

# NNLO predictions for three-jet cross sections at the LHC

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in collaboration with Michal Czakon and Alexander Mitov

based on: [[1907.12911](#)] and [[2106.05331](#)]

LEVERHULME  
TRUST



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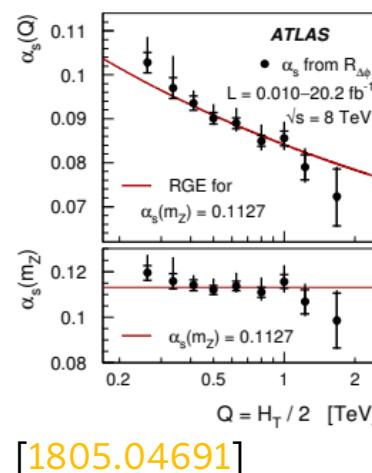
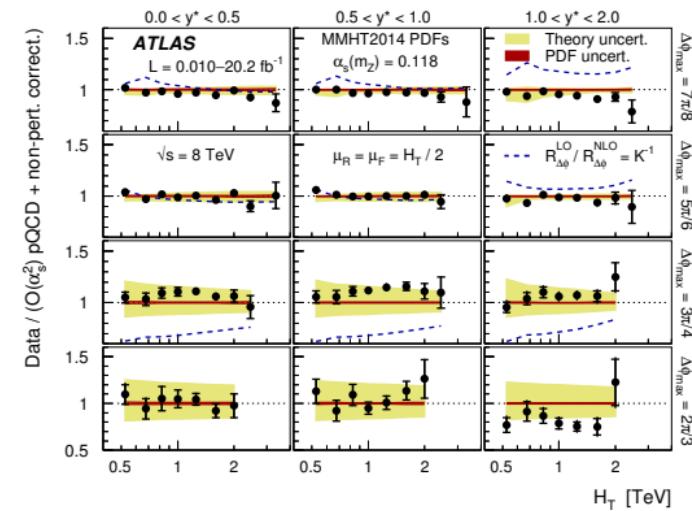


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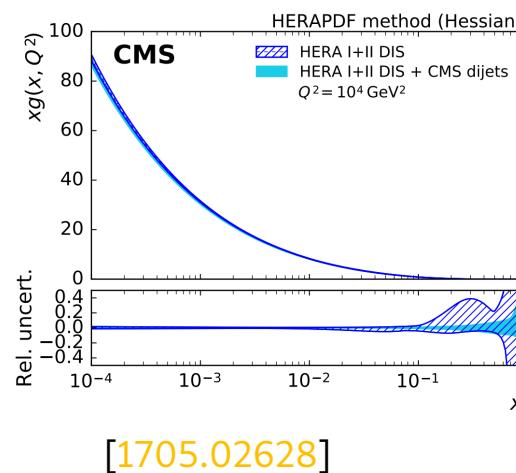
# Jet observables at the LHC

Tests of pQCD,  $\alpha_s$  extraction:  
R32 ratios, event-shapes



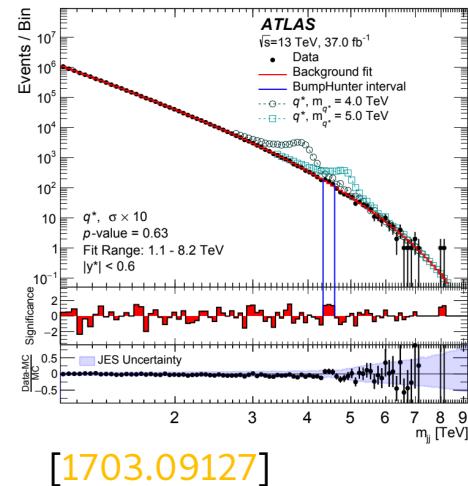
Precision theory required!

PDF determination:  
Single inclusive,  
Multi-differential dijet



[1705.02628]

NP searches:  
dijet mass



[1703.09127]

Data driven

# Precision predictions

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Fixed order  
perturbation theory

Resummation

Parton-showers

Precision theory predictions

Parametric input:  
PDFs and alphaS

Soft physics:  
MPI, colour reconnection,  
...  
...

