Techniques and phenomenology of NNLO QCD calculations for LHC processes

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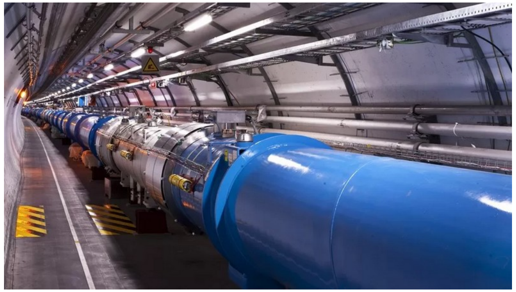
Outline

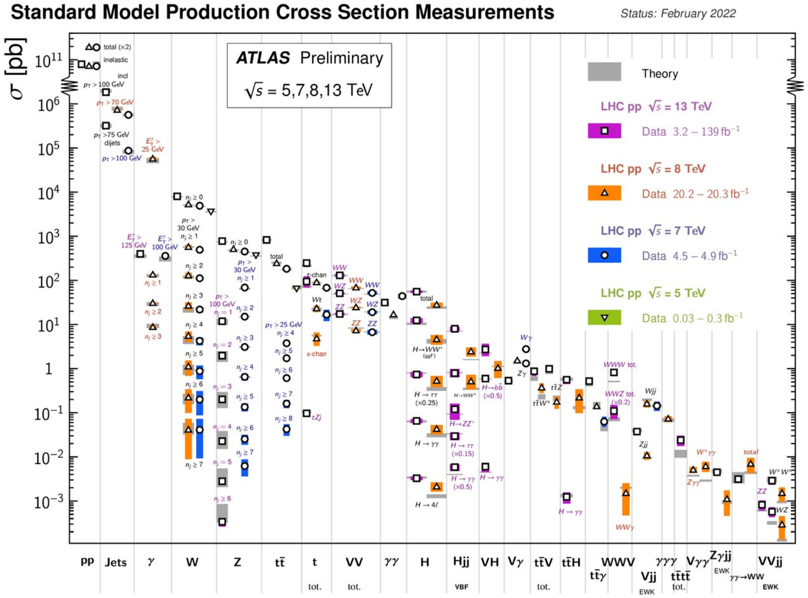
- Introduction
- Sector-improved residue subtraction
- Two-loop five-point amplitudes
- Pheno @ LHC:
  → Three-jet production through NNLO QCD
  → HighTEA
- Summary and Outlook

What are the fundamental building blocks of matter?

Scattering experiments

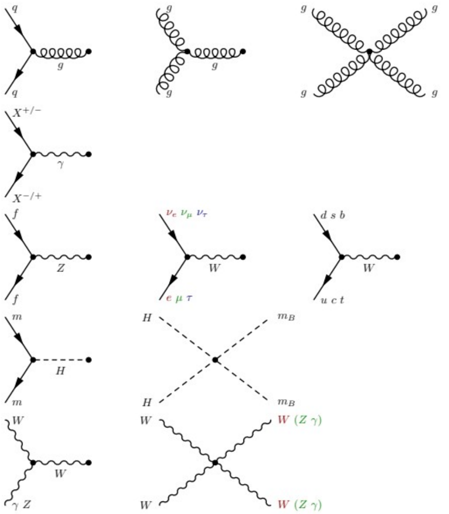


Credit: CERN



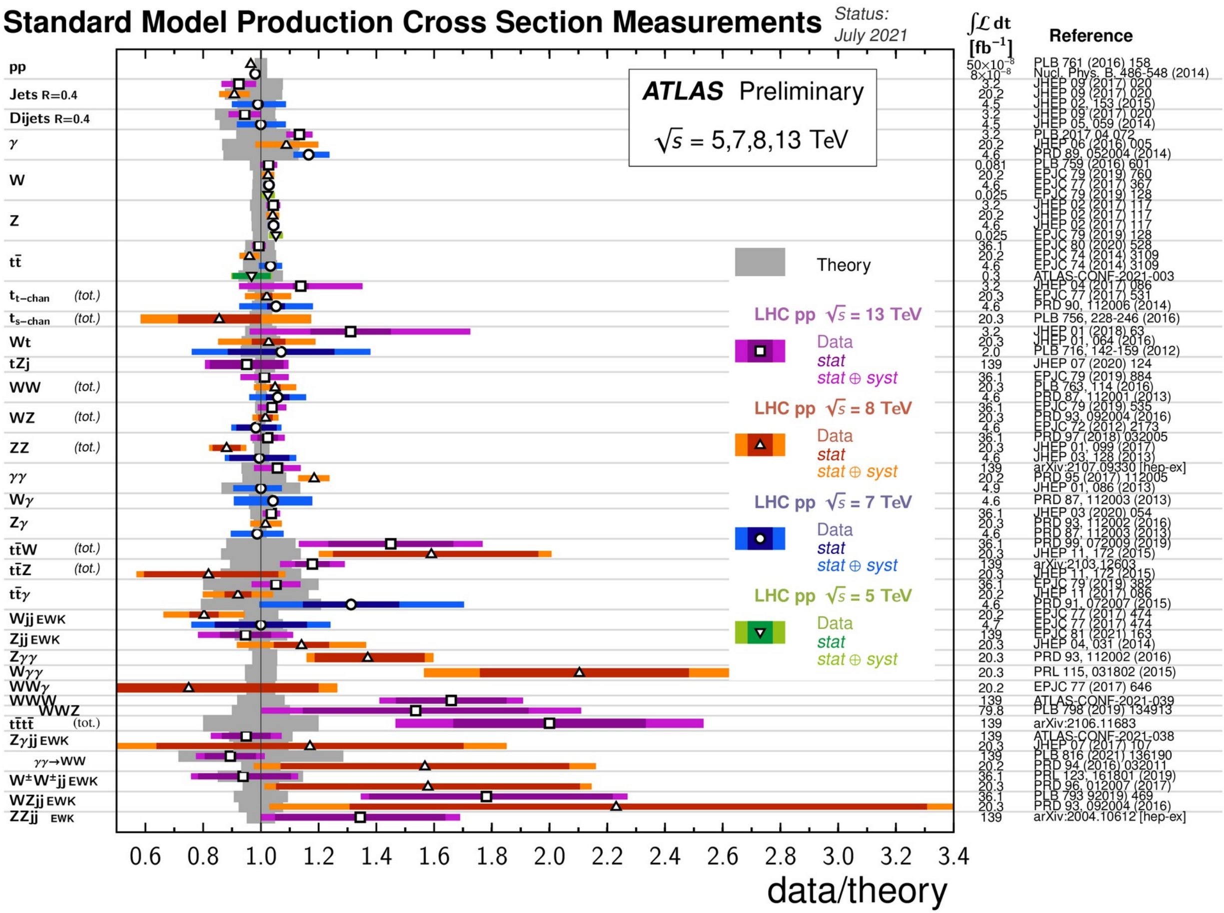
Credit: ATLAS

Theory/Model



Credit: Jack Lindon, CERN

SM measurements at the LHC



Precise measurements
<->
Precise theory

- improved SM understanding
- search for indirect NP signals

Precision predictions

Resummation

Fixed order perturbation theory

Parton-showers

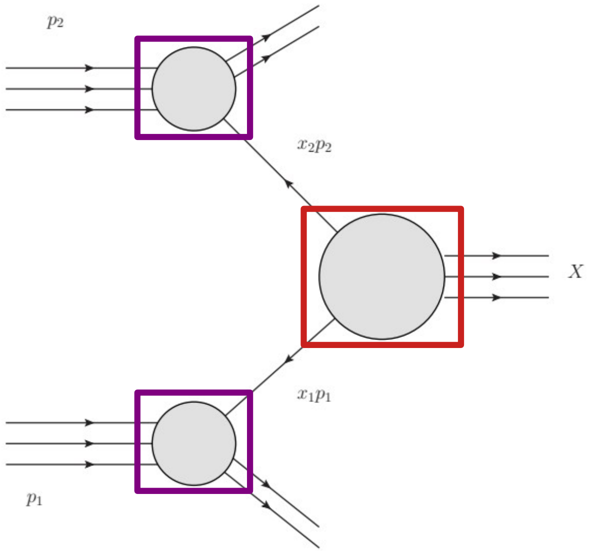
Precision theory predictions

Parametric input:
PDFs, couplings (α_s), ...

Soft physics:
MPI, colour reconnection,
...

Fragmentation/hadronisation

Perturbative QCD



Hadronic cross section:

$$\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat{\sigma}_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$$

Parton distribution functions: δ~1-3%

Perturbative expansion of partonic cross section:

$$\hat{\sigma}_{ab\to X} = \hat{\sigma}^{(0)}_{ab\to X} + \hat{\sigma}^{(1)}_{ab\to X} + \hat{\sigma}^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$$

Typical uncertainties from scale variations: δLO O(~100%) δNLO O(~10%) δNNLO (~1%)
(estimate for corrections from missing higher orders)

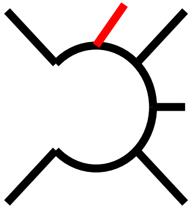
Next-to-leading order case

$$\hat{\sigma}^{(1)}_{ab} = \hat{\sigma}^{\mathrm{R}}_{ab} + \hat{\sigma}^{\mathrm{V}}_{ab} + \hat{\sigma}^{\mathrm{C}}_{ab}$$

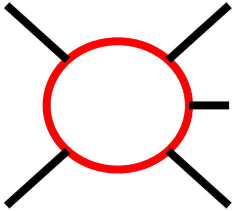
KLN theorem
sum is finite for sufficiently inclusive observables and regularization scheme independent

Each term separately infrared (IR) divergent:

Real corrections:



Virtual corrections:



$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

Phasespace integration over unresolved configurations

$$\hat{\sigma}^{\mathrm{V}}_{ab} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n}\, 2\mathrm{Re} \left\langle \mathcal{M}^{(0)}_{n} \middle| \mathcal{M}^{(1)}_{n} \right\rangle \mathrm{F}_{n}$$

Integration over loop-momentum

Collinear factorization: $\hat{\sigma}^{\mathrm{C}}_{ab} = (\text{single convolution})\, \mathrm{F}_n$

Slicing and Subtraction

Central idea: Divergences arise from infrared (IR, soft/collinear) limits → Factorization!

