

# Precision QCD phenomenology for multi-scale processes at the Large-Hadron-Collider

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IFJ PAN seminar 25<sup>th</sup> April 2024



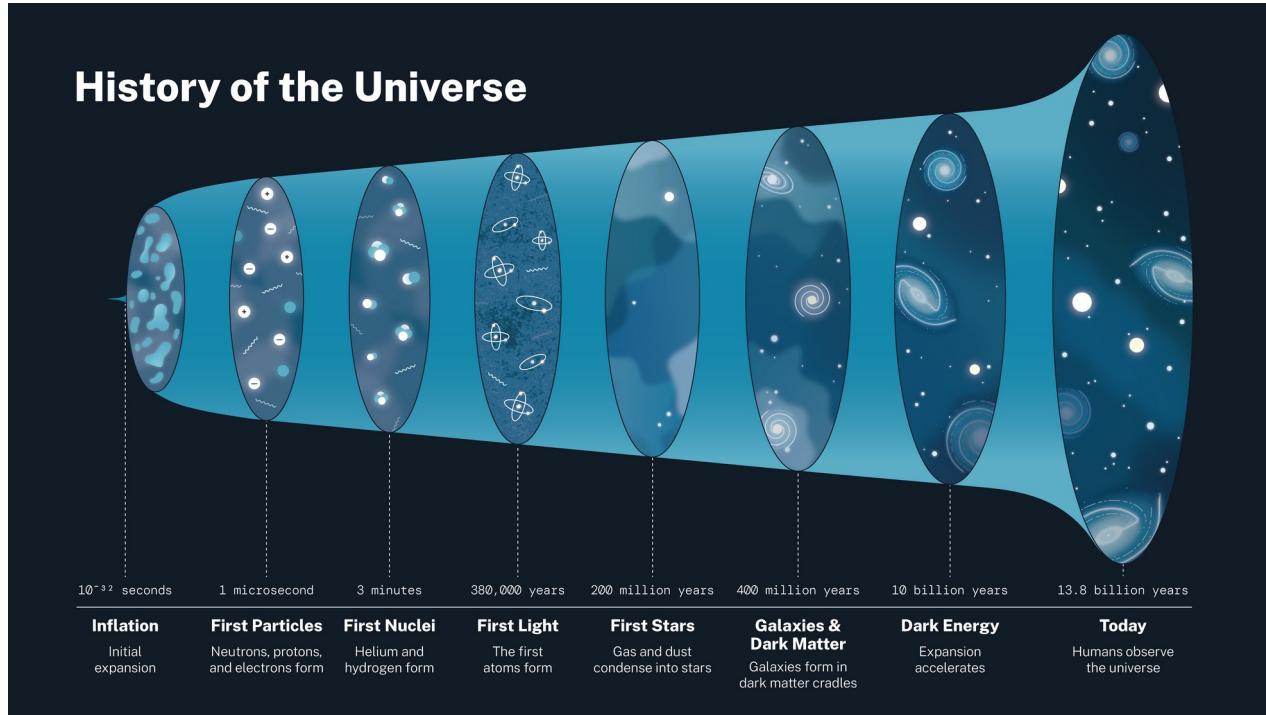
THE HENRYK NIEWODNICZAŃSKI  
INSTITUTE OF NUCLEAR PHYSICS  
POLISH ACADEMY OF SCIENCES

# Outline

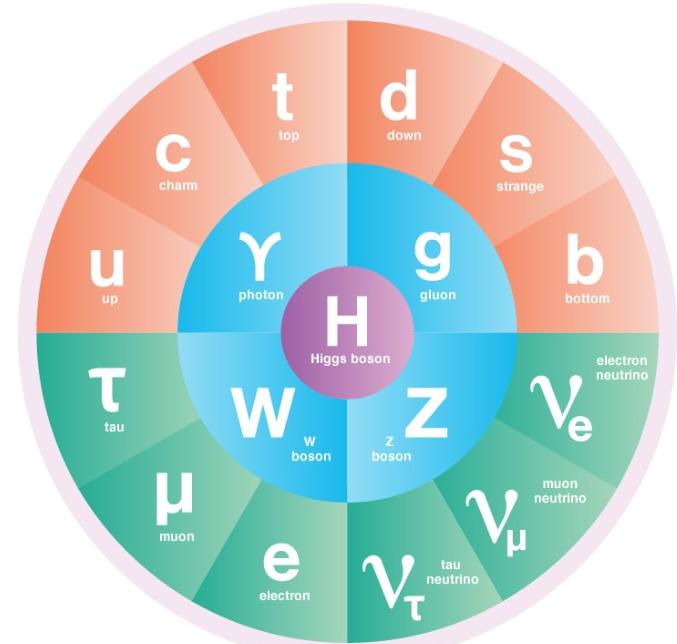
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- Precision phenomenology at the Large Hadron Collider
- Theory predictions with higher-order corrections
- Phenomenology for  $2 \rightarrow 3$  processes
- Summary and Outlook

# What is the universe made of and where does it come from?



[Credit: NASA]

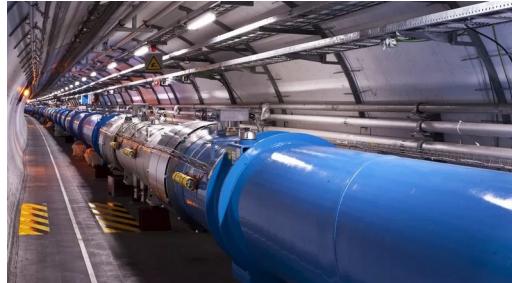


[Credit: SymmetryMagazine]

● QUARKS   ● LEPTONS   ● BOSONS   ● HIGGS BOSON

# What are the fundamental building blocks of matter?

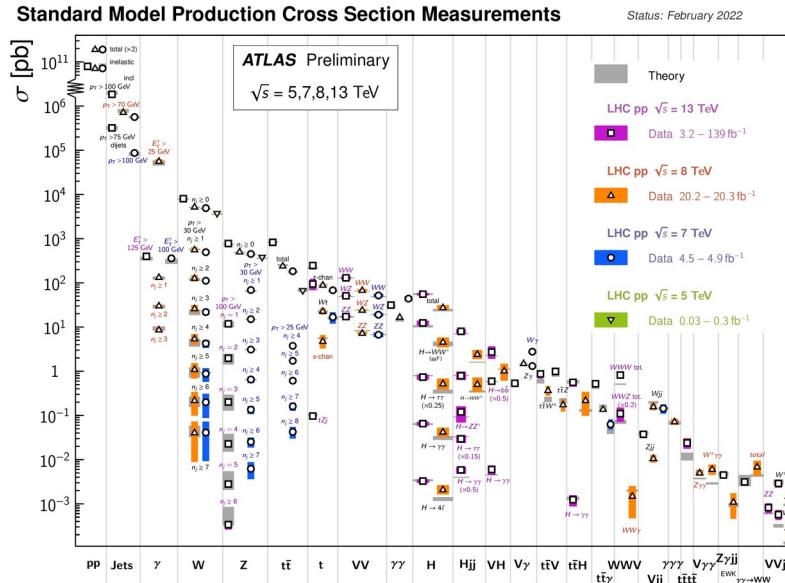
Scattering experiments



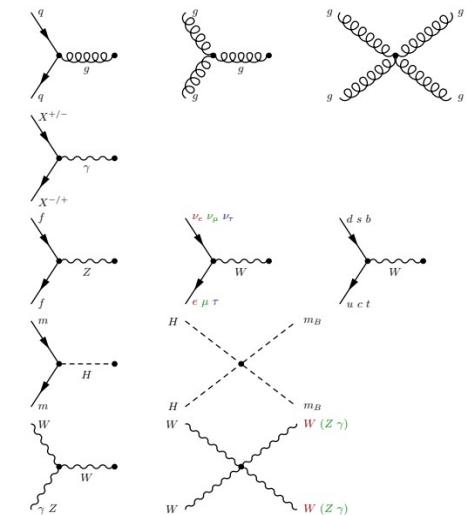
[Credit: CERN]



## Collider phenomenology

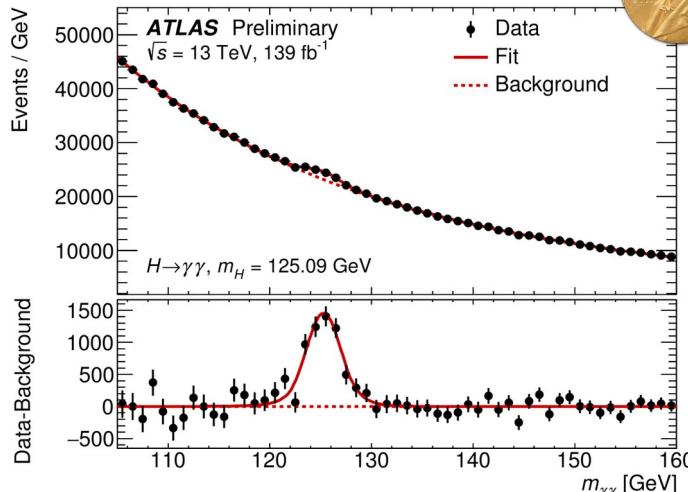


## Theory/Model



# Standard Model of Particle Physics and beyond

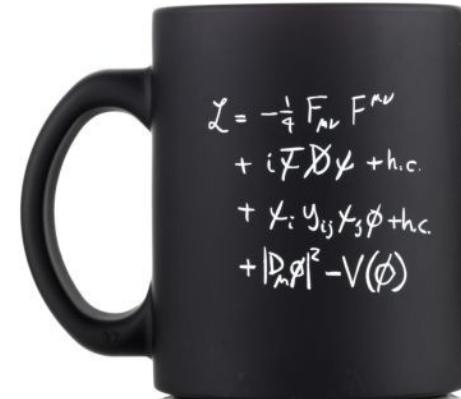
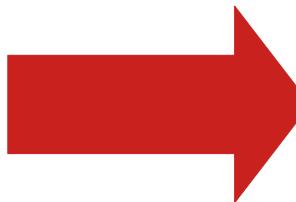
## Higgs discovery 2012



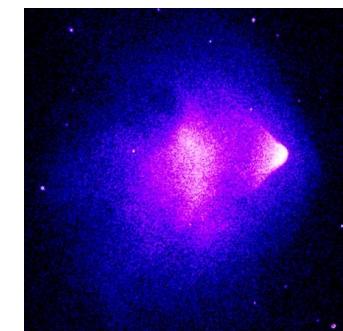
[Credit: ATLAS]

**BUT:**

- Is the Higgs a fundamental scalar?
- What is dark matter?
- Why is there a matter-anti-matter asymmetry?
- ...

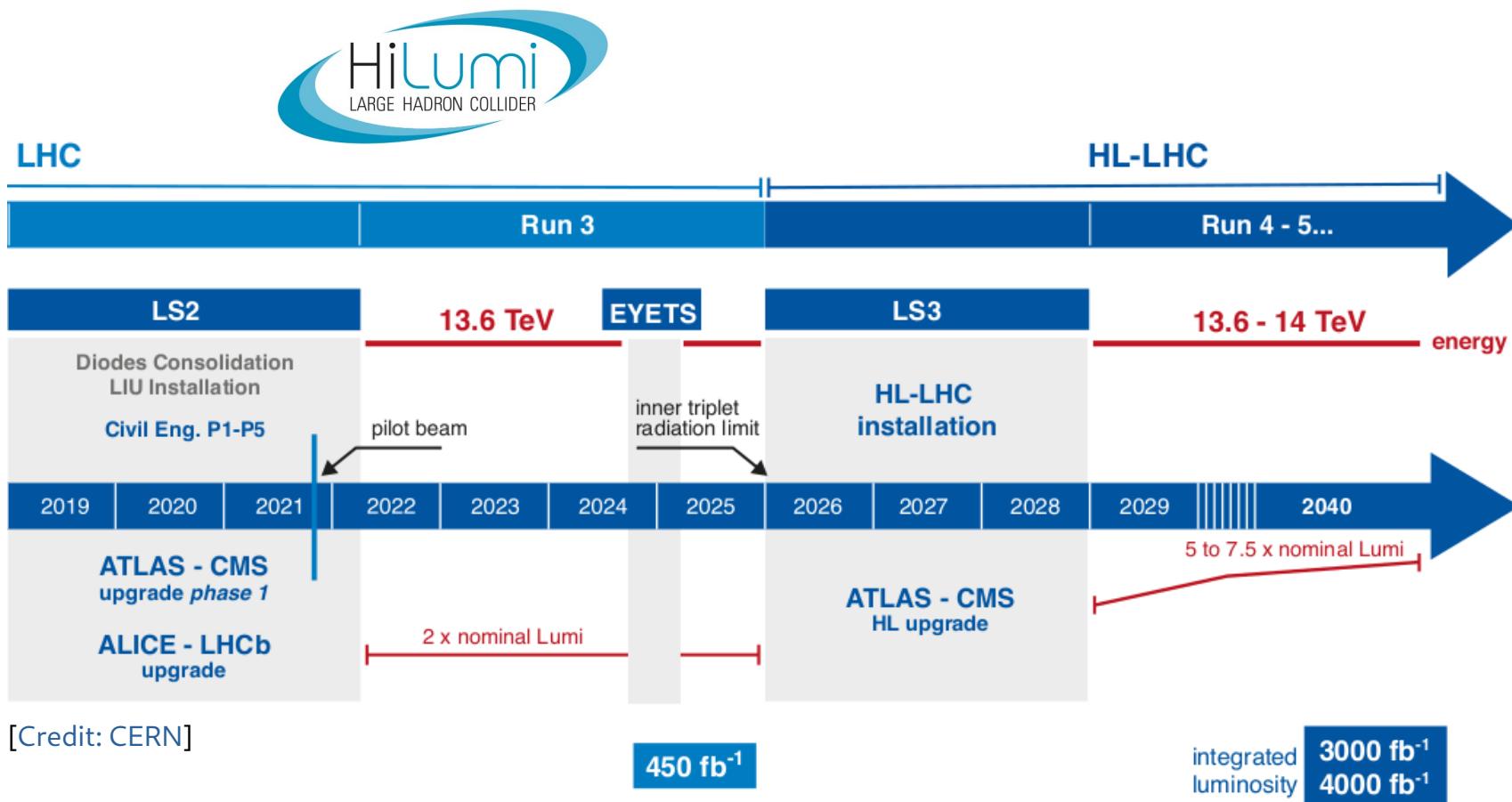


[Credit: CERN]



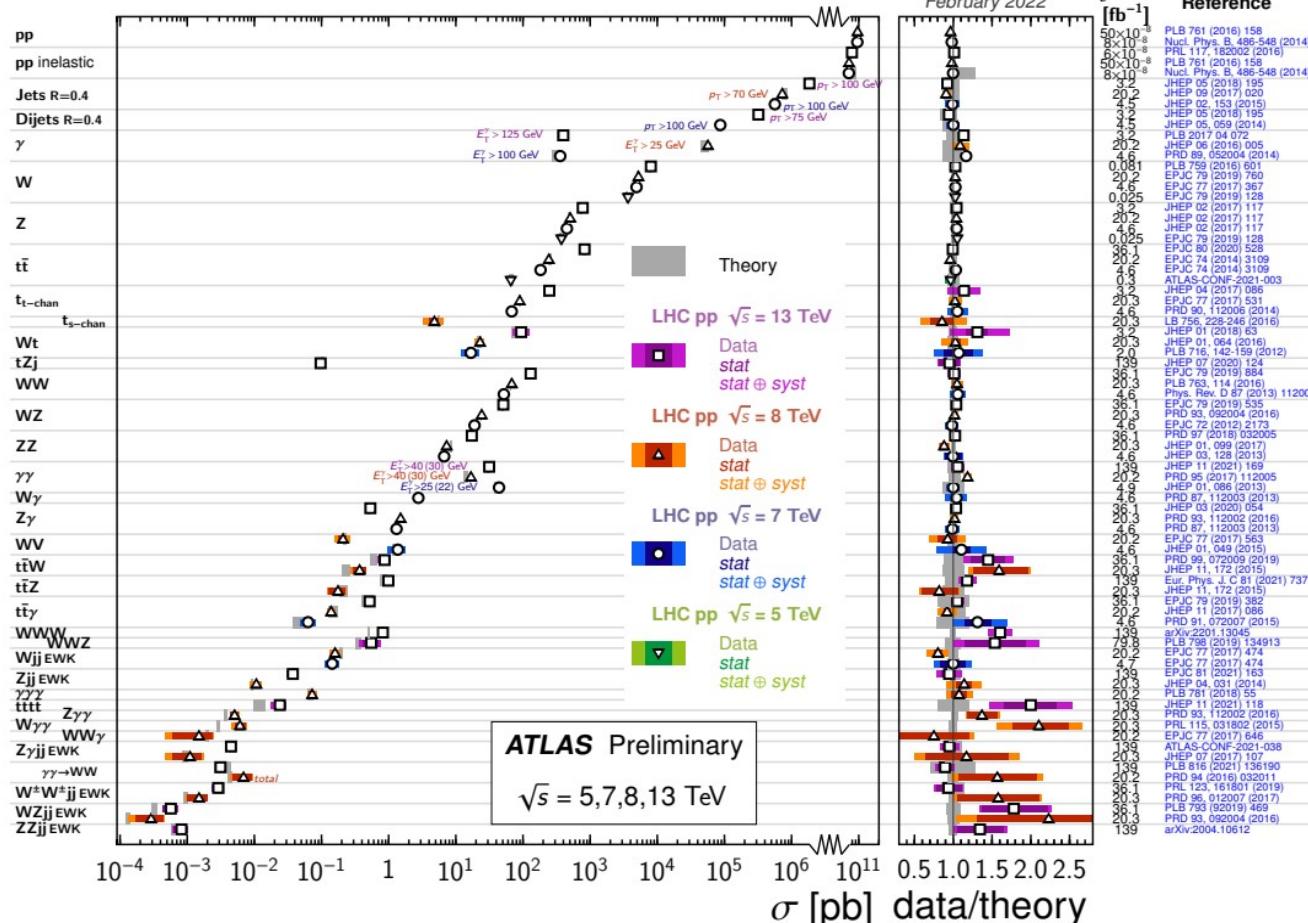
[Credit: NASA]