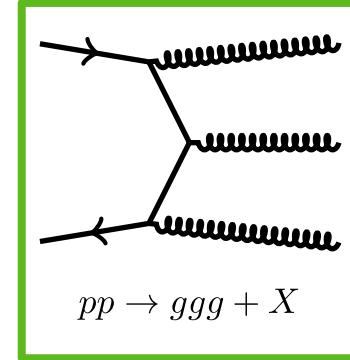
Fragmentation/hadronisation

# Three jet production

Advances in perturbative QCD allow to tackle the most complicated 2→3 process

## Bottlenecks:

- Handling of real radiation:
  - Sector-improved residue subtraction [Czakon'10'14'19]
  - Computationally very challenging! → O(1M CPUh)
- Double virtual amplitudes in leading colour approximation [Abreu'21]
  - Fast numerical evaluation → very small contribution to computational cost



**Only** Approximation made:

→ taken from [Abreu'21]

$$\mathcal{R}^{(2)}(\mu_R^2) = 2 \operatorname{Re} \left[ \mathcal{M}^{\dagger(0)} \mathcal{F}^{(2)} \right] (\mu_R^2) + |\mathcal{F}^{(1)}|^2(\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^4 c_i \ln^i \left( \frac{\mu_R^2}{s_{12}} \right)$$

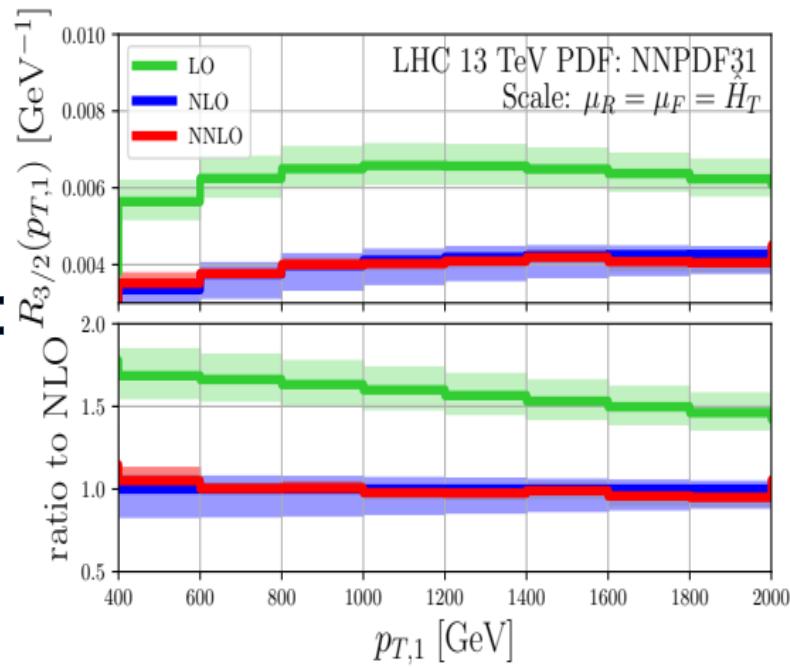
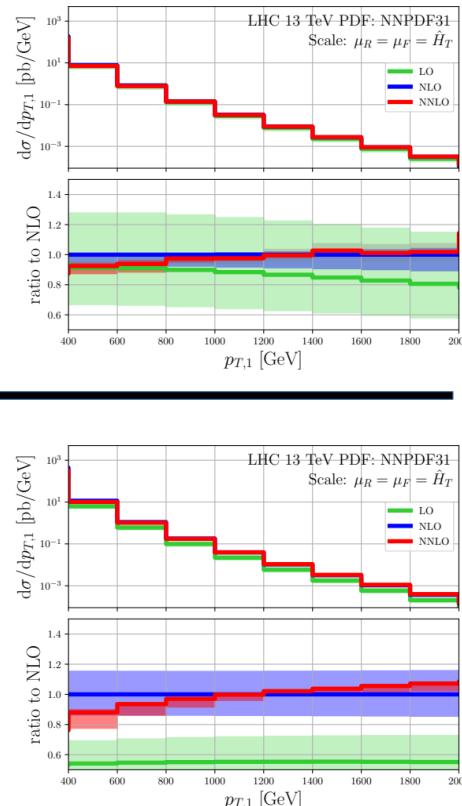
$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