Slicing

$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int_{\delta(\Phi)\geq\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} + \frac{1}{2\hat{s}} \int_{\delta(\Phi)<\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1}$$

$$\approx \frac{1}{2\hat{s}} \int_{\delta(\Phi)\geq\delta_c} \mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} + \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_n\, \tilde{M}(\delta_c)\mathrm{F}_n + \mathcal{O}(\delta_c)$$

$$\ldots + \hat{\sigma}^{\mathrm{V}}_{ab} = \text{finite}$$

- Conceptually simple
- Recycling of lower computations
- Non-local cancellations/power-corrections
  → computationally expensive

Subtraction

$$\hat{\sigma}^{\mathrm{R}}_{ab} = \frac{1}{2\hat{s}} \int \left(\mathrm{d}\Phi_{n+1} \left\langle \mathcal{M}^{(0)}_{n+1} \middle| \mathcal{M}^{(0)}_{n+1} \right\rangle \mathrm{F}_{n+1} - \mathrm{d}\tilde{\Phi}_{n+1}\, \mathcal{S}\mathrm{F}_n \right) + \frac{1}{2\hat{s}} \int \mathrm{d}\tilde{\Phi}_{n+1}\, \mathcal{S}\mathrm{F}_n$$

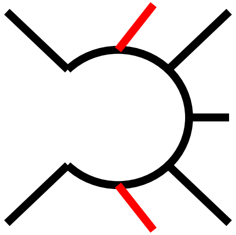
$$\frac{1}{2\hat{s}} \int \mathrm{d}\tilde{\Phi}_{n+1}\, \mathcal{S}\mathrm{F}_n = \frac{1}{2\hat{s}} \int \underline{\mathrm{d}\Phi_n\, \mathrm{d}\Phi_1}\, \mathcal{S}\mathrm{F}_n$$

Phasespace factorization
→ momentum mappings

- Conceptually more difficult
- Local subtraction → efficient
- Better numerical stability

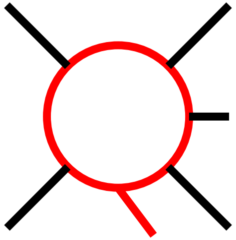
Partonic cross section beyond NLO

$$\hat{\sigma}_{ab}^{(2)} = \hat{\sigma}_{ab}^{\mathrm{VV}} + \hat{\sigma}_{ab}^{\mathrm{RV}} + \hat{\sigma}_{ab}^{\mathrm{RR}} + \hat{\sigma}_{ab}^{\mathrm{C2}} + \hat{\sigma}_{ab}^{\mathrm{C1}}$$



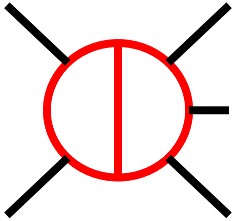
Real-Real

$$\hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle \mathrm{F}_{n+2}$$



Real-Virtual

$$\hat{\sigma}_{ab}^{\mathrm{RV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle \mathrm{F}_{n+1}$$



Virtual-Virtual

$$\hat{\sigma}_{ab}^{\mathrm{VV}} = \frac{1}{2\hat{s}} \int \mathrm{d}\Phi_{n} \left(2\mathrm{Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) \mathrm{F}_{n}$$

$\hat{\sigma}_{ab}^{\mathrm{C2}} = (\text{double convolution})\, \mathrm{F}_n$ $\qquad$ $\hat{\sigma}_{ab}^{\mathrm{C1}} = (\text{single convolution})\, \mathrm{F}_{n+1}$

Slicing and Subtraction

Central idea: Divergences arise from infrared (IR, soft/collinear) limits → Factorization!

NNLO QCD schemes

Slicing

qT-slicing [Catain'07],
N-jettiness slicing [Gaunt'15/Boughezal'15]

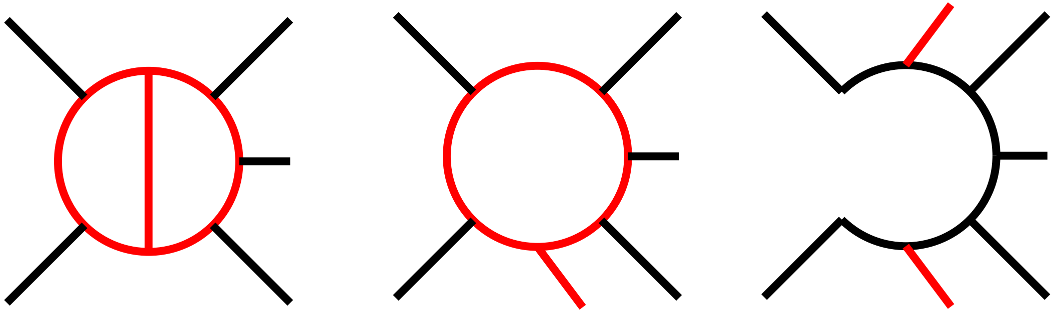
- Conceptually simple
- Recycling of lower computations
- Non-local cancellations/power-corrections
  → computationally expensive

Subtraction

Antenna [Gehrmann'05-'08], Colorful [DelDuca'05-'15],
Projetction [Cacciari'15], Geometric [Herzog'18],
Unsubtraction [Aguilera-Verdugo'19],
Nested collinear [Caola'17],
Sector-improved residue subtraction [Czakon'10-'14'19]

- Conceptually more difficult
- Local subtraction → efficient
- Better numerical stability

Sector-improved residue subtraction



Sector decomposition I

Considering working in CDR:

→ Virtuals are usually done in this regularization: $\hat{\sigma}_{ab}^{VV} = \sum_{i=-4}^{0} c_i \epsilon^i + \mathcal{O}(\epsilon)$

→ Can we write the real radiation as such expansion?

→ Difficult integrals, analytical impractical (except very simple observables)!

→ Numerics not possible, integrals are divergent → ε-poles!

How to extract these poles? → Sector decomposition!

Divide and conquer the phase space:

$$1 = \sum_{i,j}\left[\sum_{k}\mathcal{S}_{ij,k} + \sum_{k,l}\mathcal{S}_{i,k;j,l}\right] \quad \Rightarrow \quad \hat{\sigma}_{ab}^{\mathrm{RR}} = \frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2} \sum_{i,j}\left[\sum_{k}\mathcal{S}_{ij,k} + \sum_{k,l}\mathcal{S}_{i,k;j,l}\right]\left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)}\right\rangle \mathrm{F}_{n+2}$$

Sector decomposition II

Divide and conquer the phase space

- Each $\mathcal{S}_{i,k}$ (NLO), $\mathcal{S}_{ij,k}/\mathcal{S}_{i,k;j,l}$ (NNLO) has simpler divergences:
  - Soft limits of partons i and j
  - Collinear w.r.t partons k (and l) of partons i and j

$$\mathcal{S}_{i,k} = \frac{1}{D_1 d_{i,k}} \qquad D_1 = \sum_{ik} \frac{1}{d_{i,k}} \qquad d_{i,k} = \frac{E_i}{\sqrt{s}}(1-\cos\theta_{ik})$$

- Parametrization w.r.t. reference parton makes divergences explicit

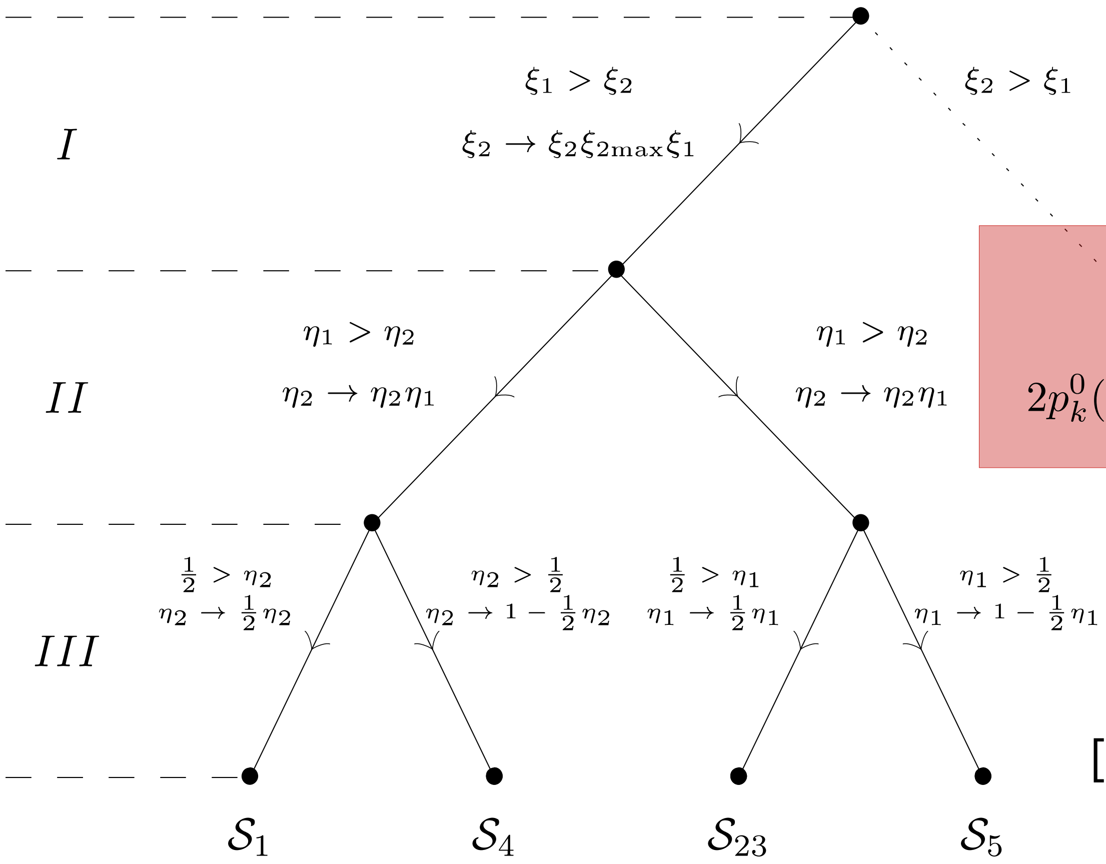
$$\hat{\eta}_i = \tfrac{1}{2}(1-\cos\theta_{ik}) \in [0,1] \qquad \hat{\xi}_i = \frac{u_i^0}{u_{\max}^0} \in [0,1]$$

