

NNLO predictions for three-jet cross sections at the LHC

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in collaboration with Michal Czakon and Alexander Mitov

based on: 1907.12911 and 2106.05331

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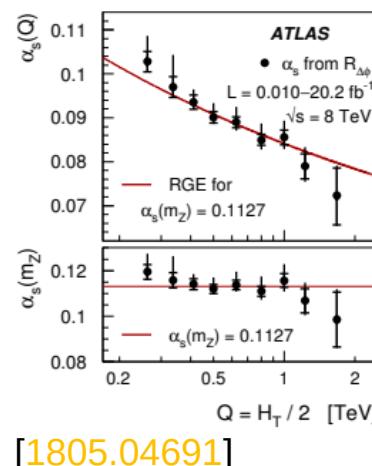
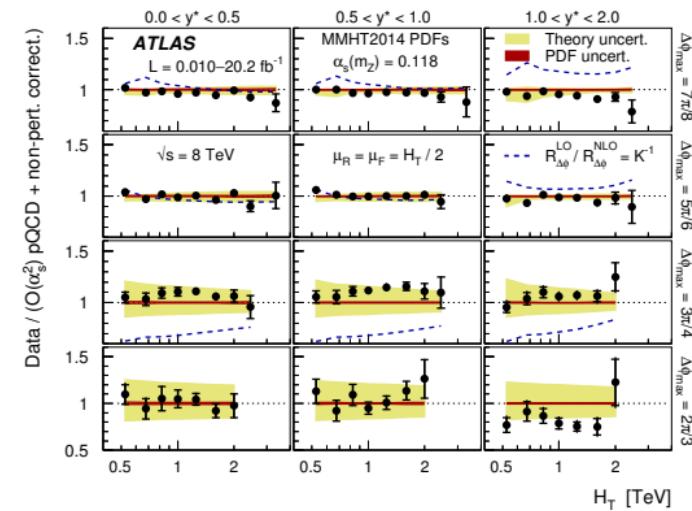
Jet observables at the LHC

The LHC produces jets abundantly → many phenomenological applications

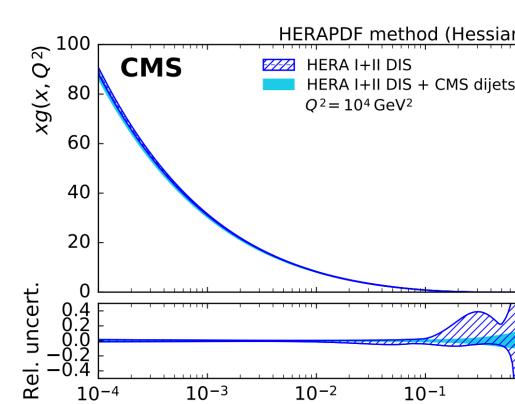
Tests of pQCD, α_s extraction:
R32 ratios, event-shapes

PDF determination:
Single inclusive,
Multi-differential dijet

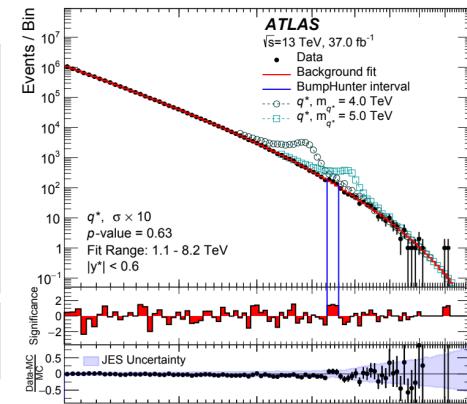
BSM searches:
dijet mass



Precision theory required!



[1705.02628]



[1703.09127]

