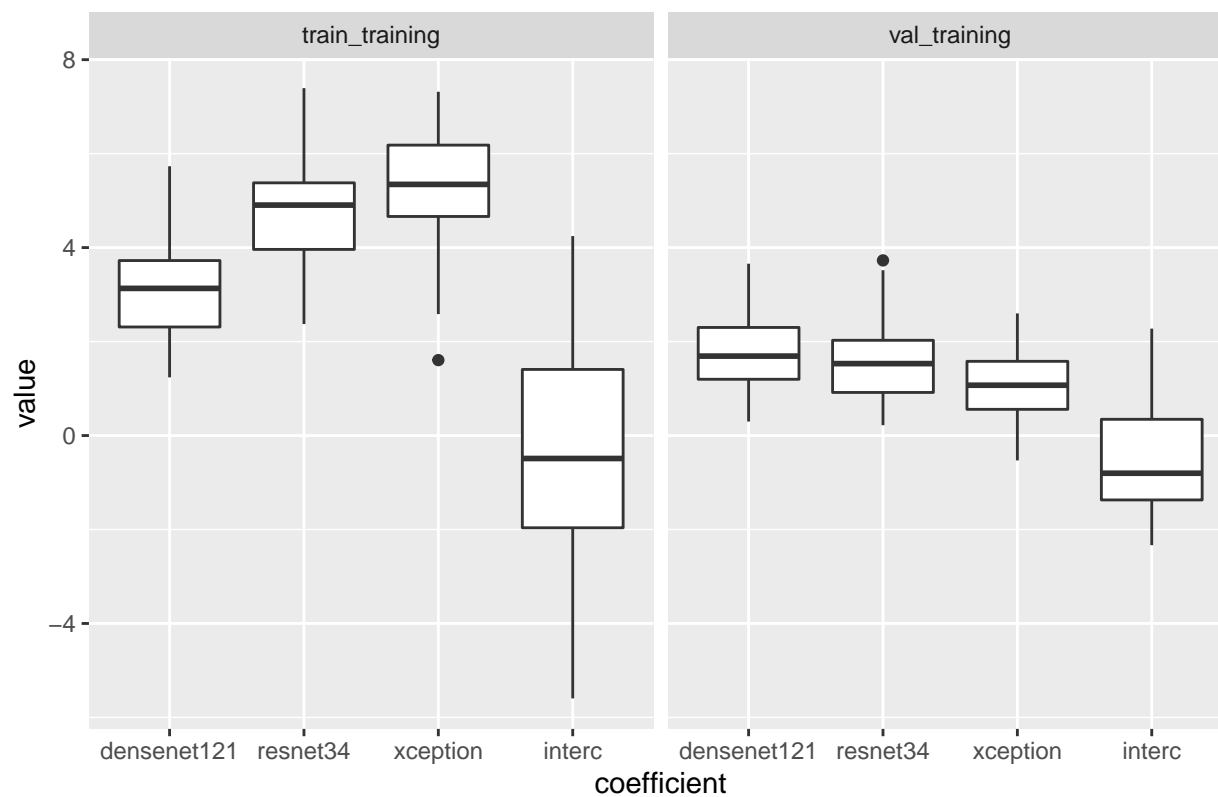
Coefficients for class 6 vs 9



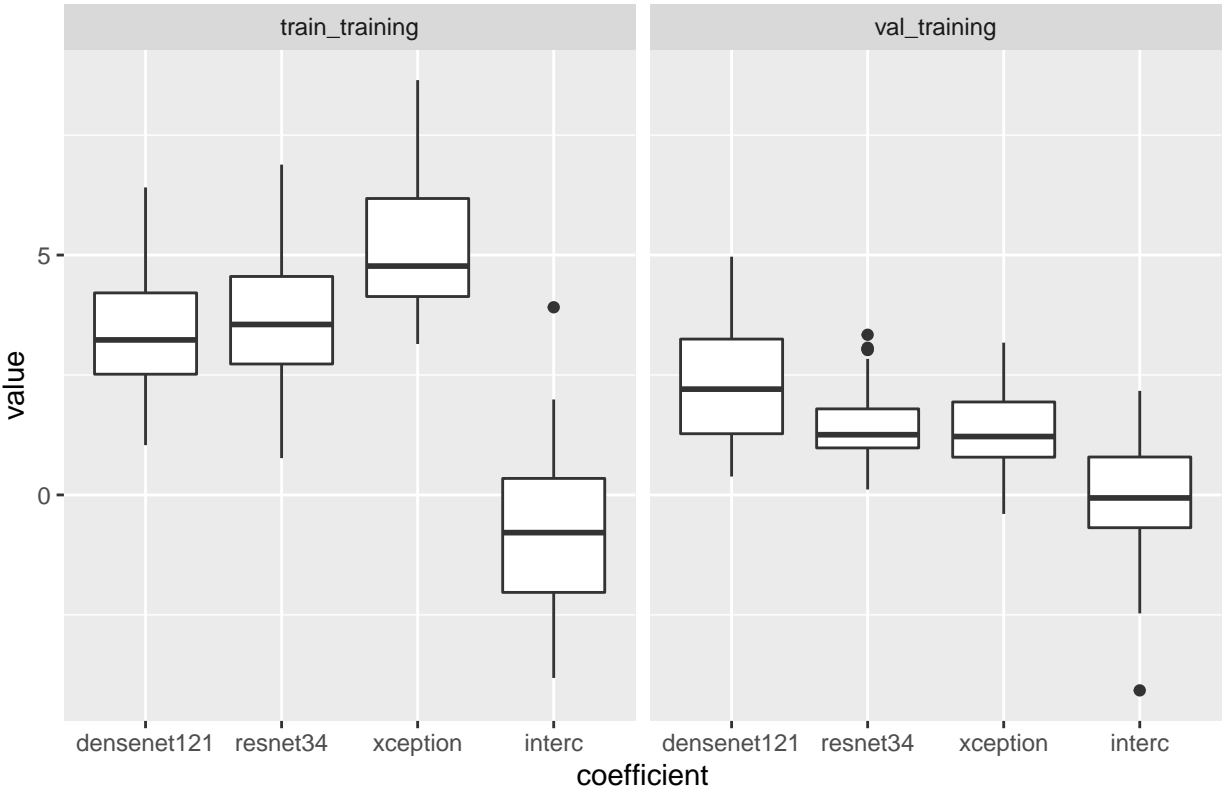
Coefficients for class 6 vs 10



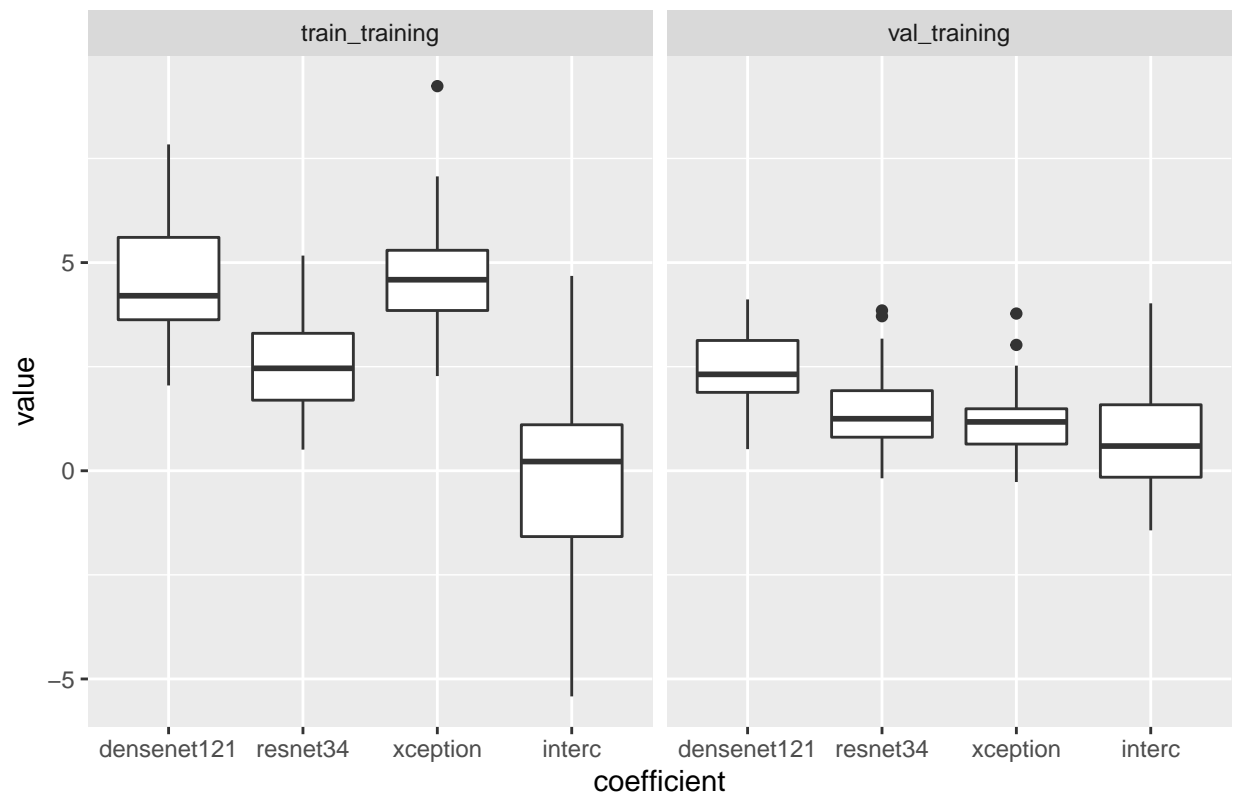
Coefficients for class 7 vs 8



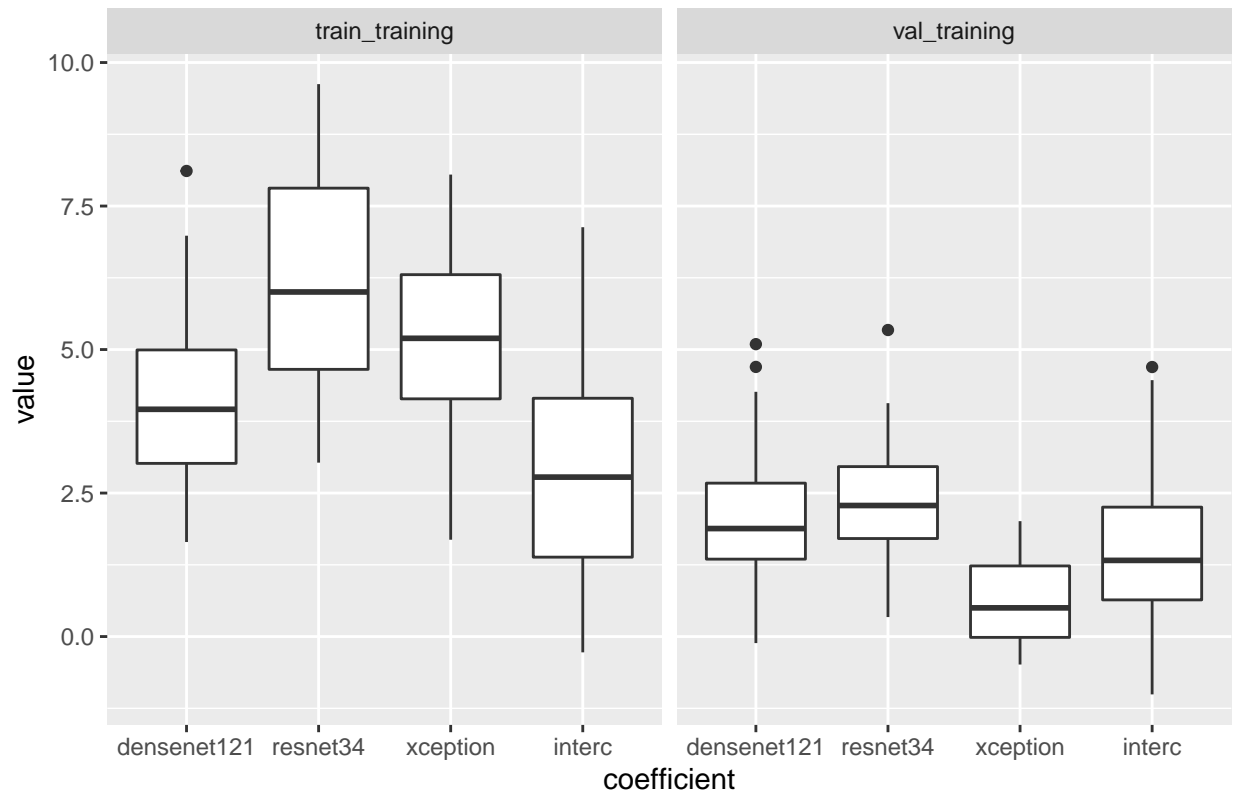
Coefficients for class 7 vs 9