# Three jet production - R32(pT1)

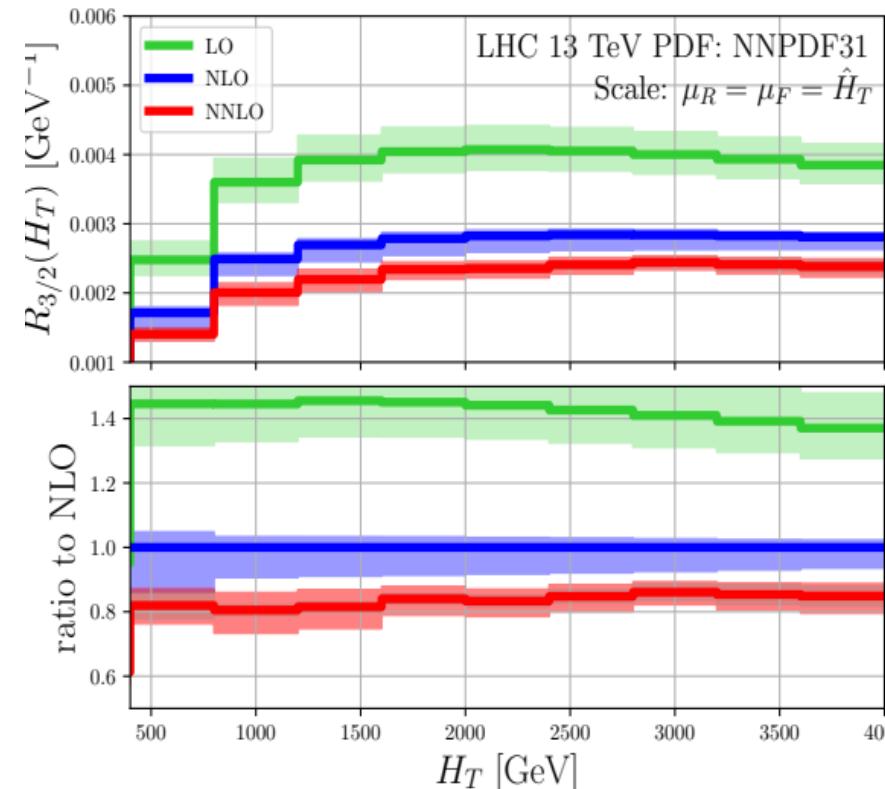
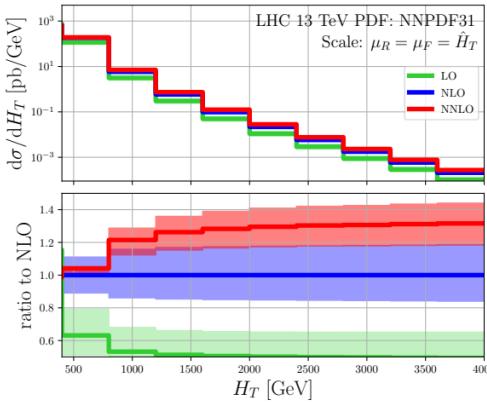
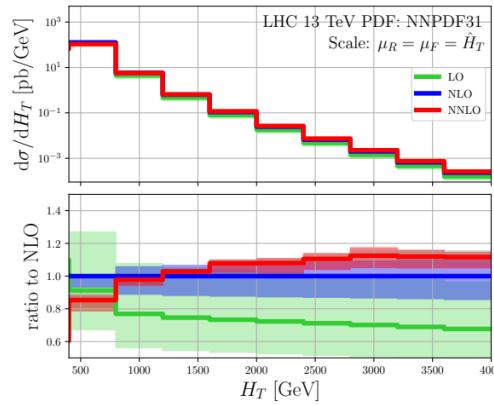
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
  - $p_T(j) > 60$  GeV and  $|y(j)| < 4.4$
  - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$  GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX}$$



# Three jet production - R32(HT)



$$H_T = \sum_{\text{jets}} p_T$$

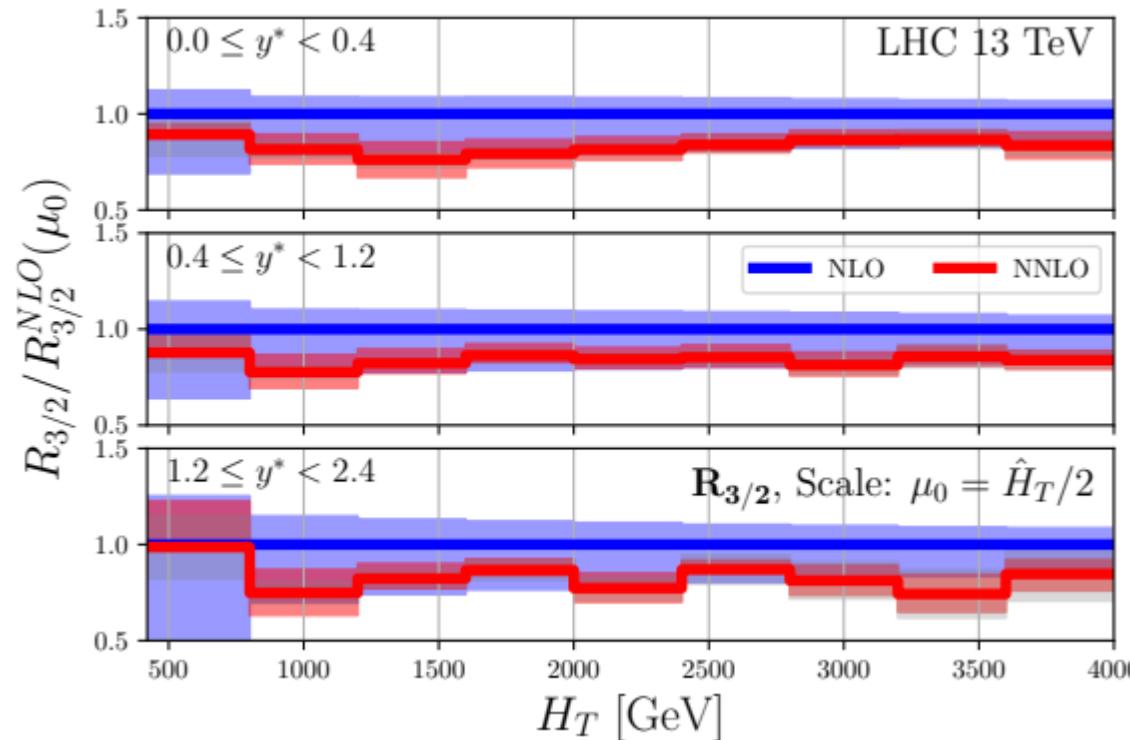
Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

