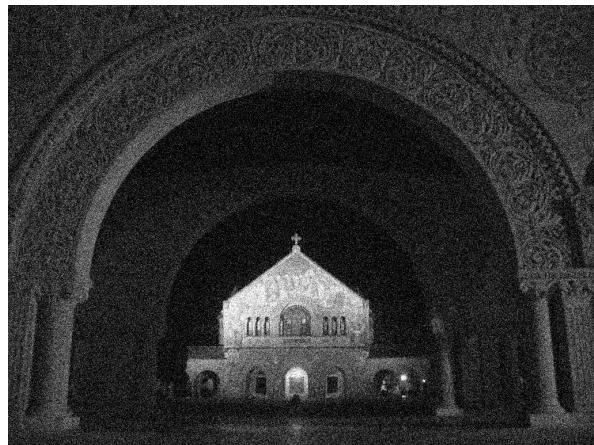


# Point operations for combining images

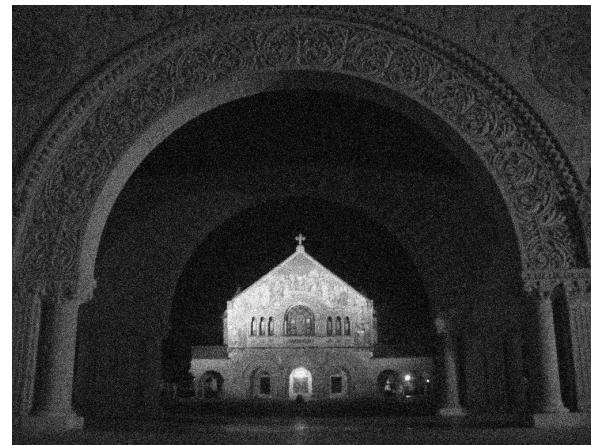
- Image averaging for noise reduction
- Combination of different exposures for high-dynamic range imaging
- Image subtraction for change detection
- Need for accurate alignment
- Displacement estimation

