INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2017

INSTRUCTIONS
GRADE 9-10

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.

2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred here by the end of the contest.

3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.

4. Calculators and graph paper are not permitted. You are allowed to use rough paper for draft work.

5. The figures are not drawn to scale. They should be used only for illustration.

6. Remember, you have about 2-3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to the other problems.

7. At the end of the allotted time, please **submit the response form to the contest supervisor**. Please do not forget to pick up your Certificate of Participation!

Good luck! *Canadian Math Kangaroo Contest team*

Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. In this diagram each number is the sum of the two numbers below. Which number must be in the cell marked with "?" ?
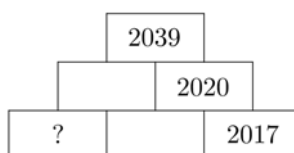
(A) 15   (B) 16   (C) 17   (D) 18   (E) 19

2. My new printer prints 24 pages in 1.5 minutes. How many pages does it print in 5 minutes?
(A) 60   (B) 72   (C) 80   (D) 96   (E) 100

3. Angela made a decoration by overlaying alternating grey and white star-like shapes. The areas of the shapes are 16 cm^2, 9 cm^2, 4 cm^2 and 1 cm^2. What is the total area of the visible grey regions?

(A) 9 cm^2 (B) 10 cm^2 (C) 11 cm^2 (D) 12 cm^2 (E) 13 cm^2

4. Maria has 24 dollars. Every one of her three siblings has 12 dollars. How many dollars does she have to give to each of her siblings, so that each of the four siblings has the same amount?
(A) 1   (B) 2   (C) 3   (D) 4   (E) 6

5. What is the value of $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$?

(A) 2   (B) $\sqrt[8]{16}$ (C) $\sqrt[8]{2}$ (D) $2\sqrt{2}$ (E) 4

6. Some girls were dancing in a circle. Antonia was the fifth to the left of Bianca and the eighth to the right of Bianca. How many girls were in the group?
(A) 11   (B) 12   (C) 13   (D) 14   (E) 15

7. A circle of radius 1 rolls along a straight line from the point K to the point L, where $\overline{KL} = 11\pi$ (see the figure). What does the circle look like in the end position at L?

(A)   (B)   (C)   (D)   (E)
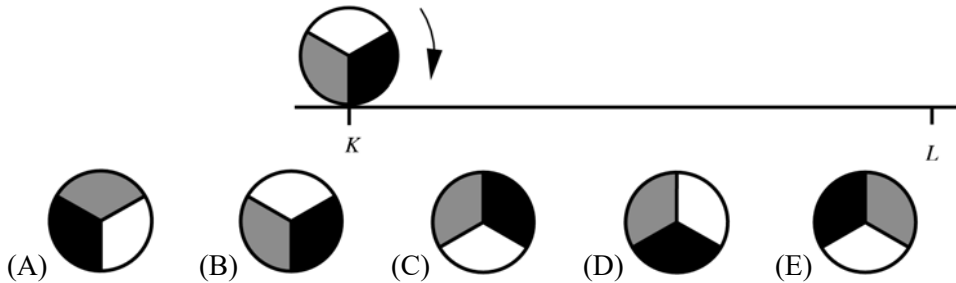
8. Martin participates in a chess tournament. He played 15 games, out of which he won nine. He has five more games to play. What will his success rate be if he wins all five remaining games?
(A) 60 %   (B) 65 %   (C) 70 %   (D) 75 %   (E) 80 %

9. One eighth of the guests of a wedding were children. Three sevenths of the adult guests were men. What fraction of the wedding guests were women?
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{1}{7}$ (E) $\frac{3}{7}$

10. My math teacher has a box containing marbles: 203 red, 117 white and 28 blue. She asked the students to draw marbles from the box, one at a time and without looking. What is the minimum number of marbles which students should take from the box to ensure that at least three of them are of the same colour?
(A) 3   (B) 6   (C) 7   (D) 28   (E) 203

Part B: Each correct answer is worth 4 points

11. ABCD is a trapezoid with side AB parallel to CD. The length of AB is 50 and the length of CD is 20. Point E is on the side AB and has the property that the segment DE divides the trapezoid ABCD into two parts of equal area (see the figure). Calculate the length of AE.

