ST3233: Tutorial 2

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1 Prediction

Consider a stationary time series $\{X_k\}_{k\geq 0}$ with mean zero and correlation function $\rho(\ell) = \mathrm{Corr}(x_t, x_{t+\ell})$. Suppose that we observe the series between time $t = 0$ and $t = k$ and would like to make a prediction in the future at time $t = k + \ell$. We consider a prediction of the type

$$\widehat{x}_{k+\ell} = A\, x_k$$

for some constant A. Compute $\mathbf{MSE}(A) \equiv \mathbb{E}\left[(\widehat{x}_{k+\ell} - x_{k+\ell})^2\right]$ and show that it is minimized for $A_\star = \rho(\ell)$.

2 Random Walk

Consider a white noise process $\{W_k\}_{k\geq 0}$ with variance σ_W^2 and a random walk time series $\{X_k\}_{k\geq 0}$ defined as $X_0 = 0$ and $X_k = X_{k-1} + W_k$. Prove that the correlation between X_k and X_{k+1} converges to one as $k \to \infty$.

3 Trend

Consider a white noise process $\{W_k\}_{k\geq 0}$ with variance σ_W^2. Consider the time series $X_k = \alpha + \beta\, k + W_k$ for two constants $\alpha, \beta \in \mathbb{R}$. We would like to estimate the trend with a moving average of the type

$$\widehat{x}_k = \frac{1}{2q+1} \sum_{j=-q}^{j=q} x_j.$$

Find the mean and variance of $\widehat{x}_k$.

4 Simulation

Consider a white noise process $\{W_k\}_{k\geq 0}$ with variance $\sigma_W^2 = 1$. Consider the time series $X_k = \sin(k) + W_k$. Simulate a realization of length $n = 500$ from this time series and use R to display the first 20 autocorrelation coefficients.

5 Oil and Gas

Load the dataset contained in `tsa3.rda` and plot the `oil` and `gas` time series representing the respective price of oil and gas in between 2000 and 2011.

1. Think about an interesting way to plot the two time series on the same plot by trying to put them on the same scale first.

2. In economics, it is often the percentage in price rather than the absolute price that is of interest. For a time series x_t, the log-returns are defined as $\log(x_k/x_{k-1}) \approx (x_k - x_{k-1})/x_{k-1}$. Define two new time series `poil` and `pgas` that contain the oil and gas log-returns. Plot them.

3. Display the firt few autocorrelation coefficient. Is there a lot of structure in these time series?

4. Compute the correlation between the two time series.

5. In order to find out if one can use the price of the gas to predict the price of the oil, draw a scatter plot of `poil`(k) versus `pgas`$(k-\ell)$ for $\ell = 1, 2, 3$.