# Image averaging for noise reduction

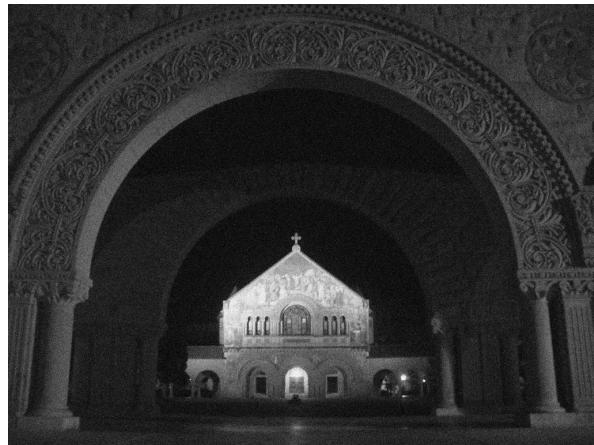
1 image



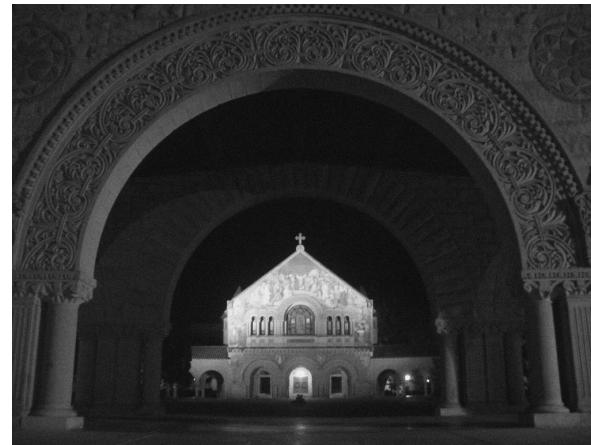
2 images



8 images



32 images



# High-dynamic range imaging



-8 f-stops



-2 f-stops



+2 f-stops



+4 f-stops



Blended image from  
Exposure Fusion

*[Tom Mertens et al. 2007]*

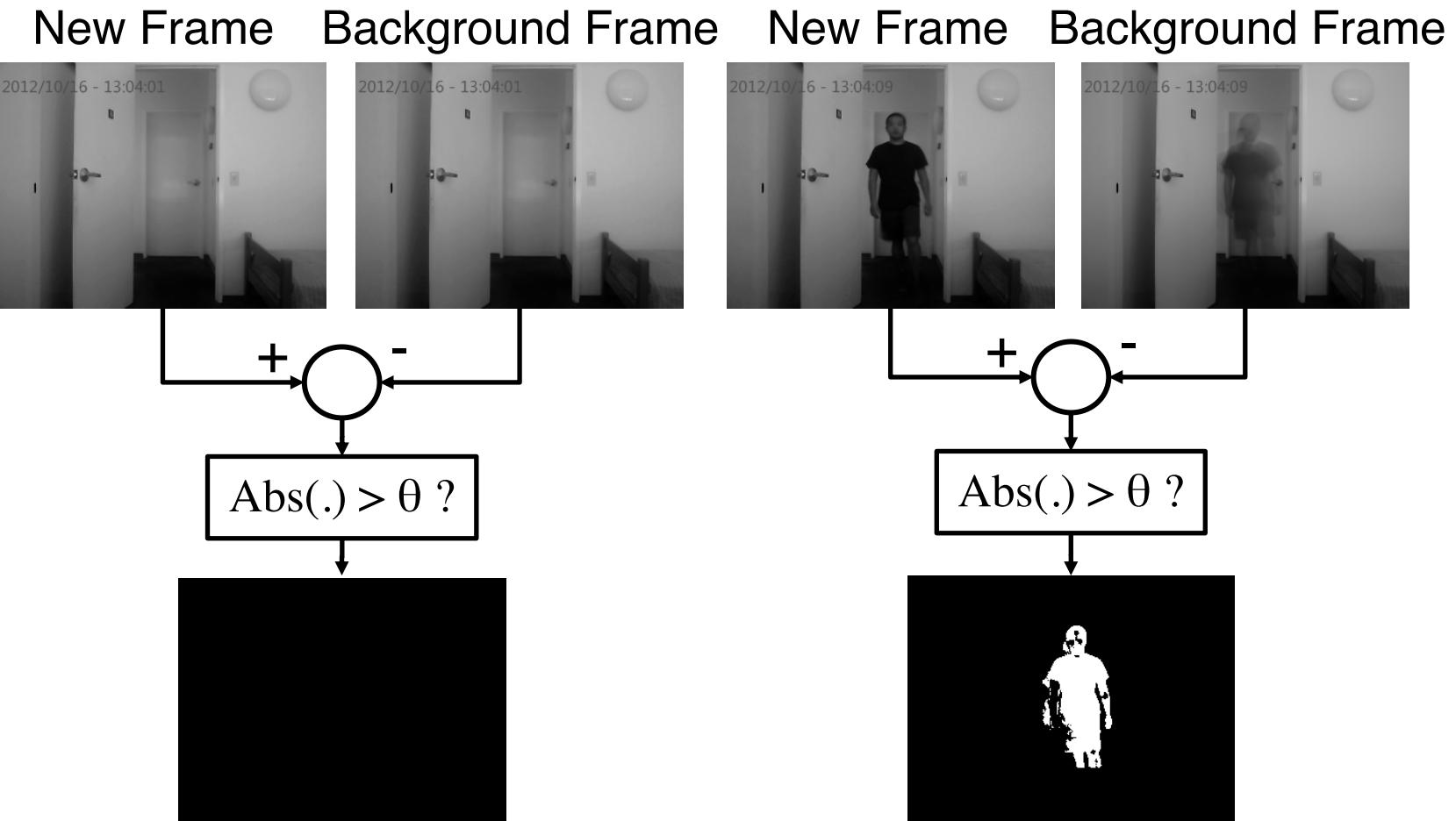


# Image subtraction

- Find differences/changes between 2 mostly identical images
- Example: digital subtraction angiography



# Video background subtraction

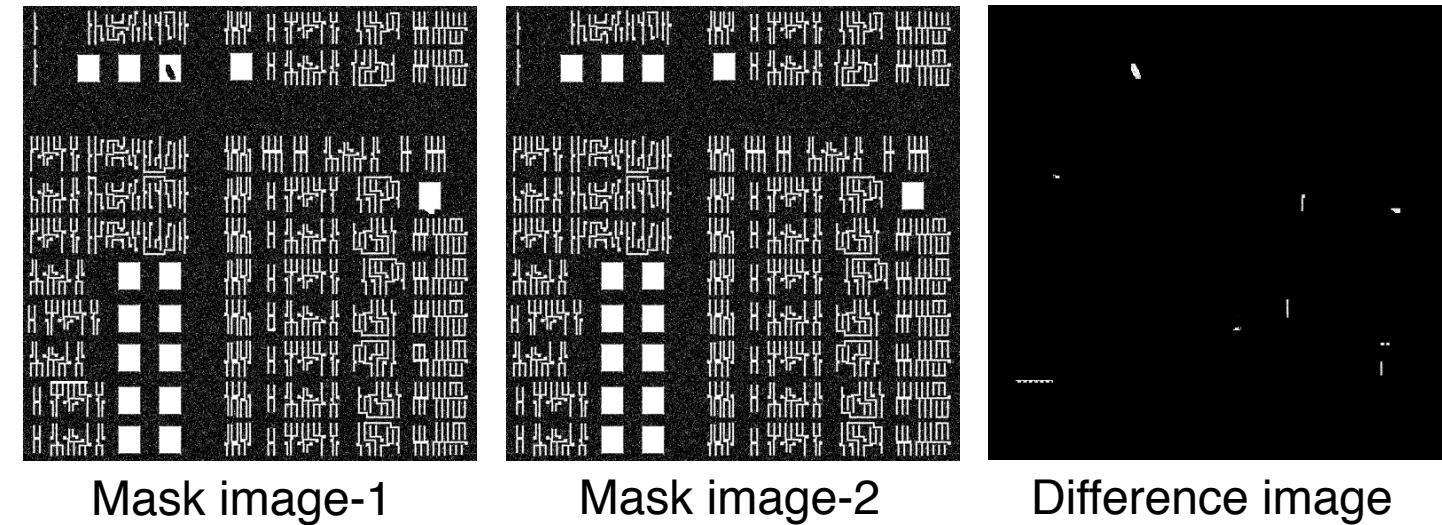
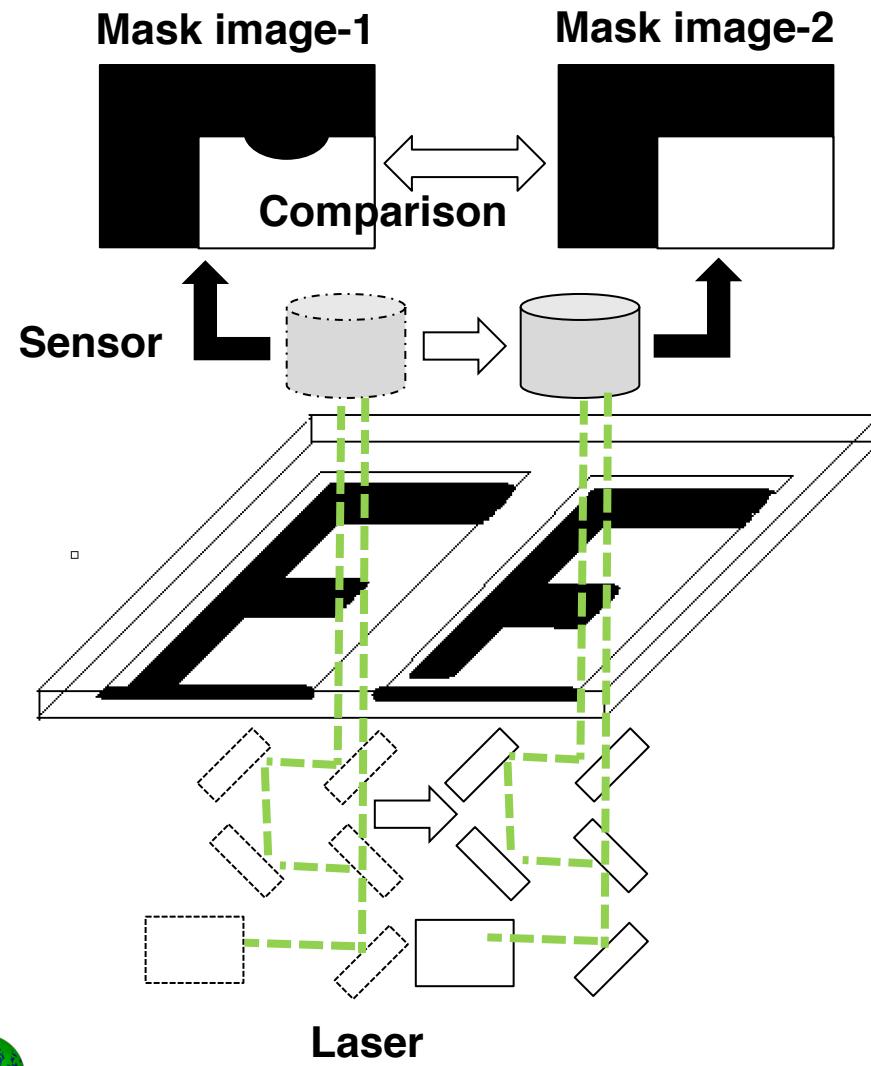


