Application of Newton's methods for solving load flow problems

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***Abstract*—Electricity distributors are responsible for the operation and planning of the transmission line expansion, and need to have real-time knowledge of the point of operation of their networks. For this, it is necessary to calculate several load flows, in order to always stay updated with the network events. The lack of knowledge of the network can cause power loss and physical damage to the networks and possibly to the equipment of its users. There are several types of methods to solve this problem, having as a criterion of choice the precision of the result, or the computational time that it requires. This work presents the implementation of two of these methods and compares the results found, it used the bars of the IEEE14 system as the topology. It conclude that in small systems, it had minimal differences, but with significant computational gain.**

***Index Terms*—Reproducibility, Power Flow, Python, Newton's Method, Fast Decoupled Method.**

I. INTRODUCTION

THE calculations of the power flow are indispensable for the planning and operation of electric power networks, this is due to the importance of having knowledge of the state of the network, the voltage and phase levels. Usually, these networks are represented in a simplified, single-line manner, and calculations are performed in p.u. Another relevant aspect to this analysis is the choice of the method used, with some prioritizing computational time, but not guaranteeing the global optimum (heuristic methods) and others that prioritize the result, but consuming a lot of processing (linear).

The work of [1], presents a proposal for calculating the power flow, based on the radial electrical energy distribution system. The problem was formulated to minimize losses and was approached in a second order cone programming. The models with second order cone programming, have a convergence arduous process, thus requesting a high computational price, but accomplishing the global optimum of the system. The work demonstrates the evolution of the need for interactions, increasing the system used, and consequently having a extend in the computational time used to find the solution. Another paper that also discusses this relation is [2], showing a possible approach to Newton's method, but discussing the variation of the error factor and the number of iterations necessary to find a solution. Newton's method is an iterative process that can converge or diverge from the optimum. The divergence normally occurs in unbalanced systems, since this model does not have characteristics to identify or work around this problem.

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The author's [3] research presents a very detailed theoretical explanation of Newton's method, demonstrating the possible divergence that the problem can assume, by the inversion of the Jacobian matrix. Adapting this method for large systems, where the inversion of the Jacobian matrix, in addition to the extremely heavy problem, can lead to the computational time suffering a divergence in an iteration, throwing out the whole process until that moment. The Jacobian matrix is responsible for having all the power flows, with this matrix being divided into 4 others, two active and two reactive. In this way, it is possible to cover all the power flows of the system, in a single matrix, but as the remarks of the author, it can have problems, since it is necessary to invert it in each iteration, requesting a high computational price. The problem found in the Jacobian matrix is also being discussed in [4], which analyzes a new way of calculating, in a non-linear way, the problem of operation of the electric power transmission.

In this work will be presented the Newton's method and Fast Decoupled method, applied in the resolution of the load flow. The theory behind the methodology will be presented, carrying out tests and justifying which one is the fastest. Thus, the objective of this work is to compare these two methods, with both results, demonstrating mathematically which one is faster. The tests are performed in an electric power transmission system.

The work is organized as follows: in the Chapter 2 is the explanation and simplifications adopted for the Decoupled Fast Method and Newton's Method. In Chapter 3, the test performed, using a transmission system and discussing the results found. In Chapter 4, is the conclusion of the work.

II. METHODOLOGY

This section will present the methodology used in this work. The workflow is initially explained, showing where the codes are applied, and where they are executed. Then, the methodology of Newton's methods is presented, with a theoretical explanation of the problem.

A. Workflow

The workflow is presented in Fig. 1, where the green and red blocks represents the start and end of the workflow, respectively. The blue blocks are the data input, the yellow are the Python codes and in purple the solutions found.

The "Demand data" is from [5], he made a model with variation over time, contacting the author it was possible to collect the data used in this paper. Using a "preprocessing" Python code it was possible to calculate the absolute value of

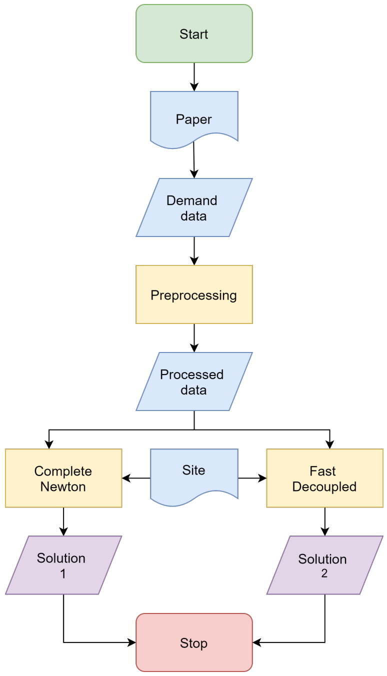

Fig. 1. Workflow showing steps for calculating the operating point using two methods.

each bar, achieving the "Processed data", this was necessary because the models use a static demand.

