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# Quantum Field Theory

## Exercises 2

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## 2 Interaction Picture

1. *Time-evolution operator*

   (a) Consider the time-evolution operator

   $$U(t,t_0) = e^{iH_0(t-t_0)}e^{-iH(t-t_0)} \, , \tag{2.1}$$

   where $H_0$ is the free field hamiltonian, $H = H_0 + H_{\text{int}}$ the interacting Hamiltonian and $t_0$ the reference time in the definition of the interaction-picture field:

   $$\phi_I(t,x) = e^{iH_0(t-t_0)}\phi(t_0,\vec{x})e^{-iH_0(t-t_0)} \, . \tag{2.2}$$

   For a general time argument $t'$ we have

   $$U(t,t') = e^{iH_0(t-t_0)}e^{-iH(t-t')}e^{-iH_0(t'-t_0)} \, . \tag{2.3}$$

   Show that this operator obeys the identities

   $$U(t_1,t_2)U(t_2,t_3) = U(t_1,t_3) \quad \text{and} \quad U(t_1,t_3)\left[U(t_2,t_3)\right]^\dagger = U(t_1,t_2) \, . \tag{2.4}$$

   (b) Assuming $H_{\text{int},I}(t) = \int \mathrm{d}^3x \frac{\lambda}{4!}\phi_I^4(x)$, show that

   $$U(t,t_0) = \mathbb{1} + (-i)\int_{t_0}^{t} \mathrm{d}t_1 H_{\text{int},I}(t_1) + (-i)^2 \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 H_{\text{int},I}(t_1)H_{\text{int},I}(t_2) + \dots \tag{2.5}$$

   is a perturbative solution in $\lambda$ of

   $$i\frac{\partial}{\partial t}U(t,t_0) = H_{\text{int},I}(t)U(t,t_0) \, , \tag{2.6}$$

   by computing the $\lambda^n$ coefficient of both sides of the equation.