# LHC Precision era and future experiments



# SM measurements at the LHC

## Standard Model Production Cross Section Measurements



# Theory picture of hadron collision events

## Factorization

"What you see depends on the energy scale"

In Quantum Chromodynamics (QCD):

$$Q \sim \Lambda_{\text{QCD}}$$

Strong coupling

- Realm of confined states
- non-perturbative physics

$$Q \gtrsim \Lambda_{\text{QCD}}$$

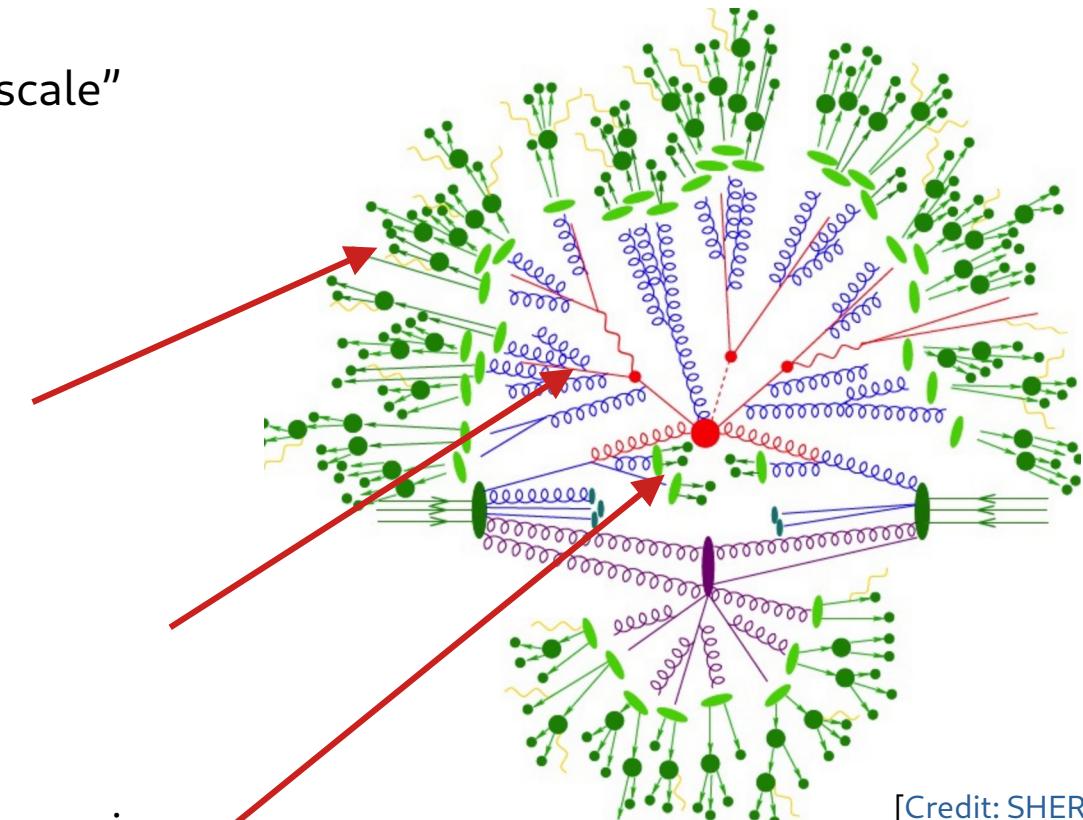
Transition region

- Parton-shower
- Resummation
- DGLAP / PDF evolution

$$Q \gg \Lambda_{\text{QCD}}$$

Small coupling  $\rightarrow$  perturbative regime

- Scattering of individual partons



[Credit: SHERPA]

# Precision predictions

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**Fixed order  
perturbation theory**

- Core element of event simulation
- Describes high Q regime

Soft physics:  
MPI, colour reconnection,  
...

Resummation

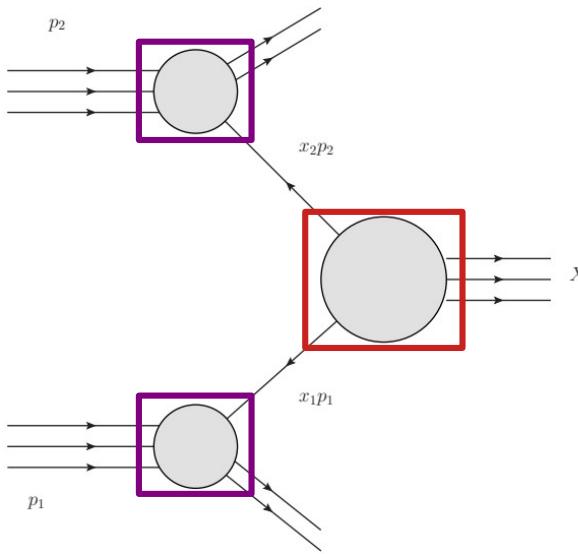
Precision theory predictions

Parton-showers

Parametric input:  
PDFs, couplings ( $\alpha_s$ ), ...

Fragmentation/hadronisation

# Perturbative QCD



Hadronic cross section in collinear factorization:

$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{ij} \int_0^1 \int_0^1 dx_1 dx_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab \rightarrow X} = \hat{\sigma}_{ab \rightarrow X}^{(0)} + \hat{\sigma}_{ab \rightarrow X}^{(1)} + \hat{\sigma}_{ab \rightarrow X}^{(2)} + \mathcal{O}(\alpha_s^3)$$

Typical uncertainties from scale variations:  $\delta\text{LO } \mathcal{O}(\sim 100\%)$ ,  $\delta\text{NLO } \mathcal{O}(\sim 10\%)$ ,  $\delta\text{NNLO } (\sim 1\%)$

(estimate for corrections from missing higher orders based on renormalisation scale invariance  $\frac{d\sigma_{h_1 h_2 \rightarrow X}}{d\mu} = 0$  )

# Example: Production of three isolated photons

$$pp \rightarrow \gamma\gamma\gamma$$

Theory to data comparison

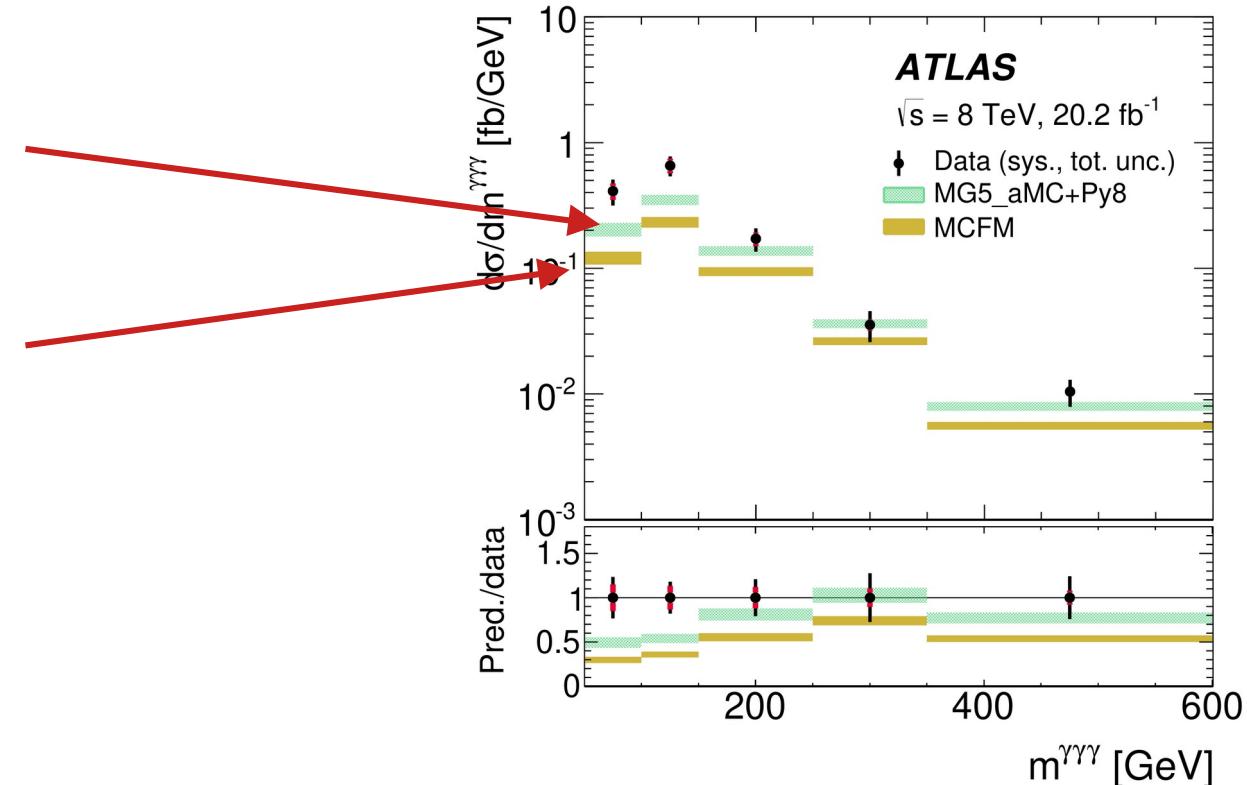
NLO QCD  
+ Parton-shower simulation

Fixed-order NLO QCD

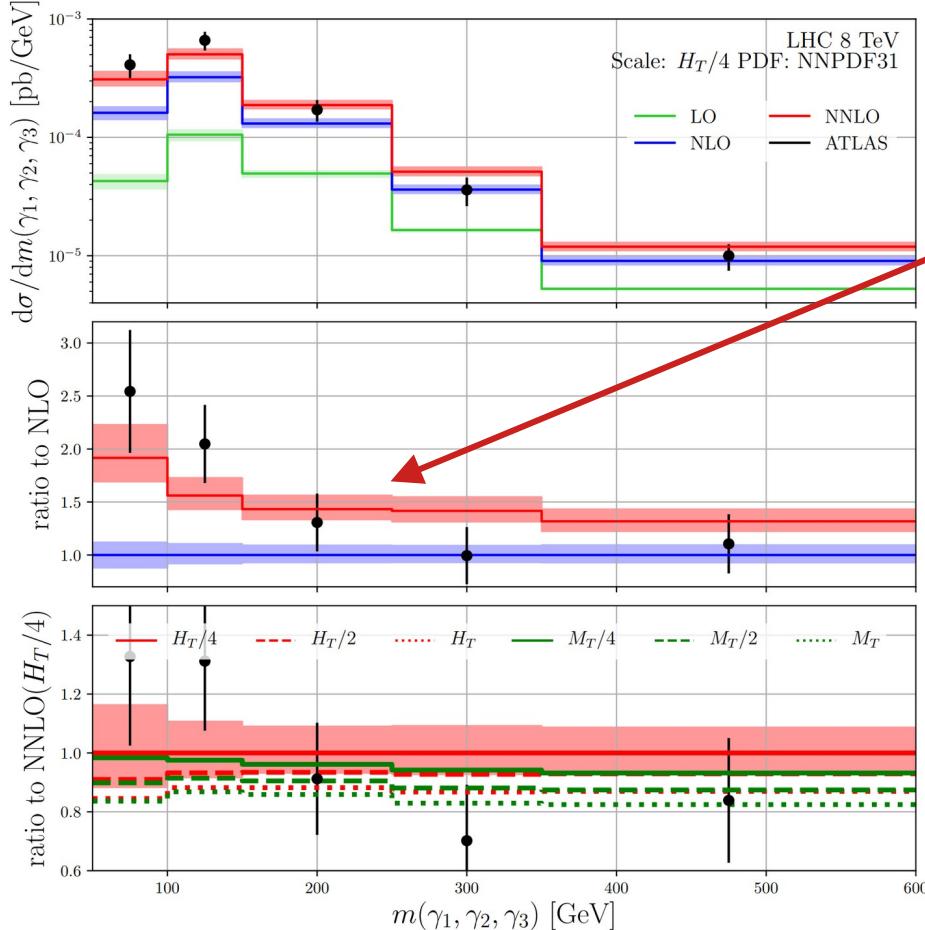
**Both fail to describe data  
(normalization and shape)**

**Why?  
→ NNLO QCD effects!**

Measurement of the production cross section of three isolated photons in  $pp$  collisions at  $\sqrt{s} = 8$  TeV using the ATLAS detector, ATLAS [1712.07291]



# NNLO QCD in three photon production



**NNLO QCD corrections to three-photon production at the LHC, Chawdhry, Czakon, Mitov, Poncelet  
[JHEP 02 (2020) 057]**

Corrections to **normalization** and **shape**

→ (Much) improved description of data

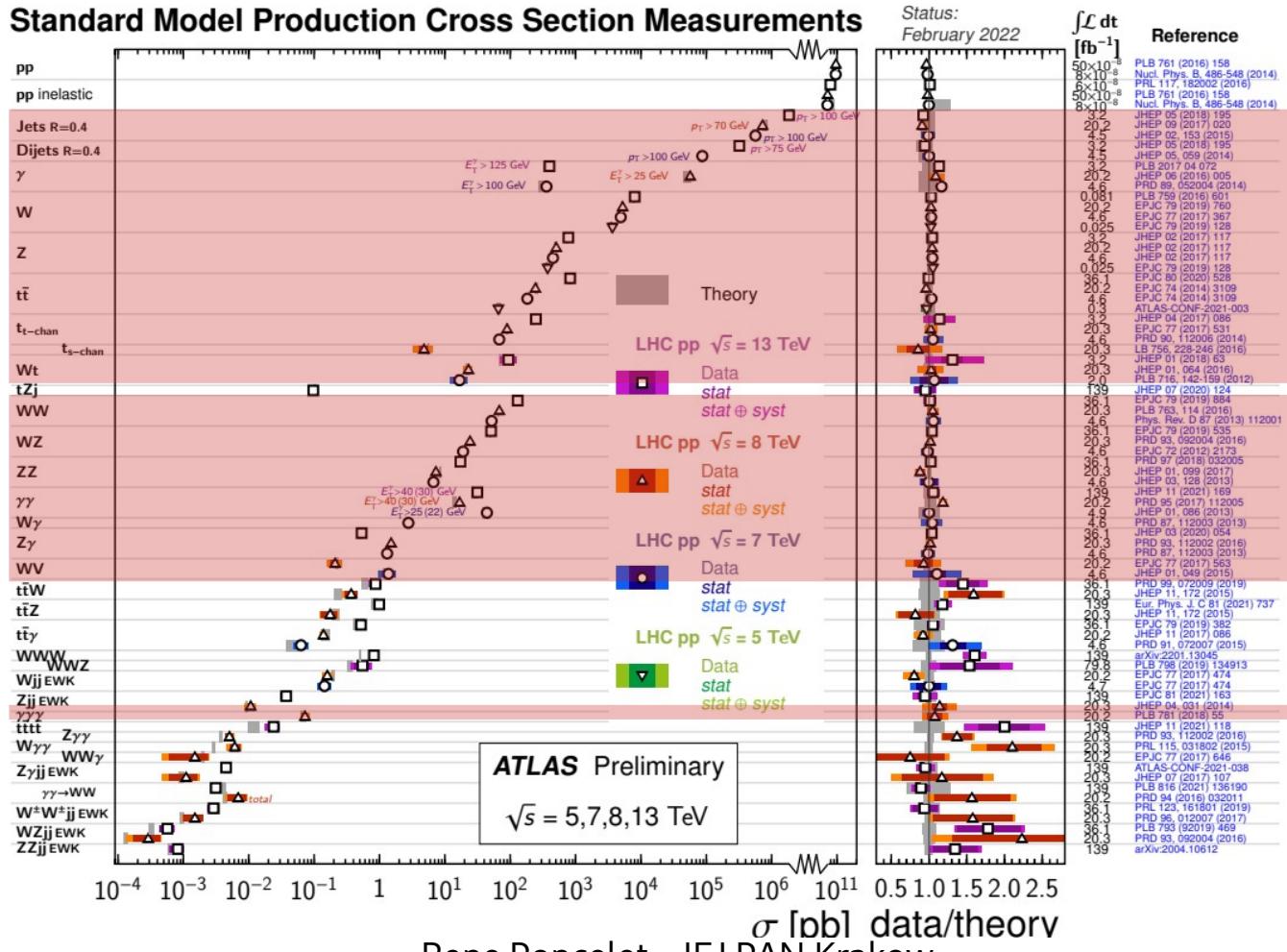
Without NNLO QCD corrections the data

- is not interpretable  
→ loss of information

or

- is misleading  
→ looks like “New Physics” = data - SM

# NNLO QCD coverage



# Theory predictions with higher-order corrections

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# Next-to-leading order case

$$\hat{\sigma}_{ab}^{(1)} = \hat{\sigma}_{ab}^R + \hat{\sigma}_{ab}^V + \hat{\sigma}_{ab}^C$$

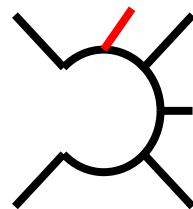


## KLN theorem

sum is finite for sufficiently inclusive observables  
and regularization scheme independent

Each term separately infrared (IR) divergent:

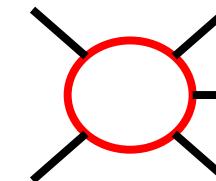
Real corrections:



$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1}$$

Phase space integration over unresolved configurations

Virtual corrections:



$$\hat{\sigma}_{ab}^V = \frac{1}{2\hat{s}} \int d\Phi_n 2\text{Re} \left\langle \mathcal{M}_n^{(0)} \middle| \mathcal{M}_n^{(1)} \right\rangle F_n$$

Integration over loop-momentum  
(UV divergences cured by renormalization)

# IR singularities in real radiation

$$\hat{\sigma}_{ab}^R = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1}$$



$$\sim \int_0 dE d\theta \frac{1}{E(1 - \cos \theta)} f(E, \cos(\theta))$$

Finite function

Divergent

Regularization in Conventional Dimensional Regularization (CDR)  $d = 4 - 2\epsilon$

$$\rightarrow \int_0 dE d\theta \frac{1}{E^{1-2\epsilon}(1 - \cos \theta)^{1-\epsilon}} f(E, \cos(\theta)) \sim \frac{1}{\epsilon^2} + \dots$$

Cancellation against similar divergences in

$$\hat{\sigma}_{ab}^V = \frac{1}{2\hat{s}} \int d\Phi_n 2\text{Re} \left\langle \mathcal{M}_n^{(0)} \middle| \mathcal{M}_n^{(1)} \right\rangle F_n$$

# How to extract these poles? Slicing and Subtraction

Central idea: Divergences arise from infrared (IR, soft/collinear) limits → Factorization!