- Example: Splitting function

$$\sim \frac{1}{s_{ik}} P(z) \qquad s_{ik} = (p_i + p_k)^2 = 2 p_k^0 u_{\max}^0 \xi_i \eta_i \qquad \sim \frac{1}{\eta_i \xi_i}$$

Sector decomposition II – triple collinear factorization

Three particle invariant does not factorize trivially:



Double soft factorization:

$$\theta(u_i^0 - u_j^0) + \theta(u_i^0 - u_j^0)$$

$$(p_k + u_i + u_j)^2 =$$
$$2p_k^0(\xi_i \eta_i u_{\max}^{0,i} + \xi_j \eta_j u_{\max}^{0,j} + \xi_i \xi_j \frac{u_{max}^{0,i} u_{max}^{0,j}}{p_k^0} \angle(u_i, u_j))$$

[Czakon'10,Caola'17]

Sector decomposition III

Factorized singular limits in each sector:

$$\frac{1}{2\hat{s}}\int \mathrm{d}\Phi_{n+2}\,\mathcal{S}_{kl,m}\left\langle \mathcal{M}^{(0)}_{n+2}\middle|\mathcal{M}^{(0)}_{n+2}\right\rangle \mathrm{F}_{n+2} = \sum_{\text{sub-sec.}}\int \mathrm{d}\Phi_n \prod \mathrm{d}x_i\,\underbrace{x_i^{-1-b_i\epsilon}}_{\text{singular}}\,\mathrm{d}\tilde{\mu}(\{x_i\})\underbrace{\prod x_i^{a_i+1}\,\langle \mathcal{M}_{n+2}|\mathcal{M}_{n+2}\rangle}_{\text{regular}}\,\mathrm{F}_{n+2}$$

$$x_i \in \{\eta_1, \xi_1, \eta_2, \xi_2\}$$

Regularization of divergences:

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$\int_0^1 \mathrm{d}x\,\left[x^{-1-b\epsilon}\right]_+ f(x) = \int_0^1 \frac{f(x)-f(0)}{x^{1+b\epsilon}}$$

Finite NNLO cross section

$$\hat{\sigma}_{ab}^{\rm RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2} \left\langle \mathcal{M}_{n+2}^{(0)} \middle| \mathcal{M}_{n+2}^{(0)} \right\rangle F_{n+2}$$

$$\hat{\sigma}_{ab}^{\rm RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1}\, 2{\rm Re} \left\langle \mathcal{M}_{n+1}^{(0)} \middle| \mathcal{M}_{n+1}^{(1)} \right\rangle F_{n+1}$$

$$\hat{\sigma}_{ab}^{\rm VV} = \frac{1}{2\hat{s}} \int d\Phi_{n} \left(2{\rm Re} \left\langle \mathcal{M}_{n}^{(0)} \middle| \mathcal{M}_{n}^{(2)} \right\rangle + \left\langle \mathcal{M}_{n}^{(1)} \middle| \mathcal{M}_{n}^{(1)} \right\rangle \right) F_{n}$$

$$\hat{\sigma}_{ab}^{\rm C1} = (\text{single convolution})\, F_{n+1}$$

$$\hat{\sigma}_{ab}^{\rm C2} = (\text{double convolution})\, F_{n}$$

↓ sector decomposition and master formula

$$x^{-1-b\epsilon} = \underbrace{\frac{-1}{b\epsilon}}_{\text{pole term}} + \underbrace{\left[x^{-1-b\epsilon}\right]_+}_{\text{reg. + sub.}}$$

$$(\sigma_F^{RR}, \sigma_{SU}^{RR}, \sigma_{DU}^{RR}) \quad (\sigma_F^{RV}, \sigma_{SU}^{RV}, \sigma_{DU}^{RV}) \quad (\sigma_F^{VV}, \sigma_{DU}^{VV}, \sigma_{FR}^{VV}) \quad (\sigma_{SU}^{C1}, \sigma_{DU}^{C1}) \quad (\sigma_{DU}^{C2}, \sigma_{FR}^{C2})$$

↓ re-arrangement of terms → 4-dim. formulation [Czakon'14, Czakon'19]

$$\underline{(\sigma_F^{RR})} \quad \underline{(\sigma_F^{RV})} \quad \underline{(\sigma_F^{VV})} \quad \underline{(\sigma_{SU}^{RR}, \sigma_{SU}^{RV}, \sigma_{SU}^{C1})} \quad \underline{(\sigma_{DU}^{RR}, \sigma_{DU}^{RV}, \sigma_{DU}^{VV}, \sigma_{DU}^{C1}, \sigma_{DU}^{C2})} \quad \underline{(\sigma_{FR}^{RV}, \sigma_{FR}^{VV}, \sigma_{FR}^{C2})}$$

separately finite: ε poles cancel

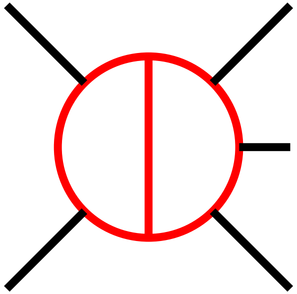
C++ framework

- Formulation allows efficient algorithmic implementation → STRIPPER
- **High degree of automation**:
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers + O(100) hardcoded:
    AvH, OpenLoops, Recola, NJET, HardCoded

    → In practice: Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

Two-loop five-point amplitudes

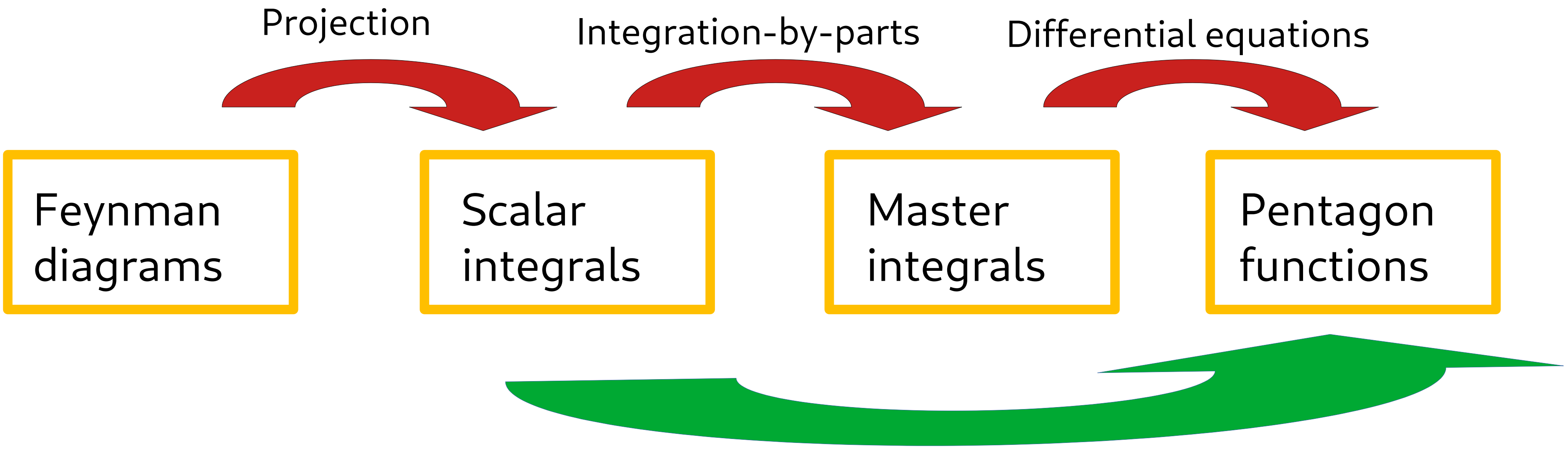
Massless:
[Chawdry'19'20'21] (3A+2j,2A+3j)
[Abreu'20'21] (3A+2j,5j)
[Agarwal'21] (2A+3j)
[Badger'21'23] (5j,gggAA,jjjjA)



1 external mass:
[Abreu'21] (W+4j)
[Badger'21'22] (Hqqgg,W4q,Wajjj)
[Hartanto'22] (W4q)

Overview

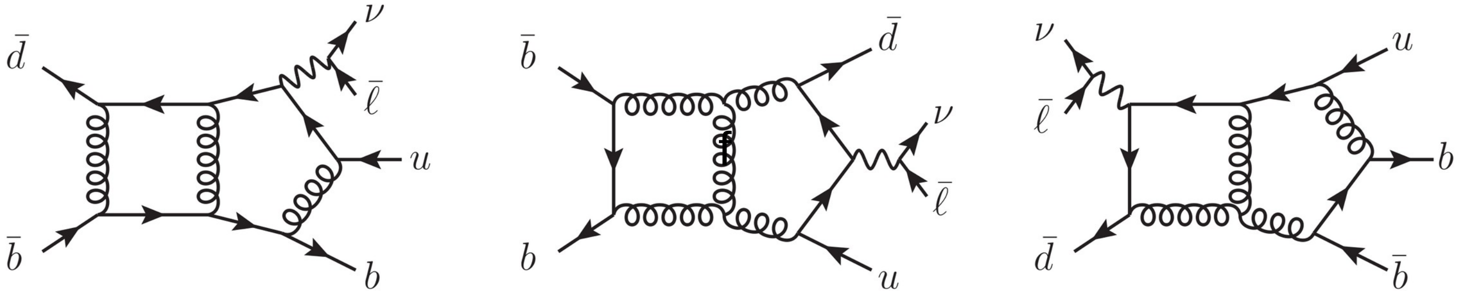
Old school approach:



Automated framework using finite fields to avoid expression swell based on FiniteFlow [Peraro'19]

Projection to scalar integrals

Generate diagrams (contributing to leading-colour) with QGRAF



Factorizing decay: $A_6^{(L)} = A_5^{(L)\mu} D_\mu P$ $\quad M_6^{2(L)} = \sum_{\text{spin}} A_6^{(0)^*} A_6^{(L)} = M_5^{(L)\mu\nu} D_{\mu\nu} |P|^2$

Projection on scalar functions (FORM+Mathematica):
→ anti-commuting γ_5 + Larin prescription

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}$$

$$\Rightarrow \quad a_i^{(L)} = a_i^{(L),\text{even}} + \text{tr}_5 a_i^{(L),\text{odd}} \qquad a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