Data driven

Precision predictions

Fixed order
perturbation theory

Resummation

Parton-showers

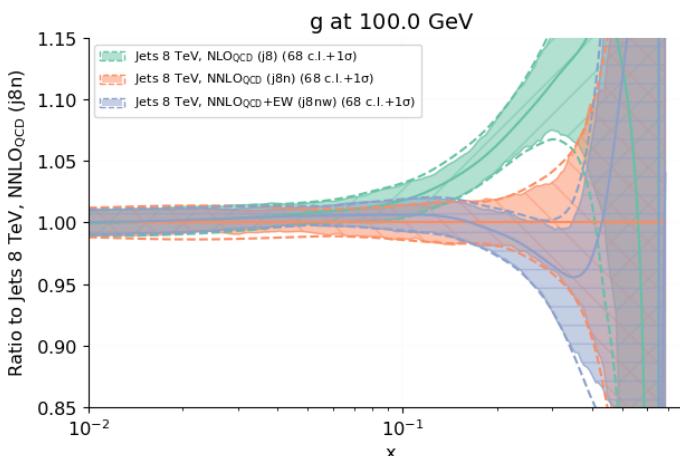
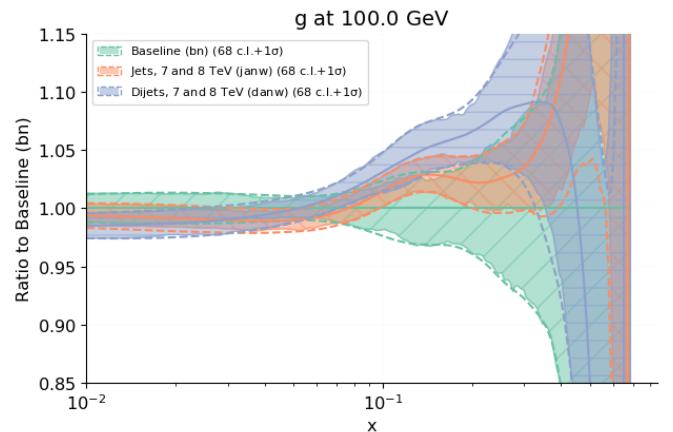
Precision theory predictions

Parametric input:
PDFs and alphaS

Soft physics:
MPI, colour reconnection,
...
...

Fragmentation/hadronisation

Higher order pQCD: PDF fits with jets



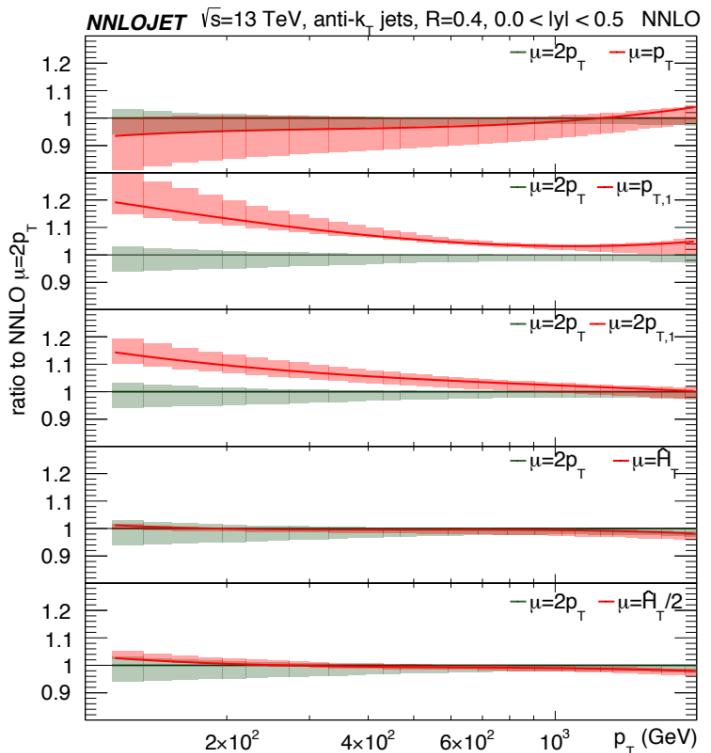
Idea (quite old actually [[Giele'94](#)]):

Here by a collaboration of NNLOJet and NNPDF [[Khalek'20](#)]:

Combine single inclusive and dijet triple differential measurements by ATLAS and CMS to constrain the large gluon-x

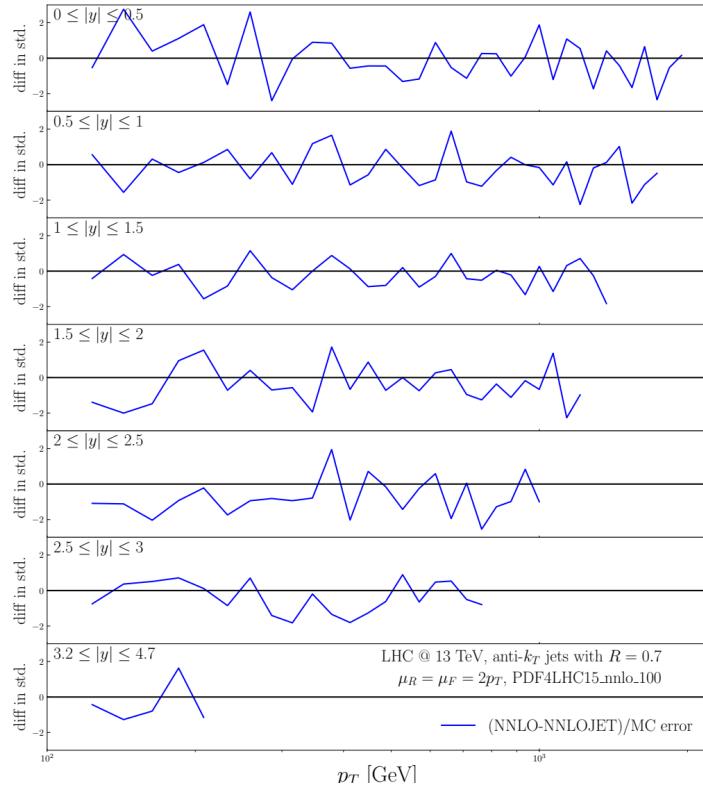
- Reduced uncertainty in large-x gluon PDF
- **NNLO QCD corrections crucial** to obtain consistent results between data sets
- NLO EW [[Dittmaier'12](#)] or full NLO corrections [[Frederix'17, Reyer'19](#)]