## Slicing

$$\begin{aligned}\hat{\sigma}_{ab}^R &= \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi) < \delta_c} d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1} \\ &\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi) \geq \delta_c} d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1} + \frac{1}{2\hat{s}} \int d\Phi_n \tilde{M}(\delta_c) F_n + \mathcal{O}(\delta_c)\end{aligned}$$

$$\dots + \hat{\sigma}_{ab}^V = \text{finite}$$

## Subtraction

$$\begin{aligned}\hat{\sigma}_{ab}^R &= \frac{1}{2\hat{s}} \int \left( d\Phi_{n+1} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(0)} \right\rangle F_{n+1} - d\tilde{\Phi}_{n+1} \mathcal{S} F_n \right) + \frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S} F_n \\ &\quad \frac{1}{2\hat{s}} \int d\tilde{\Phi}_{n+1} \mathcal{S} F_n = \frac{1}{2\hat{s}} \int \underline{d\Phi_n d\Phi_1} \mathcal{S} F_n\end{aligned}$$

Phase space factorization  
→ momentum mappings

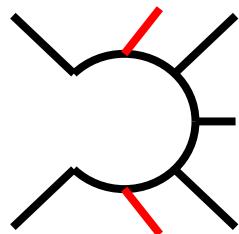
Most popular  
NLO QCD schemes:  
CS [[hep-ph/9605323](#)],  
FKS [[hep-ph/9512328](#)]

→ Basis of modern  
event simulation

# Partonic cross section beyond NLO

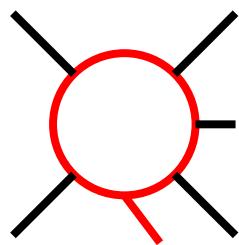
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$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$



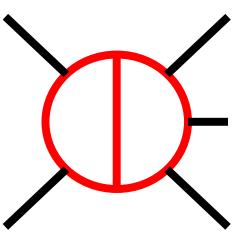
Real-Real

$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle F_{n+2}$$



Real-Virtual

$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle F_{n+1}$$



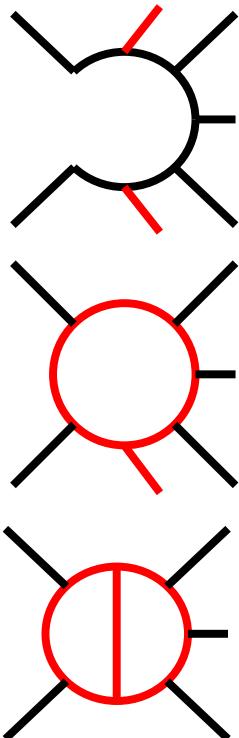
Virtual-Virtual

$$\hat{\sigma}_{ab}^{\text{VV}} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \left\langle \mathcal{M}_n^{(0)} \middle| \mathcal{M}_n^{(2)} \right\rangle + \left\langle \mathcal{M}_n^{(1)} \middle| \mathcal{M}_n^{(1)} \right\rangle \right) F_n$$

$$\hat{\sigma}_{ab}^{\text{C2}} = (\text{double convolution}) F_n \quad \hat{\sigma}_{ab}^{\text{C1}} = (\text{single convolution}) F_{n+1}$$

# Partonic cross section beyond NLO

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\text{VV}} + \hat{\sigma}_{ab}^{\text{RV}} + \hat{\sigma}_{ab}^{\text{RR}} + \hat{\sigma}_{ab}^{\text{C2}} + \hat{\sigma}_{ab}^{\text{C1}}$$



Real-Real

Technically substantially more complicated!

Main bottlenecks:

- Real - real  $\rightarrow$  overlapping singularities  
Many possible limits  $\rightarrow$  good organization principle needed
- Real - virtual  $\rightarrow$  stable matrix elements
- Virtual - virtual  $\rightarrow$  complicated case-by-case analytic treatment

$$\hat{\sigma}_{ab}^{\text{C2}} = (\text{double convolution}) F_n$$

$$\hat{\sigma}_{ab}^{\text{C1}} = (\text{single convolution}) F_{n+1}$$

$$\hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \left\langle M_{n+2}^{(0)} | M_{n+2}^{(0)} \right\rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{\text{RV}} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \left\langle M_{n+1}^{(0)} | M_{n+1}^{(1)} \right\rangle F_{n+1}$$

$$\hat{\sigma}_{ab} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \left\langle M_n^{(0)} | M_n^{(1)} \right\rangle + \left\langle M_n^{(0)} | M_n^{(0)} \right\rangle \right) F_n$$

# Slicing and Subtraction

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## Slicing

- Conceptually simple
- Recycling of lower computations
- Non-local cancellations/power-corrections  
→ computationally expensive

## NNLO QCD schemes

qT-slicing [[Catani'07](#)],  
N-jettiness slicing [[Gaunt'15/Boughezal'15](#)]

## Subtraction

- Conceptually more difficult
- Local subtraction → efficient
- Better numerical stability
- Choices:
  - Momentum mapping
  - Subtraction terms
  - Numerics vs. analytic

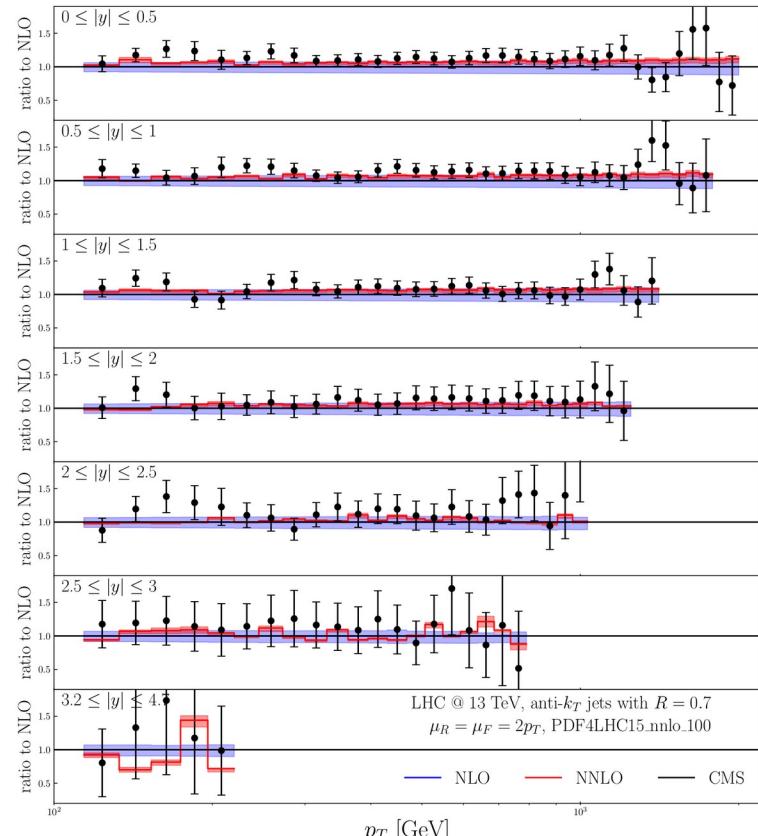
Antenna [[Gehrmann'05-'08](#)],  
Colorful [[DelDuca'05-'15](#)],  
**Sector-improved residue subtraction** [[Czakon'10-'14'19](#)]  
Projection [[Cacciari'15](#)],  
Nested collinear [[Caola'17](#)],  
Geometric [[Herzog'18](#)],  
Unsubtraction [[Aguilera-Verdugo'19](#)],  
...

# Minimal sector-improved residue subtraction

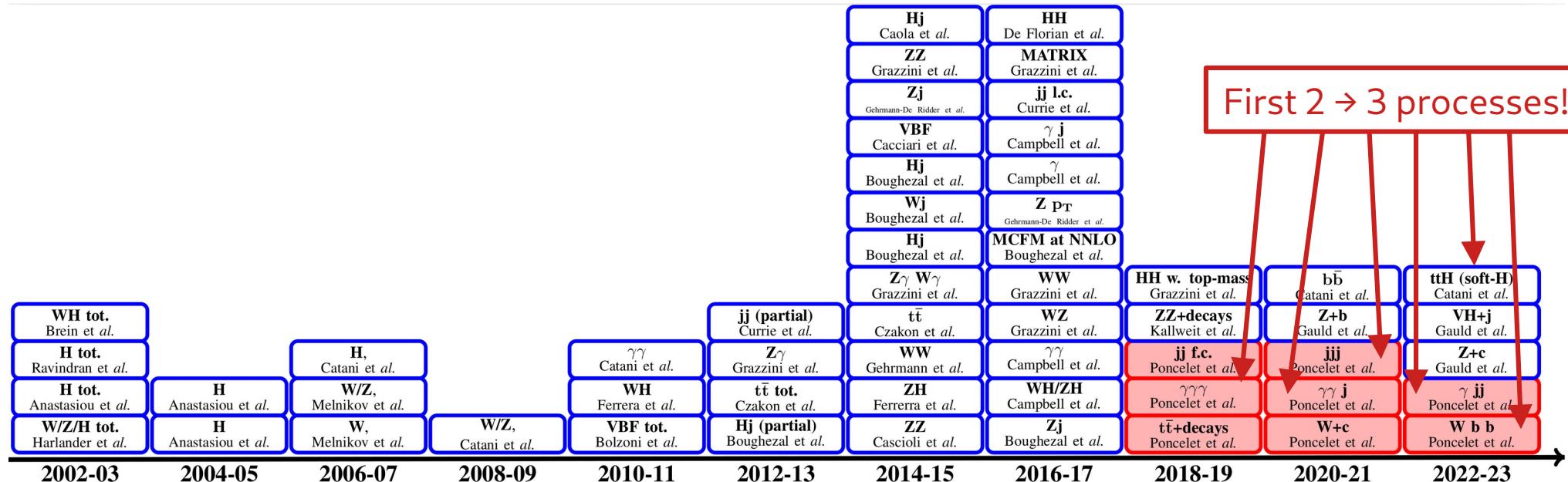
**Single-jet inclusive rates with exact color at  $\mathcal{O}(\alpha_s^4)$**   
Czakon, Hameren, Mitov, Poncelet, JHEP 10 (2019), 262

Refined formulation of the  
sector-improved residue subtraction

- New phase space parametrisation  
→ minimization of subtraction kinematics  
→ improved computational efficiency/stability
- Improved sector decomposition
- New 4 – dimensional formulation
- First application: inclusive jet production  
→ demonstrates that the **scheme is complete**  
→ no approximations

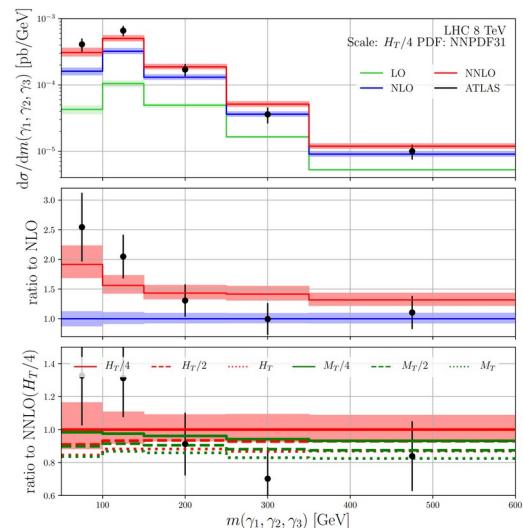


# The NNLO QCD revolution

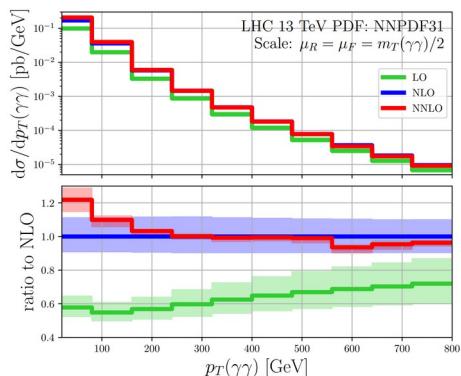


# NNLO QCD for massless $2 \rightarrow 3$ processes

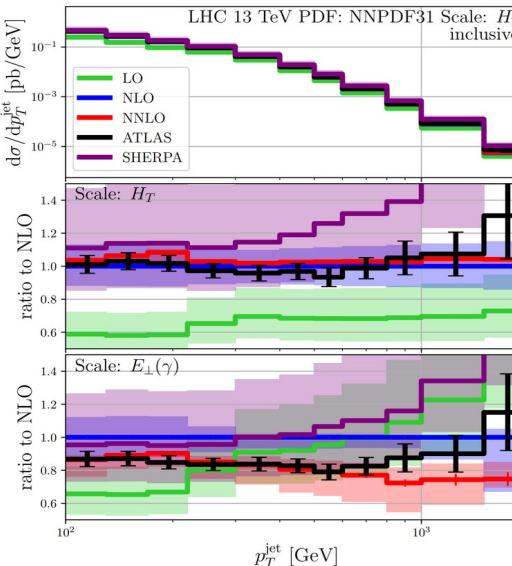
$pp \rightarrow \gamma\gamma\gamma$



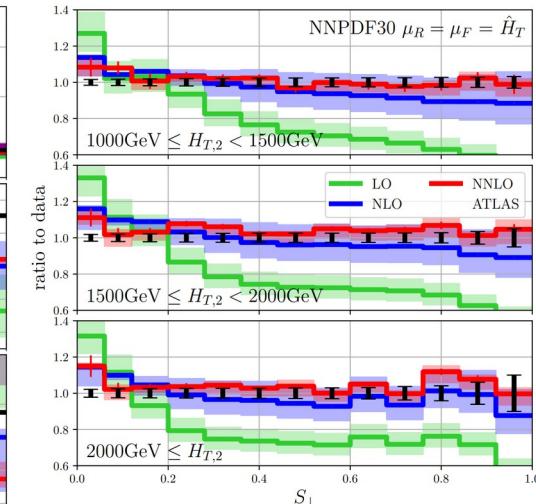
$pp \rightarrow \gamma\gamma j$



$pp \rightarrow \gamma jjj$



$pp \rightarrow jjj$



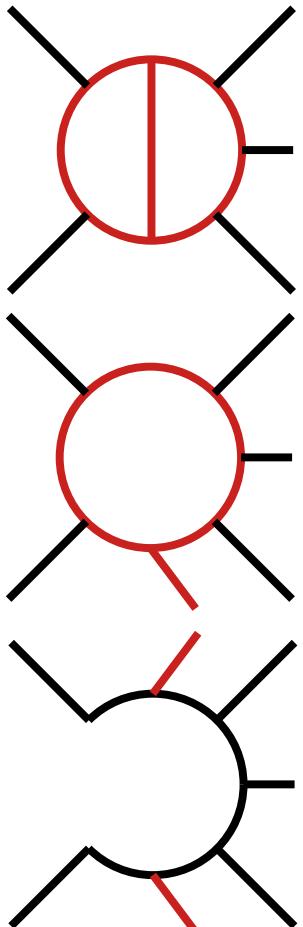
Chawdhry, Czakon, Mitov,  
**Poncelet** [[1911.00479](#)]  
Kallweit, Sotnikov,  
Wiesemann [[2010.04681](#)]

Chawdhry, Czakon, Mitov,  
**Poncelet** [[2103.04319](#)]

Badger, Czakon, Hartanto,  
Moodie, Peraro, **Poncelet**,  
Zoia [[2304.06682](#)]

Czakon, Mitov, **Poncelet**  
[[2106.05331](#)]  
+ Alvarez, Cantero, Llorente  
[[2301.01086](#)]

# NNLO QCD for 2→3 processes - inputs



## Two-loop amplitudes

- (Non-) planar 5 point massless [Chawdry'19'20'21, Abreu'20'21'23, Agarwal'21, Badger'21'23]  
→ triggered by efficient MI representation [Chicherin'20]

## One-loop amplitudes → OpenLoops [Buccioni'19]

- Many legs and IR stable (soft and collinear limits)

## Double-real Born amplitudes → AvHlib[Bury'15]

- IR finite cross-sections → NNLO subtraction schemes
  - qT-slicing [Catani'07], N-jettiness slicing [Gaunt'15/Boughezal'15], Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15], Projection [Cacciari'15], Geometric [Herzog'18], Unsubtraction [Aguilera-Verdugo'19], Nested collinear [Caola'17], Local Analytic [Magnea'18], **Sector-improved residue subtraction** [Czakon'10-'14,'19]

# Phenomenology for $2 \rightarrow 3$ processes

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# Multi-jet observables

## Test of pQCD and extraction of strong coupling constant

NLO theory unc. (MHO) > experimental unc.