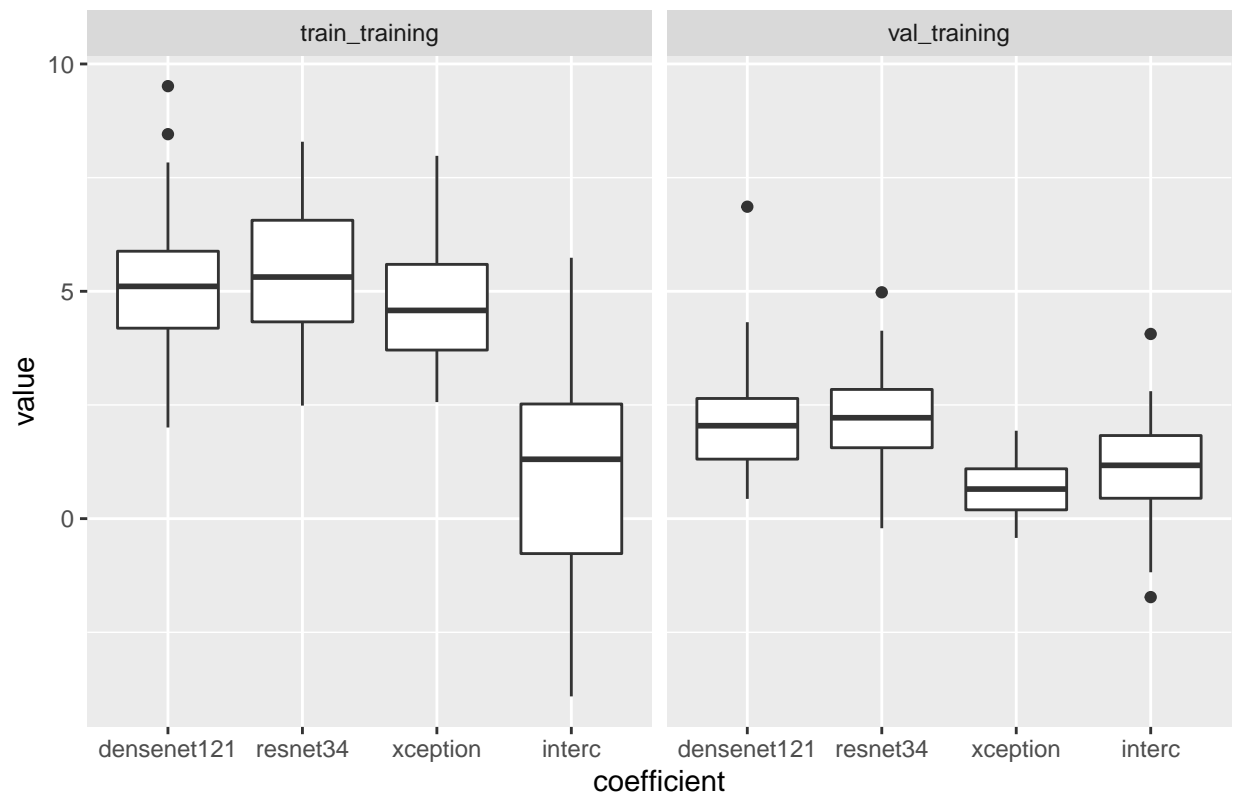
Coefficients for class 7 vs 10



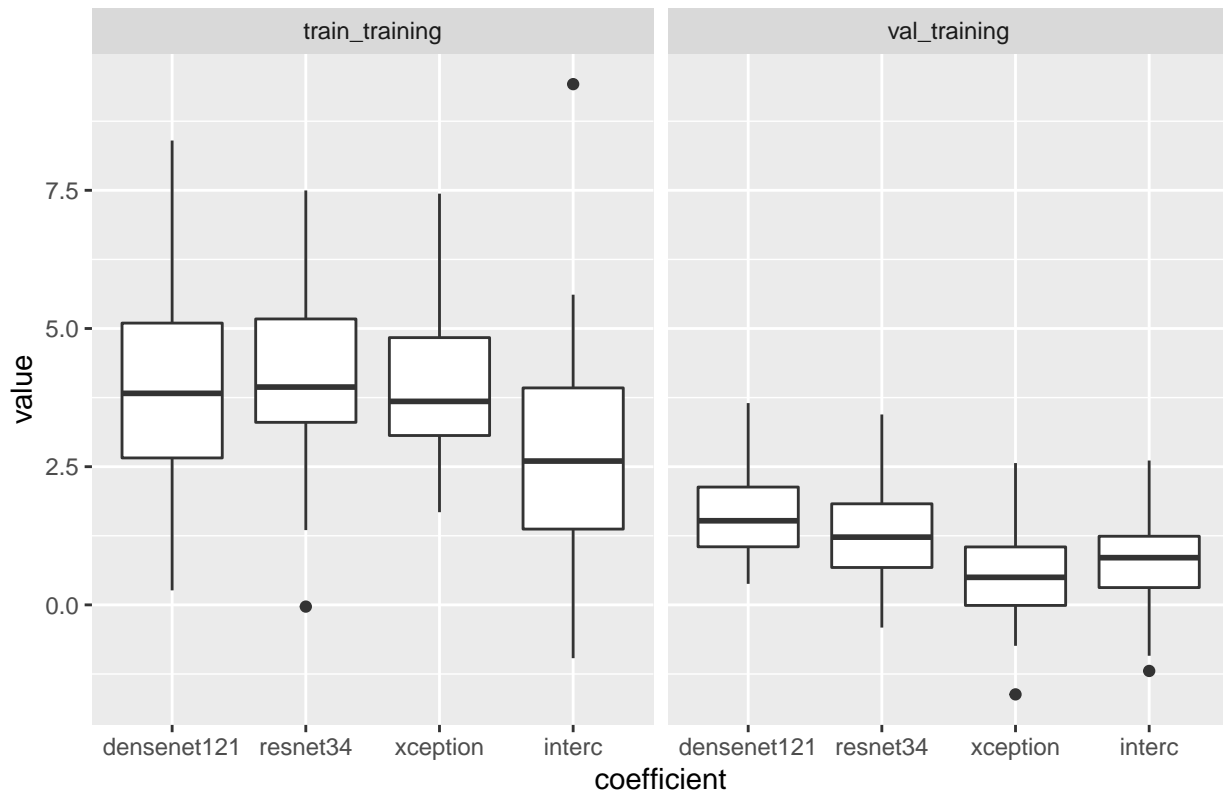
Coefficients for class 8 vs 9



Coefficients for class 8 vs 10



Coefficients for class 9 vs 10



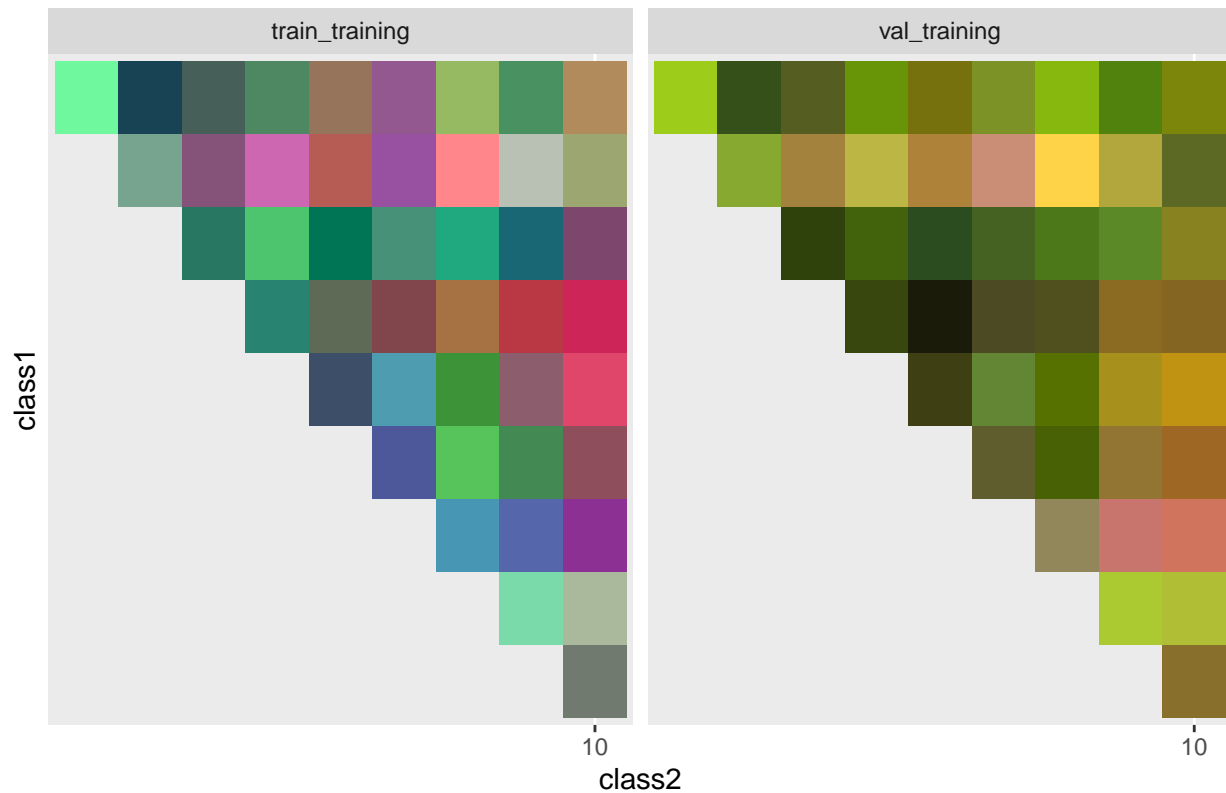
```
avg_lda_coefs <- lda_coefs %>% filter(coefficient != "interc") %>% group_by(class1, class2, precision, train_type)