Higher order pQCD: lessons from dijet



Detailed studies of
scale dependence:
Event-based choices vs.
single jet choices
[Currie'18]

Study of
sub-leading colour
effects
in quark channels:
smaller than $O(1\%)$
[Czakon'19]

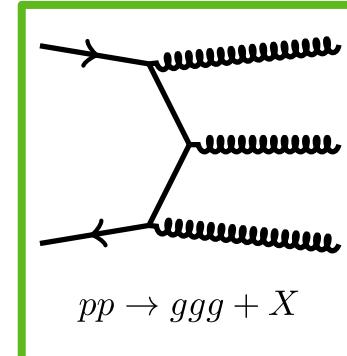


Three jet production

Advances in perturbative QCD allow to tackle the most complicated $2 \rightarrow 3$ process

Bottlenecks:

- Double virtual amplitudes in leading colour approximation [Abreu'21]
- Handling of real radiation:
 - Sector-improved residue subtraction [Czakon'10'14'19] conceptually capable
 - Computationally very challenging! $\rightarrow O(1M \text{ CPUh})$



Only Approximation made: $\mathcal{R}^{(2)}(\mu_R^2) = 2 \operatorname{Re} [\mathcal{M}^{\dagger(0)} \mathcal{F}^{(2)}] (\mu_R^2) + |\mathcal{F}^{(1)}|^2 (\mu_R^2) \equiv \mathcal{R}^{(2)}(s_{12}) + \sum_{i=1}^4 c_i \ln^i \left(\frac{\mu_R^2}{s_{12}} \right)$
 \rightarrow taken from [Abreu'21]

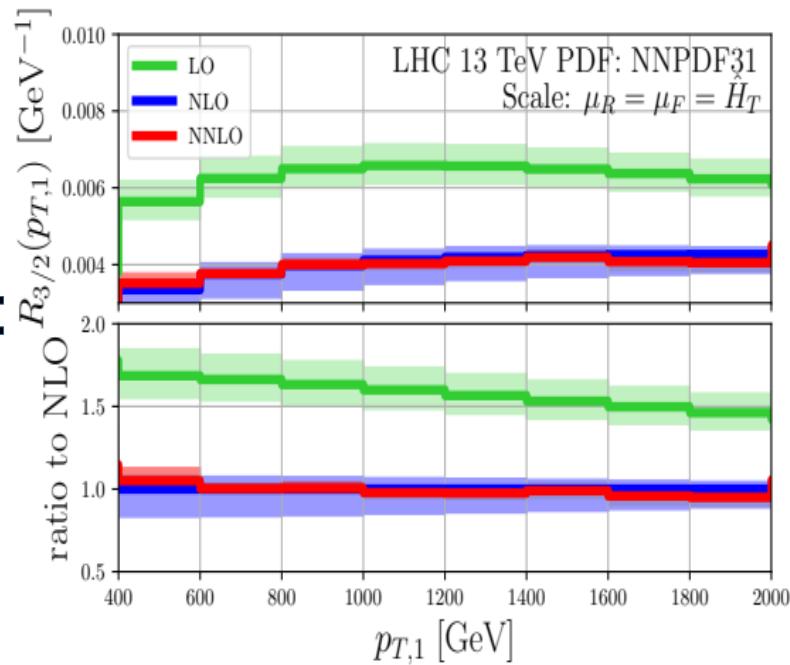
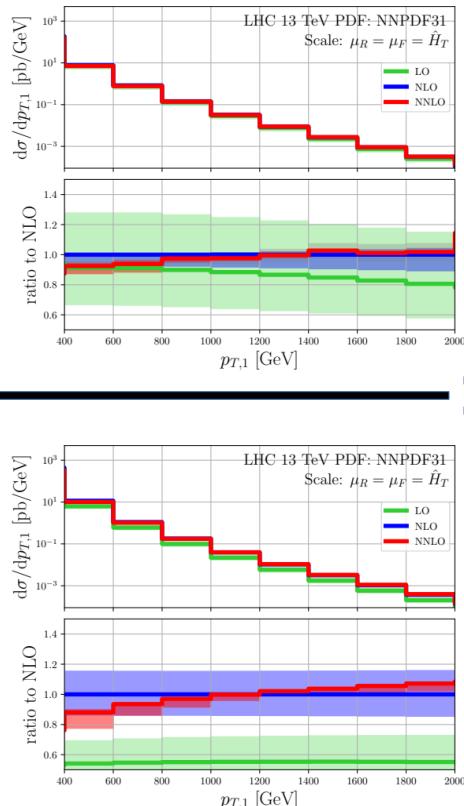
$$\mathcal{R}^{(2)}(s_{12}) \approx \mathcal{R}^{(2)l.c.}(s_{12})$$

