National University of Singapore
ST3233: Applied Time Series

# Assignment 1

Semester 1: 2016-2017

1. Please, write your group number as well as the matriculation number and name of each member of the group.
2. The assignment is to be uploaded on the IVLE by the **9th of October**, 11:59pm.
3. Late assignment will not be accepted. Only typed **pdf** files can be submitted.
4. if your group number is XX, please name your file:

   `assignment_1_XX.pdf`

   For example, if you are submitting a report for the group 13, your file should be named

   `assignment_1_13.pdf`

## Exercise 1 (Electricity forecast) [Pts = 20]

The file `electricity_load.dat` contains the hourly electricity demand in Poland during a few weeks of the year 1997.

1. Load the data and use the command `ts(...)` to convert it into a time series object.
2. Display the time series. What is an appropriate seasonal period?
3. Decompose the time series into the superposition of a trend / seasonal / remainder pattern.
4. Use a seasonplot to display the seasonal pattern. Explain why the time series is not perfectly seasonal.
5. Use a triple exponential smoothing approach to make a forecast for the next 3 weeks. To do so, you will use the command `hw(..., initial="simple", seasonal="additive", ...)`.

## Exercise 2 (Bootstrap estimate) [Pts = 20]

Consider a MA(2) model of the type $X_k = W_k + \alpha\, W_{k-1} + \beta\, W_{k-2}$ for a white noise process $\{W_k\}_{k \geq 0}$. The purpose of this exercise is to investigate the accuracy of the estimation procedure provided by the software R.

1. Consider the MA(2) process

$$X_k = W_k - (5/6)\, W_{k-1} + (1/6)\, W_{k-2}. \tag{1}$$

   Is this MA(2) process invertible?

2. Use the command `arima.sim(...)` to simulate a trajectory of length $T = 10^3$ from this process and use the command `arima(..., order = c(0,0,2)))` to fit an MA(2) model to your simulated time series. Double check that the estimation procedure provides a good estimate of $\alpha = -5/6$ and $\beta = 1/6$.
3. We would like to estimate the accuracy of the estimation procedure for short time series. To this end, simulate 1000 time series of length $T = 40$, each one of them following the MA(2) process (1). For each one of them, compute an estimate $\widehat{\beta}$ of the coefficient $\beta$; you will thus obtain a sequence of estimates $\{\widehat{\beta}_1, \widehat{\beta}_2, \ldots, \widehat{\beta}_{1000}\}$.
4. Plot a histogram of all these estimates. Finally, compute the mean / variance of all these estimates.
5. Simulate a time series of length $T = 10^4$ following the process $X_k = W_k + \alpha\, W_{k-1}$ with $\alpha = 3$. Use the command `arima(..., order = c(0,0,1)))` to compute an estimate $\widehat{\alpha}$ of $\alpha$: why is the value of $\widehat{\alpha}$ very different from the true value $\alpha = 3$. Explain why one obtains $\widehat{\alpha} \approx 1/3$.

## Exercise 3 (Sale forecast) [Pts = 20]

A Singaporean aircon retailer is trying to plan how many air-conditionning devices he should purchase for the month of June 2017. In June 2016, he sold 51 devices. Analyse the number of queries containing the word “aircon” using google trend and propose a method for forecasting his sales for next June.

## Exercise 4 (Yule-Walker) [Pts = 20]

The file `asg_1_MA3.dat` contains a time series of length $T = 10^4$. We would like to fit an $AR(3)$ process

$$X_k = \alpha\, X_{k-1} + \beta\, X_{k-2} + \gamma\, X_{k-3} + W_k$$

to this dataset.

1. Estimate the first three autocorrelation coefficients $\widehat{\rho}(1)$ and $\widehat{\rho}(2)$ and $\widehat{\rho}(3)$.
2. Write down the associated Yule-Walker equations for estimating $\alpha$, $\beta$ and $\gamma$.
3. Find $\widehat{\alpha}$, $\widehat{\beta}$ and $\widehat{\gamma}$ by solving the Yule-Walker equations.
   Remark: you can use R to invert matrices, if necessary.