Update:

$$\text{Background}[t] := \alpha \text{ Background}[t-1] + (1-\alpha) \text{ New}[t]$$



# Image subtraction in IC manufacturing: inspection of photomasks



# Where is the defect?

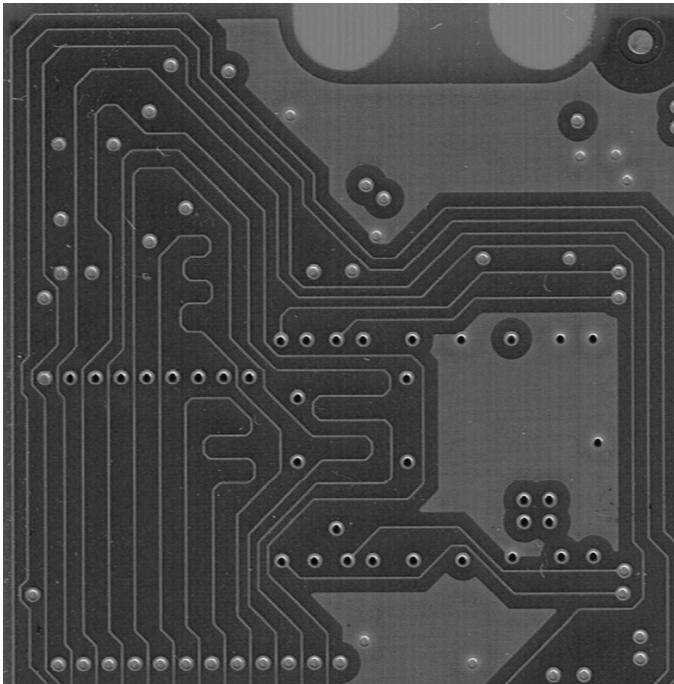


Image  $g[x,y]$  (no defect)

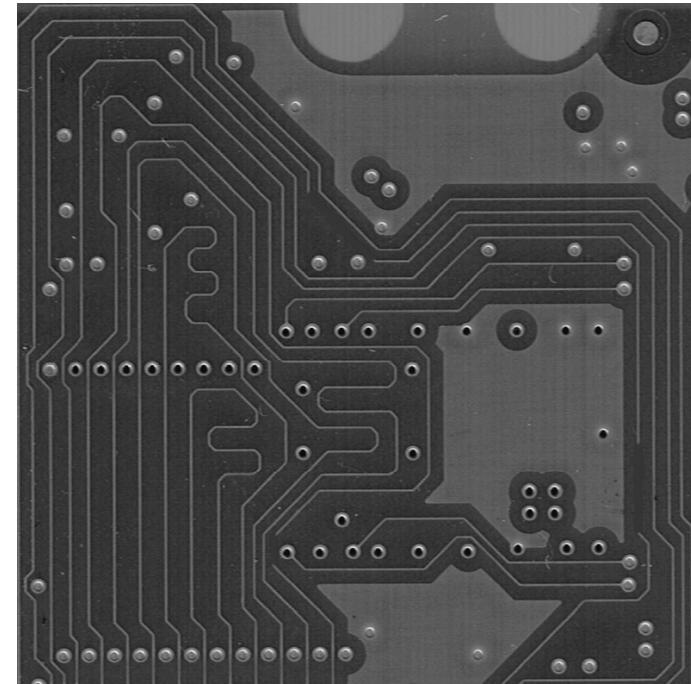
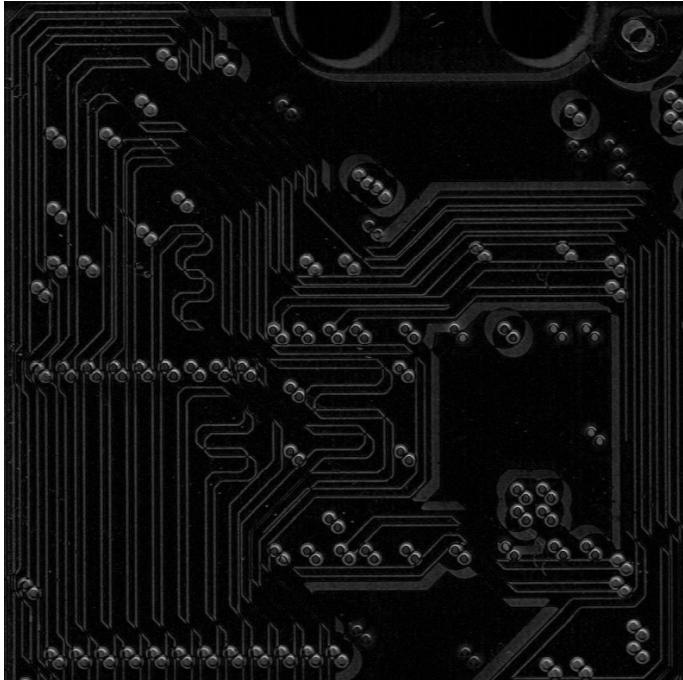