Integration-By-Parts reduction

$$a_i^{(L),p} = \sum_i c_{j,i}(\{p\},\epsilon)\mathcal{I}(\{p\},\epsilon)$$

→ prohibitively large number of integrals

$$\mathcal{I}_i(\{p\},\epsilon) \equiv \mathcal{I}(\vec{n_i},\{p\},\epsilon) = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\})$$

Integration-By-Parts identities connect different integrals → system of equations
→ only a small number of independent "master" integrals

$$0 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d}\frac{\mathrm{d}^d k_2}{(2\pi)^d} l_\mu \frac{\partial}{\partial l^\mu}\prod_{k=1}^{11} D_k^{-n_{i,k}}(\{p\},\{k\}) \quad \text{with} \quad l \in \{p\}\cap\{k\}$$

LiteRed (+ Finite Fields)

$$a_i^{(L),p} = \sum_i d_{j,i}(\{p\},\epsilon)\mathrm{MI}(\{p\},\epsilon)$$

Master integrals & finite remainder

Differential Equations: $\mathrm{d}\vec{\mathrm{MI}} = \mathrm{d}A(\{p\}, \epsilon)\vec{\mathrm{MI}}$

Canonical basis: $\mathrm{d}\vec{\mathrm{MI}} = \epsilon \mathrm{d}\tilde{A}(\{p\})\vec{\mathrm{MI}}$

[Remiddi, 97]
[Gehrmann, Remiddi, 99]
[Henn, 13]

Simple iterative solution

$$\mathrm{MI}_i = \sum_w \epsilon^w \tilde{\mathrm{MI}}_i^w \quad \text{with} \quad \tilde{\mathrm{MI}}_i^w = \sum_j c_{i,j} m_j$$

Chen-iterated integrals
"Pentagon"-functions

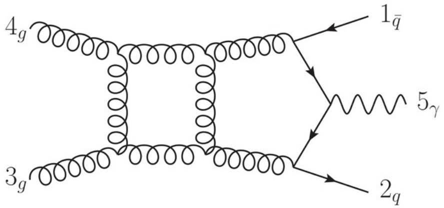
[Chicherin, Sotnikov, 20]
[Chicherin, Sotnikov, Zoia, 21]

Putting everything together (and removing of IR poles):

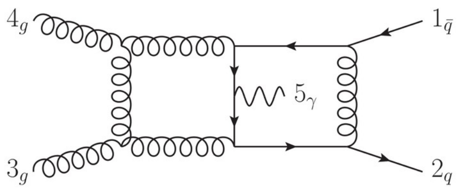
$$f_i^{(L),p} = a_i^{(L),p} - \text{poles} \qquad f_i^{(L),p} = \sum_j c_{i,j}(\{p\}) m_j + \mathcal{O}(\epsilon)$$

Reconstruction of Amplitudes

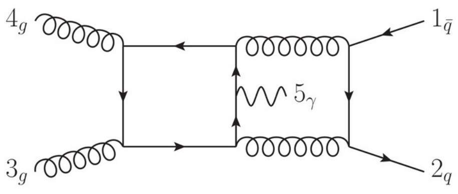
[Badger'21]



$A^{(2),N_c^2}_{34;q}$, $A^{(2),N_c}_{\delta;q}$



$A^{(2),1}_{34;q}$, $A^{(2),N_c}_{\delta;q}$, $A^{(2),1/N_c}_{\delta;q}$



$A^{(2),N_c}_{34;l}$, $A^{(2),1/N_c}_{34;l}$, $A^{(2),1/N_c^2}_{\delta;l}$

New optimizations

- Syzygy's to simplify IBPs
- Exploitation of Q-linear relations
- Denominator Ansaetze
- On-the-fly partial fractioning

amplitude	helicity	original	stage 1	stage 2	stage 3	stage 4
$A^{(2),1}_{34;q}$	$-++-+$	94/91	74/71	74/0	22/18	22/0
$A^{(2),1}_{34;q}$	$-+-++$	93/89	90/86	90/0	24/14	18/0
$A^{(2),1/N_c^2}_{34;q}$	$-++-+$	90/88	73/71	73/0	23/18	22/0
$A^{(2),1/N_c^2}_{34;q}$	$-+-++$	90/86	86/82	86/0	24/14	19/0
$A^{(2),1/N_c}_{34;l}$	$-+-++$	89/82	74/67	73/0	27/14	20/0
$A^{(2),1/N_c}_{34;l}$	$-++-+$	85/81	61/58	60/0	27/18	20/0
$A^{(2),N_c^2}_{34;q}$	$-+-++$	58/55	54/51	53/0	20/16	20/0

Massive reduction of complexity

Three-jet production through NNLO QCD

Multi-jet observables

Test of pQCD and extraction of strong coupling constant

NLO theory unc. (MHO) > experimental unc.

- NNLO QCD needed for precise theory-data comparisons

  → Restricted to two-jet data [Currie'17+later][Czakon'19]

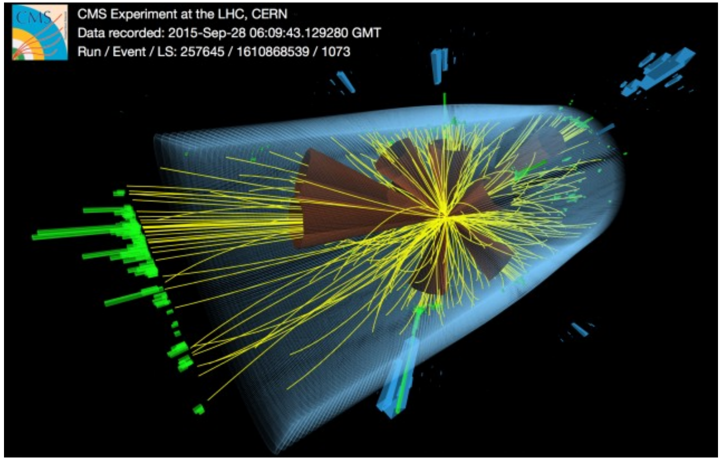
- New NNLO QCD three-jet → access to more observables
  - Jet ratios

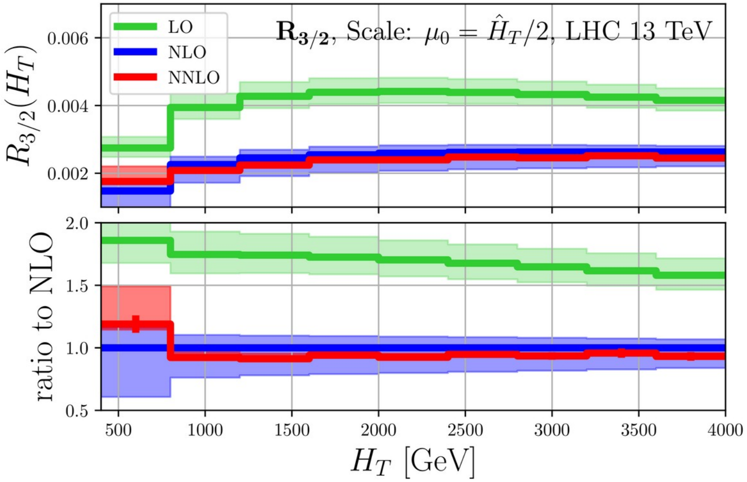
    Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
    Czakon, Mitov, **Poncelet** [2106.05331]

$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

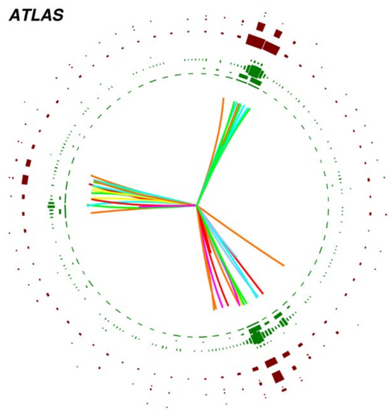
  - Event shapes

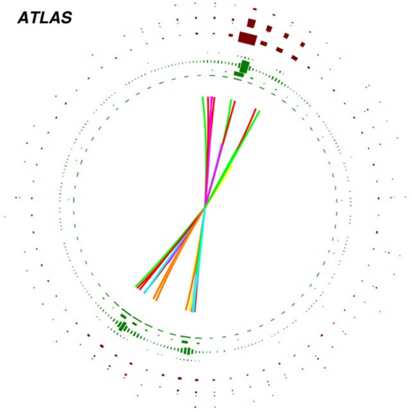
    NNLO QCD corrections to event shapes at the LHC
    Alvarez, Cantero, Czakon, Llorente, Mitov, **Poncelet** [2301.01086]





Encoding QCD dynamics in event shapes





Using (global) event information to separate different regimes of QCD event evolution:

- **Thrust & Thrust-Minor**

$$T_\perp = \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,, \quad \text{and} \quad T_m = \frac{\sum_i |\vec{p}_{T,i} \times \hat{n}_\perp|}{\sum_i |\vec{p}_{T,i}|}\,.$$

- **Energy-energy correlators**

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

Separation of energy scales: $H_{T,2} = p_{T,1} + p_{T,2}$

Ratio to 2-jet:

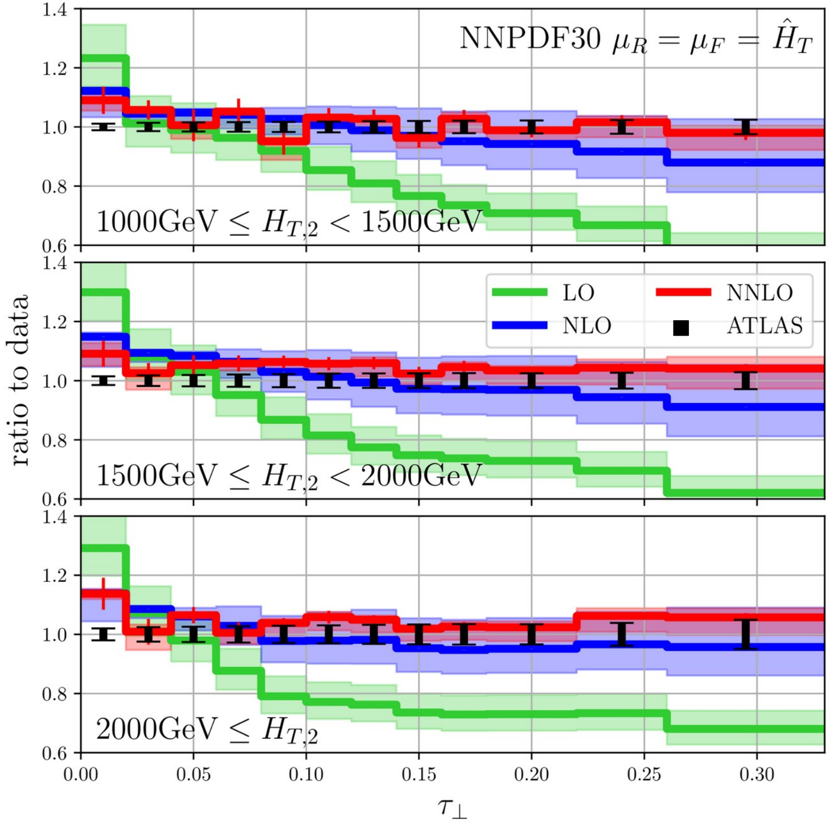
$$R^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0}) = \frac{\mathrm{d}\sigma_3^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}{\mathrm{d}\sigma_2^i(\mu_R, \mu_F, \mathrm{PDF}, \alpha_{S,0})}$$

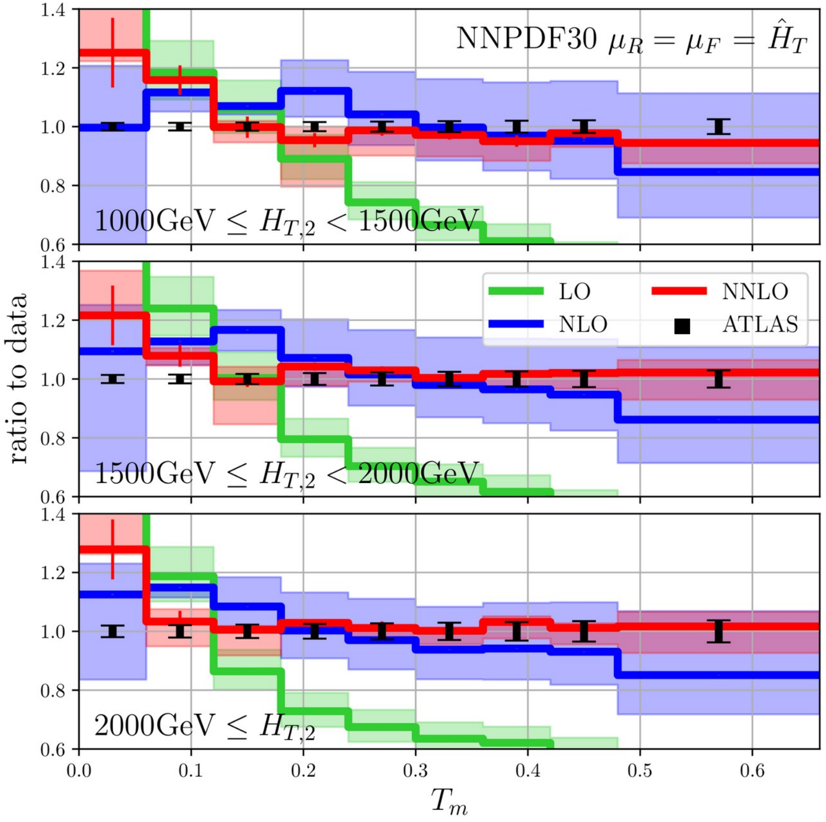
Here: use jets as input → experimentally advantageous (better calibrated, smaller non-pert.)

Transverse Thrust @ NNLO QCD

NNLO QCD corrections to event shapes at the LHC
Alvarez, Cantero, Czakon, Llorente, Mitov, Poncelet [2301.01086]

ATLAS [2007.12600]

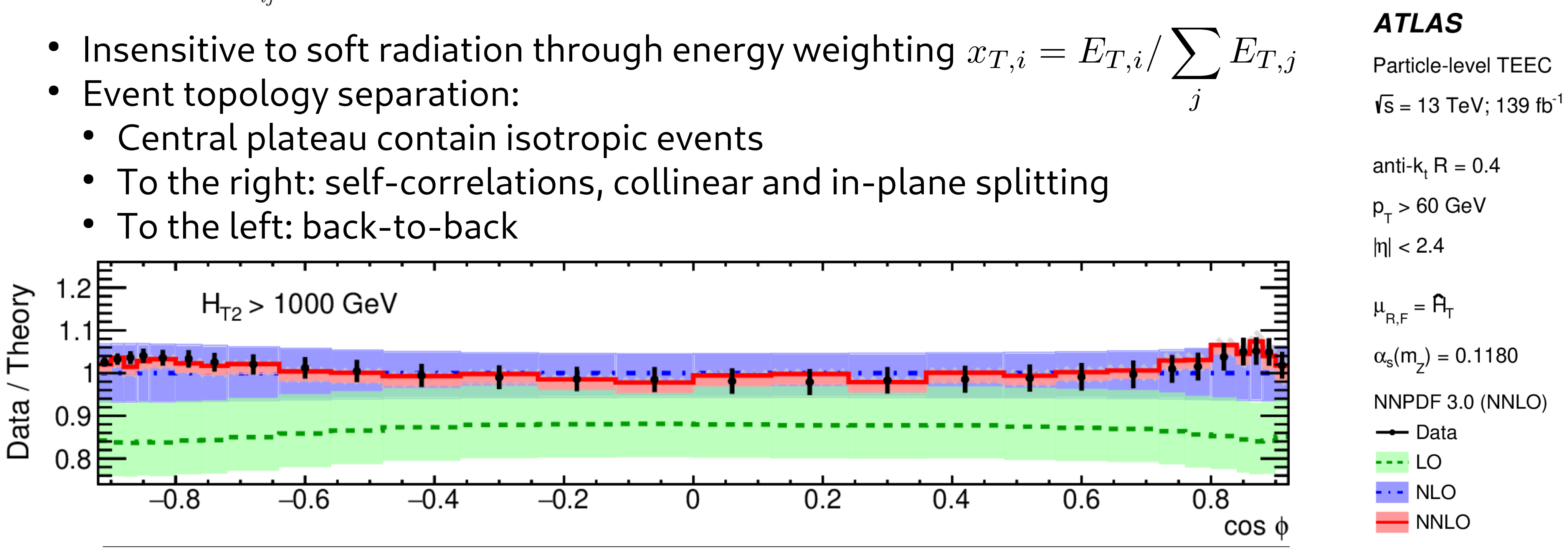




The transverse energy-energy correlator

$$\frac{1}{\sigma_2}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Delta\phi} = \frac{1}{\sigma_2}\sum_{ij}\int \frac{\mathrm{d}\sigma\; x_{\perp,i}x_{\perp,j}}{\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}}\delta(\cos\Delta\phi - \cos\Delta\phi_{ij})\mathrm{d}x_{\perp,i}\mathrm{d}x_{\perp,j}\mathrm{d}\cos\Delta\phi_{ij}\,,$$

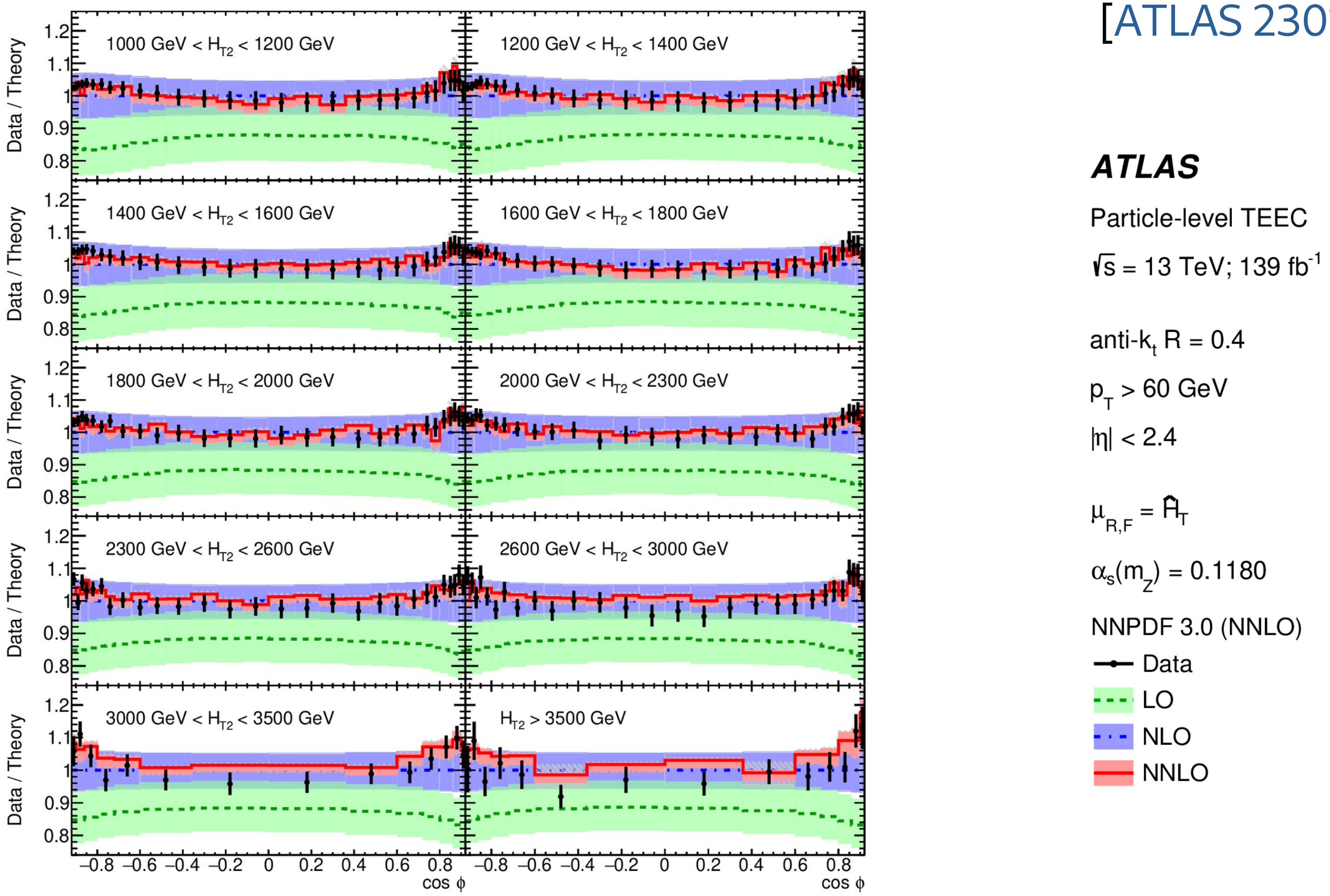
- Insensitive to soft radiation through energy weighting $x_{T,i} = E_{T,i}/\sum_j E_{T,j}$
- Event topology separation:
  - Central plateau contain isotropic events
  - To the right: self-correlations, collinear and in-plane splitting
  - To the left: back-to-back



