Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. The average of four numbers is 9. What is the fourth number if three of the numbers are 5, 9 and 12?

   (A) 6 (B) 8 (C) 9 (D) 10 (E) 36

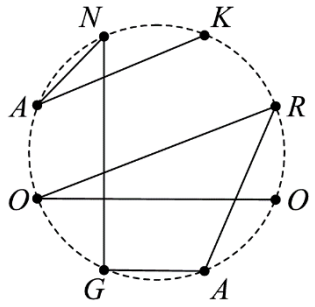
2. Which of the following numbers is the closest to the result of $\frac{17\times0.3\times20.16}{999}$?

   (A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100

3. On a test consisting of 30 questions, Ruth had 50% more right answers than she had wrong answers. Each answer was either right or wrong. How many right answers did Ruth have, assuming she answered all questions?

   (A) 10 (B) 12 (C) 15 (D) 18 (E) 20

4. Iva marked eight points on a circle and named them by letters. Then she connected the points with a broken line with seven legs, as shown in the figure.



   How many broken lines with seven legs connecting the eight points are there, such that reading the letters when moving along a broken line Iva can obtain the word *KANGAROO*?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 6

5. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

   (A) 4 (B) 3 (C) 2 (D) 1 (E) 0

6. Football fans were travelling to a game in 32 minibuses. There was an equal number of people in each of them. Eight minibuses broke down on the way and the fans from these buses got on the remaining ones. After that there were two more fans in every minibus. How many fans were travelling to the game?

   (A) 48 (B) 144 (C) 192 (D) 256 (E) 384

7. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the negative sign (–) in front. Counting backwards from +3, he would write: 3, 2, 1, 0, 00, 000, 0000, ... What is the result of 000 + 0000 in his notation?

(A) 1 (B) 00000 (C) 000000 (D) 0000000 (E) 00000000

8. There are 2016 kangaroos, each of them is either grey or red, at least one of them is grey and at least one is red. For every kangaroo K we compute the fraction of the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as K (including the kangaroo K). Find the sum of the fractions of all 2016 kangaroos.

(A) 2016 (B) 1344 (C) 1008 (D) 672 (E) more information is needed

9. What is the minimum number of swaps of any two adjacent letters needed to convert the word VELO into the word LOVE?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

10. Sven wrote five different one-digit positive integers on a blackboard. He discovered that no sum of any two numbers is equal to 10. Which of the following numbers did Sven definitely write on the blackboard?

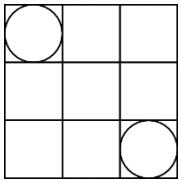
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Part B: Each correct answer is worth 4 points

11. Let $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which of the numbers a, b, c, d is the greatest?

(A) a (B) b (C) c (D) d (E) impossible to determine

12. A 3 × 3 table is divided into 9 unit squares, and two circles are inscribed in two of them (see the figure).



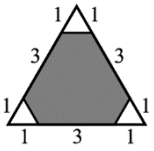
What is the distance between the two circles? (A distance between figures is the shortest distance between any two points on the two figures.)

(A) $2\sqrt{2} - 1$ (B) $\sqrt{2} + 1$ (C) $2\sqrt{2}$ (D) 2 (E) 3

13. In a tennis tournament's playoffs, six of the results of the quarter-finals, the semi-finals and the final were (not necessarily in this order): Bella beat Ann; Celine beat Donna; Gina beat Holly; Gina beat Celine; Celine beat Bella; and Emma beat Farah. Which result is missing?

(A) Gina beat Bella (B) Celine beat Ann (C) Emma beat Celine
(D) Bella beat Holly (E) Gina beat Emma

14. What percent of the area of the triangle in the figure is shaded?

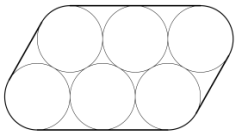


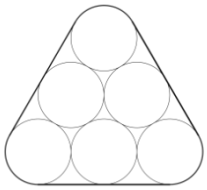
(A) 80% (B) 85% (C) 88% (D) 90%
(E) impossible to determine

15. At which of the given times do the two hands of a watch form the smallest angle?

(A) 2:11 (B) 4:22 (C) 6:33 (D) 8:44 (E) 10:55

16. Jack wants to hold six circular pipes together by a rubber band, each pipe with a diameter 2 cm. He considered the two options shown below.





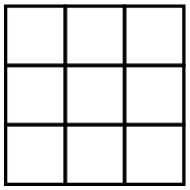
Which of the following is true about the lengths of the rubber bands?