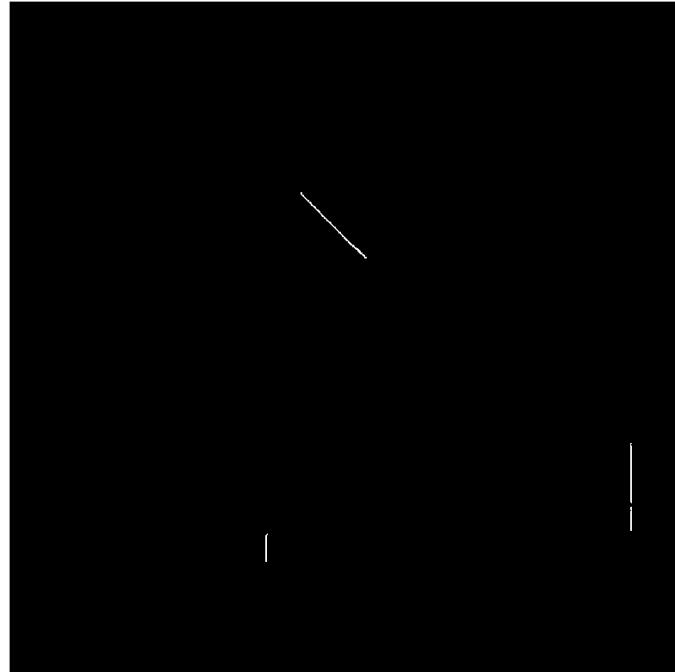
Image  $f[x,y]$  (w/ defect)



# Absolute difference between two images



$|f-g|$  w/o alignment



$|f-g|$  w/ alignment



# Displacement estimation by block matching

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

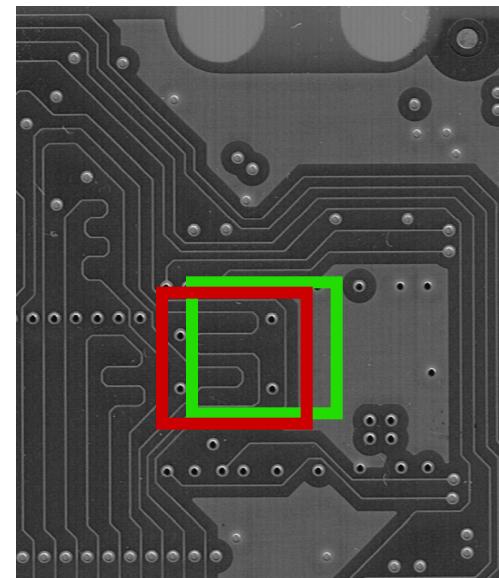


Image  $g[x,y]$

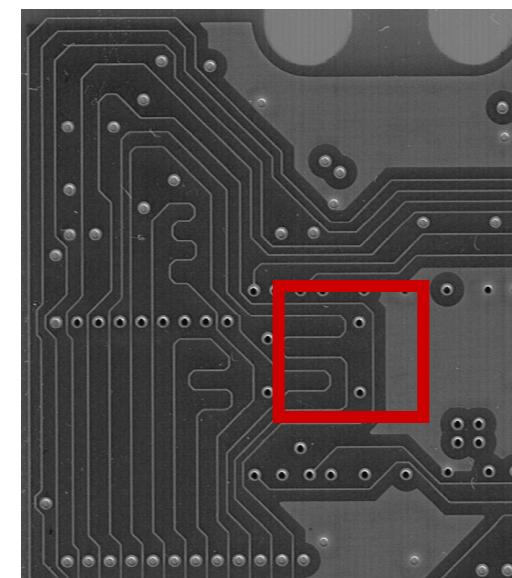


Image  $f[x,y]$

Rectangular array of pixels is selected as a measurement window

# Displacement estimation by block matching

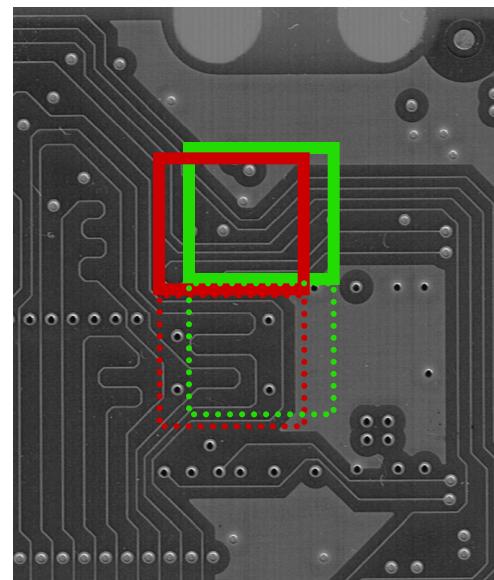


Image  $g[x,y]$

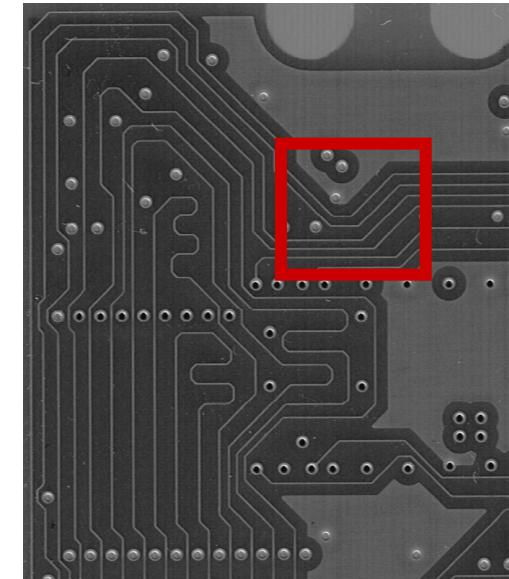


Image  $f[x,y]$

. . . process repeated for another  
measurement window position.