This data, along with data from the system, was applied in two different power flow methods, using the topology from [6]. The Solution 1 from Newton's method and the Solution 2 from Fast Decoupled method were compared, thus ending the workflow.

B. POWER FLOW MODEL

Usually, the representation of a transmission model uses a single-phase system, with a bar diagram with each bar representing a connection point with the flow, demand and generation.

Electric power transmission systems are usually described as shown in Fig. 2, demonstrating a three-bar system k, i, j, which has a voltage $V_{i,d}$, the d component represents the period these variables are in. Between the bars there is a transmission line, with resistances, impedances and reactances. Above that line, it is the direction of the flow, the active and reactive powers, and its current. In each bar, at the bottom, an injection and demand consumption is displayed, with the direction of the arrow representing these characteristics. The transmission lines have only one differential from this illustration, the shunt capacitors. As they are miles long, it is possible to find a capacity between lines and ground.

To calculate the power flow between the bars i, j, is used the relation of Equations 1. Where the first equation $P_{i,j}$ represents the direction from i to j and $P_{j,i}$, the opposite direction. The V is the voltage of each bar and θ the phase, those components signals (positive or negative) depends on the direction of the flow being calculated. The variables g and b, are the real and imaginary components of the line connected in each bar, respectively.

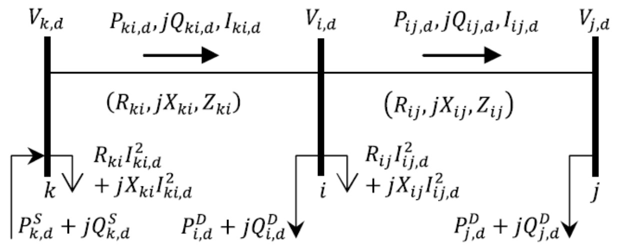

Fig. 2. Electricity distribution model with three bars.
Source: Retrieved from [5].

$$\begin{cases} P_{i,j} = V_i^2 g_{i,j} - V_i V_j (g_{i,j} cos\theta_{i,j} + b_{i,j} sen\theta_{i,j}) \\ P_{j,i} = V_j^2 g_{i,j} - V_i V_j (g_{i,j} cos\theta_{i,j} - b_{i,j} sen\theta_{i,j}) \end{cases} \tag{1}$$

The equation mentioned above calculates only the local values, and it is necessary to calculate the system as a whole, to know if there will be a voltage outside of acceptable parameters on any bus, and what the power supplied by the substation should be, to provide the demand of the system. To carry out this analysis, Newton's method was adapted to the charge flow problem, thus originating the methodology. This method calculates the system components, in iterations, finding the mismatches Δ (errors) for each one: of power ΔP and ΔQ; voltage ΔV and phase $\Delta\theta$. Thus, in each iteration the value of ΔP and ΔQ is found, after performing this calculation, it is possible to find $\Delta\theta$ and ΔV, which are added in the voltage and phase. In this way, a value closer to the optimum is seen in each iteration. It is necessary to invert the matrices in each iteration, having dimensions of the number of bars by number of bars, and demanding a high computational cost to perform these calculations.

The electric power transmission system has a voltage range, in alternating current, from 138 kV to 765 kV, within this range the voltage classes vary from 230 kV, 345 kV, 440 kV and 500 kV. As the tension levels are very high, in this type of system, it is possible to consider certain simplifications, adopting:

- $\theta_{i,j} \approx 0$, therefore $cos(\theta_{i,j}) \approx 1$.
- $B_{i,i}V_i^2 >> Q_i$
- $V_k \approx 1pu$

The Fast Decoupled method is based on the previous method, with an implementation of the simplifications mentioned above, with the flowchart of this process being represented in Figure 3. Therefore, first the method calculate the physical parameters of the system, and the first mismatch of the active power ΔP. The next step is the start of the iterative process, where is made comparison of the highest mismatch value of the active power ΔP with the error factor ζ. If it is bigger, it is calculated the mismatch of phase $\Delta\theta$, the phase in the following iteration is updated, increasing p, matching $KQ = 1$ and calculating the mismatch of the reactive power ΔQ. But, if the value of ΔP is lower than the value of ζ, the stopping criterion KP is equal to zero and the value of KQ is compared. If it is equal to zero, the problem found a solution, if not, the problem goes to the calculation of ΔQ.