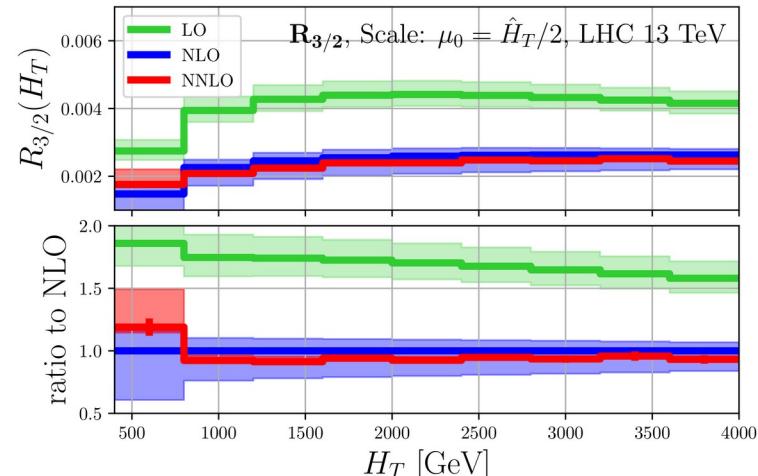
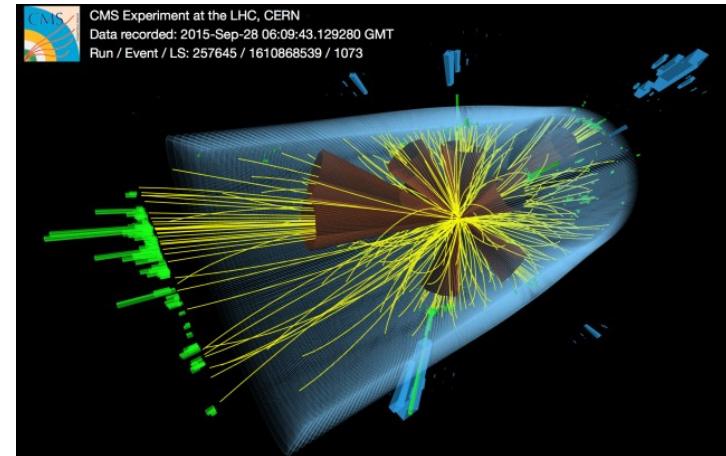
- NNLO QCD needed for precise theory-data comparisons  
→ Restricted to two-jet data [Currie'17+later][Czakon'19]
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

**Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC**  
Czakon, Mitov, Poncelet [Phys.Rev.Lett. 127 \(2021\) 15, 152001](#)

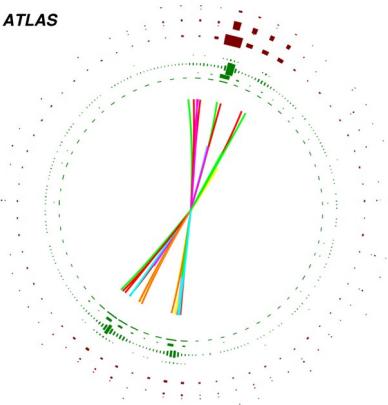
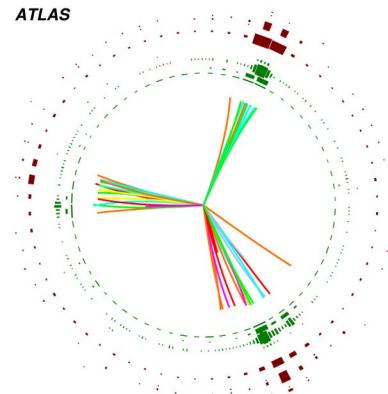
$$R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$$

- Event shapes

**NNLO QCD corrections to event shapes at the LHC**  
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [JHEP 03 \(2023\) 129](#)



# Encoding QCD dynamics in event shapes



Using (global) event information to separate different regimes of QCD event evolution:

## Energy-energy correlators

$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} x_{\perp,i} x_{\perp,j} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

Separation of energy scales:  $H_{T,2} = p_{T,1} + p_{T,2}$

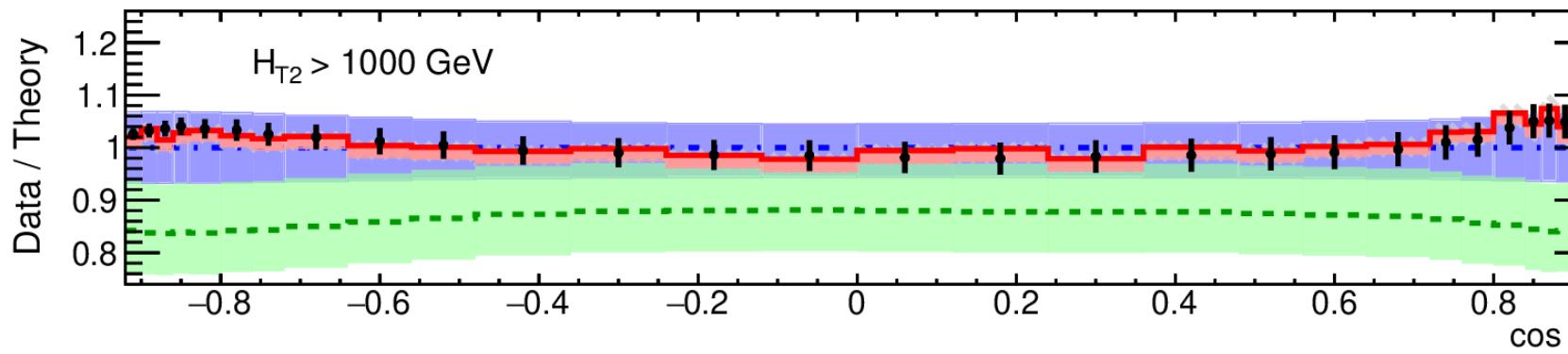
Ratio to 2-jet:  $R^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0}) = \frac{d\sigma_3^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}{d\sigma_2^i(\mu_R, \mu_F, \text{PDF}, \alpha_{S,0})}$

Here: jets as input → experimentally advantageous  
(better calibrated, smaller non-pert.)

# The transverse energy-energy correlator

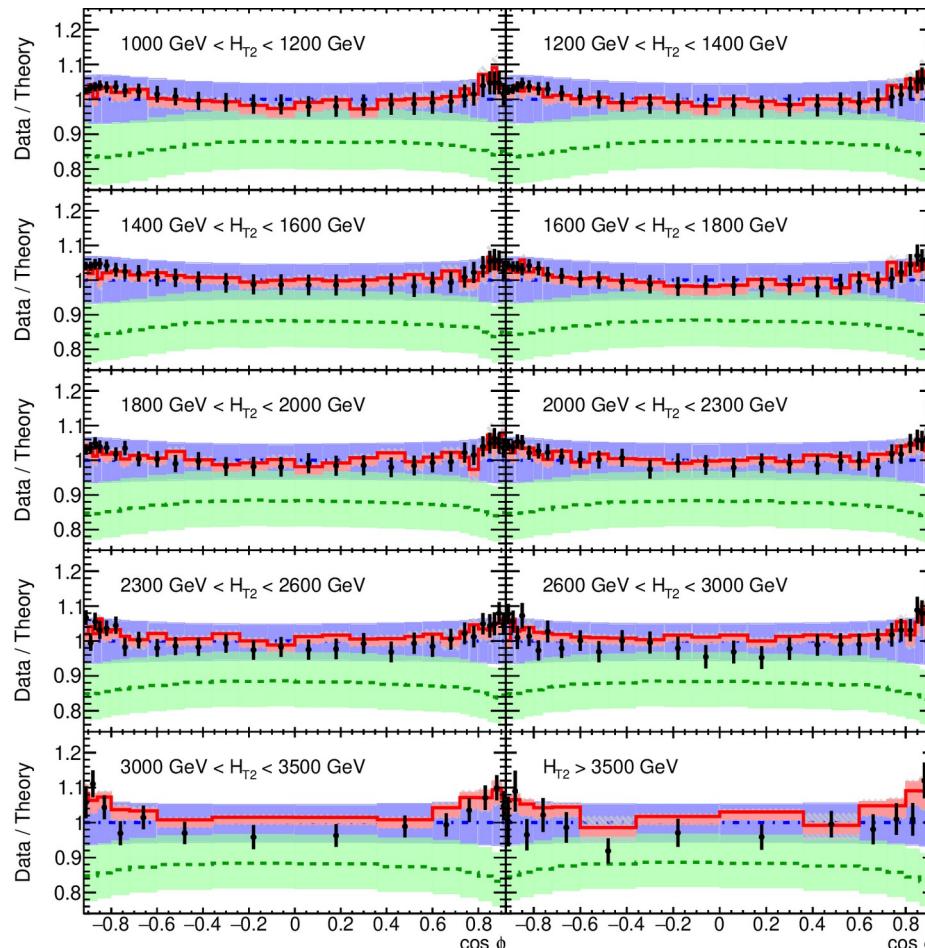
$$\frac{1}{\sigma_2} \frac{d\sigma}{d \cos \Delta\phi} = \frac{1}{\sigma_2} \sum_{ij} \int \frac{d\sigma x_{\perp,i} x_{\perp,j}}{dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij}} \delta(\cos \Delta\phi - \cos \Delta\phi_{ij}) dx_{\perp,i} dx_{\perp,j} d \cos \Delta\phi_{ij},$$

- Insensitive to soft radiation through energy weighting  $x_{T,i} = E_{T,i} / \sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

# Double differential TEEC



[ATLAS 2301.09351]

ATLAS

Particle-level TEEC

$\sqrt{s} = 13 \text{ TeV}; 139 \text{ fb}^{-1}$

$\text{anti-}k_t R = 0.4$

$p_T > 60 \text{ GeV}$

$|\eta| < 2.4$

$$\mu_{R,F} = \hat{\mu}_T$$

$$\alpha_s(m_Z) = 0.1180$$

NNPDF 3.0 (NNLO)

— Data

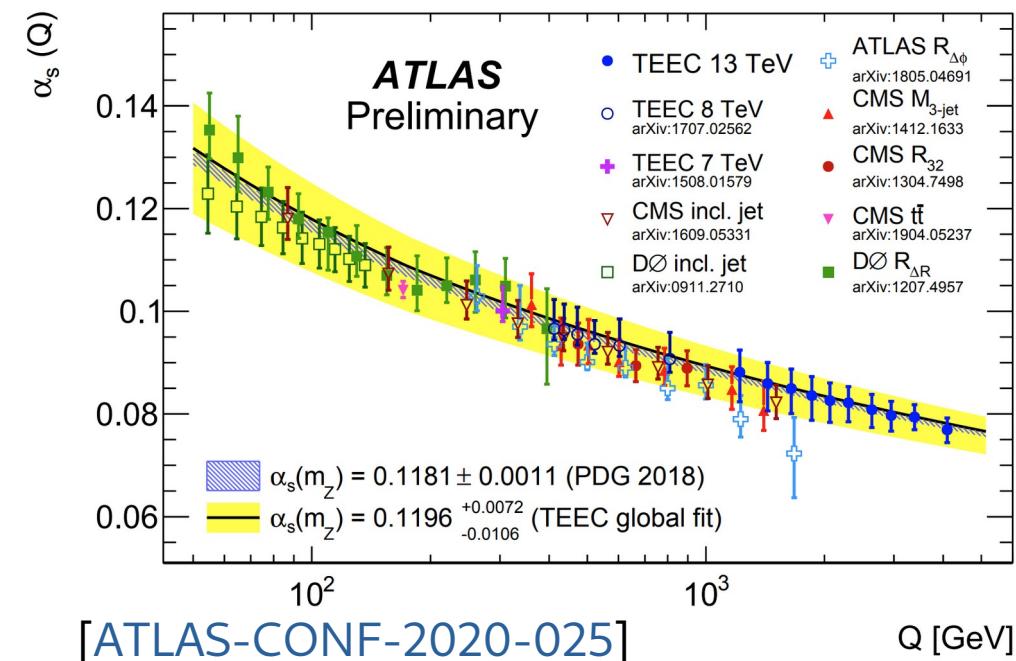
— LO

— NLO

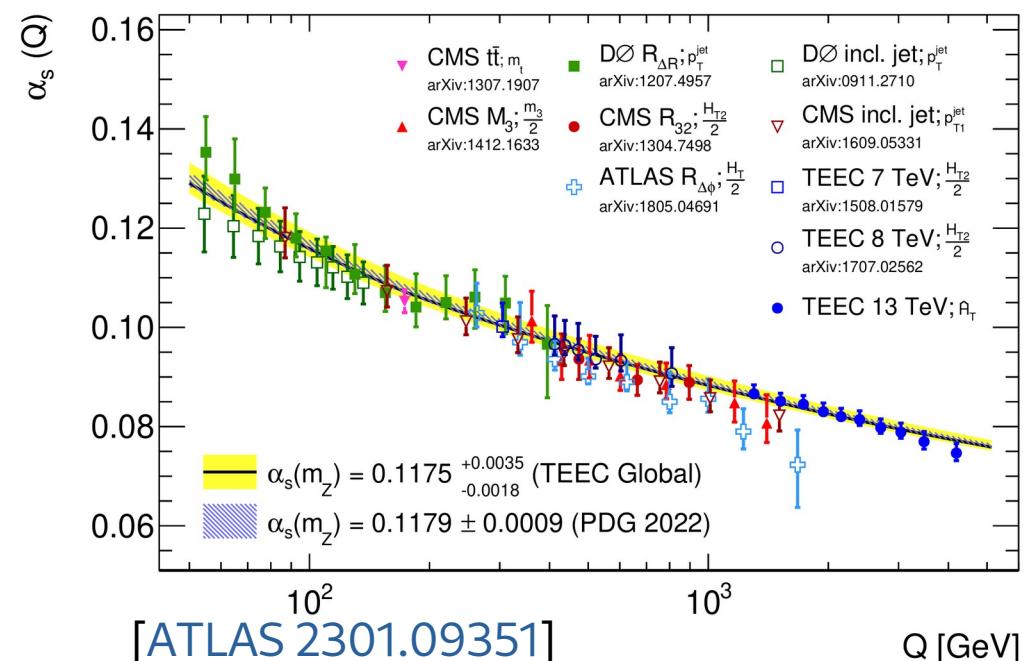
— NNLO

# Running of $\alpha_s$

NLO QCD

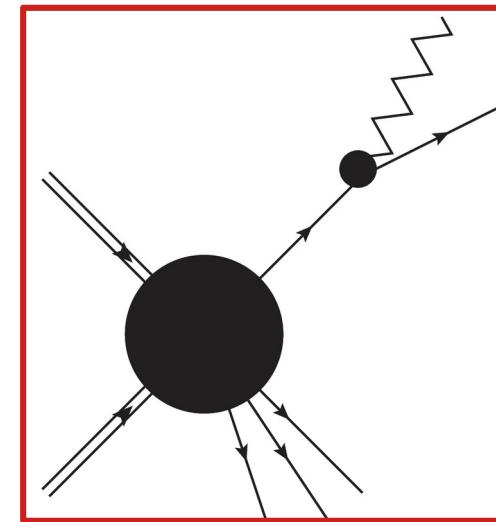
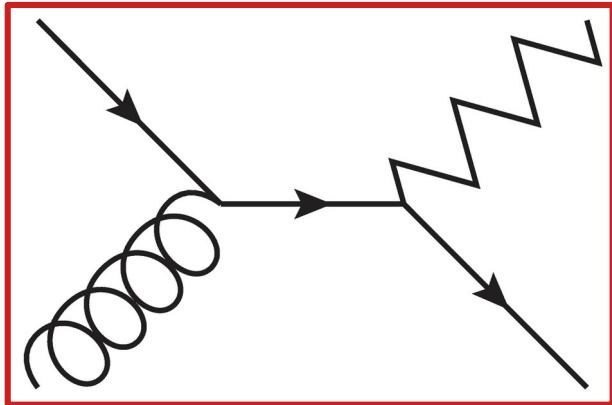


NNLO QCD



# Prompt photon production

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## Direct production

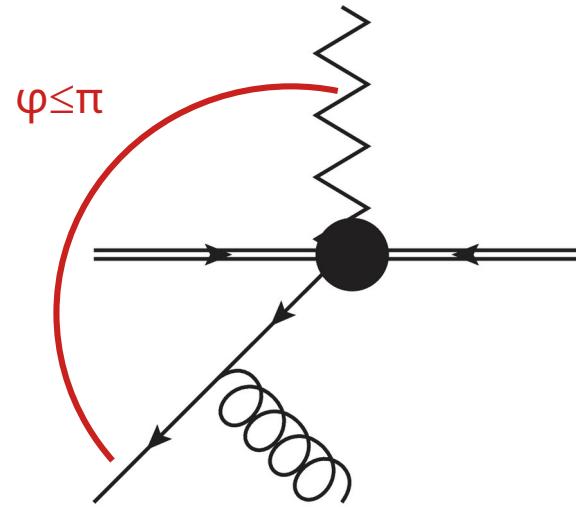
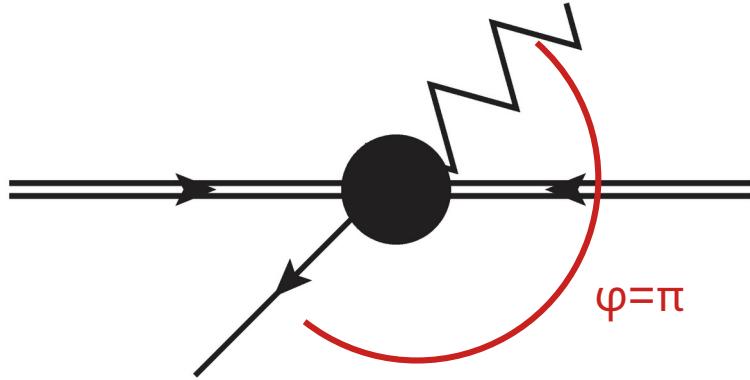
- Test of perturbative QCD
- Gluon PDF sensitivity
- Estimates for BSM backgrounds