# Integer pixel shifts

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

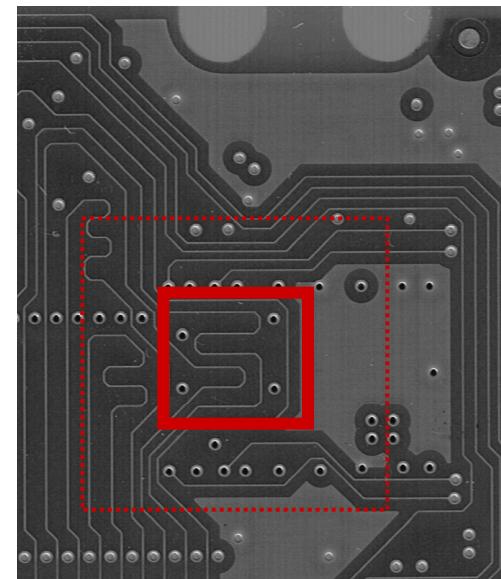


Image  $g[x,y]$

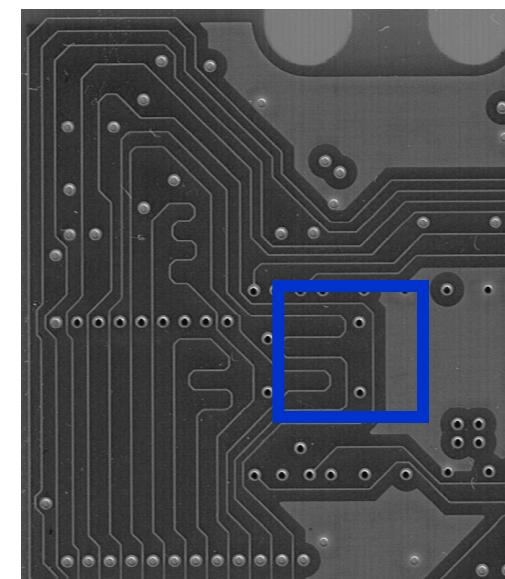
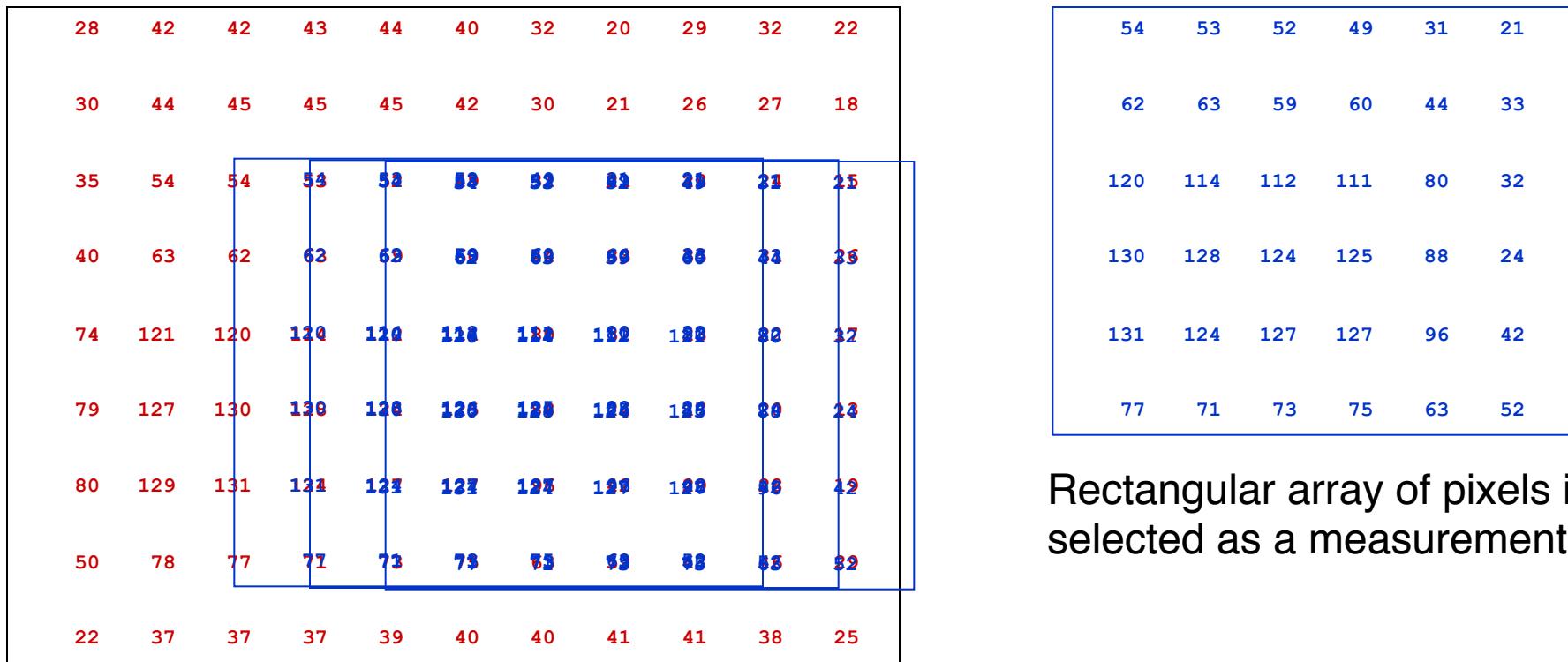


Image  $f[x,y]$

Rectangular array of pixels is selected as a measurement window

# Integer pixel shifts



Rectangular array of pixels is selected as a measurement window

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

# Error metric

- *Sum of Squared Differences*

$$SSD[\Delta_x, \Delta_y] = \sum_{[x,y] \in \text{msmnt window}} (f[x, y] - g[x + \Delta_x, y + \Delta_y])^2$$

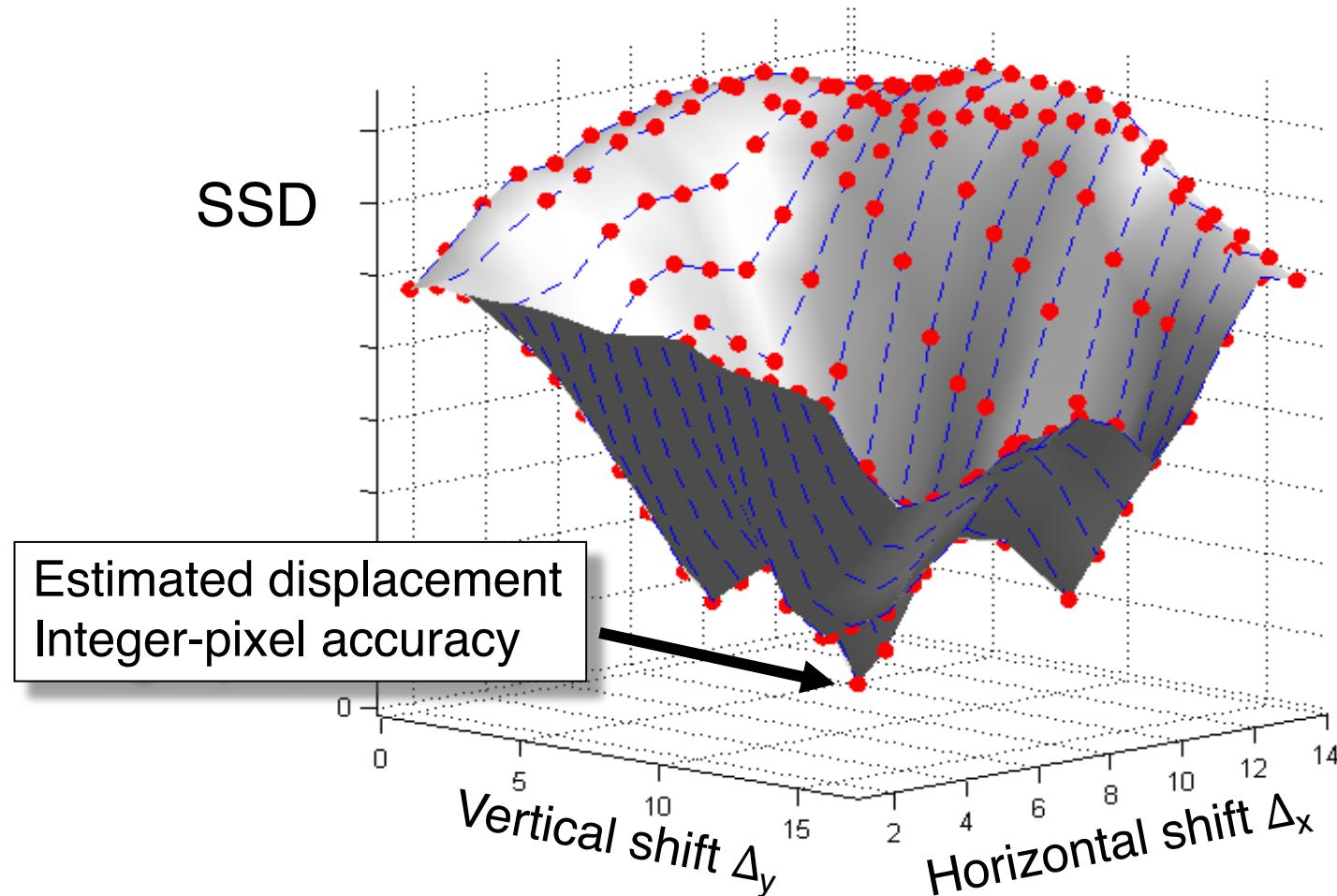
Sum all values  
in measurement  
window