This process is called a half active iteration. The second part, called reactive half iteration, has the same characteristics

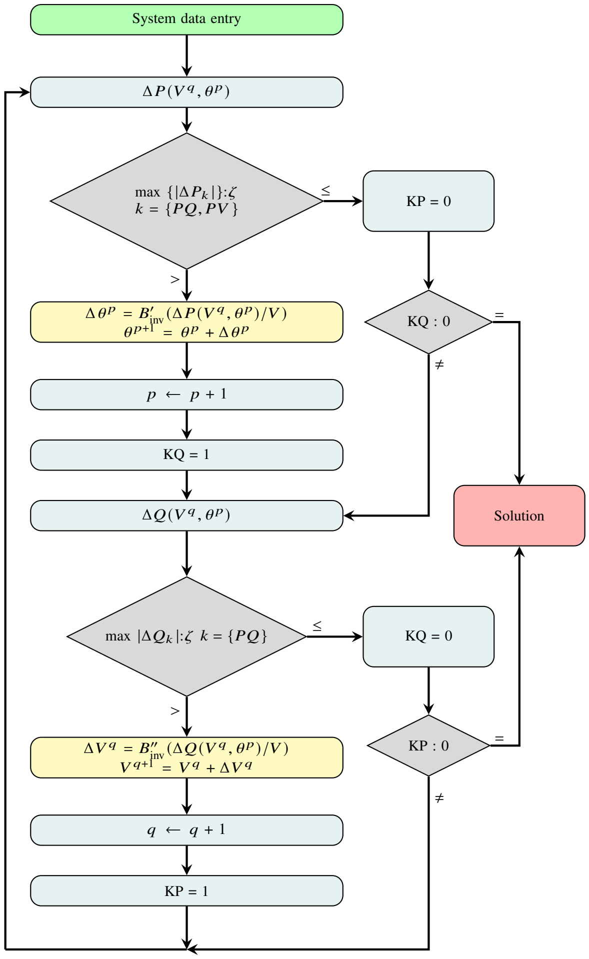

Fig. 3. Flowchart of the Fast Decoupled Newton Method.

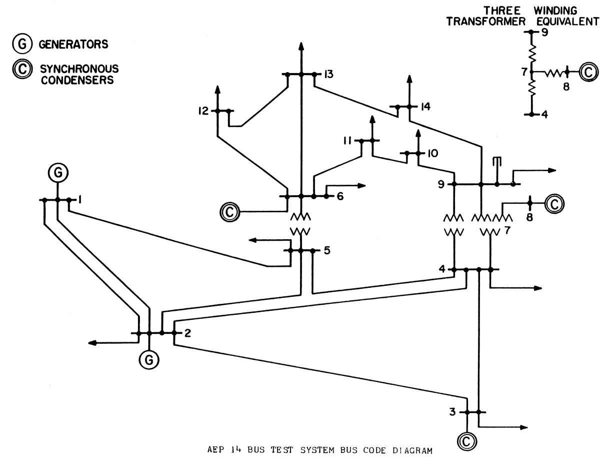

Fig. 4. IEEE14 system topology.

Source: Retrieved from [6]

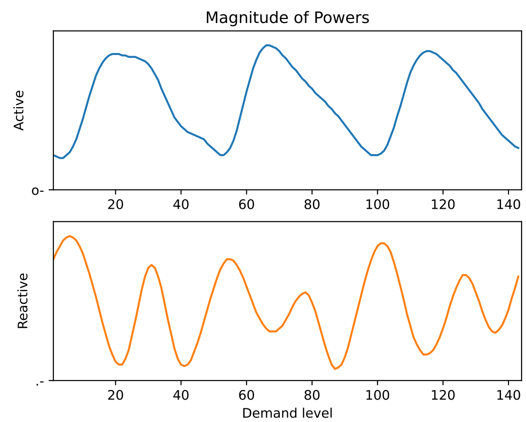

Fig. 5. Behavior of powers in 72 hours.

Source: Retrieved from [5]

as the previous one, but now the ΔQ is compared with the ζ, if it is bigger, the ΔV is calculated and the voltage is updated for the next iteration, increasing the q iteration counter and making it equal to KP, going back to the beginning of the iterative process and recalculating the ΔP. If the highest value of ΔQ is lower than ζ, the process equals $KQ = 0$, and checks whether the value is $KP = 0$, if yes, a solution has been found, if not, the process recalculates the ΔP.

III. TESTS AND RESULTS

The codes were developed in Python, using the version Python 3.6.5, employing a Acer Predator Helios 300 computer, with an i7-7700HQ processor and a frequency of 2.8 GHz. The libraries implemented were Numpy and Matplotlib, it was also used Jupyter Notebook for the documentation. The code and its versions are located in a repository on GitHub [7], where there is the history and an explanation of the code, including the database used. For the tests, the IEEE14 system was implemented, shown by Fig. 4. This system was retrieved from [6], with its data, having 14 bars, with 20 lines. For the test, $V^{base} = 230kV$, $S^{base} = 100MVA$, $\zeta = 10^{-5}$ were used and the iterations p and q were limited to 200.