Three jet production - R32($\rho T1$)

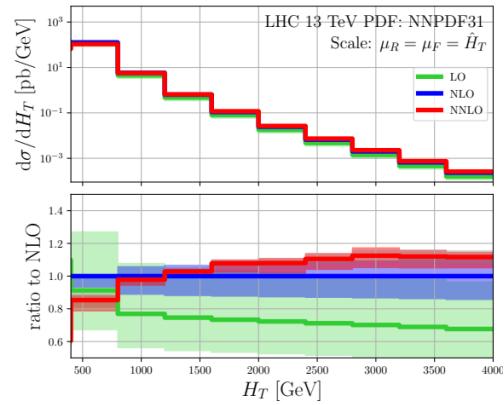
- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
 - $p_T(j) > 60$ GeV and $|y(j)| < 4.4$
 - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$ GeV
- Scales:

$$\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$$

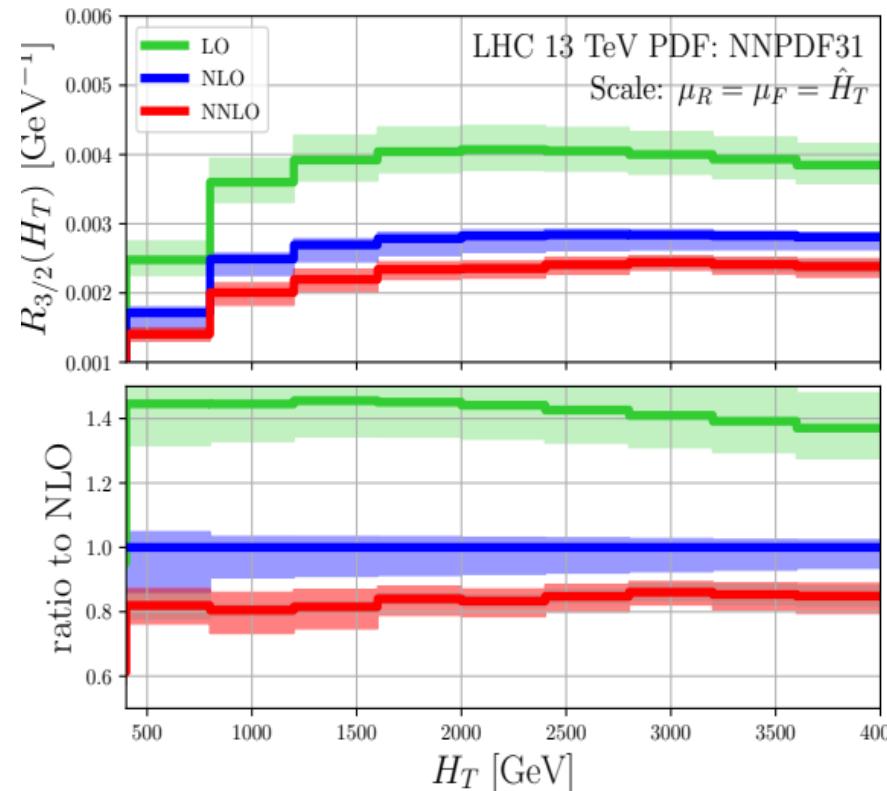
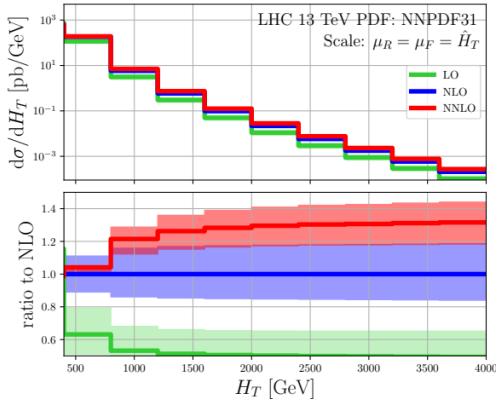
$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX}$$