(A) The left rubber band is π cm shorter.
(B) The left rubber band is 4 cm shorter.
(C) The right rubber band is π cm shorter.
(D) The right rubber band is 4 cm shorter.
(E) Both rubber bands have the same length.

17. Eight unmarked envelopes contain the numbers 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses several envelopes randomly. Alice takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alice's sum. How many envelopes did Eve take?

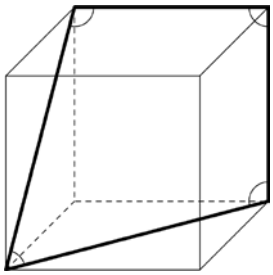
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

18. Peter wants to colour the cells of a 3 × 3 square in such a way that each of the rows, the columns and both diagonals have three cells of three different colours. What is the least number of colours Peter could use?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. The figure shows a cube with four marked angles. What is the sum of these angles?



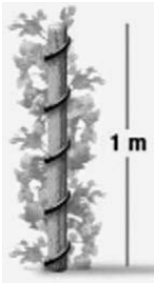
(A) 315° (B) 330° (C) 345° (D) 360° (E) 375°

20. I have some strange dice: the faces show the numbers 1 to 6 as usual, except that the odd numbers are negative (–1, –3, –5 in place of 1, 3, 5). If I throw two such dice, which of these totals cannot be achieved?

(A) 3 (B) 4 (C) 5 (D) 7 (E) 8

Part C: Each correct answer is worth 5 points

21. A plant wound itself exactly 5 times around a pole with height 1 m and circumference 15 cm as shown in the picture.



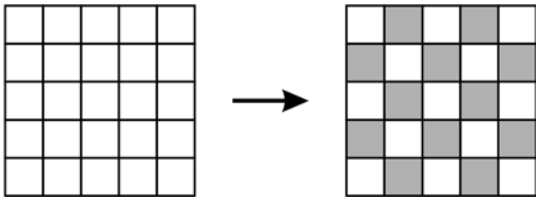
As it climbed, its height increased at a constant rate. What is the length of the plant?

(A) 0.75 m (B) 1.0 m (C) 1.25 m (D) 1.5 m (E) 1.75 m

22. What is the largest possible remainder that can be obtained when a two-digit number is divided by the sum of its digits?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

23. A 5 × 5 square is divided into 25 cells. Initially all its cells are white, as shown on the left.



Neighbouring cells are those that share a common edge. On each move exactly two neighbouring cells have their colours changed to the opposite colour (e.g. white cells become black and black ones become white). What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?

(A) 10 (B) 12 (C) 13 (D) 14 (E) 24

24. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles?

(A) 5 (B) 10 (C) 12 (D) 20 (E) 24

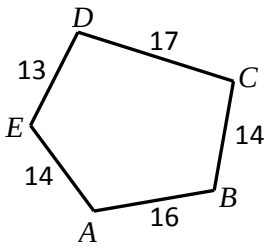
25. In the Kangaroo Republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. Two of the altitudes of a triangle are 10 cm and 11 cm. Which of the following cannot be the length of the third altitude?

(A) 5 cm (B) 6 cm (C) 7 cm (D) 10 cm (E) 100 cm

27. The diagram shows a pentagon. Sepideh draws five circles with centres A, B, C, D, E such that the two circles on each side of the pentagon touch. The lengths of the sides of the pentagon are given. Which point is the centre of the largest circle that she draws?



(A) A (B) B (C) C (D) D (E) E

28. The unit square is divided into a 9-square grid (fig. 1). Four line segments are drawn (fig. 2).

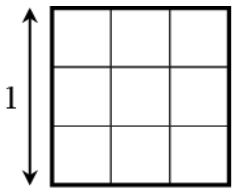


fig. 1

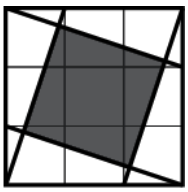


fig. 2

What is the area of the shaded square?

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{3\sqrt{2}}{10}$ (D) $\frac{\sqrt{3}}{4}$ (E) $\frac{4}{9}$

29. Dates can be written in the form DD.MM.YYYY. For example, today's date is 20.03.2016. A date is called "surprising" if all 8 digits in its written form are different. In what month will the next surprising date occur?

(A) March (B) June (C) July (D) August (E) December

30. At a conference, the 2016 participants are registered from P1 to P2016. Each participant from P1 to P2015 shook hands with exactly the same number of participants as the one on their registration number. How many hands did the 2016th participant shake?

(A) 1 (B) 504 (C) 672 (D) 1008 (E) 2015