

**IFJ PAN**

**Theory Division – Particle Theory**

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## QUANTUM FIELD THEORY

### EXERCISES 4

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#### 4 Cross-sections

1. *Total (leading-order) cross-section for  $\phi\phi \rightarrow \phi\phi$*

Compute the differential cross section for the process  $\phi(p_A)\phi(p_B) \rightarrow \phi(p_1)\phi(p_2)$  in  $\phi^4$  scalar theory at  $\mathcal{O}(\lambda^2)$  in perturbation theory. Start from the formula for the differential cross section

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \left( \prod_{f=1}^2 \frac{d^3 p_f}{(2\pi)^3} \frac{1}{E_f} \right) (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2) |\mathcal{M}(p_A, p_B \rightarrow p_1 p_2)|^2$$

and

- (a) show that the final state phase space can be written as

$$d\Pi_2 \equiv \left( \prod_{f=1}^2 \frac{d^3 p_f}{(2\pi)^3} \frac{1}{E_f} \right) (2\pi)^4 \delta^{(4)}(p_A + p_B - p_1 - p_2) = d\cos\Theta \frac{1}{16\pi} \frac{2|\vec{p}_1|}{E_{cm}^2}$$

where  $E_{cm}$  and  $\Theta$  are the energy and the angle with respect to the scattering axis in the center-of-mass frame respectively.

- (b) show that the matrix element reduces to

$$|\mathcal{M}(p_A, p_B \rightarrow p_1 p_2)|^2 = \lambda^2$$

- (c) show that the total cross section is

$$\sigma = \frac{\lambda^2}{32\pi E_{cm}^2}$$