[ATLAS 2301.09351]

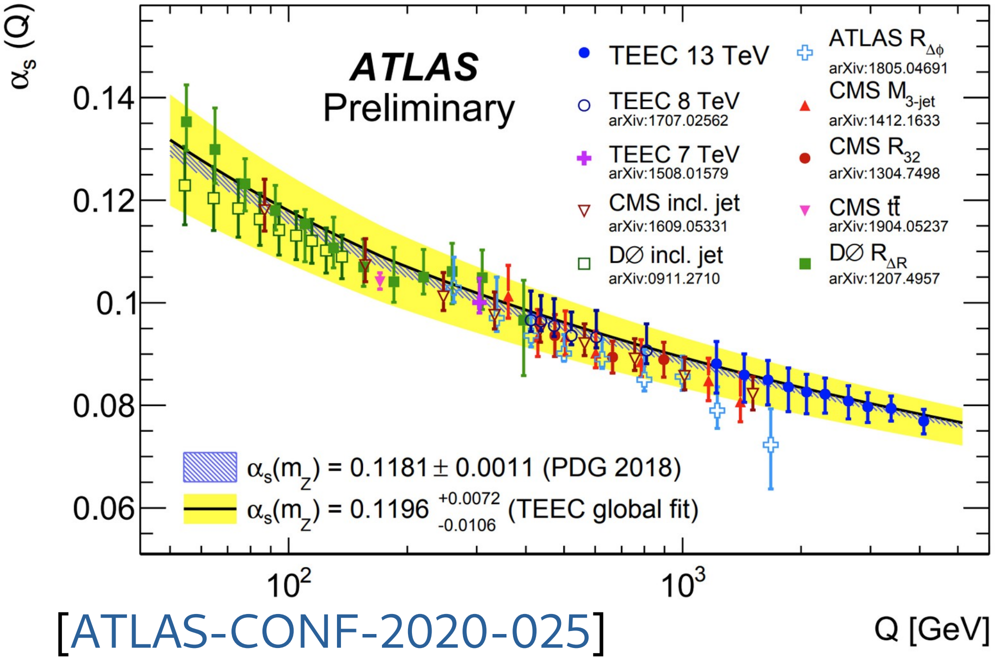
Double differential TEEC

[ATLAS 2301.09351]



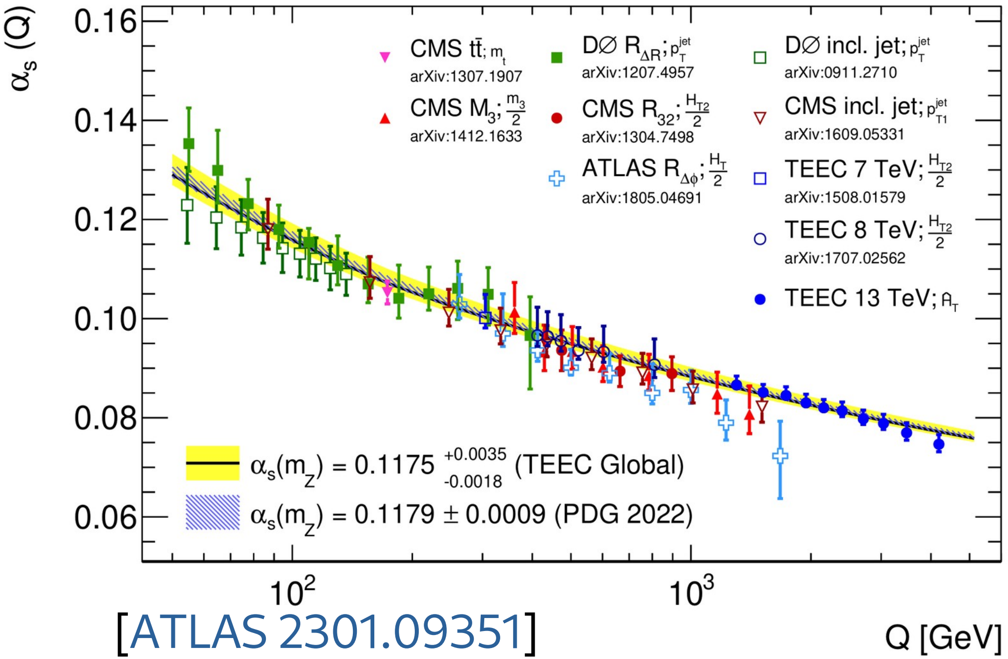
Running of α_S

NLO QCD



[ATLAS-CONF-2020-025]

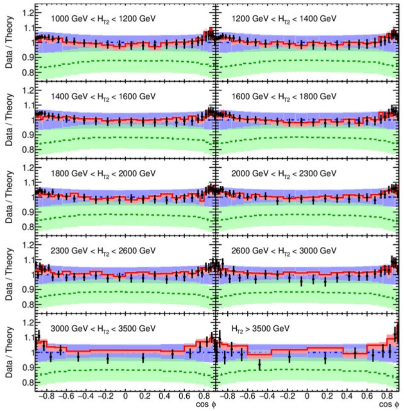
NNLO QCD



[ATLAS 2301.09351]

HighTEA

HighTEA



= ~100 MCPUh

How to make this more efficient/environment-friendly/accessible/faster?

high tea

for your freshly brewed analysis

https://www.precision.hep.phy.cam.ac.uk/hightea

HighTEA: **High energy Theory Event Analyser**
[2304.05993]

Michał Czakon,[a] Zahari Kassabov,[b] Alexander Mitov,[c] Rene Poncelet,[c] Andrei Popescu[c]

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[b] *DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom*
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Basic idea

- Database of precomputed “Theory Events”
  - **Equivalent to a full fledged computation**
  - Currently this means partonic fixed order events
  - Extensions to included showered/resummed/hadronized events is feasible
  - (Partially) Unweighting to increase efficiency
- Analysis of the data through an user interface
  - Easy-to-use
  - Fast
  - Flexible:
    - Observables from basic 4-momenta
    - Free specification of bins
    - Renormalization/Factorization Scale variation
    - PDF (member) variation
    - Specify phase space cuts

Not so new idea:
LHE [Alwall et al ‘06],
Ntuple [BlackHat ‘08’13],

Factorizations

Factorizing renormalization and factorization scale dependence:

$$w_s^{i,j} = w_{\mathrm{PDF}}(\mu_F, x_1, x_2) w_{\alpha_s}(\mu_R) \left(\sum_{i,j} c_{i,j} \ln\left(\mu_R^2\right)^i \ln\left(\mu_F^2\right)^j \right)$$

PDF dependence:

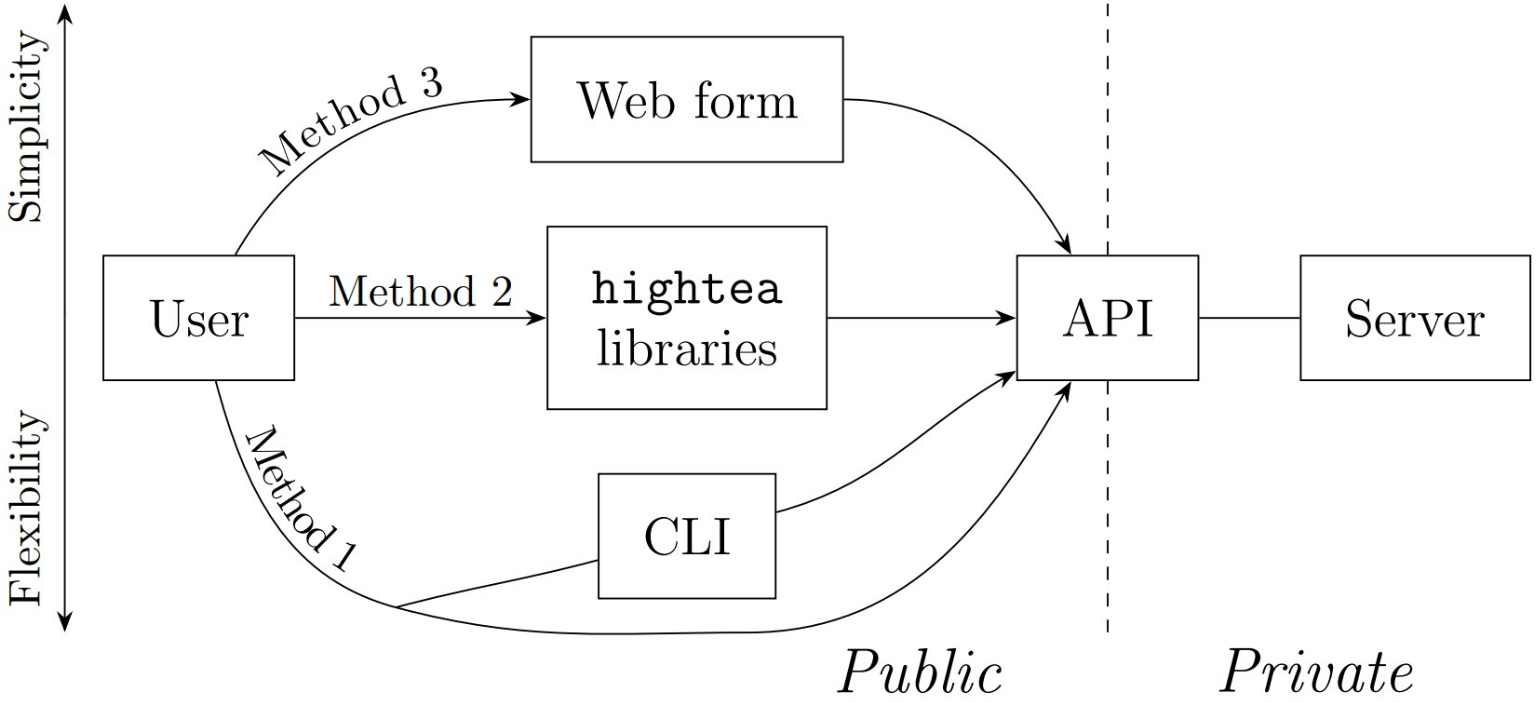
$$w_{\mathrm{PDF}}(\mu, x_1, x_2) = \sum_{ab \in \mathrm{channel}} f_a(x_1, \mu) f_b(x_2, \mu)$$