As explained in the methodology chapter, the demand data varies with time, but for the calculation of the transmission system operation, these data must be static. For this, a pre-processing is used in these data, in order to find the peak of absolute demand, for each type of power, active and reactive. These demand data can be seen in Fig. 5, in which the first graph shows data on active power and the second, data on reactive power. It was used 144 datas, for each bar, and for each type of power. These values represent the demand for a transmission network, in 72 hours, with each interval representing half an hour.

In both tests, the result of the voltage was the same, generating Fig. 6. It shows the voltage level in p.u, on each of the bars. This unit, represents the division of the value of each of the bars, by the value of the base voltage. This is done to decrease the value to be calculated, changing, for example, the data from 230 kV to 1 p.u, using the same basis of the tests. There was no variation in the voltage level between the two models because it is a model that does not have problems in two transmission lines and has a demand compatible with that supplied. Thus, the values found are close to 1 p.u, consistent with the average of a transmission system.

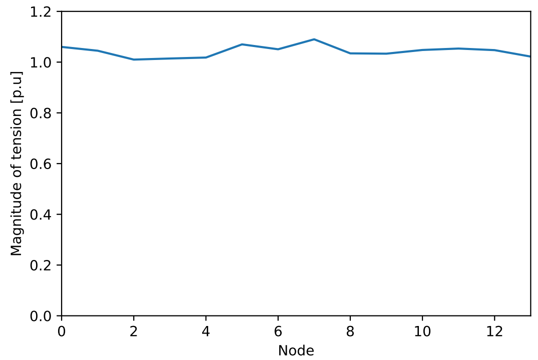

Fig. 6. Electricity distribution model with three bars.

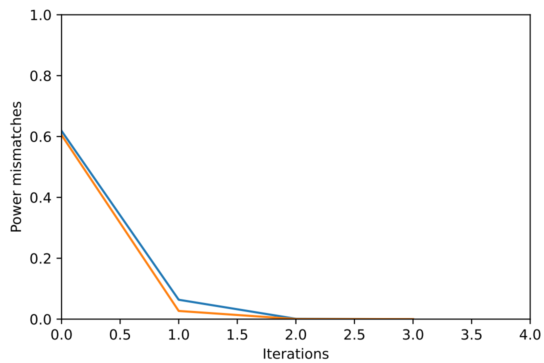

Fig. 7. Evolution of power mismatches for the Newton's method.

In each interaction there are power mismatches represented by Fig. 7 and 8, for the Newton's method and Fast Decoupled method, respectively. As explained earlier, the Newton method tends to have fewer iterations, but it takes more computational time, so for this test it had a duration of 0.0126 seconds, requiring 4 iterations. And for the Fast Decoupled Method it took an iteration more, but a computational time of 0.0069 seconds, about a 55% reduction between the two. This gain is found for small systems, as commented in theory, this difference increases with the size of the system, bigger the system, bigger the difference. This is due to the need to invert, in each iteration, the Jacobian matrix, and in the case of the Fast Decoupled method, only once.

IV. CONCLUSIONS

This work presented two methodologies for solving the load flow problem. As explained earlier, this problem is very important for the operation and planning of the electricity transmission system. Each of the methods presented has its advantages and disadvantages. Making the choice of which to use important, taking into account the accuracy of the results and the time it takes to calculate.

Performing the tests, it was possible to find that for the 14-bar system, there was a 55% decrease in the computational time required to find the system's operating point. For this system, the results found were the same. This would not

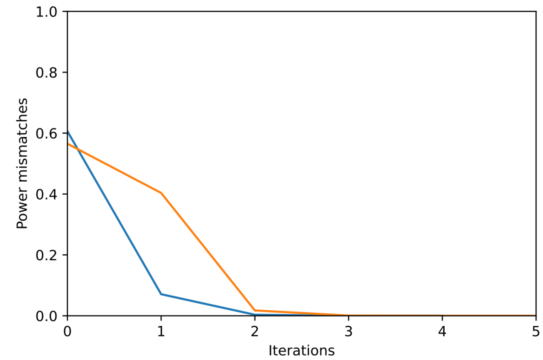

Fig. 8. Evolution of power mismatches for the Fast Decoupled method.

happen for larger systems, since the computational differences and errors due to the adopted approaches would be more discrepant.

In the future, one of the lines of research would be to compare more methods, with heavier systems. In this way, it could be possible to find a correlation between the size of the system, the computational time spent and the proximity of the solution to the global optimum. Thus being able to create scenarios with particular problems, such as voltage drop and systems without sufficient demand supplied, being able to expand this work to other research scopes.

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