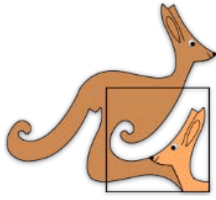INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA, 2018

INSTRUCTIONS
GRADE 5-12

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, circle only one of the proposed five choices. If you circle more than one choice, your response will be marked as wrong.

2. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.

3. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11 -20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.

4. The use of external material or aid of any kind is not permitted.

5. The figures are not drawn to scale. They should be used only for illustration purposes.

6. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.

7. At the end of the allotted time, please give the response form to the contest supervisor.

8. Do not forget to pick up your Certificate of Participation on your way out!

Good luck!

Canadian Math Kangaroo Contest team

Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. In my family each child has at least two brothers and at least one sister.
   What is the smallest possible number of children in my family?
   (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

2. What is the value of x such that $(x + 2^{2018})^2 - (x - 2^{2018})^2 = 2^{2019}$?
   (A) 1   (B) 2^2 (C) 2   (D) 2^{2020} (E) 2^{-1}

3. What is the number of summands of the sum $6 + 6 + \cdots + 6$ if the sum equals 6^3?
   (A) 216   (B) 6   (C) 18   (D) 36   (E) 108

4. The distance from the top of the cat sleeping on the
   floor to the top of the cat sitting on the table is 150
   cm. The distance from the top of the cat sitting on
   the floor to the top on the cat sleeping on the table
   is 110 cm. What is the height of the table?
   (A) 110 cm   (B) 120 cm   (C) 130 cm
   (D) 140 cm   (E) 150 cm
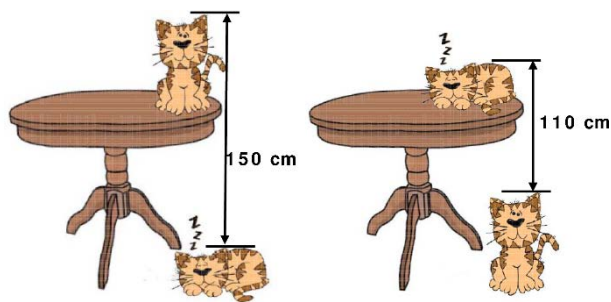
5. The sum of 5 consecutive integers is 10^{2018}. What is the middle number?
   (A) 10^{2013} (B) 5^{2017} (C) 10^{2017} (D) 2^{2018} (E) $2 \cdot 10^{2017}$

6. Given three congruent regular hexagons, we call A, B, C the
   total area of the shaded zones in each one of the figures.
   Which of the following statements is true?
   (A) $A = B = C$ (B) $B = C \neq A$
   (C) $C = A \neq B$ (D) $A = B \neq C$ (E) Each of the three areas A, B, C has a different value.

7. Mary has collected 42 apples, 60 apricots and 90 cherries. She wants to divide them into identical piles
   using all of the fruit and then give a pile to each of her friends.
   What is the largest number of piles she can make?
   (A) 3   (B) 6   (C) 10   (D) 14   (E) 42

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  P 4 5
+ Q R S
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  6 5 4
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8. Some of the digits in the following correct addition have been replaced by the letters
   P, Q, R and S, as shown. How much is P+Q+R+S?
   (A) 14   (B) 15   (C) 16   (D) 17   (E) 24

9. What is the sum of 25% of 2018 and 2018% of 25?
   (A) 1009   (B) 2016   (C) 2018   (D) 3027   (E) 5045

10. The lengths of the two sides of the triangle are 5 and 2, and the length of the third side is an odd integer number. Find the length of the third side.
    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

Part B: Each correct answer is worth 4 points

11. Two buildings are located on one street at a distance of 250 metres from each other. There are 100 students living in the first building, and there are 150 students living in the second building. Where should a bus stop be built so that the total distance that all residents of both buildings have to walk from this bus stop to their buildings would be the least possible?
    (A) in front of the first building   (B) 100 metres from the first building
    (C) 100 metres from the second building   (D) in front of the second building
    (E) anywhere between the buildings

12. There are 105 numbers written in a row: $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots$ (Each number n is written exactly n times). How many of these numbers are divisible by 3?
    (A) 4   (B) 12   (C) 21   (D) 30   (E) 45

13. Eight congruent semicircles are drawn inside a square of length 4. What is the area of the non-shaded part of the square?
    (A) 2π (B) 8   (C) $6 + \pi$ (D) $3\pi - 2$ (E) 3π

14. On a certain day 40 trains each travelled between two of the towns M, N, O, P and Q. 12 trains travelled either from or to M. 12 trains travelled either from or to N. 12 trains travelled either from or to O. 12 trains travelled either from or to P. How many trains travelled from or to Q?
    (A) 12   (B) 24   (C) 32   (D) 36   (E) 40

15. At the University of Humanities, you can study languages, history and philosophy. 35% of students that study languages, study English. 13% of the university students study a language other than English. No student studies more than one language. What percentage of the university students study languages?
    (A) 13%   (B) 20%   (C) 22%   (D) 48%   (E) 65%

16. Peter wanted to buy a book, but he didn't have any money. He bought it with the help of his father and his two brothers. His father gave him half of the amount given by his brothers. His elder brother gave him one third of what the others gave. The younger brother gave him $10. What was the price of the book?
    (A) $24   (B) $26   (C) $28   (D) $30   (E) $32