Three jet production - R32(HT)



==



$$H_T = \sum_{\text{jets}} p_T$$

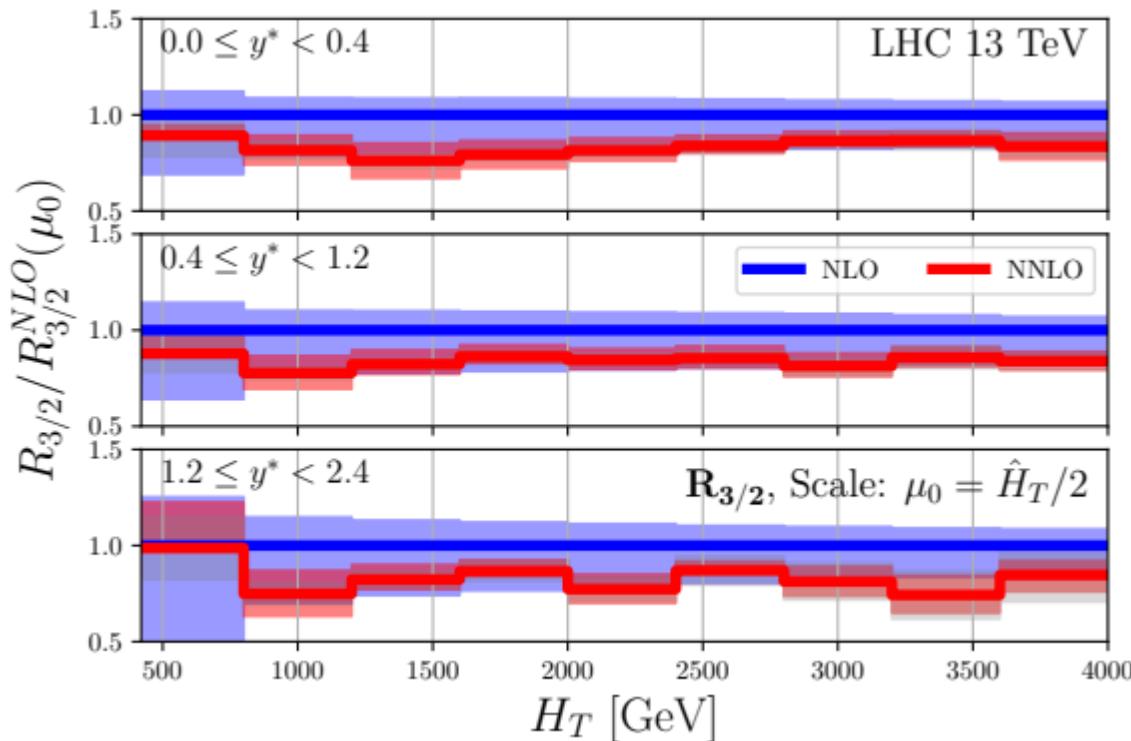
Scale dependence correlated in ratio

→ reduction of scale dependence

→ flat k-factor

→ scale bands in ratio barely overlap

Three jet production – R₃₂(HT, y^{*})



Double differential w.r.t. $y^* = |y(j_1) - y(j_2)|/2$

Different central scale choice: $\hat{H}_T/2$

Three jet production – azimuthal decorrelation

Kinematic constraints on the azimuthal separation between the two leading jets (φ_{12})

φ_{12} sensitive to the jet multiplicity:

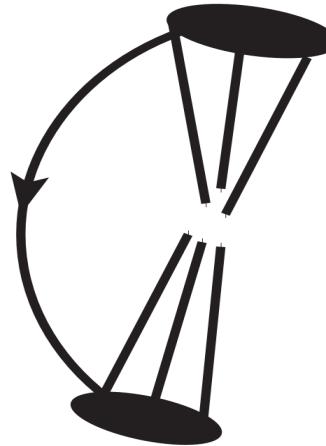
2j: $\varphi_{12} = \pi$

3j: $\varphi_{12} > 2/3\pi$

4j: unconstrained

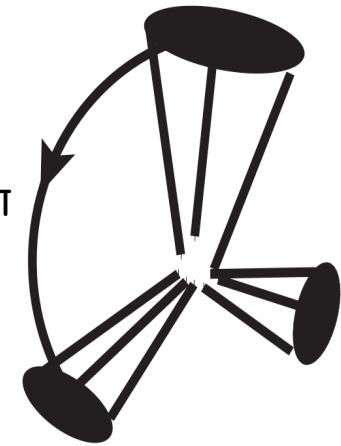
Dijet:

$$\varphi_{12} = \pi$$



Trijet:

$$\varphi_{12} > 2/3\pi$$



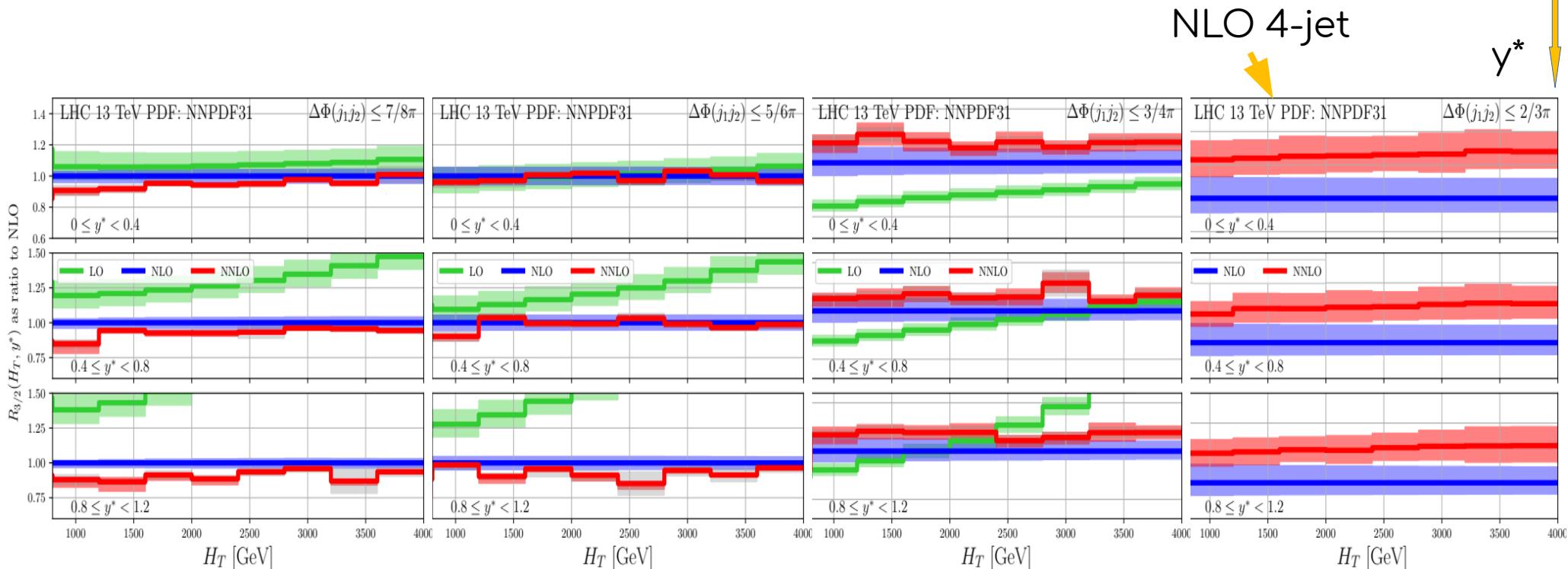
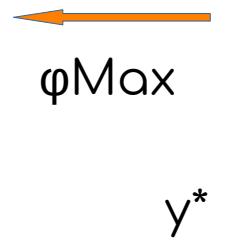
Study of the ratio

$$R_{32}(HT, y^*, \varphi_{Max}) = \frac{(d\sigma_3(\varphi < \varphi_{Max}) / dHT/dy^*)}{(d\sigma_2 / dHT/dy^*)}$$