## Fragmentation

- Depends on non-perturbative fragmentation functions
- Separation from “direct” not unique

# Why photon plus a jet pair?

---



- Non-back-to-back Born configurations  
→ access to angular correlations between the photon and jets
- Access to different kinematic regimes through distinguishable photon  
→ enhance direct, high- or low-z fragmentation
- Background process for BSM:  $pp \rightarrow \gamma + Y(\rightarrow jj)$

# Photon plus jet pair

Measurement of isolated-photon plus two-jet production in pp collisions at  $\sqrt{s} = 13$  TeV with the ATLAS detector [1912.09866]

<b>Requirements on photon</b>	$E_T^\gamma > 150$ GeV, $ \eta^\gamma  < 2.37$ (excluding $1.37 <  \eta^\gamma  < 1.56$ ) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 4.8$ GeV (reconstruction level) $E_T^{\text{iso}} < 0.0042 \cdot E_T^\gamma + 10$ GeV (particle level)		
<b>Requirements on jets</b>	at least two jets using anti- $k_t$ algorithm with $R = 0.4$ $p_T^{\text{jet}} > 100$ GeV, $ y^{\text{jet}}  < 2.5$ , $\Delta R^{\gamma\text{-jet}} > 0.8$		
<b>Phase space</b>	<b>total</b>	<b>fragmentation enriched</b> $E_T^\gamma < p_T^{\text{jet}2}$	<b>direct enriched</b> $E_T^\gamma > p_T^{\text{jet}1}$
<b>Number of events</b>	755 270	111 666	386 846

Modelled with hybrid isolation

$$E_\perp(r) \leq E_{\perp\max}(r) = 0.1 E_\perp(\gamma) \left( \frac{1 - \cos(r)}{1 - \cos(R_{\max})} \right)^2 \quad \text{for } r \leq R_{\max} = 0.1$$



$$E_\perp(r) \leq E_{\perp\max} = 0.0042 E_\perp(\gamma) + 10 \text{ GeV} \quad \text{for } r \leq R_{\max} = 0.4$$



No fragmentation contribution  
→ Purely pQCD through NNLO  
→ focus on “inclusive” and “direct” PS

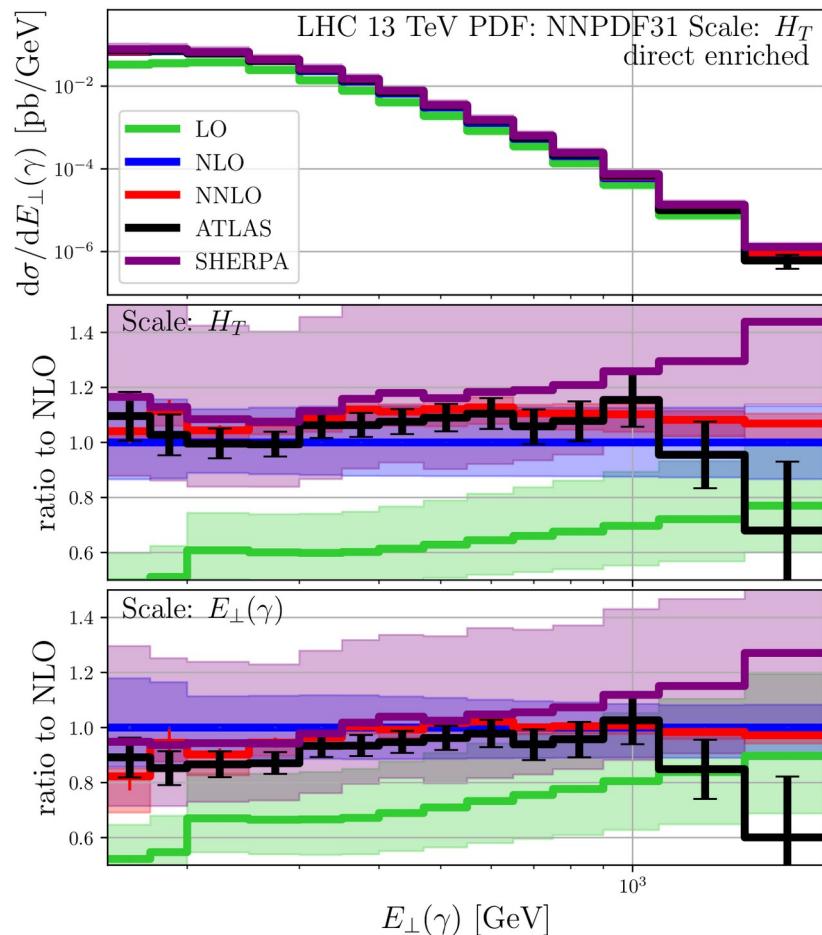
# Theory - data comparisons

## NNLO QCD

- Describes data well
- Improvements on the shape
- Small corrections
- Small remaining scale dependence

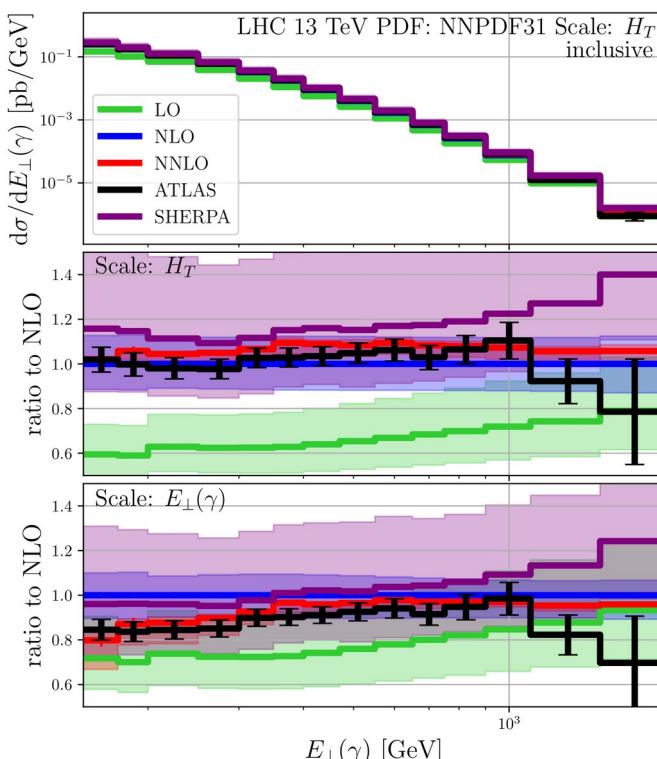
## Comment on the SHERPA predictions

- Large NLO scale uncertainties
- The shape is not well described
- Maybe an artefact of multi-jet merging?

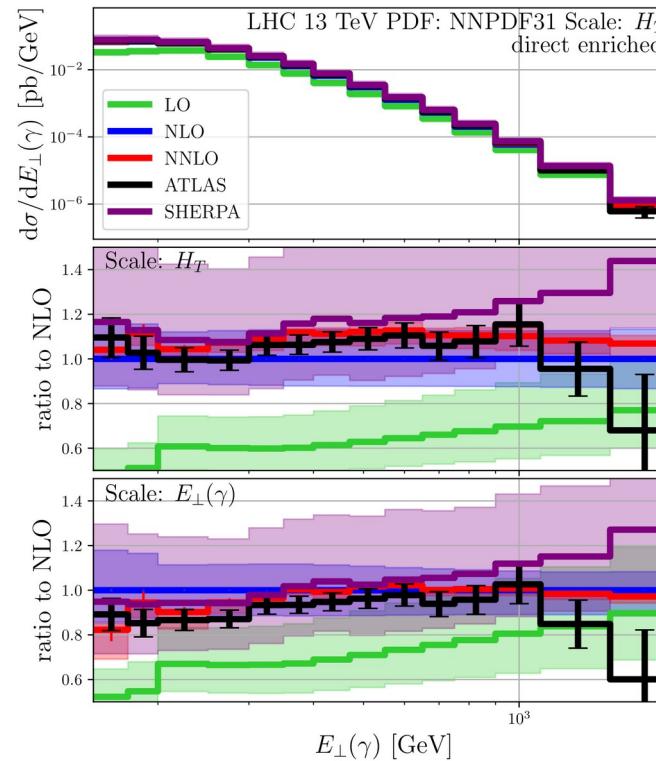


# Inclusive vs. direct vs. fragmentation

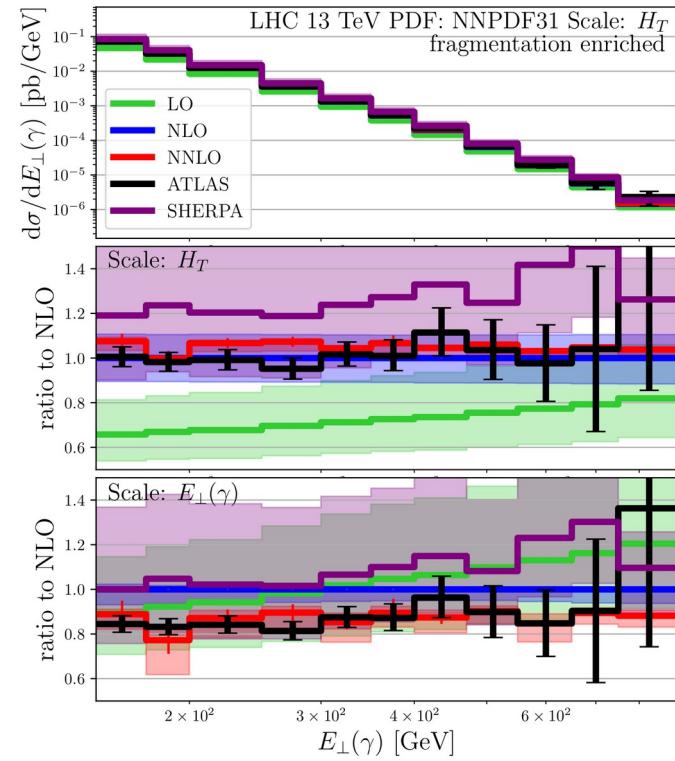
## Inclusive



## Direct-enriched



## Fragmentation



## Transverse photon energy

# Scale choice

$$\mu_R = \mu_F = H_T = E_{\perp}(\gamma) + p_T(j_1) + p_T(j_2)$$

*Full tree kinematics*

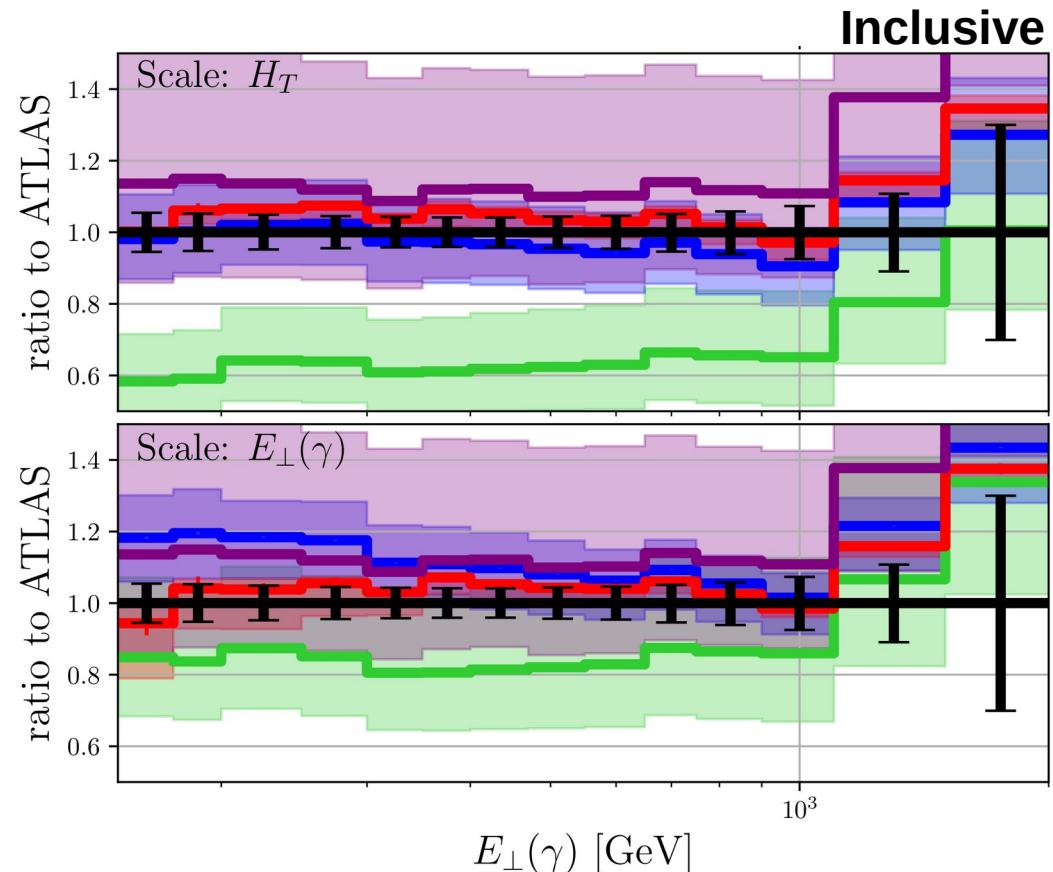
$$\mu_R = \mu_F = E_{\perp}(\gamma),$$

*Only photon*

## Perturbative convergence

NNLO result similar **but**  $E_{\perp}(\gamma)$

- Larger (negative) NNLO corrections
- Larger scale dependence (for jet obs.)



# Scale choice

*Full tree kinematics*

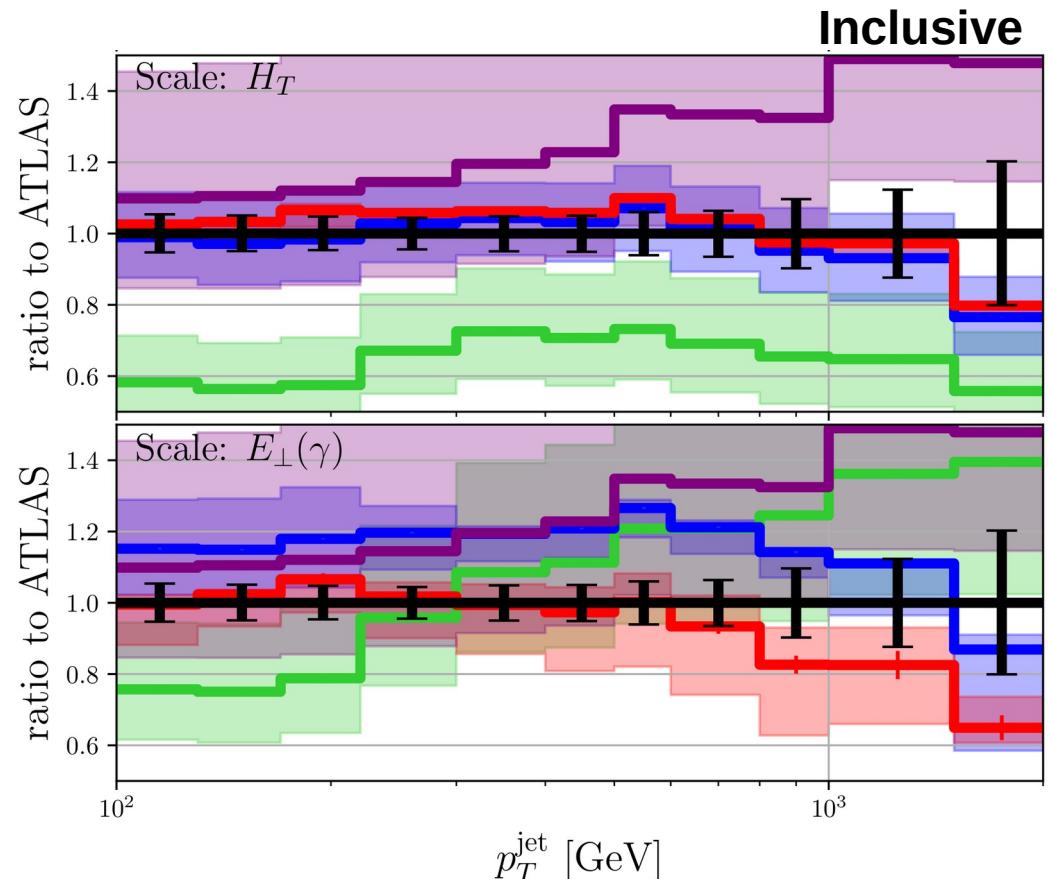
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$\mu_R = \mu_F = E_{\perp}(\gamma)$ ,  
*Only photon*