17. How many 3-digit numbers are there with the property that the 2-digit number obtained by deleting the middle digit is equal to one ninth of the original 3-digit number?
    (A) 1   (B) 2   (C) 3   (D) 4   (E) 5
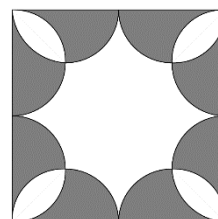
18.   In the calculation shown, how many times does the term 2018^2 appear inside the square root to make the calculation correct? $\sqrt{2018^2 + 2018^2 + \ldots + 2018^2} = 2018^{10}$
        (A) 5   (B) 8   (C) 18   (D) 2018^8 (E) 2018^{18}

19.   On an island there are P penguins and D dingoes. How many animals will there be on the island if each dingo eats 5 penguins?
        (A) $P - 4D$ (B) $D - 4P$ (C) $D + P - 5$ (D) $P + D - 5PD$ (E) $D - 5P$

20.   There are two diagonals drawn in a regular 2018-gon with its vertices numbered from 1 to 2018. One diagonal connects the vertices with the numbers 18 and 1018, the other connects the vertices with the numbers 1018 and 2000. How many vertices do the resulting three polygons have?
        (A) $38, 983, 1001$ (B) $37, 983, 1001$ (C) $38, 982, 1001$
        (D) $37, 982, 1000$ (E) $37, 983, 1002$

Part C: Each correct answer is worth 5 points

21.   Several integers are written on a blackboard, including the number 2018. The sum of all these integers is 2018. The product of these integers is also 2018. Which of the following could be the number of integers written on the blackboard?
        (A) 2016   (B) 2017   (C) 2018   (D) 2019   (E) 2020

22.   Four positive numbers are given. You choose three of them, calculate their arithmetic mean and then add the fourth number. This can be done in four different ways. The results are $17, 21, 23$ and 29, respectively. What is the largest of the given four numbers?
        (A) 12   (B) 15   (C) 21   (D) 24   (E) 29

23.   The points $A_0, A_1, A_2, \ldots$ lie on a line such that $A_0 A_1 = 1$ and the point A_n is the midpoint of the segment $A_{n+1}A_{n+2}$ for every non-negative integer n. What is the length of the segment A_0A_{11}?
        (A) 171   (B) 341   (C) 512   (D) 587   (E) 683

24.   Two concentric circles of radii 1 and 9 make a ring. In the interior of this ring n circles are drawn without overlapping, each being tangent to both of the circles of the ring (an example of such shape for $n = 1$ and different radii is shown in the picture). What is the largest possible value for n?
        (A) 1   (B) 2   (C) 3   (D) 4   (E) 5
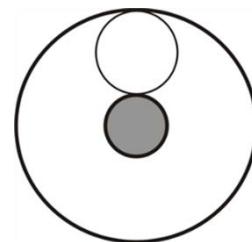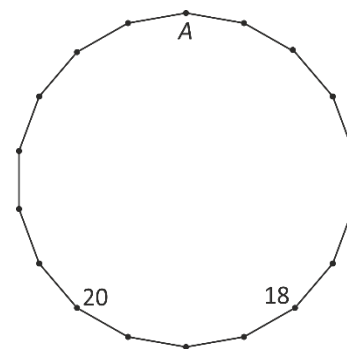
25. At each vertex of the 18-gon in the picture, a number should be written which is equal to the sum of the numbers at the two adjacent vertices. Two of the numbers are given. What number should be written at the vertex A?

    (A) 2018   (B) −20   (C) 18   (D) 38   (E) −38

26. Diana draws a rectangular grid of 12 squares on squared paper. Some of the squares are painted black.

    In each blank square she writes the number of black squares that share a side with it. The figure shows an example. Now she does the same in a rectangular grid with 2018 squares. What is the maximum value that she can obtain as the result of the sum of all the numbers in the grid?

    (A) 1262   (B) 2016   (C) 2018   (D) 3025   (E) 3027

27. Let M be the sum of all odd positive integers less than 100, and let K be the sum of all even positive numbers less than or equal to 100. What is $K - M$?

    (A) 50   (B) 100   (C) 0   (D) −100   (E) −50

28. Each number from the set {1,2,3,4,5,6} is written exactly into one cell of a 2×3 table.
    In how many ways can this be done such that in each row and in each column the sum of the numbers is divisible by 3?

    (A) 24   (B) 36   (C) 42   (D) 45   (E) 48

29. Ed made a large cube by gluing together a number of small identical cubes and then he painted some of the faces of the large cube. His sister Nicole dropped the cube and it broke into the original small cubes. 45 of these small cubes didn't have any painted faces. How many faces of the large cube did Ed paint?

    (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

30. Two chords AB and AC are drawn in a circle with diameter AD. The angle $\angle BAC = 60^0$, $BE \perp AC$, $AB = 24$ cm, $EC = 3$ cm. What is the length of the chord BD?

    (A) $\sqrt{3}$ cm   (B) 2 cm   (C) 3 cm
    (D) $2\sqrt{3}$ cm   (E) $3\sqrt{2}$ cm