# Three jet production – R<sub>32</sub>(HT,y\*)



Double differential w.r.t.  $y^* = |y(j_1) - y(j_2)|/2$

Different central scale choice:  $\hat{H}_T/2$

# Three jet production – azimuthal decorrelation

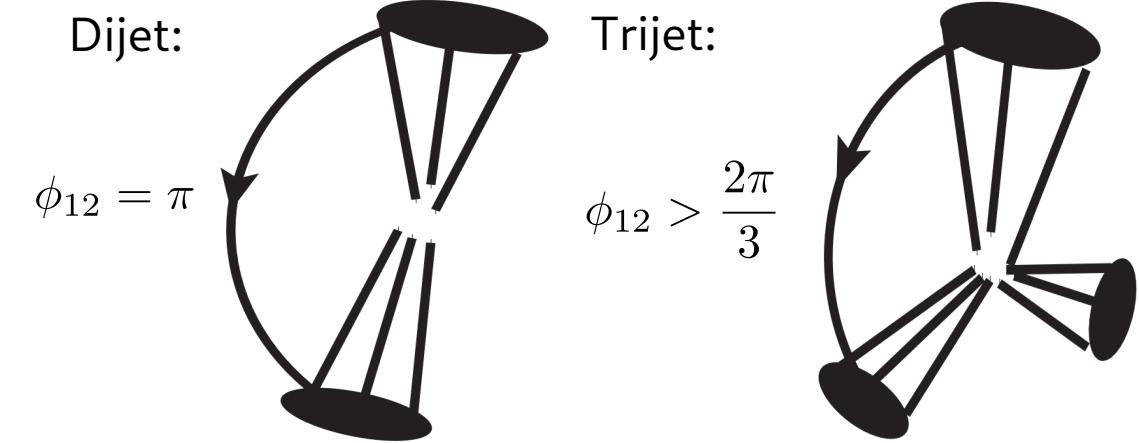
Kinematic constraints on the azimuthal separation between the two leading jets ( $\phi_{12}$ )

$\phi_{12}$  sensitive to the jet multiplicity:

$$2j: \phi_{12} = \pi$$

$$3j: \phi_{12} > \frac{2\pi}{3}$$

4j: unconstrained

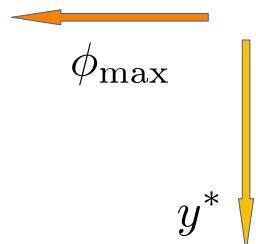


Study of the ratio:

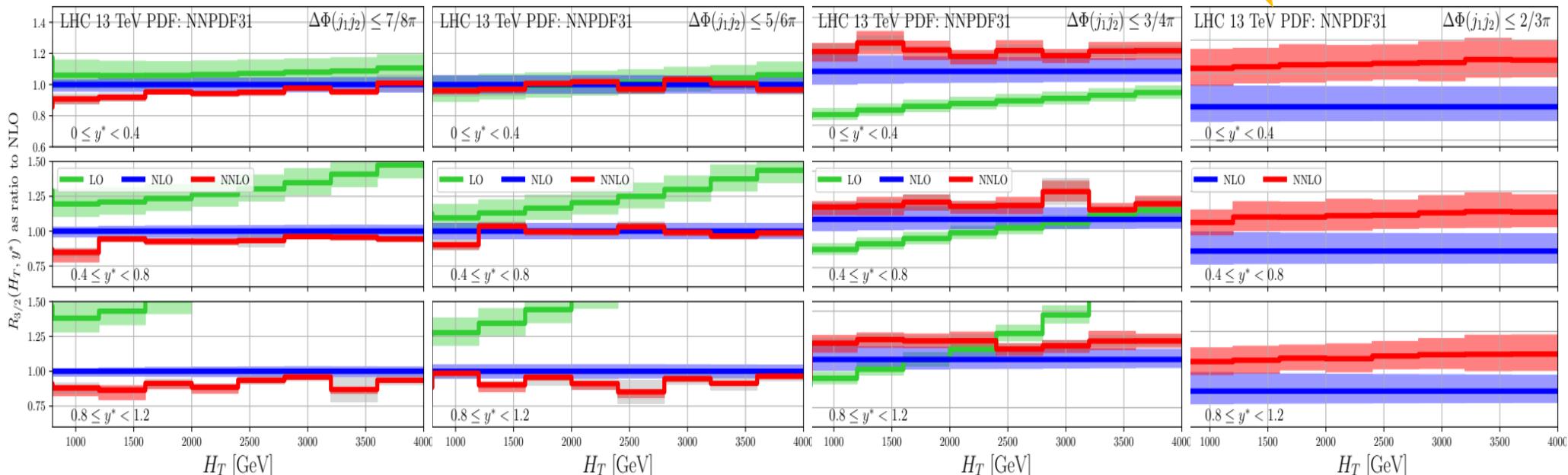
$$R_{32}(H_T, y^*, \phi_{\max}) = \frac{d\sigma_3(H_T, y^*, \phi_{12} < \phi_{\max})}{d\sigma_2(H_T, y^*)}$$

# Three jet production - azimuthal decorrelation

NNLO/NLO K-factor smaller than NLO/LO  
Scale dependence is reduced

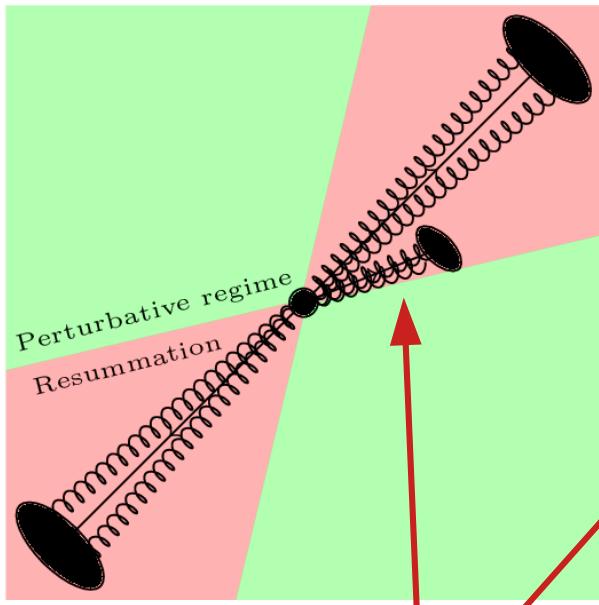


Work in progress: phasespace in [1805.04691]