Horizontal  
displacement

Vertical  
displacement

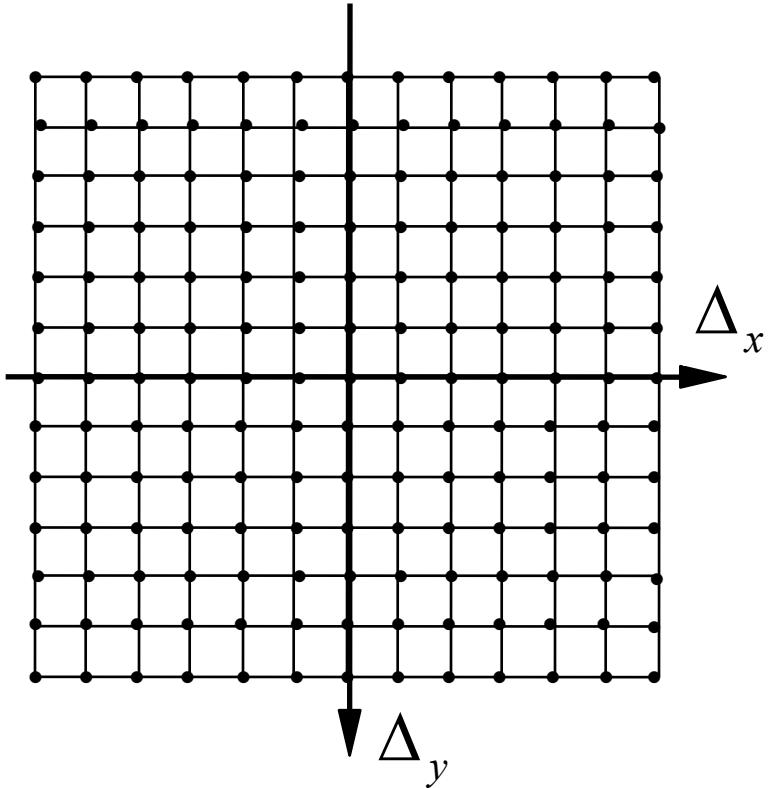
- Alternatives: SAD (*Sum of Absolute Differences*), cross correlation, mutual information . . .
- Robustness against outliers: sum of saturated squared differences, median of squared differences . . .

# SSD values resulting from block matching



# Block matching: search strategies

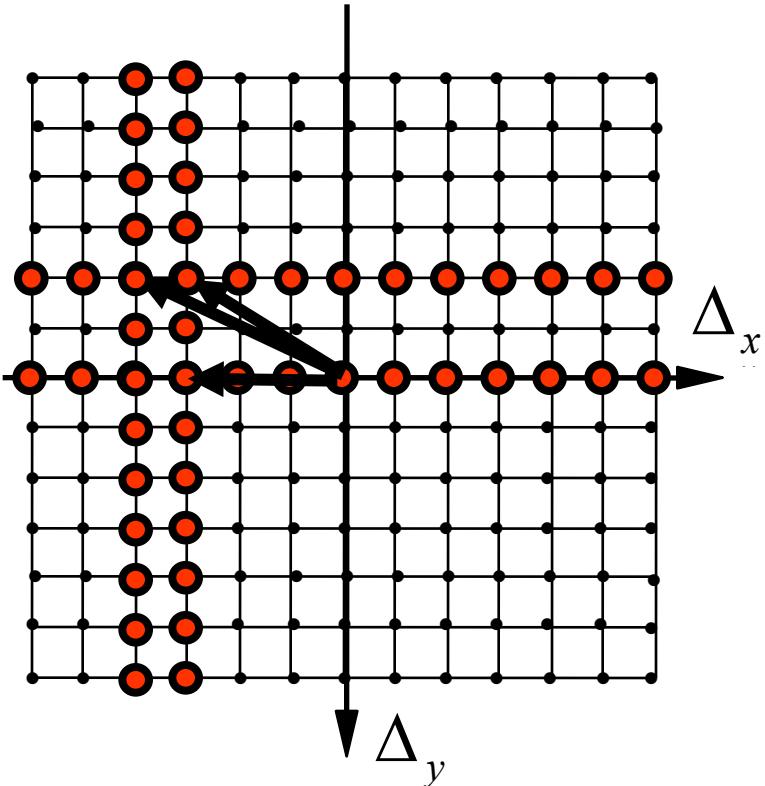
## Full search



- All possible displacements within the search range are compared.
- Computationally expensive
- Highly regular, parallelizable