$$\text{With } y^* = |y_1 - y_2|/2$$

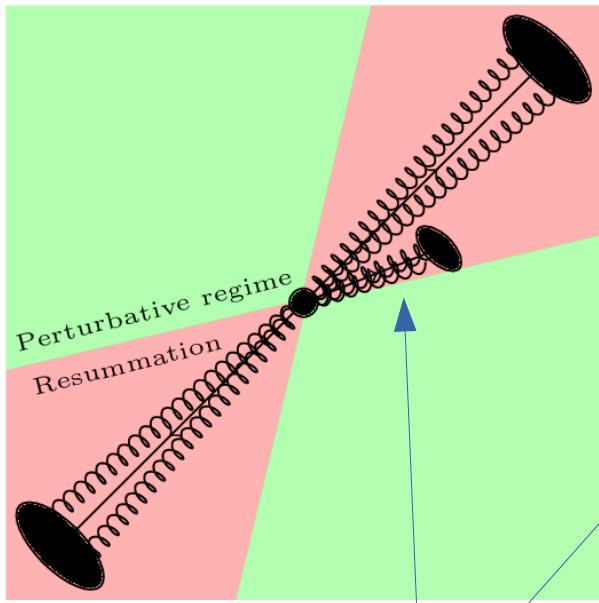
Three jet production – R₃₂(HT, y*, φMax)

NNLO/NLO K-factor smaller than NLO/LO
 Scale dependence is reduced

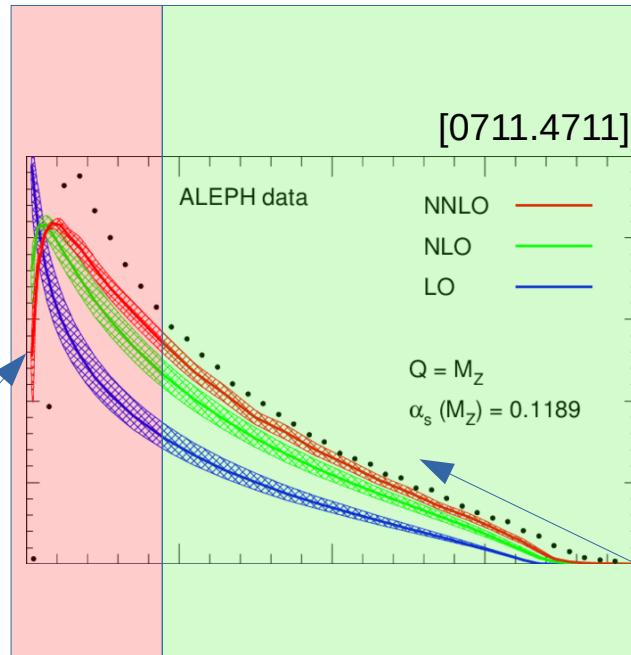


Event-shapes regimes

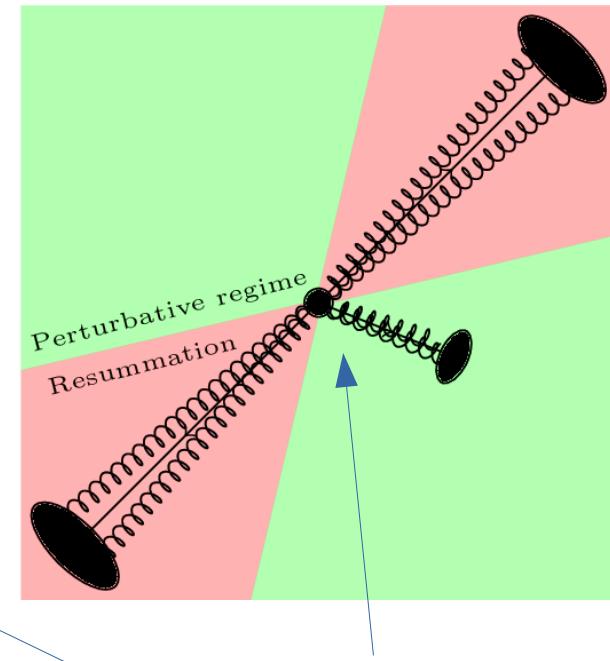
Typically event-shapes measure departure from N hard jet case