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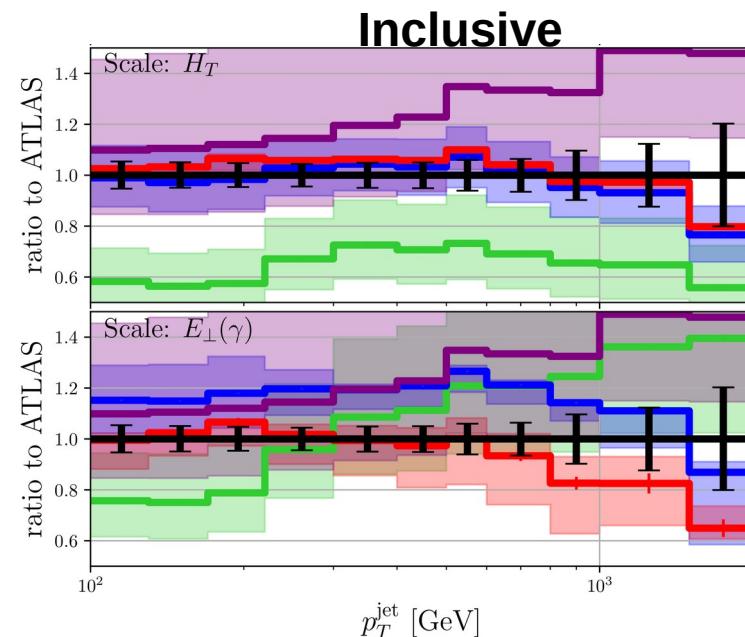
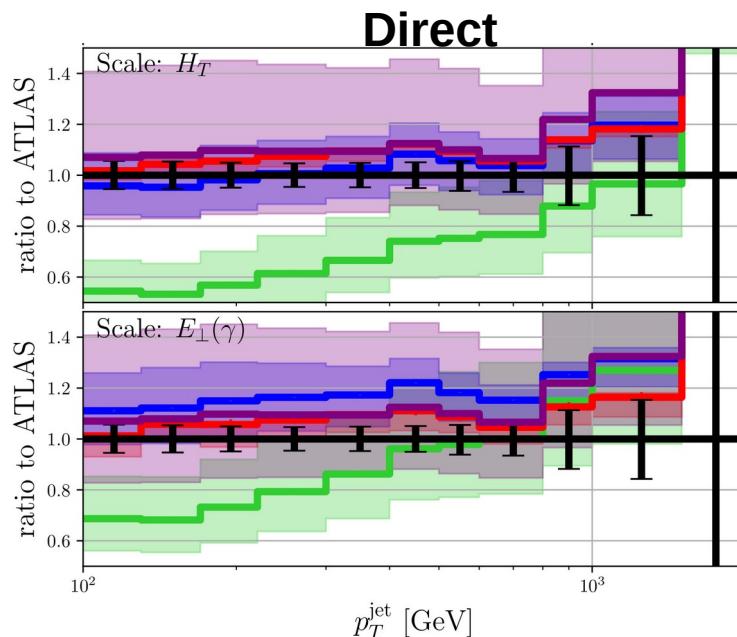
# Scale choice



**$E_{\perp}(\gamma)$  does not capture relevant scales for  $pp \rightarrow \gamma + 2j$**

- Better for “direct” enriched phase space  $p_T(\gamma) > p_T(j_1)$   
 $\rightarrow E_{\perp}(\gamma)$  closer to  $H_T = p_T(\gamma) + p_T(j_1) + p_T(j_2)$

**NNLO QCD needed  
for this conclusion**



## Summary & Outlook

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# Overview $2 \rightarrow 3$ massless cross sections

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$pp \rightarrow \gamma\gamma\gamma$   
Sector-improved RS [1911.00479]  
MATRIX [2010.04681]

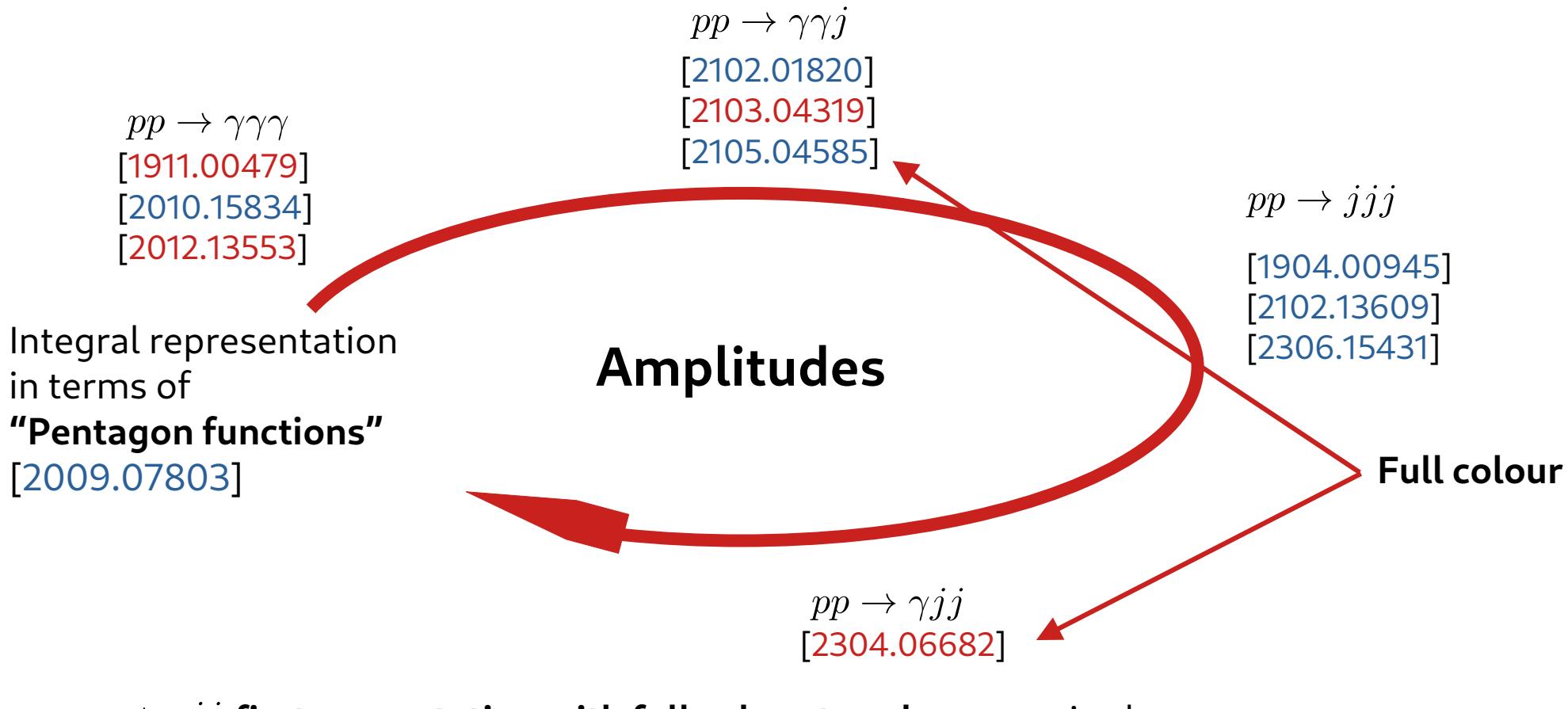
$pp \rightarrow \gamma\gamma j$   
Sector-improved RS [2105.06940]

$pp \rightarrow jjj$   
Sector-improved RS  
[2106.05331]  
[2301.01086]  
NNLOJET (gluons only)  
[2203.13531]

## Cross sections

$pp \rightarrow \gamma jj$   
Sector-improved RS  
[2304.06682]

# Overview $2 \rightarrow 3$ massless amplitudes



# Take home messages

---

- Precision phenomenology is stable of LHC physics
  - but requires higher-order corrections!
  - NNLO QCD or even higher orders are needed to keep up with experimental precision
- Completion of **massless 2→3** processes at hadron colliders through NNLO QCD

$$pp \rightarrow \gamma\gamma\gamma \quad pp \rightarrow \gamma\gamma j \quad pp \rightarrow \gamma jj \quad pp \rightarrow jjj$$

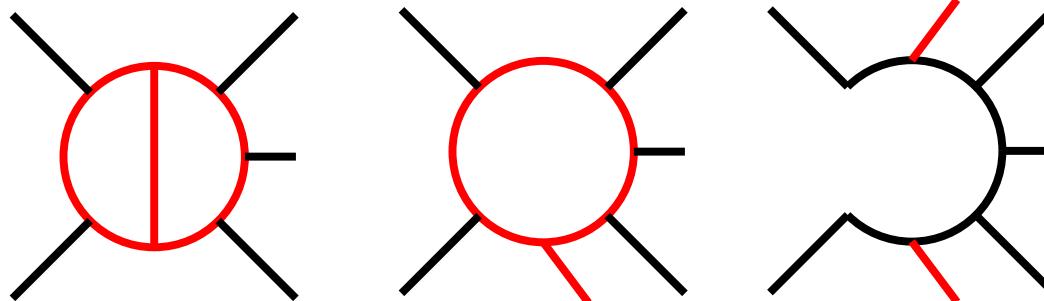
- Most important bottlenecks from theory side:
  - Real radiation contributions  
(subtraction, Monte Carlo methods, efficiency, automation,...)
  - Two-loop amplitudes  
(including external/internal masses are the current frontier)

# Backup

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## Sector-improved residue subtraction

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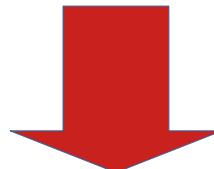
# Sector decomposition I

---

Considering working in CDR:

- Virtuals are usually done in this regularization:  $\hat{\sigma}_{ab}^{VV} = \sum_{i=-4}^0 c_i \epsilon^i + \mathcal{O}(\epsilon)$
- Can we write the real radiation as such expansion?

- Difficult integrals, analytical impractical (except very simple observables)!
- Numerics not possible, integrals are divergent →  $\epsilon$ -poles!



How to extract these poles? → Sector decomposition!

**Divide and conquer** the phase space:

$$1 = \sum_{i,j} \left[ \sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \quad \xrightarrow{\text{red arrow}} \quad \hat{\sigma}_{ab}^{\text{RR}} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \sum_{i,j} \left[ \sum_k \mathcal{S}_{ij,k} + \sum_{k,l} \mathcal{S}_{i,k;j,l} \right] \langle \mathcal{M}_{n+2}^{(0)} | \mathcal{M}_{n+2}^{(0)} \rangle F_{n+2}$$

# Sector decomposition II

---

Divide and conquer the phase space

- Each  $\mathcal{S}_{i,k}$  (NLO),  $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$  (NNLO) has simpler divergences:

- Soft limits of partons i and j
- Collinear w.r.t partons k (and l) of partons i and j

$$\mathcal{S}_{i,k} = \frac{1}{D_1 d_{i,k}} \quad D_1 = \sum_{ik} \frac{1}{d_{i,k}} \quad d_{i,k} = \frac{E_i}{\sqrt{s}} (1 - \cos \theta_{ik})$$

- Parametrization w.r.t. reference parton makes divergences explicit

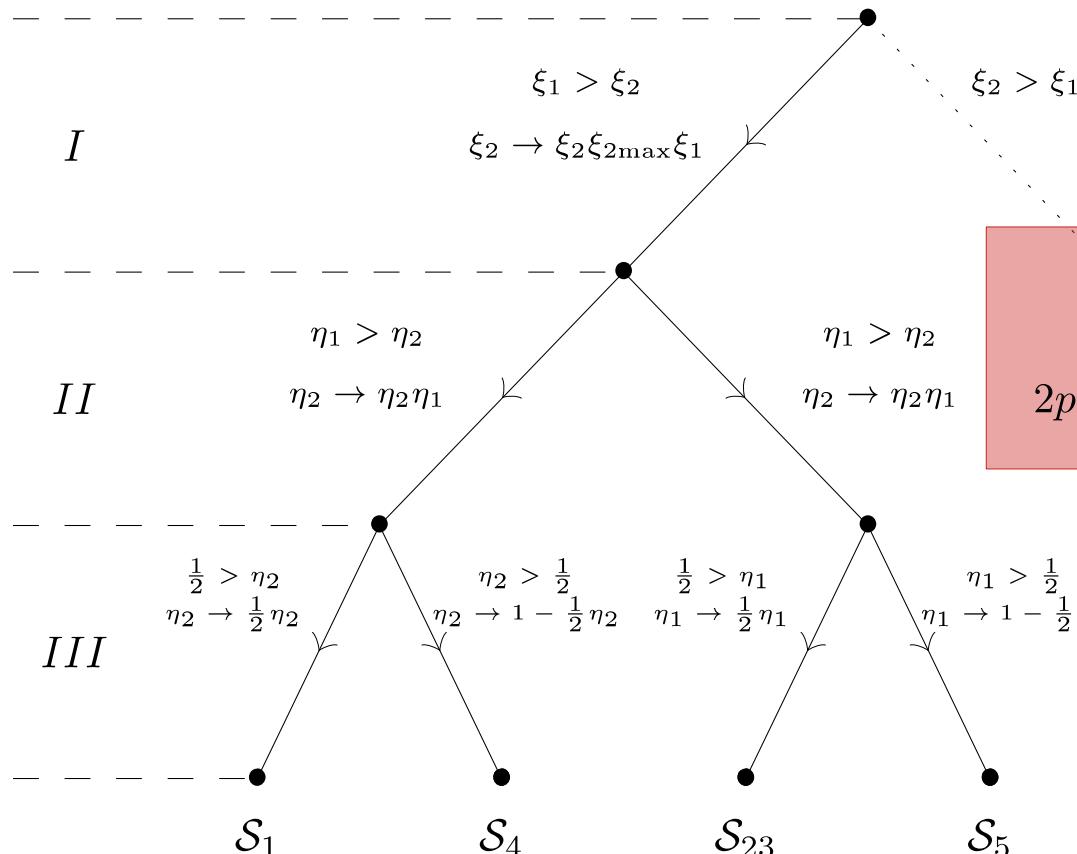
$$\hat{\eta}_i = \frac{1}{2}(1 - \cos \theta_{ik}) \in [0, 1] \quad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0, 1]$$

- Example: Splitting function

$$\sim \frac{1}{s_{ik}} P(z) \quad s_{ik} = (p_i + p_k)^2 = 2p_k^0 u_{\max}^0 \xi_i \eta_i \sim \frac{1}{\eta_i \xi_i}$$

# Sector decomposition II – triple collinear factorization

Three particle invariant does not factorize trivially:



Double soft factorization:

$$\theta(u_i^0 - u_j^0) + \theta(u_i^0 - u_j^0)$$

$$(p_k + u_i + u_j)^2 = \\ 2p_k^0 (\xi_i \eta_i u_{\max}^{0,i} + \xi_j \eta_j u_{\max}^{0,j} + \xi_i \xi_j \frac{u_{\max}^{0,i} u_{\max}^{0,j}}{p_k^0} \angle(u_i, u_j))$$

[Czakon'10,Caola'17]

# Sector decomposition III

---

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}} \int d\Phi_{n+2} \mathcal{S}_{kl,m} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle F_{n+2} = \sum_{\text{sub-sec.}} \int d\Phi_n \prod dx_i \underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}} d\tilde{\mu}(\{x_i\}) \underbrace{\prod x_i^{a_i+1} \left\langle \mathcal{M}_{n+2} | \mathcal{M}_{n+2} \right\rangle F_{n+2}}_{\text{regular}}$$
$$x_i \in \{\eta_1, \xi_1, \eta_2, \xi_2\}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 dx [x^{-1-b\epsilon}]_+ f(x) = \int_0^1 \frac{f(x) - f(0)}{x^{1+b\epsilon}}$$

# Finite NNLO cross section

$$\hat{\sigma}_{ab}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle F_{n+2}$$

$\hat{\sigma}_{ab}^{C1} = (\text{single convolution}) F_{n+1}$

$$\hat{\sigma}_{ab}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} 2\text{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle F_{n+1}$$

$\hat{\sigma}_{ab}^{C2} = (\text{double convolution}) F_n$

$$\hat{\sigma}_{ab}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left( 2\text{Re} \left\langle \mathcal{M}_n^{(0)} \middle| \mathcal{M}_n^{(2)} \right\rangle + \left\langle \mathcal{M}_n^{(1)} \middle| \mathcal{M}_n^{(1)} \right\rangle \right) F_n$$



sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{[x^{-1-b\epsilon}]_+}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$



re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

separately finite:  $\epsilon$  poles cancel

# Improved phase space generation

---

Phase space cut and differential observable introduce

*mis-binning*: mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [[Czakon'19](#)]:

Minimization of # of different subtraction kinematics in each sector

# Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from  $n+2$  to  $n$  particle phase space:

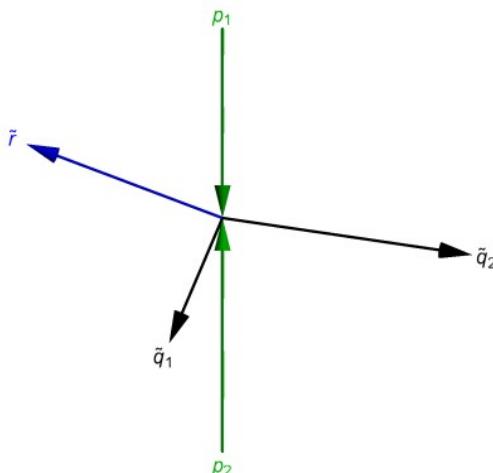
$$\{P, r_j, u_k\} \rightarrow \{\tilde{P}, \tilde{r}_j\}$$

Requirements:

- Keep direction of reference  $r$  fixed
- Invertible for fixed  $u_i$ :  $\{\tilde{P}, \tilde{r}_j, u_k\} \rightarrow \{P, r_j, u_k\}$
- Preserve Born invariant mass:  $q^2 = \tilde{q}^2, \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons  $u_i$
- Rescale reference momentum  $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration



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New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

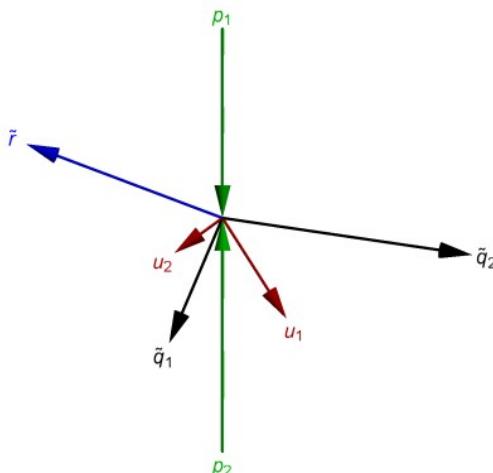
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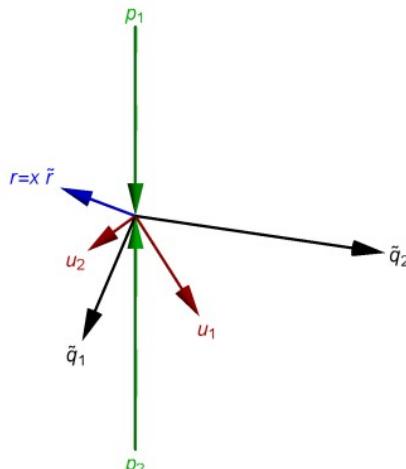
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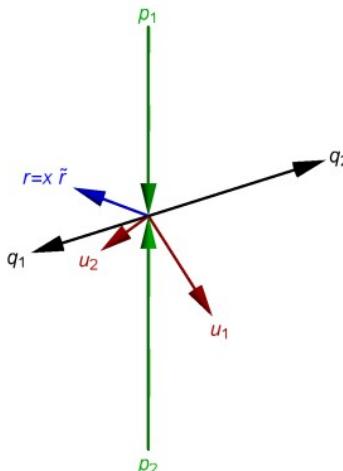
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# t'HV corrections