```

'summarise()' has grouped output by 'class1', 'class2', 'precision', 'train_type'. You can override with 'ungroup()'.

```
avg_lda_coefs_vt <- avg_lda_coefs %>% filter(train_type=="val_training")
avg_lda_coefs_tt <- avg_lda_coefs %>% filter(train_type=="train_training")
avg_lda_coefs_vt$value <- avg_lda_coefs_vt$value - min(avg_lda_coefs_vt$value)
avg_lda_coefs_vt$value <- avg_lda_coefs_vt$value / max(avg_lda_coefs_vt$value)
avg_lda_coefs_tt$value <- avg_lda_coefs_tt$value - min(avg_lda_coefs_tt$value)
avg_lda_coefs_tt$value <- avg_lda_coefs_tt$value / max(avg_lda_coefs_tt$value)
avg_lda_coefs <- rbind(avg_lda_coefs_vt, avg_lda_coefs_tt)
avg_lda_c_w <- pivot_wider(avg_lda_coefs, names_from = coefficient, values_from = value)
avg_lda_c_w[, c("class1", "class2")] <- lapply(avg_lda_c_w[, c("class1", "class2")], as.factor)
avg_lda_c_w$top_net <- factor(c("densenet121", "resnet34", "xception")[max.col(as.matrix(avg_lda_c_w[, c("class1", "class2")])])])
```

```
raster_plot <- ggplot(avg_lda_c_w) +
  geom_tile(aes(x=class2, y=class1, fill=rgb(densenet121, resnet34, xception))) +
  scale_y_discrete(limits=rev, breaks=seq(0,classes, 10)) + scale_x_discrete(breaks=seq(0,classes, 10))
raster_plot
```