# Block matching: search strategies

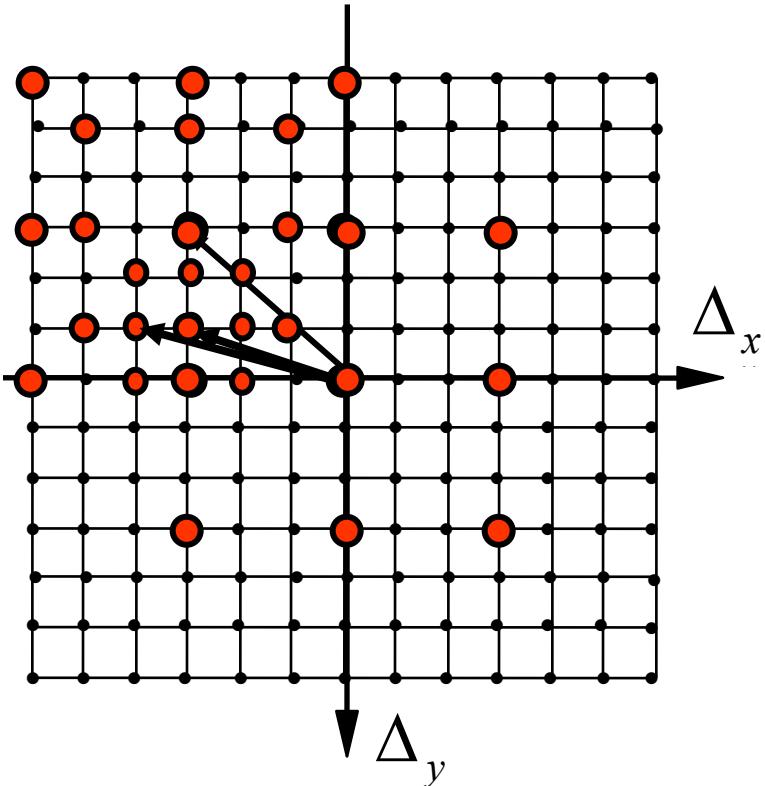
## Conjugate direction search



- Alternate search in x and y directions
- Stop when there is no further improvement

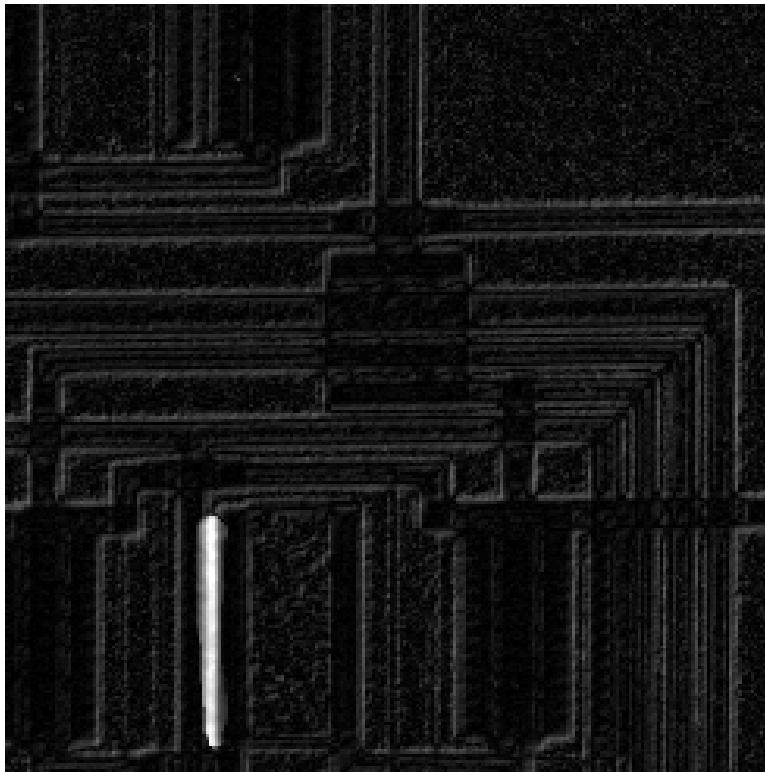
# Block matching: search strategies

## Coarse-to-fine

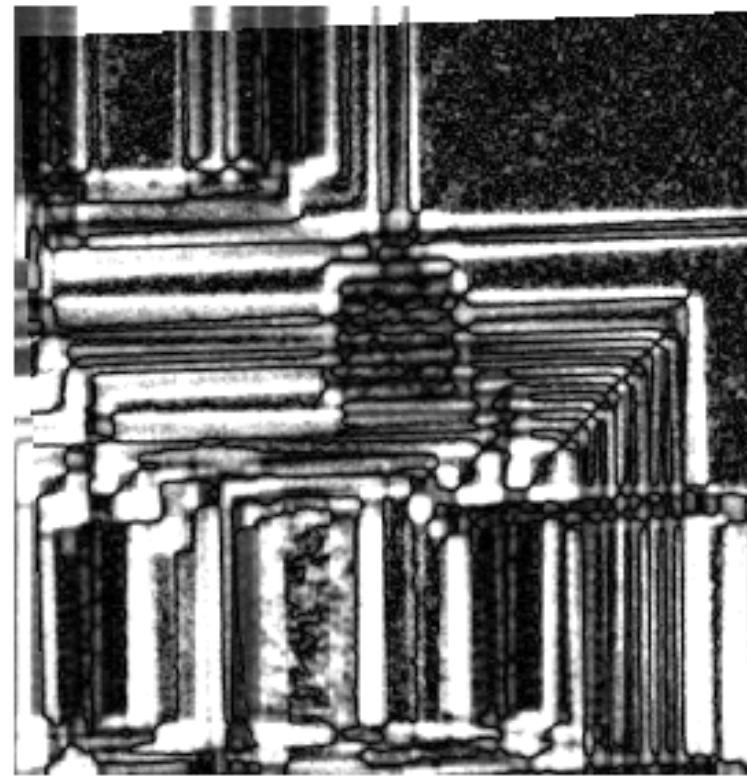


- Start with coarsely spaced candidate displacements
- Smaller pattern when best match is in the middle
- Stop when desired displacement accuracy is reached