Observables: Implemented by infrared safe measurement function (MF)  $F_m$

Infrared property in STRIPPER context:

- $\{x_i\} \rightarrow 0 \leftrightarrow$  single unresolved limit  
 $\Rightarrow F_{n+2} \rightarrow F_{n+1}$
- $\{x_i\} \rightarrow 0 \leftrightarrow$  double unresolved limit  
 $\Rightarrow F_{n+2} \rightarrow F_n$   
 $\Rightarrow F_{n+1} \rightarrow F_n$

Tool for new formulation in the 't Hooft Veltman scheme:

Parameterized MF  $F_{n+1}^\alpha$

- $F_n^\alpha \equiv 0$  for  $\alpha \neq 0$   
(NLO MF)
- 'arbitrary'  $F_n^0$   
(NNLO MF)
- $\alpha \neq 0 \Rightarrow$  DU = 0 and SU separately finite

Example:  $F_{n+1}^\alpha = F_{n+1} \Theta_\alpha(\{\alpha_i\})$   
with  $\Theta_\alpha = 0$  if some  $\alpha_i < \alpha$

# t'HV corrections

---

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \quad \text{where} \quad \sigma_{SU}^c = \int d\Phi_{n+1} (I_{n+1}^c F_{n+1} + I_n^c F_n)$$

NLO measurement function ( $\alpha \neq 0$ ):

$$\int d\Phi_{n+1} (I_{n+1}^{RR} + I_{n+1}^{RV} + I_{n+1}^{C1}) F_{n+1}^\alpha = \text{finite in 4 dim.}$$

All divergences cancel in  $d$ -dimensions:

$$\sum_c \int d\Phi_{n+1} \left[ \frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^\alpha \equiv \sum_c \mathcal{I}^c = 0$$

# t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\begin{aligned} \sigma_{SU}^c - \mathcal{I}^c &= \int d\Phi_{n+1} \left\{ \left[ \frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[ \frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} + I_n^{c,(0)} \right] F_n \right\} \\ &\quad - \int d\Phi_{n+1} \left[ \frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1} \Theta_\alpha(\{\alpha_i\}) \\ &= \int d\Phi_{n+1} \left[ \frac{I_{n+1}^{c,(-2)} F_{n+1} + I_n^{c,(-2)} F_n}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_n^{c,(-1)} F_n}{\epsilon} \right] (1 - \Theta_\alpha(\{\alpha_i\})) \\ &\quad + \int d\Phi_{n+1} \left[ I_{n+1}^{c,(0)} F_{n+1} + I_n^{c,(0)} F_n \right] + \int d\Phi_{n+1} \left[ \frac{I_n^{c,(-2)}}{\epsilon^2} + \frac{I_n^{c,(-1)}}{\epsilon} \right] F_n \Theta_\alpha(\{\alpha_i\}) \end{aligned}$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha \rightarrow 0} + \underbrace{C^c}_{\text{no divergencies}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \rightarrow \text{DU}}$$

# t'HV corrections

Looks like slicing, but it is slicing *only* for divergences  
→ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension

- all  $N_k^c(\alpha)$  regular in  $\alpha$
- start expression independent of  $\alpha \Rightarrow$  all logs cancel
- only  $N_0^c(0)$  relevant



SU contribution:  $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$   
original expression  $\sigma_{SU}$  in 4-dim  
without poles, no further  $\epsilon$  pole cancellation

# C++ framework

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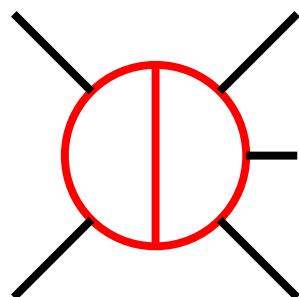
- Formulation allows efficient algorithmic implementation → STRIPPER
- **High degree of automation:**
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers + O(100) hardcoded:  
AvH, OpenLoops, Recola, NJET, HardCoded
    - In practice: Only **two-loop matrix** elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

## Two-loop five-point amplitudes

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Massless:

- [Chawdry'19'20'21] ( $3A+2j, 2A+3j$ )
- [Abreu'20'21] ( $3A+2j, 5j$ )
- [Agarwal'21] ( $2A+3j$ )
- [Badger'21'23] ( $5j, gggAA, jjjjA$ )



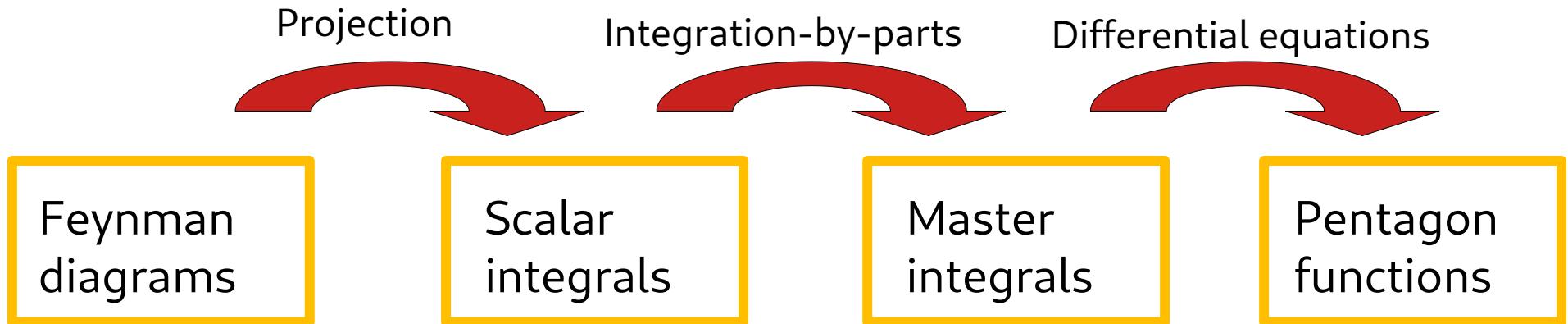
1 external mass:

- [Abreu'21] ( $W+4j$ )
- [Badger'21'22] ( $Hqqgg, W4q, Wajjj$ )
- [Hartanto'22] ( $W4q$ )

# Overview

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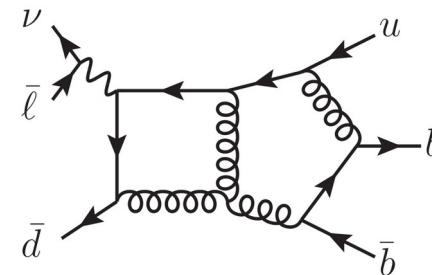
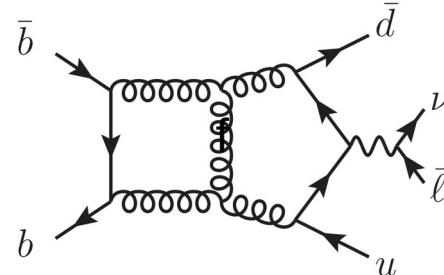
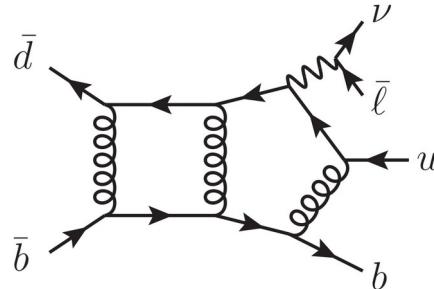
Old school approach:



Automated framework using finite fields  
to avoid expression swell based on  
FiniteFlow [Peraro'19]

# Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



$$\text{Factorizing decay: } A_6^{(L)} = A_5^{(L)\mu} D_\mu P$$

$$M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)*} A_6^{(L)} = M_5^{(L)\mu\nu} D_{\mu\nu} |P|^2$$

Projection on scalar functions (FORM+Mathematica):  
→ anti-commuting  $\gamma_5$  + Larin prescription

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$



$$a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}}$$

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon)$$

# Integration-By-Parts reduction

---

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\}, \epsilon) \mathcal{I}(\{p\}, \epsilon)$$

→ prohibitively large number of integrals

$$\mathcal{I}_i(\{p\}, \epsilon) \equiv \mathcal{I}(\vec{n}_i, \{p\}, \epsilon) = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\})$$

Integration-By-Parts identities connect different integrals → system of equations  
→ only a small number of independent “master” integrals

$$0 = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} l_\mu \frac{\partial}{\partial l^\mu} \prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\}, \{k\}) \quad \text{with} \quad l \in \{p\} \cap \{k\}$$

LiteRed (+ Finite Fields)



$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\}, \epsilon) \text{MI}(\{p\}, \epsilon)$$

# Master integrals & finite remainder

---

Differential Equations:  $d\vec{\text{MI}} = dA(\{p\}, \epsilon)\vec{\text{MI}}$

[Remiddi, 97]

[Gehrmann, Remiddi, 99]

[Henn, 13]

Canonical basis:  $d\vec{\text{MI}} = \epsilon d\tilde{A}(\{p\})\vec{\text{MI}}$

Simple iterative solution



$$\text{MI}_i = \sum_w \epsilon^w \tilde{\text{MI}}_i^w \quad \text{with} \quad \tilde{\text{MI}}_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals

"Pentagon"-functions

[Chicherin, Sotnikov, 20]

[Chicherin, Sotnikov, Zoia, 21]

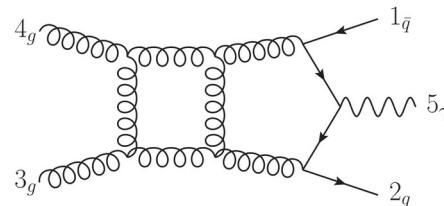
Putting everything together (and removing of IR poles):

$$f_i^{(L),p} = a_i^{(L),p} - \text{poles}$$

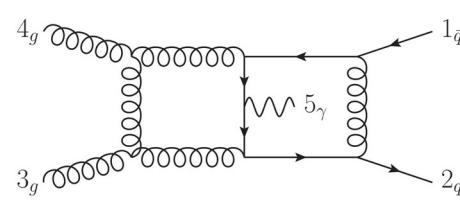
$$f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

# Reconstruction of Amplitudes

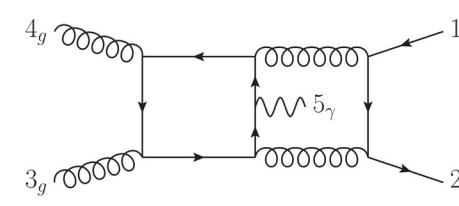
[Badger'21]



$$A_{34;q}^{(2),N_c^2}, A_{\delta;q}^{(2),N_c}$$



$$A_{34;q}^{(2),1}, A_{\delta;q}^{(2),N_c}, A_{\delta;q}^{(2),1/N_c}$$



$$A_{34;l}^{(2),N_c}, A_{34;l}^{(2),1/N_c}, A_{\delta;l}^{(2),1/N_c^2}$$

## New optimizations

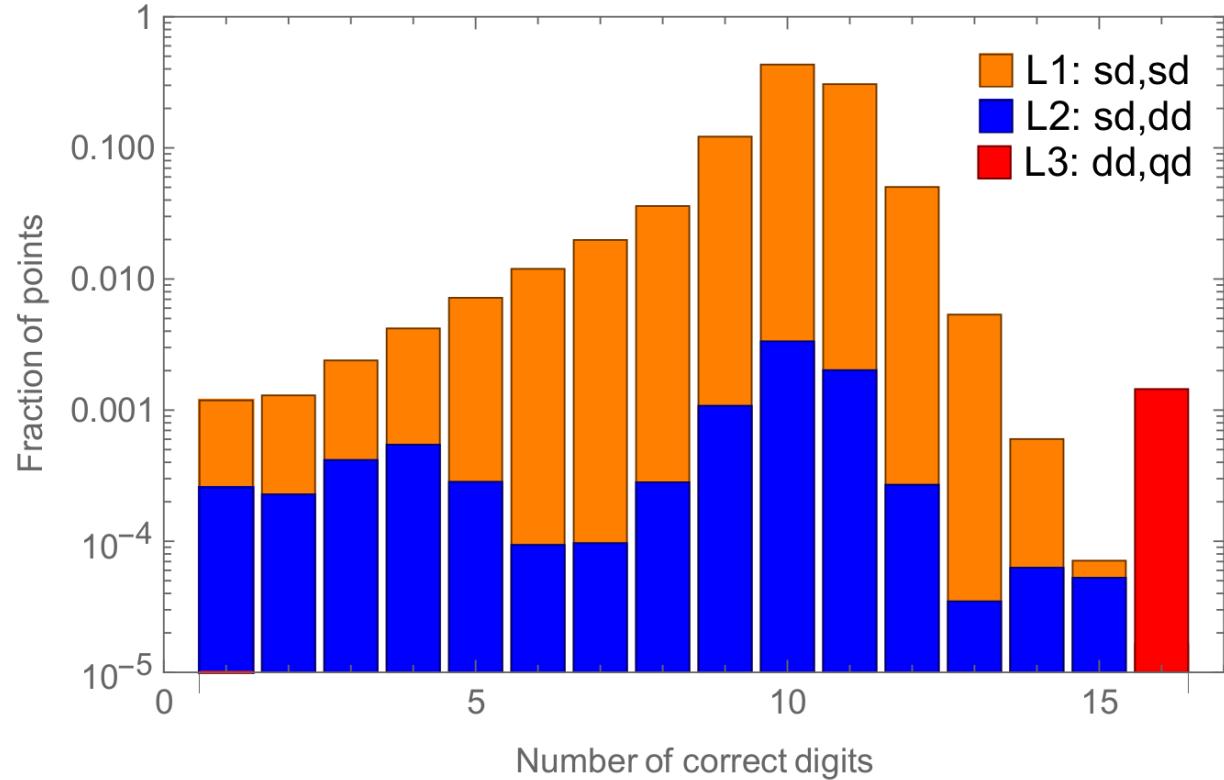
- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansaetze
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A_{34;q}^{(2),1}$	- + + - +	94/91	74/71	74/0	22/18	22/0
$A_{34;q}^{(2),1}$	- + - + +	93/89	90/86	90/0	24/14	18/0
$A_{34;q}^{(2),1/N_c^2}$	- + + - +	90/88	73/71	73/0	23/18	22/0
$A_{34;q}^{(2),1/N_c^2}$	- + - + +	90/86	86/82	86/0	24/14	19/0
$A_{34;l}^{(2),1/N_c}$	- + - + +	89/82	74/67	73/0	27/14	20/0
$A_{34;l}^{(2),1/N_c}$	- + + - +	85/81	61/58	60/0	27/18	20/0
$A_{34;q}^{(2),N_c^2}$	- + - + +	58/55	54/51	53/0	20/16	20/0

Massive reduction of complexity

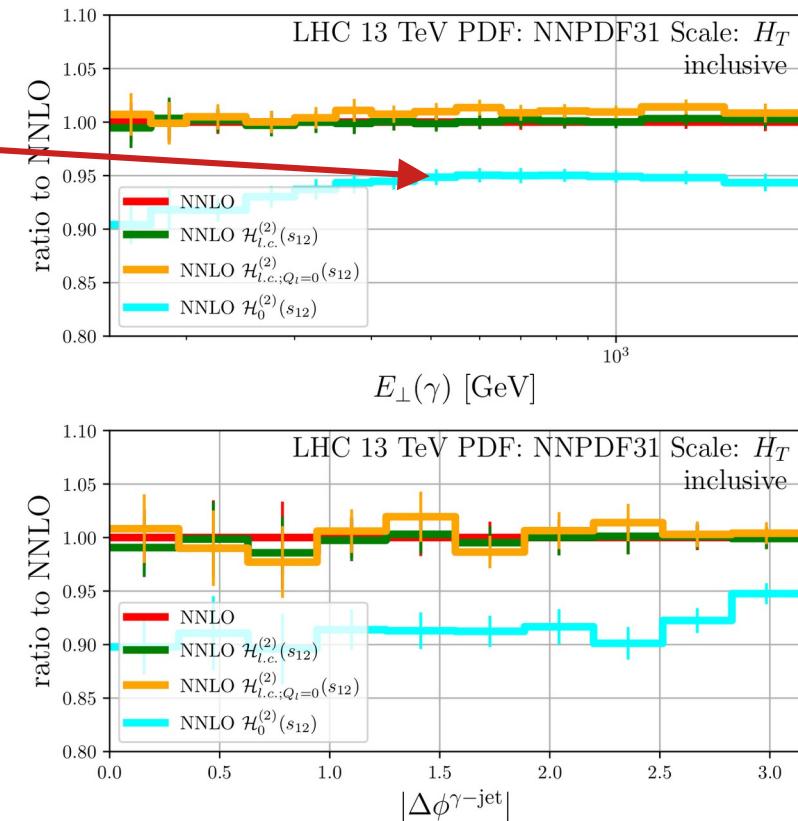
# Two-loop matrix element stability

- Stable evaluation requires high floating point precision for rational functions
- In rarer cases higher precision “Pentagon” functions necessary
- 2.2 million events needed  
→ fast evaluation essential

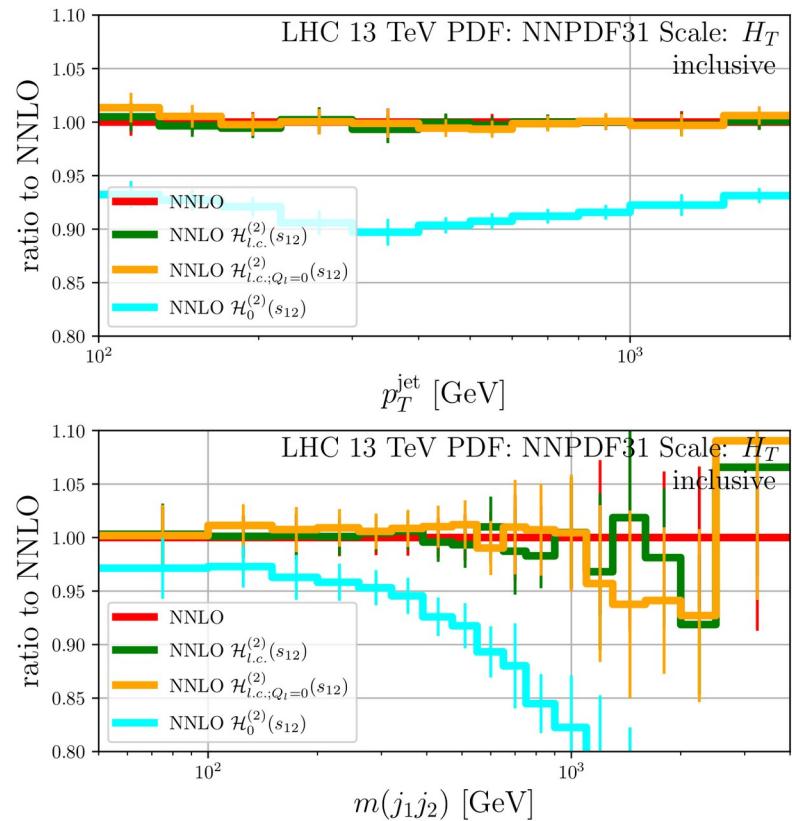


# Quality of leading colour the approximation

Two-loop contribution  
 ~ 5-10%  
 wrt. full NNLO  
 (scheme dep.)