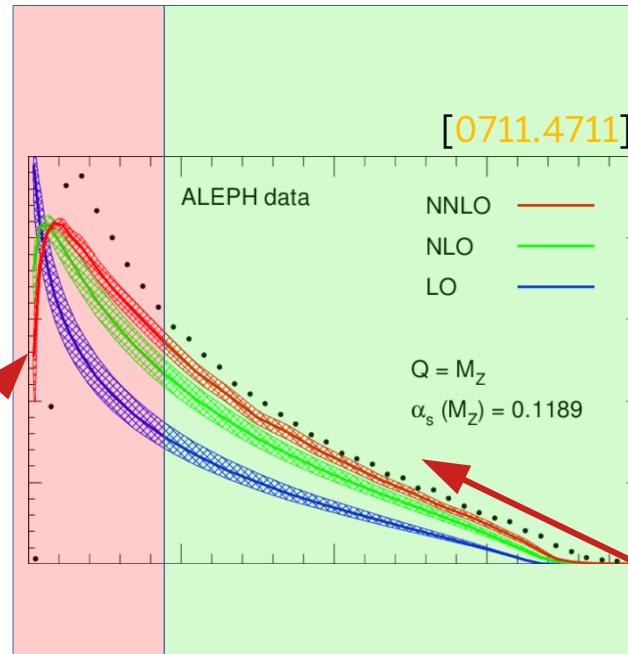
# Event-shapes regimes

Typically event-shapes measure departure from dijet topologies

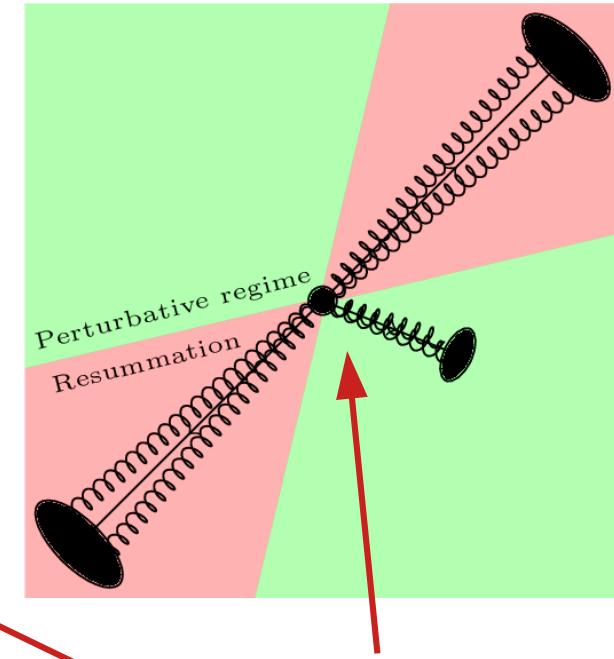


Anisotropic, dijet like

Sensitivity to resummation,  
non-perturbative effects



Example: 1-Thrust at LEP



Isotropic, multi-jet

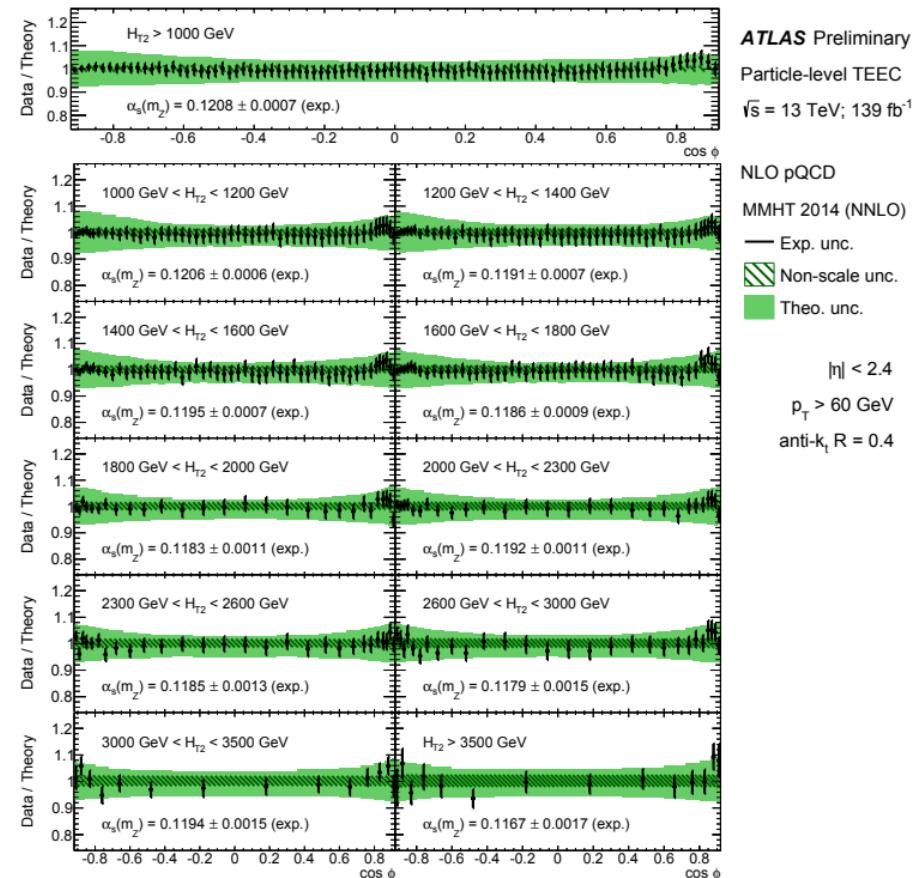
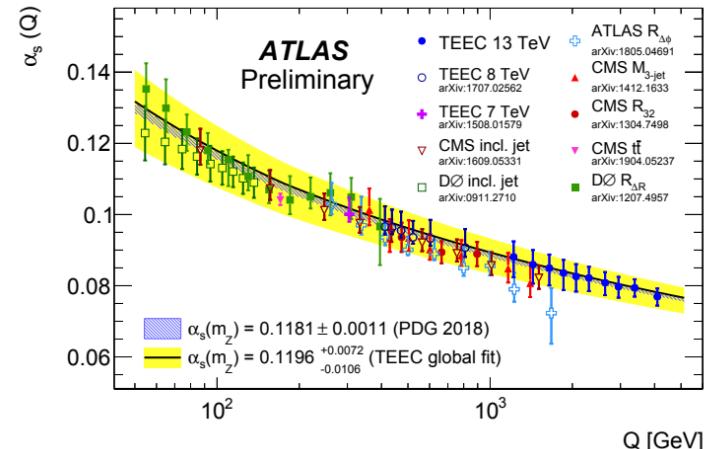
Sensitive to hard  
matrix elements