# Absolute difference between images

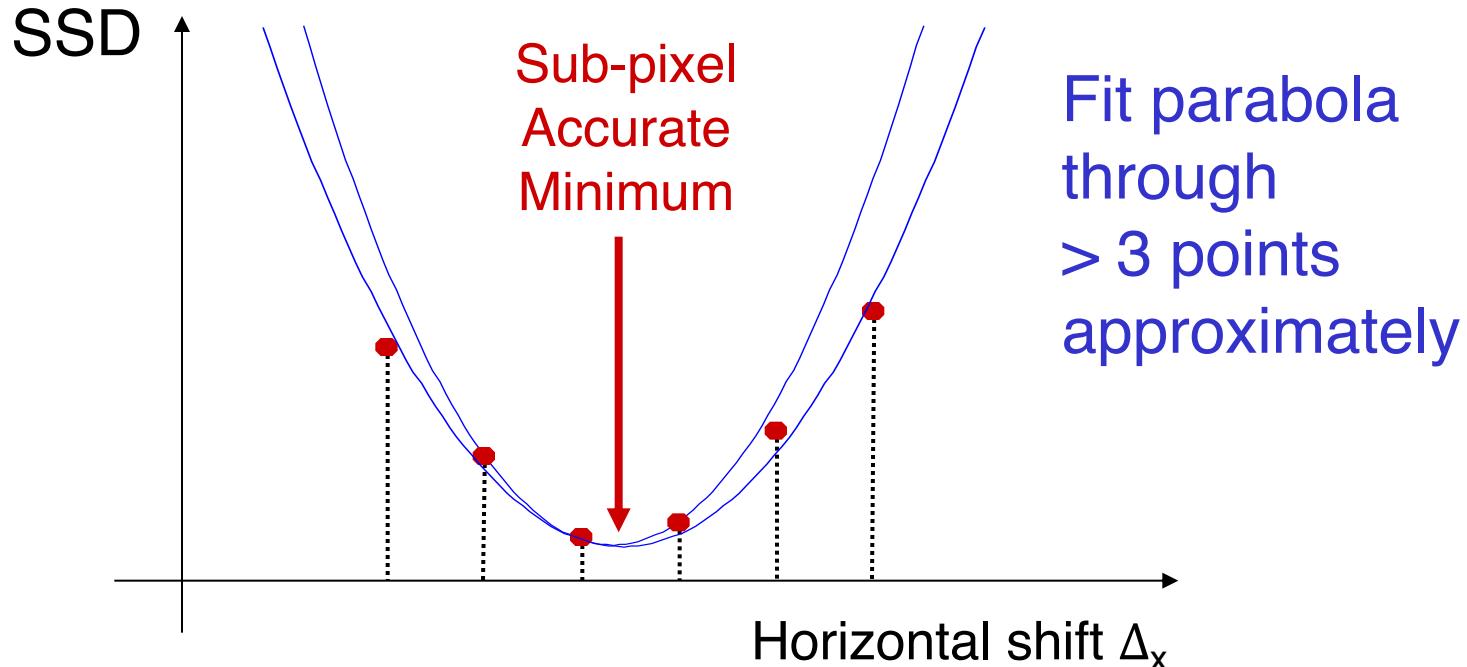


w/ integer-pixel alignment

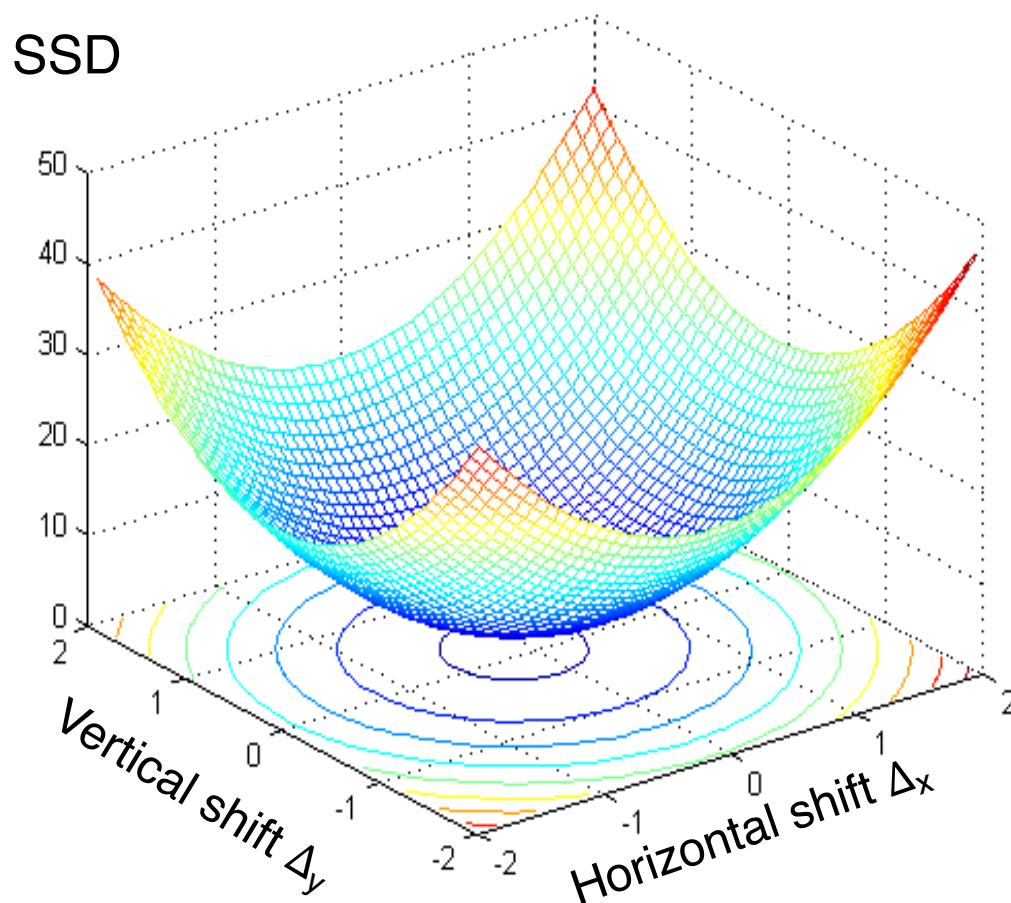


w/o alignment

# Interpolation of the SSD Minimum



# 2-d Interpolation of SSD Minimum



## Paraboloid

- Perfect fit through 6 points
- Approximate fit through  
 $> 6$  points

# Sub-pixel accuracy

- Interpolate pixel raster of the reference image to desired sub-pixel accuracy (e.g., by bi-linear or bi-cubic interpolation)
- Straightforward extension of displacement vector search to fractional accuracy
- Example: half-pixel accurate displacements