Anisotropic, 2-prong like
Sensitivity to resummation,
non-perturbative effects



Example: 1-Thrust at LEP



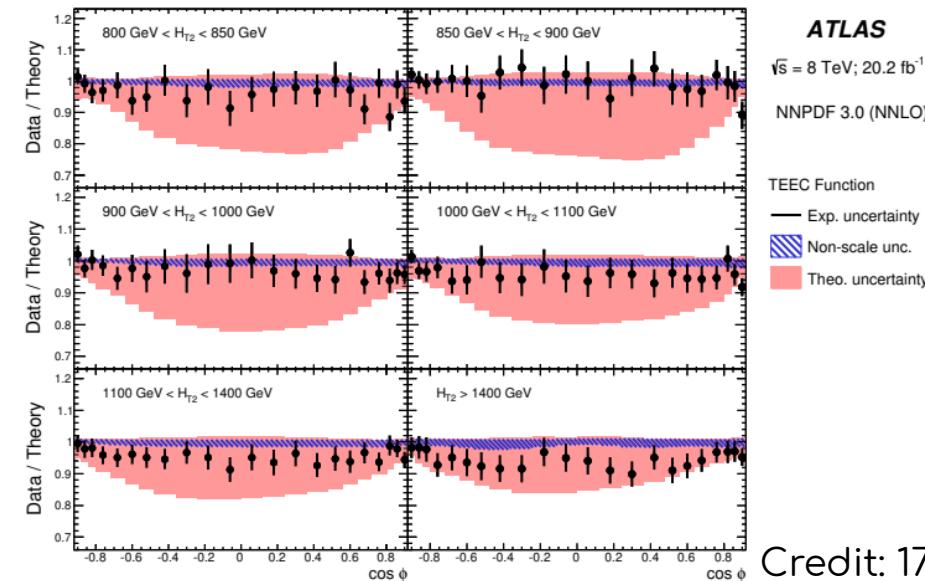
Isotropic, multi-jet
Sensitive to hard
matrix elements

Event-shapes at the LHC

Event-shapes are measured using multi-jet events
 → three jet is the leading contribution

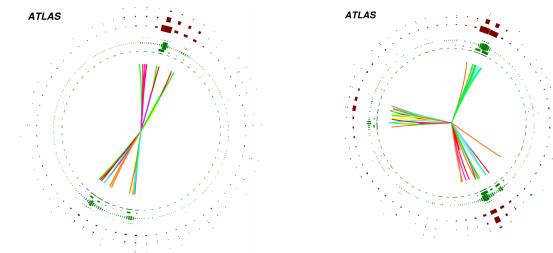
TEEC (Transverse Energy-Energy Correlation)

$$\frac{1}{\sigma} \frac{d\Sigma}{d \cos \phi} = \frac{1}{N} \sum_{A=1}^N \sum_{ij} \frac{E_{\perp,i}^A E_{\perp,j}^A}{\left(\sum_k E_{T,k}^A \right)^2} \delta(\cos \phi - \cos \phi_{ij})$$

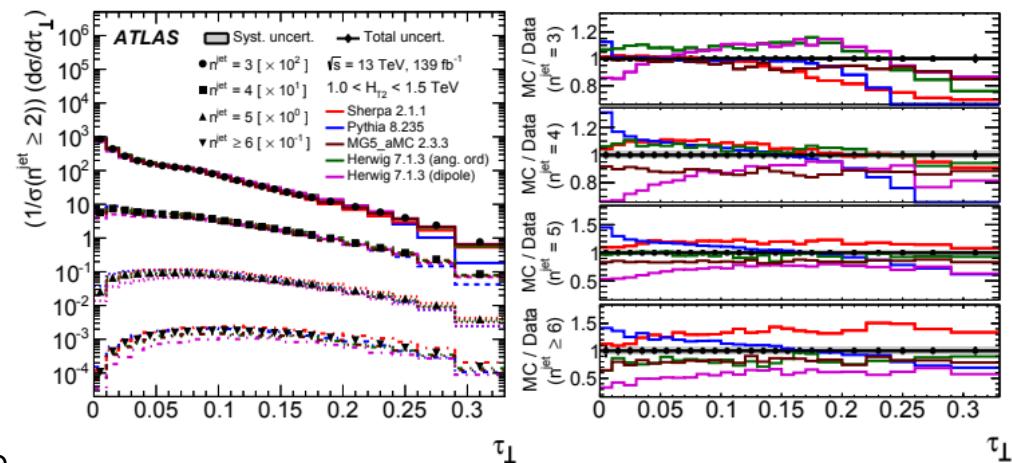


Credit: 1707.02562

Credit: ATLAS 2007.12600



Transverse Thrust: $\tau_T = 1 - \frac{\sum_i |\vec{p}_{T,i} \cdot \hat{n}|}{\sum_i |\vec{p}_{T,i}|}$



Summary and Outlook

Jet rates with the sector-improved residue subtraction framework

- Full NNLO QCD predictions for di-jet production available
 - scale choice important
 - sub-leading colour contributions small
- Three jets @ the LHC:
 - First predictions available with approximate two-loop contribution!
 - improved scale dependence and stabilized K-factors
 - ρT spectra, HT, double differential
 - Real radiation for $2 \rightarrow 3$ can be handled.
But efficiency is a concern and needs some attention!

Many interesting applications ahead! → Event-shapes
Stay tuned!

Thank you for your attention!