# Event-shapes at the LHC

Strong coupling measurements from event-shapes:  
 → three jet is leading contribution  
 → normalization through dijet rates

TEEC: Transverse Energy-Energy Correlation

$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} = \frac{1}{N} \sum_{A=1}^N \sum_{ij} \frac{E_{\perp,i}^A E_{\perp,j}^A}{\left( \sum_k E_{T,k}^A \right)^2} \delta(\cos \phi - \cos \phi_{ij})$$



[ATLAS-CONF-2020-025]

# Event-shapes at the LHC

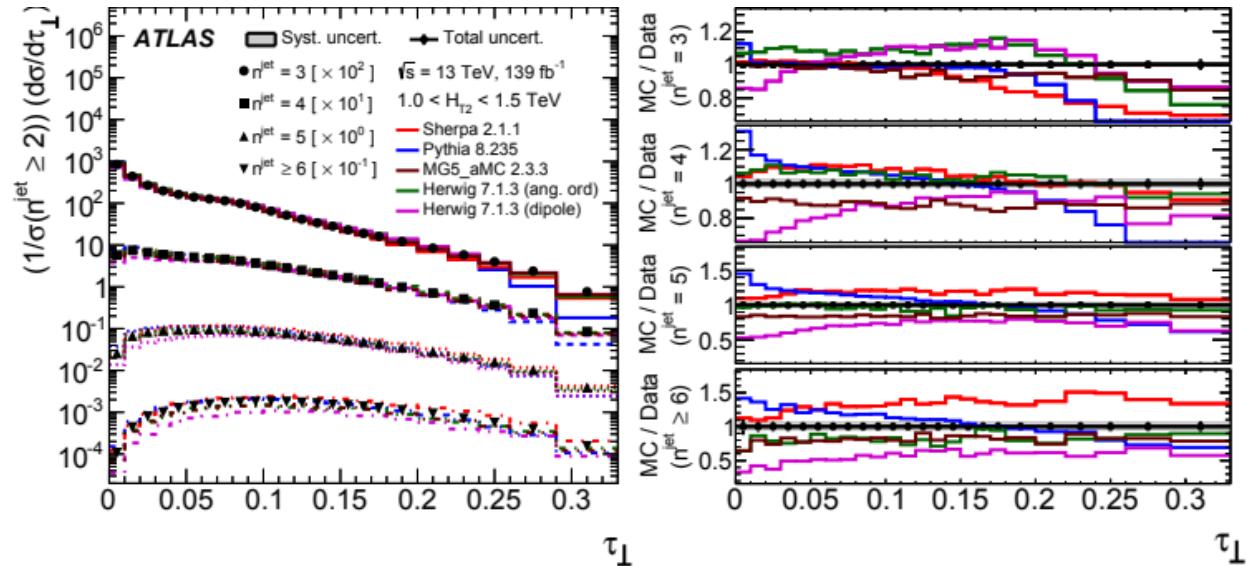
ATLAS measurement of event shapes [2007.12600]

Transverse Thrust:

$$\tau_T = 1 - \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}|}{\sum |\vec{p}_{T,i}|}$$

Sphericity tensor:

$$\mathcal{M}_{xyz} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{1}{|\vec{p}_i|} \begin{pmatrix} p_{x,i}^2 & p_{x,i}p_{y,i} & p_{x,i}p_{z,i} \\ p_{y,i}p_{x,i} & p_{y,i}^2 & p_{y,i}p_{z,i} \\ p_{z,i}p_{x,i} & p_{z,i}p_{y,i} & p_{z,i}^2 \end{pmatrix}$$



# Summary and Outlook

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R32 ratios with the sector-improved residue subtraction framework

- Full NNLO QCD predictions for dijet production available
  - scale choice important
  - sub-leading colour contributions small
- Three jets @ the LHC:
  - First predictions available with approximate two-loop contribution!
    - improved scale dependence and stabilized K-factors
    - $pT$  spectra, HT, double differential
  - Real radiation for  $2 \rightarrow 3$  can be handled.  
But efficiency is a concern and needs some attention!