(A) 25   (B) 30   (C) 35   (D) 40   (E) 45
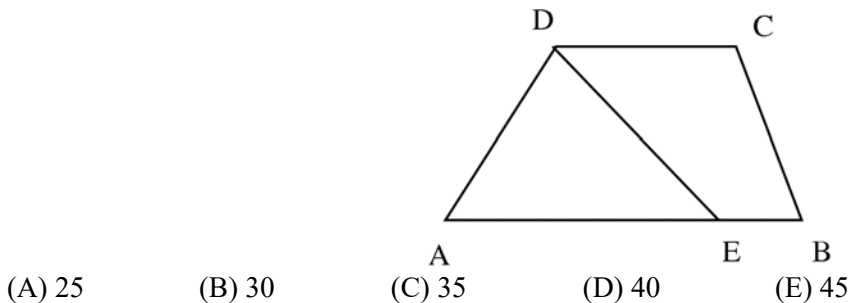
12. How many pairs (a, b) of digits a and b satisfy the condition that the product $\overline{30a} \cdot \overline{7b8}$ of the three-digit numbers $\overline{30a}$ and $\overline{7b8}$ is positive and divisible by 15?
(A) 1   (B) 4   (C) 10   (D) 14   (E) 100

13. Six perpendiculars to the sides are drawn from the midpoints of the sides of a regular triangle (see figure). What fraction of the area of the initial triangle does the resulting hexagon cover?
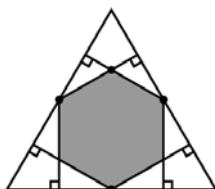
(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

14. The sum of the squares of three consecutive positive integers is 770. Which is the largest of these integers?
(A) 15   (B) 16   (C) 17   (D) 18   (E) 19

15. A belt drive system consists of the wheels A, B and C, which rotate without slippage. The wheel B turns four full rounds when A turns five full rounds, and B turns six full rounds when C turns seven full rounds. Find the perimeter of A if the perimeter of C is 30 cm.
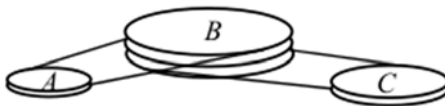
(A) 27 cm   (B) 28 cm   (C) 29 cm   (D) 30 cm   (E) 31 cm

16. Tycho wants to prepare a schedule for his jogging. Every week, he wants to jog on the same days of the week. He never wants to jog on two consecutive days. He wants to jog three times per week. How many such schedules are possible?

(A) 6   (B) 7   (C) 9   (D) 10   (E) 35

17. Four brothers have different heights. Tobias is shorter than Victor by the same length by which he is taller than Peter. Oscar is shorter than Peter by the same length as well. Tobias is 184 cm tall and the average height of all the four brothers is 178 cm. How tall is Oscar?

(A) 160 cm   (B) 166 cm   (C) 172 cm   (D) 184 cm   (E) 190 cm

18. Pere would like to determine who earns more money and who earns less among his three colleagues, Francis, Maria and Joana, from the following two facts. First, he knows that if Francis is not the best paid among the three of them, it must be Joana. Second, he knows that if Joana is not the worst paid, then Maria is the one who earns the most. Who is the one who earns the most, and the one who earns the least, in this order?

(A) Maria, Joana   (B) Francis, Joana   (C) Joana, Maria   (D) Maria, Francis   (E) Francis, Maria

19.   Jenny decided to enter numbers into the cells of the 3 × 3 table so that the sums of the numbers in all four 2 × 2 squares are the same. The three numbers in the corner cells have already been written as shown in the figure. Which number should she write in the fourth corner cell marked with the "?"?

3		1
2		?

(A) 5   (B) 4   (C) 1   (D) 0   (E) impossible to determine

20.   Seven natural numbers a, b, c, d, e, f, g are written in a row. The sum of all of them equals 2017; any two neighbouring numbers differ by ± 1. Which of the numbers can be equal to 286?
(A) only a or g (B) only b or f (C) only c or e (D) only d (E) any of them

Part C: Each correct answer is worth 5 points

21.   There are four children of different integer ages under 18. The product of their ages is 882. What is the sum of their ages?
(A) 23   (B) 25   (C) 27   (D) 31   (E) 33

22.   On the six faces of a given die the following numbers are written: –3; –2; –1; 0; 1; 2. All six numbers are equally likely to appear. Richard rolled the die twice and multiplied the outcomes that appeared. What is the probability that the product is a negative number?
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{11}{36}$ (D) $\frac{13}{36}$ (E) $\frac{1}{3}$