α_s dependence:

$$w_{\alpha_s}(\mu) = (\alpha_s(\mu))^m$$

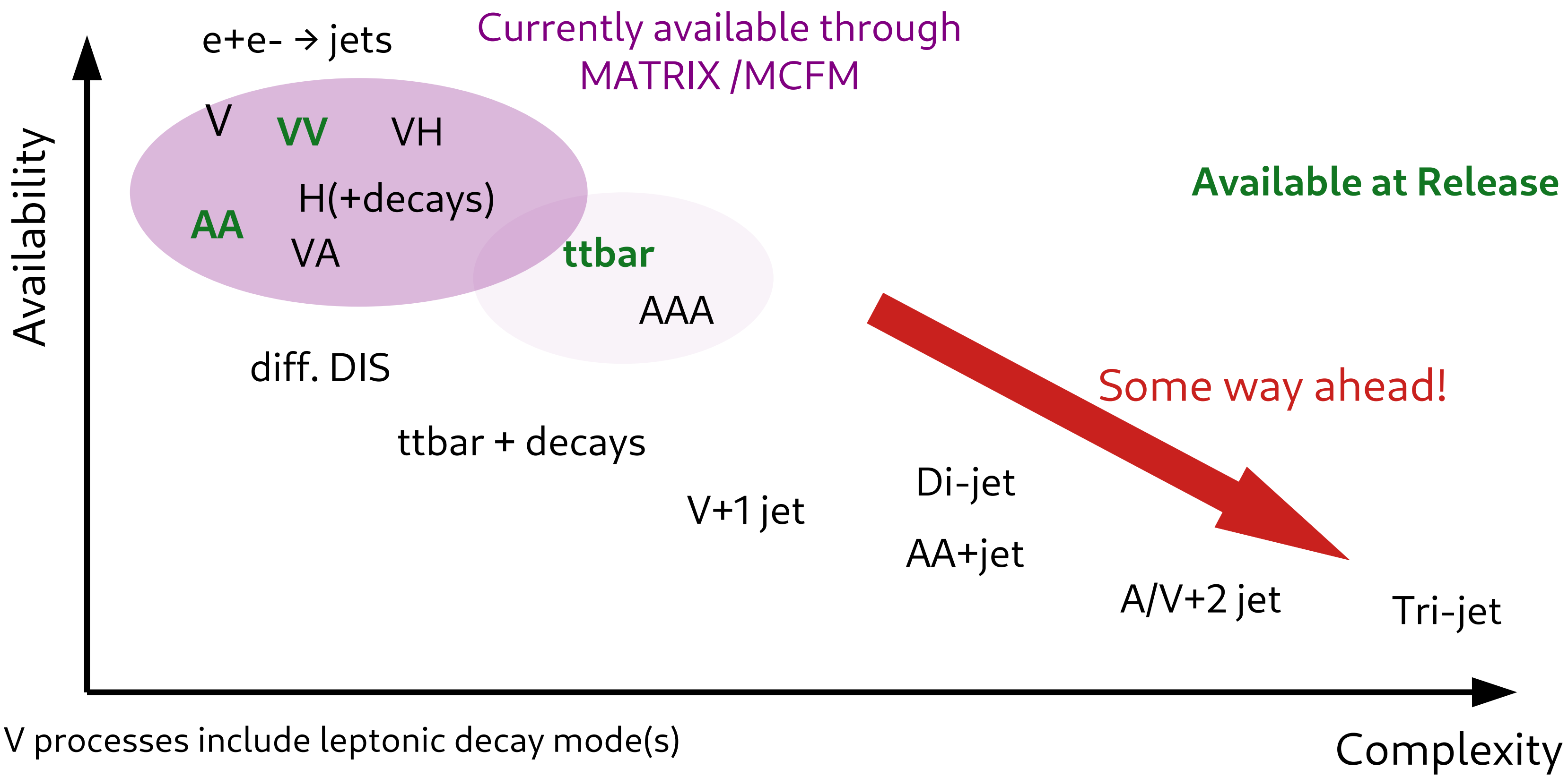
Allows **full control over scales and PDF**

HighTEA interface



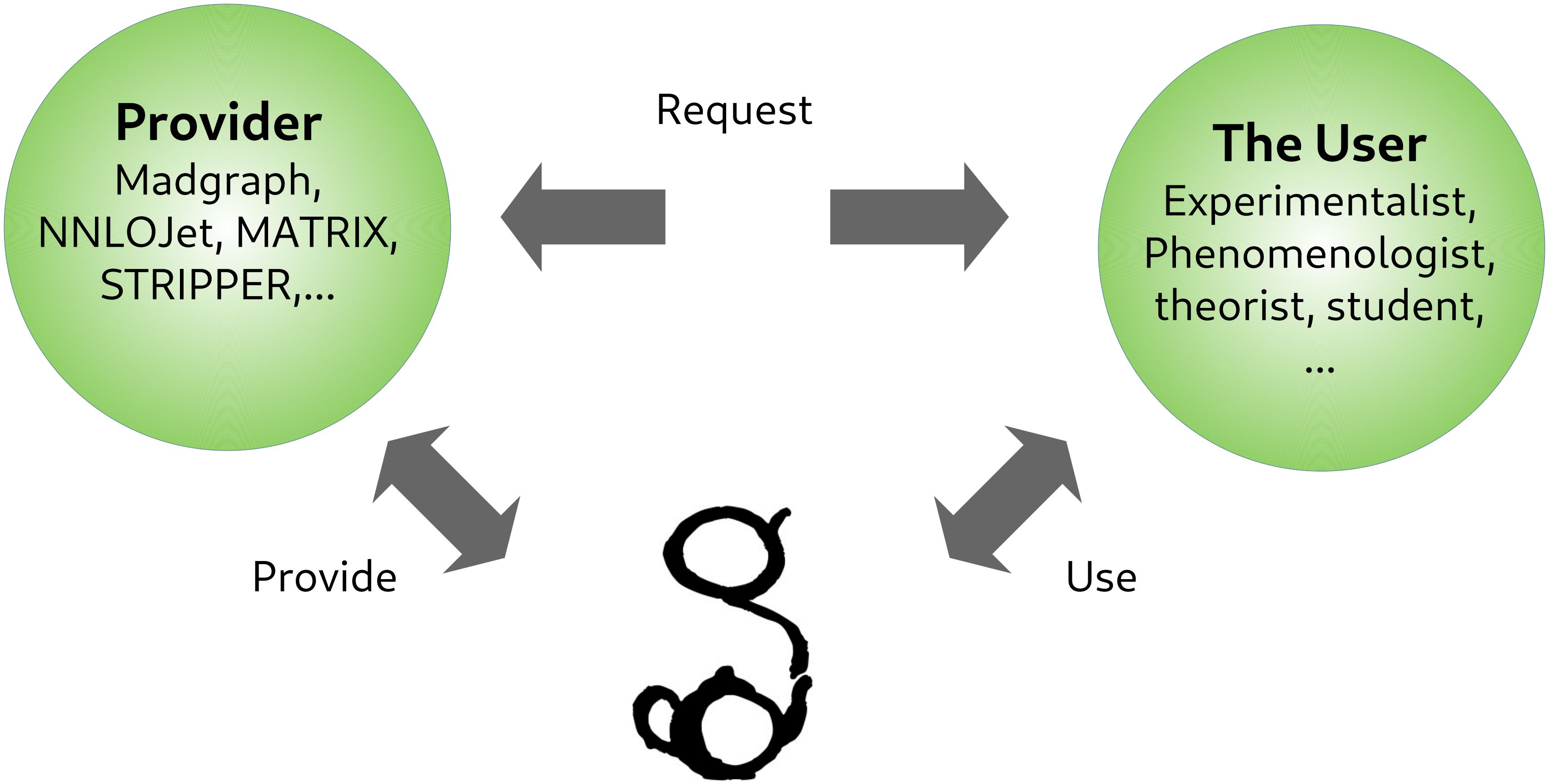
Available Processes

Processes currently implemented in our STRIPPER framework through NNLO QCD



* V processes include leptonic decay mode(s)

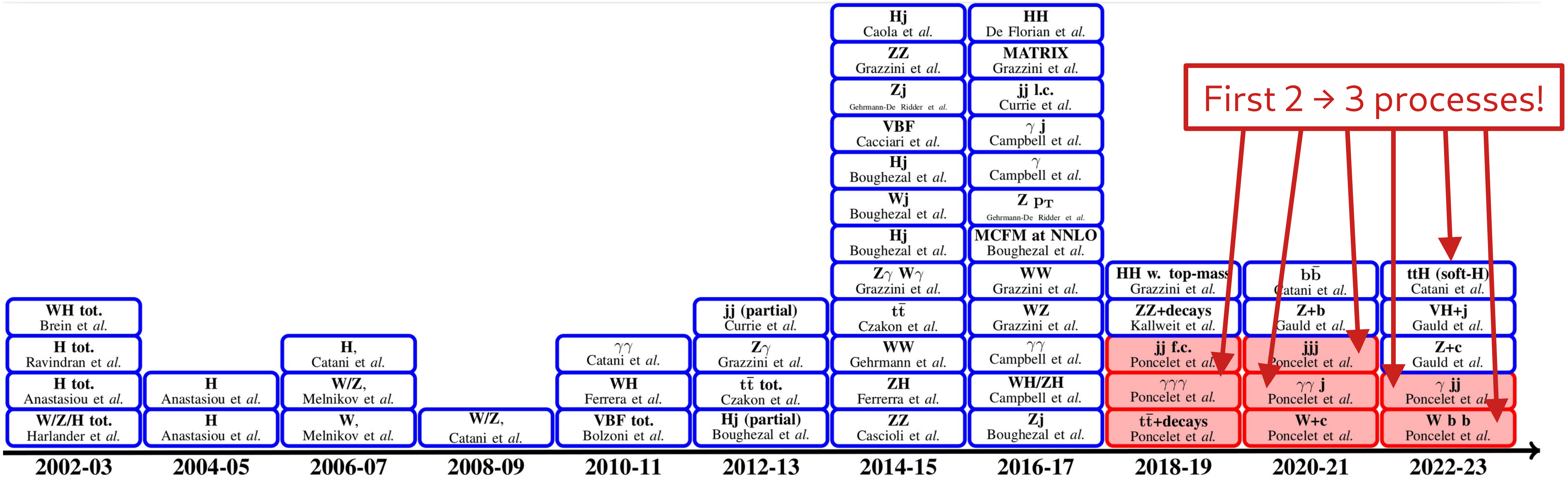
The Vision



Provider
Madgraph, NNLOJet, MATRIX, STRIPPER,...
Request
The User
Experimentalist, Phenomenologist, theorist, student, ...
Provide
Use