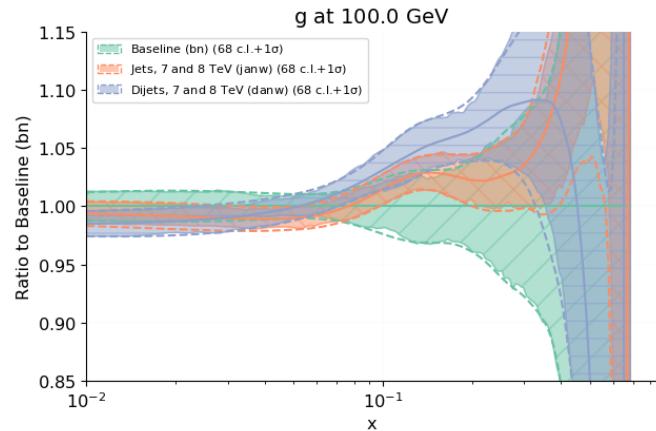
Many interesting applications ahead! → Event-shapes  
Stay tuned!

Thank you for your attention!

# Backup

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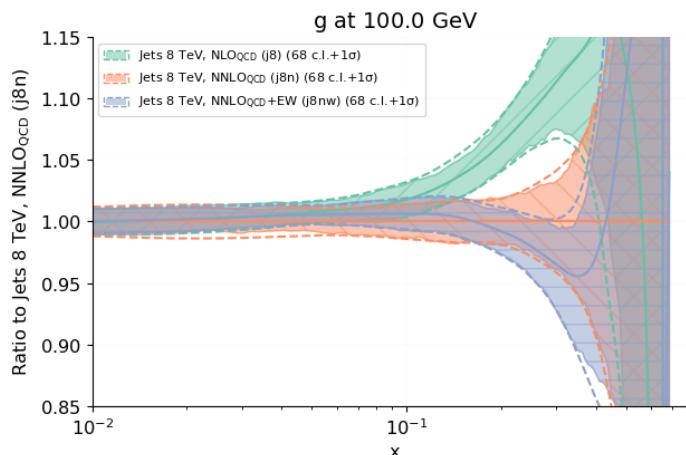
# Higher order pQCD: PDF fits with jets



Idea (quite old actually [[Giele'94](#)]):

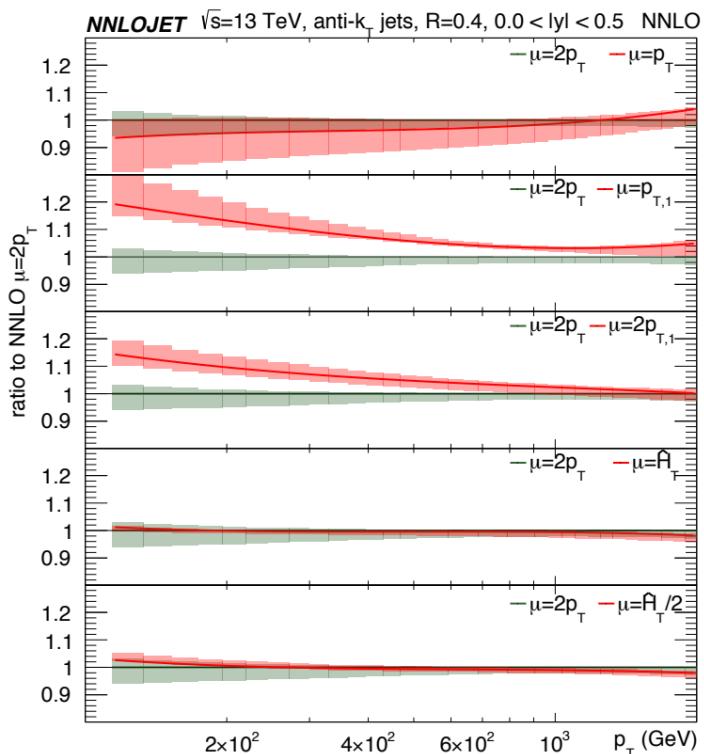
Here by a collaboration of NNLOJet and NNPDF [[Khalek'20](#)]:

Combine single inclusive and dijet triple differential measurements by ATLAS and CMS to constrain the large gluon-x



- Reduced uncertainty in large-x gluon PDF
- **NNLO QCD corrections crucial** to obtain consistent results between data sets
- NLO EW [[Dittmaier'12](#)] or full NLO corrections [[Frederix'17, Reyer'19](#)]

# Higher order pQCD: lessons from dijet



Detailed studies of  
scale dependence:  
Event-based choices vs.  
single jet choices  
[Currie'18]

Study of  
sub-leading colour  
effects  
in quark channels:  
smaller than  $O(1\%)$   
[Czakon'19]