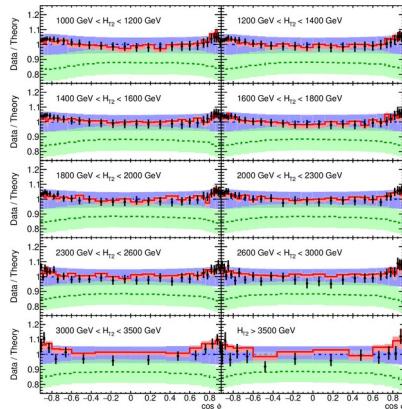
"Leading colour"  
 Approximation  
 "Error" = O(~1%)  
 wrt full NNLO



# HighTEA

---

# HighTEA



= ~100 MCPUh

How to make this more  
efficient/environment-friendly/  
accessible/faster?

high tea  
for your freshly brewed analysis

<https://www.precision.hep.phy.cam.ac.uk/hightea>

Rene Poncelet – IFJ PAN Krakow

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# Basic idea

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→ Database of precomputed “Theory Events”

- **Equivalent to a full fledged computation**
- Currently this means partonic fixed order events
- Extensions to included showered/resummed/hadronized events is feasible
- (Partially) Unweighting to increase efficiency

Not so new idea:  
LHE [[Alwall et al '06](#)],  
Ntuple [[BlackHat '08'13](#)],

→ Analysis of the data through an user interface

- Easy-to-use
- Fast
  - Observables from basic 4-momenta
  - Free specification of bins
- Flexible:
  - Renormalization/Factorization Scale variation
  - PDF (member) variation
  - Specify phase space cuts

# Factorizations

---

Factorizing renormalization and factorization scale dependence:

$$w_s^{i,j} = w_{\text{PDF}}(\mu_F, x_1, x_2) w_{\alpha_s}(\mu_R) \left( \sum_{i,j} c_{i,j} \ln(\mu_R^2)^i \ln(\mu_F^2)^j \right)$$

PDF dependence:

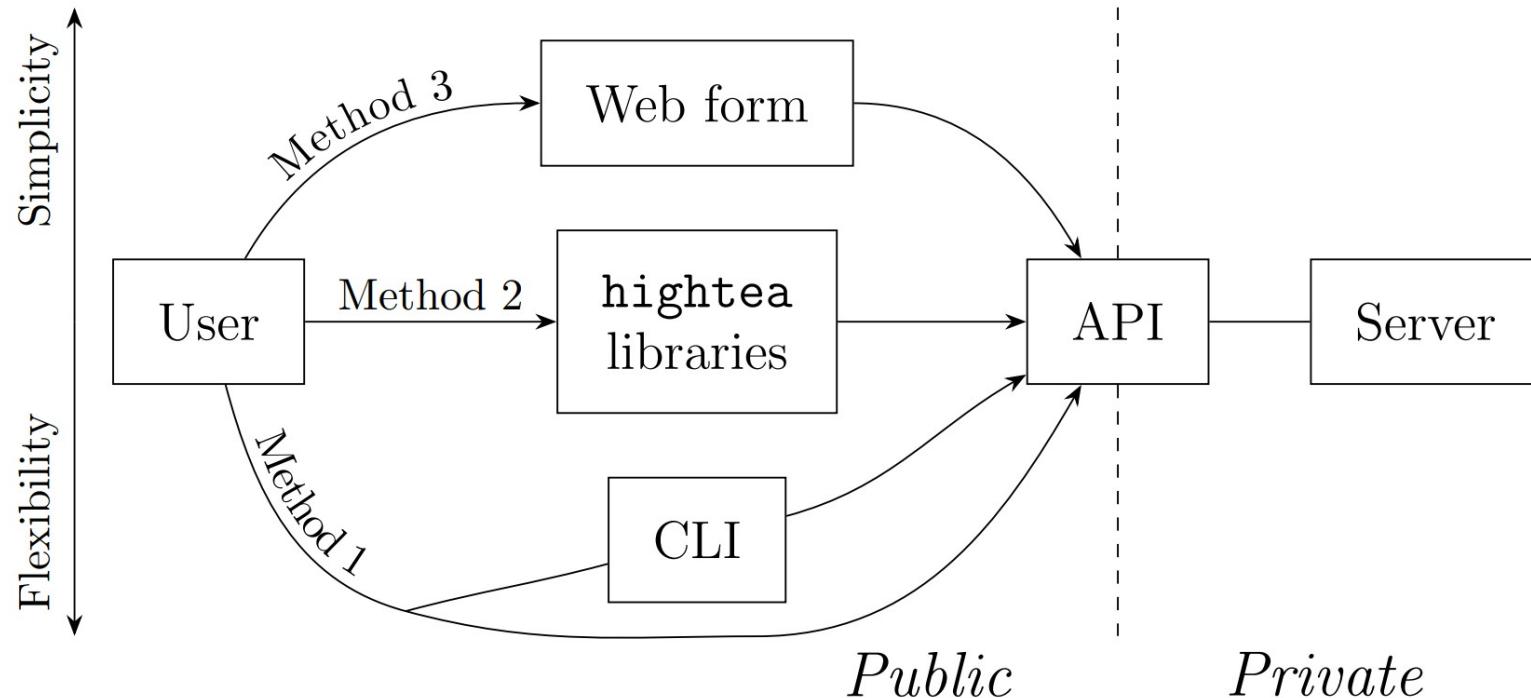
$$w_{\text{PDF}}(\mu, x_1, x_2) = \sum_{ab \in \text{channel}} f_a(x_1, \mu) f_b(x_2, \mu)$$

$\alpha_s$  dependence:

$$w_{\alpha_s}(\mu) = (\alpha_s(\mu))^m$$

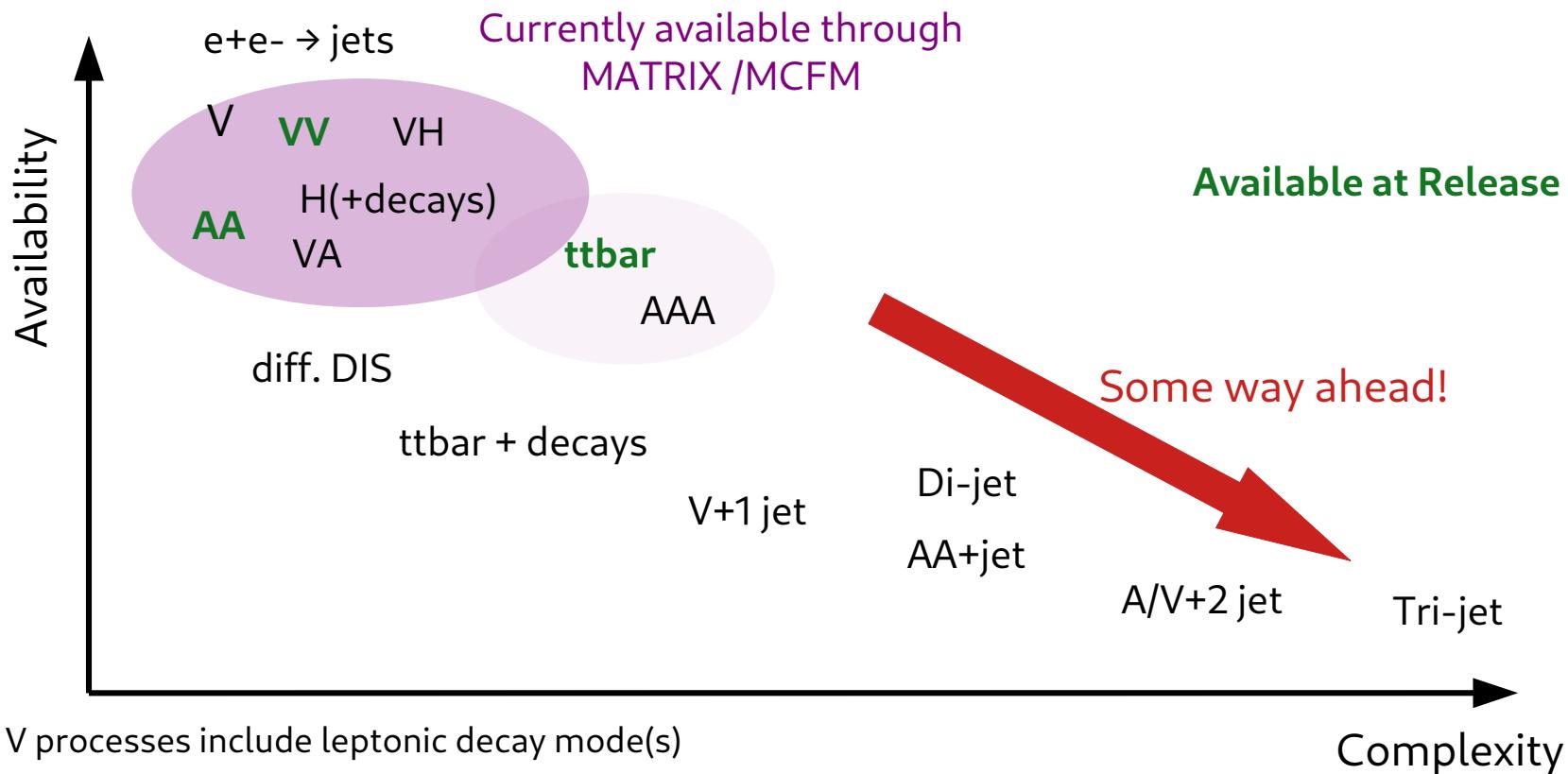
Allows **full control over scales and PDF**

# HighTEA interface



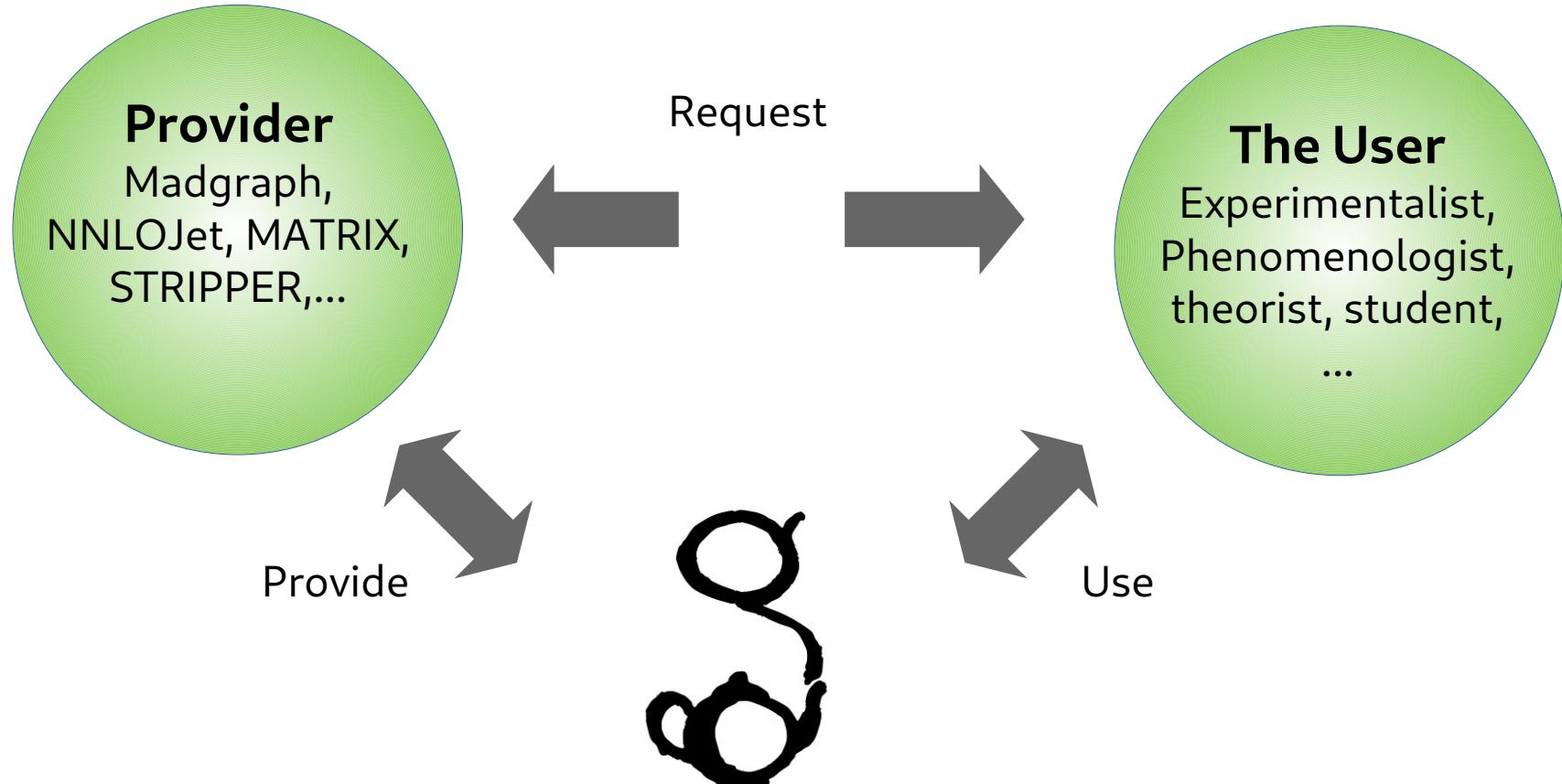
# Available Processes

Processes **currently** implemented in our STRIPPER framework through **NNLO QCD**



# The Vision

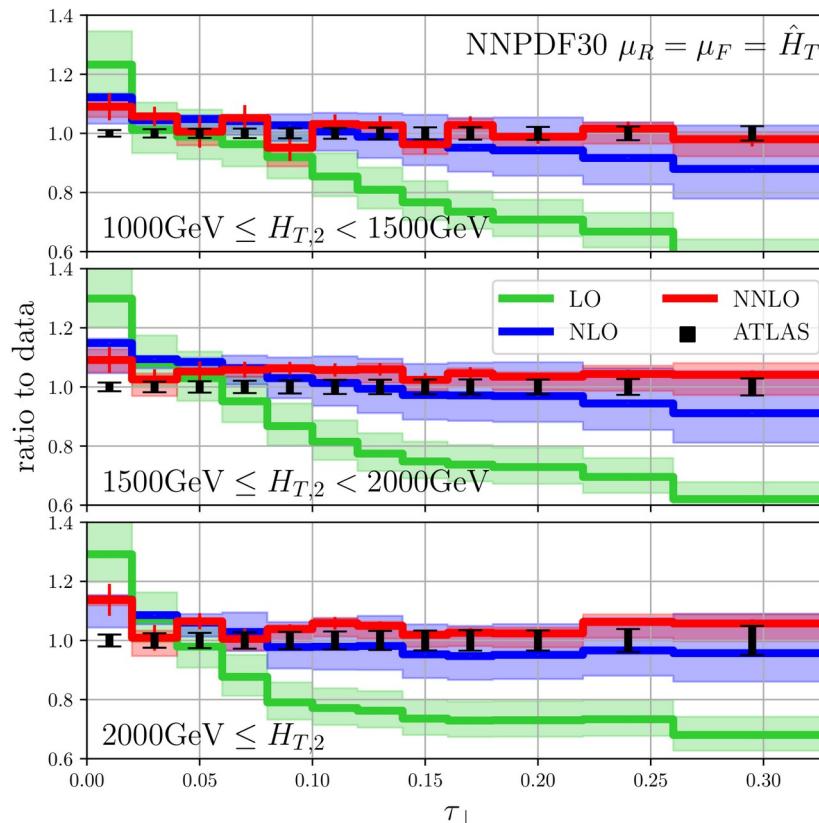
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# Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC

Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]



ATLAS [2007.12600]