Summary & Outlook

The NNLO QCD revolution



Take home message

- Subtraction at NNLO QCD gains maturity, challenges remain…
  - Efficiency
  - Still difficult to run codes
  - Database like HighTEA possible solution?
- Two-loop matrix elements for high multiplicity are the single most significant bottleneck for NNLO QCD calculations
  - 2 to 3 massless ME completed in full colour (~10 years of work of ~5 research groups)
  - 1 mass MEs next challenge…
- NNLO QCD is a staple for SM precision phenomenology
  - matching to parton-shower is important next step…

Backup

Improved phase space generation

Phase space cut and differential observable introduce

mis-binning : mismatch between kinematics in subtraction terms

→ leads to increased variance of the integrand

→ slow Monte Carlo convergence

New phase space parametrization [Czakon'19]:

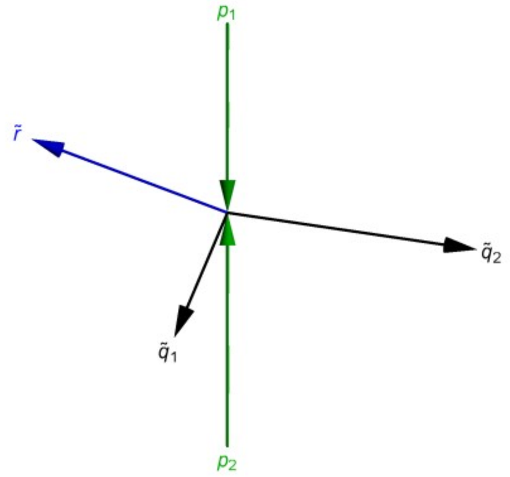
Minimization of # of different subtraction kinematics in each sector

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

Mapping from n+2 to n particle phase space: $\{P, r_j, u_k\} \to \left\{\tilde{P}, \tilde{r}_j\right\}$



Requirements:

- Keep direction of reference r fixed
- Invertible for fixed u_i: $\left\{\tilde{P}, \tilde{r}_j, u_k\right\} \to \{P, r_j, u_k\}$
- Preserve Born invariant mass: $q^2 = \tilde{q}^2\,,\;\; \tilde{q} = \tilde{P} - \sum_{j=1}^{n_{fr}} \tilde{r}_j$

Main steps:

- Generate Born configuration
- Generate unresolved partons u_i
- Rescale reference momentum $r = x\tilde{r}$
- Boost non-reference momenta of the Born configuration

Improved phase space generation

New phase space parametrization:

Minimization of # of different subtraction kinematics in each sector

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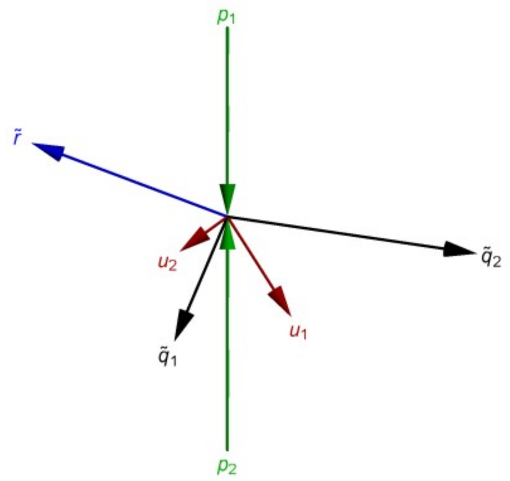

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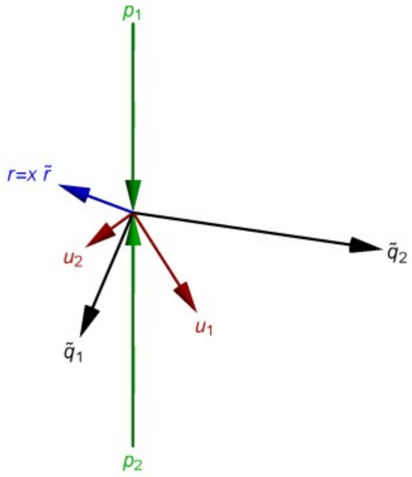

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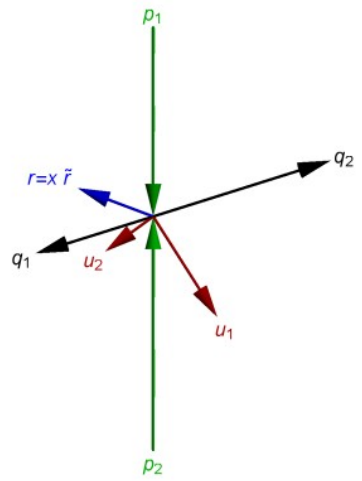

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- Boost non-reference momenta of the Born configuration

t'HV corrections

Observables: Implemented by infrared safe measurement function (MF) F_m

Infrared property in STRIPPER context:

- $\{x_i\} \to 0 \leftrightarrow$ single unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_{n+1}$
- $\{x_i\} \to 0 \leftrightarrow$ double unresolved limit
  $\Rightarrow \mathrm{F}_{n+2} \to \mathrm{F}_{n}$
  $\Rightarrow \mathrm{F}_{n+1} \to \mathrm{F}_{n}$

Tool for new formulation in the 't Hooft Veltman scheme:

Parameterized MF $\mathrm{F}^{\alpha}_{n+1}$

- $\mathrm{F}^{\alpha}_{n} \equiv 0$ for $\alpha \neq 0$
  (NLO MF)
- 'arbitrary' F^{0}_{n}
  (NNLO MF)
- $\alpha \neq 0 \Rightarrow \mathrm{DU} = 0$ and SU separately finite

Example: $F^{\alpha}_{n+1} = F_{n+1}\Theta_{\alpha}(\{\alpha_i\})$
with $\Theta_{\alpha} = 0$ if some $\alpha_i < \alpha$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \qquad \text{where} \qquad \sigma_{SU}^{c} = \int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{c}} F_{n+1} + I_{n}^{\mathrm{c}} F_{n} \right)$$

NLO measurement function ($\alpha \neq 0$):

$$\int \mathrm{d}\Phi_{n+1} \left(I_{n+1}^{\mathrm{RR}} + I_{n+1}^{\mathrm{RV}} + I_{n+1}^{\mathrm{C1}} \right) F_{n+1}^{\alpha} = \text{finite in 4 dim.}$$

All divergences cancel in d-dimensions:

$$\sum_{c} \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}^{\alpha} \equiv \sum_{c} \mathcal{I}^{c} = 0$$

t'HV corrections

$$\sigma_{SU} = \sigma_{SU}^{RR} + \sigma_{SU}^{RV} + \sigma_{SU}^{C1} \underbrace{-\mathcal{I}^{RR} - \mathcal{I}^{RV} - \mathcal{I}^{C1}}_{=0}$$

$$\begin{aligned}
\sigma_{SU}^{c} - \mathcal{I}^{c} &= \int \mathrm{d}\Phi_{n+1} \left\{ \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} + I_{n+1}^{c,(0)} \right] F_{n+1} + \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} + I_{n}^{c,(0)} \right] F_{n} \right\} \\
&\quad - \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)}}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)}}{\epsilon} \right] F_{n+1}\Theta_\alpha(\{\alpha_i\}) \\
&= \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n+1}^{c,(-2)} F_{n+1} + I_{n}^{c,(-2)} F_n}{\epsilon^2} + \frac{I_{n+1}^{c,(-1)} F_{n+1} + I_{n}^{c,(-1)} F_n}{\epsilon} \right] (1 - \Theta_\alpha(\{\alpha_i\})) \\
&\quad + \int \mathrm{d}\Phi_{n+1} \left[I_{n+1}^{c,(0)} F_{n+1} + I_{n}^{c,(0)} F_n \right] + \int \mathrm{d}\Phi_{n+1} \left[\frac{I_{n}^{c,(-2)}}{\epsilon^2} + \frac{I_{n}^{c,(-1)}}{\epsilon} \right] F_n\Theta_\alpha(\{\alpha_i\})
\end{aligned}$$

$$=: \underbrace{Z^c(\alpha)}_{\text{integrable, zero volume for } \alpha\to 0} + \underbrace{C^c}_{\text{no divergencies}} + \underbrace{N^c(\alpha)}_{\text{only } F_n \to \text{DU}}$$

t'HV corrections

Looks like slicing, but it is slicing *only* for divergences
$\rightarrow$ no actual slicing parameter in result

Powerlog-expansion:

$$N^c(\alpha) = \sum_{k=0}^{\ell_{\max}} \ln^k(\alpha) N_k^c(\alpha)$$

- all $N_k^c(\alpha)$ regular in α
- start expression independent of $\alpha \Rightarrow$ all logs cancel
- only $N_0^c(0)$ relevant

Putting parts together:

$$\sigma_{SU} - \sum_c N_0^c(0) \text{ and } \sigma_{DU} + \sum_c N_0^c(0)$$

are finite in 4 dimension

$$\Downarrow$$

SU contribution: $\sigma_{SU} - \sum_c N_0^c = \sum_c C^c$
original expression σ_{SU} in 4-dim without poles, no further ϵ pole cancellation