RGB image formed from lda coefficients for networks densenet, resnet, xception

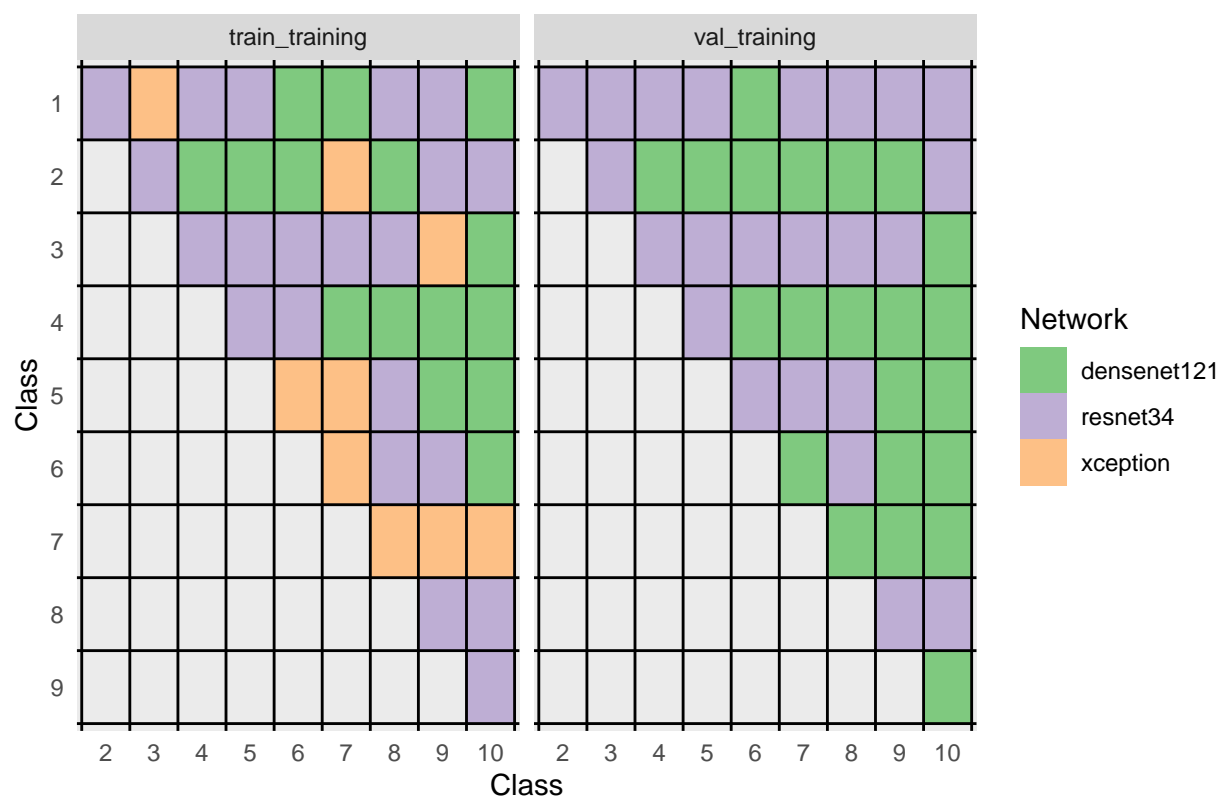


Correspondence between colors and networks is red - densenet, green - resnet, blue - xception.

```
coefs_grid <- ggplot(avg_lda_c_w, aes(x=class2, y=class1, fill=top_net)) +
  geom_raster() +
  scale_fill_brewer(type="qual") +
  facet_wrap(~train_type) +
  scale_y_discrete(limits=rev) +
  geom_vline(xintercept=seq(-0.5, 9.5, 1.0)) +
  geom_hline(yintercept=seq(-0.5, 9.5, 1.0)) +
  guides(fill=guide_legend(title="Network")) +
  xlab("Class") +
  ylab("Class") +
  ggtitle("Network with highest lda weight for class pairs") +
  theme(plot.title = element_text(hjust = 0.5),
        axis.ticks = element_blank(),
        panel.grid.major = element_blank(),
        panel.grid.minor = element_blank())
```

coefs_grid

Network with highest lda weight for class pairs



Densenet is far less dominating in this experiment than in visualizations_ensemble_outputs_CIF10. Other networks seem to be more competitive when training is done just on half of CIFAR 10 training set.