23.   An arbitrary two-digit number consists of the digits a and b. By repeating this pair of digits three times, one obtains a six-digit number. By which of the following numbers will this six-digit number be always divisible?
(A) 2   (B) 5   (C) 7   (D) 9   (E) 11

24.   My friend wants to use a special seven-digit password. The digits of the password occur exactly as many times as its digit value. And the same digits of this number are always written consecutively, for example, 4444333 or 1666666. How many such passwords are possible?
(A) 6   (B) 7   (C) 10   (D) 12   (E) 13

25.   Paul wants to write a natural number in each box in the diagram such that each number is the sum of the two numbers in the boxes immediately underneath. At most how many odd numbers can Paul write?
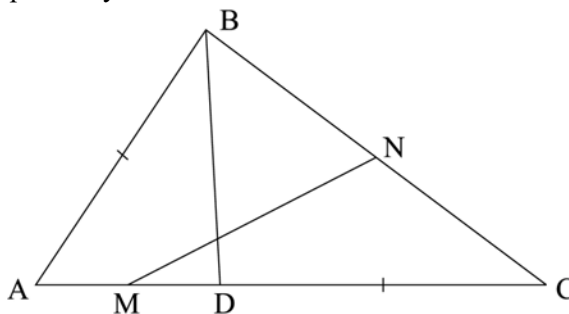(A) 13   (B) 14   (C) 15   (D) 16   (E) 17
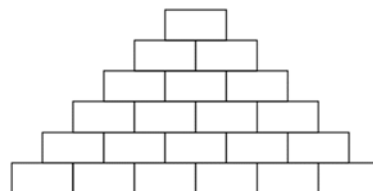
26.   A sequence of 2017 digits is written. It is known that all two-digit numbers, which consist of two consecutive digits in the sequence (in the order in which they are written) are divisible by 17 or by 23. The last digit in the sequence is 1. What is the first digit in the sequence?
        (A) 2   (B) 3   (C) 4   (D) 6   (E) 9

27.   When you swap the digits of a two-digit code, you get the mirrored code: e.g. 80 becomes 08 and the other way around. Look at all the possible codes from 00 up to 99. How many of them differ by a perfect square from their mirrored code? (Note 0 is a perfect square as well!)
        (A) 18   (B) 28   (C) 32   (D) 40   (E) 42

28.   On a balance scale, three different weights are put at random on each pan and the result is shown in the picture. The weights are of 101, 102, 103, 104, 105 and 106 grams. What is the probability that the 106-gram weight stands on the heavier (right) pan?
        (A) 75%   (B) 80%   (C) 90%   (D) 95%   (E) 100%

29.   Points A and B are on the circle with centre M. PB is tangent to the circle at B, therefore perpendicular to radius MB. The distances $\overline{PA}$ and $\overline{MB}$ are integers, $\overline{PB} = \overline{PA} + 6$. How many possible values are there for $\overline{MB}$?
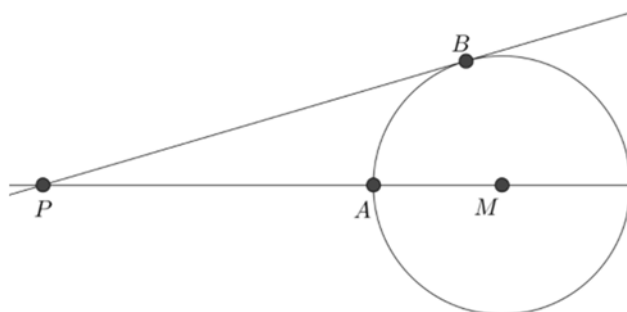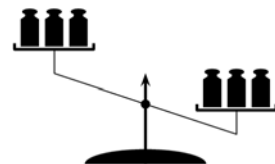
        (A) 0   (B) 2   (C) 4   (D) 6   (E) 8

30.   Point D is chosen on the side AC of triangle ABC so that $\overline{DC} = \overline{AB}$. Points M and N are the midpoints of the segments AD and BC, respectively. If $\angle NMC = \alpha$. What is the measure of $\angle BAC$?

        (A) 2α (B) $90^o - \alpha$ (C) $45^o + \alpha$ (D) $90^o - \frac{\alpha}{2}$ (E) 60°