Mid-Term

Time Series Analysis ECON - 6376

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I INTRODUCTION

The datasets used in this assignment will explore the realm of time series data using economic indicators from the Federal Reserve Economic Data (FRED). FRED is a trusted source of economic data maintained by the Federal Reserve Bank of St. Louis, providing insights into the U.S. economy. We will analyze four key datasets: the Consumer Price Index for New Vehicles and all items, along with the Producer Price Index for Finished Consumer Foods and Finished Goods. These datasets provide valuable insights into consumer behavior, price dynamics, and production costs. We will apply time series analysis techniques to uncover patterns and trends within these datasets.

1.1 TIME SERIES DATA

Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average (CUUR0000SETA01):

This series is provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Consumer Price Index (CPI). The Consumer Price Index is a widely used economic indicator that measures the average change over time in the prices paid by urban consumers for a market basket of consumer goods and services. This dataset tracks changes in the prices of newly manufactured vehicles, including cars, trucks, and other motor vehicles. It reflects the price movements of these new vehicles in urban areas across the United States. This data is not seasonally adjusted**[1]**

Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (CPIAUCSL):

This series is a fundamental economic indicator provided by the U.S. Bureau of Labor Statistics (BLS). This series measures the average change in prices over time of goods and services purchased by households in urban areas of the United States. The series is based on prices of food, clothing, shelter, fuels, transportation fares, charges for doctors' and dentists' services, drugs, and other goods and services that people buy for day-to-day living. **[2]**

Producer Price Index by Commodity: Final Demand: Finished Consumer Foods (WPSFD4111):

This series is provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Producer Price Index (PPI). This dataset is focused on tracking the changes in the prices of finished consumer food products at the production or wholesale level. This dataset specifically monitors the price changes for finished consumer food products. These products include a wide range of items that are ready for consumption by consumers, such as packaged food, beverages, and other consumable goods. This dataset considers final demand, which means it tracks prices for goods that are ready for sale to consumers, not intermediate goods used in the production process. **[3]**

Producer Price Index by Commodity: Final Demand: Finished Goods (WPSFD49207):

series provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Producer Price Index (PPI). This dataset focuses on tracking the changes in prices for finished goods at the production or wholesale level. It encompasses a wide range of finished goods, including manufactured products that are ready for distribution and sale. These can range from electronics, appliances, and clothing to machinery, vehicles, and various consumer products. Changes in the prices of finished goods can provide insights into the overall direction of economic activity. **[4]**

1.2 Data Collection process

During the data collection process, according to the FRED website documentation, out of the 4 datasets taken, 3 of them are seasonally adjusted, the three datasets are:

- WPSFD4111
- WPSFD49207
- CPIAUCSL

These series are adjusted by the Bureau of Labor Statistics (BLS) using the X-13ARMIA-SEATS, this seasonal adjustment in the data might lead to a loss of detail, as these may smooth out the data, which leads to a loss in specific seasonal patterns. This might also lead to potential errors in the data, as these adjustments are based out of certain assumptions and models.

1.3 Literature Survey

The significance of time series analysis in forecasting is underlined in this study, particularly in relation to projecting future stock prices. The investigation was focused on the US Consumer Price Index (CPI), and it used data from Yahoo Finance, a dependable source, with monthly frequency, spanning a decade. The main goal is to choose the best forecasting model, taking into account covariance stationarity, linear and exponential trends, seasonality, and any notable changes in the time series. This analysis highlights the effectiveness of time series techniques in deriving insightful knowledge from past data to guide present and future investment plans. **[5]**

The Producer Price Index (PPI), a useful indicator that captures price changes from the seller's perspective across multiple sectors, is examined in depth in this paper. PPI is a useful instrument for assessing short-term inflation patterns as well as a source of analytical information for businesses and scholars. International agencies like the IMF and Eurostat, which use it for economic analysis and cross-country comparisons, also find it to be relevant. This study uses a dataset of quarterly PPI data from the United States covering the years 1960 to 2002 to demonstrate its real-world application. The Jenkins Box approach to ARIMA modeling is used in the methodology. By assessing stationarity through the Augmented Dickey Fuller Test, differencing, and analyzing Autocorrelation and Partial Autocorrelation Functions, an optimal ARIMA model was identified. **[6]**

In this article, we examine the differences between the Consumer Price Index (CPI) and the Producer Price Index (PPI), two important economic indicators. The CPI largely focuses on the price of goods and services purchased for personal consumption by urban Americans, including imports. The PPI, on the other hand, provides a wider view by taking into account the full output of American producers, including the goods and services that both producers and consumers have purchased. It is noteworthy that the PPI does not include sales and taxes in producer returns, whereas the CPI does, showing the direct effect on consumer expenses. The PPI assists in determining real economic growth through adjustments for inflation, whereas the CPI makes it easier to measure changes in the cost of living by taking into account both income and expenditure sources.The importance of CPI and PPI in shedding light on various aspects of economic dynamics and policy implications is emphasized in this article. **[7]**

This paper examines the crucial contributions made by the Producer Price Index (PPI) and Consumer Price Index (CPI) to the analysis and comprehension of economic trends and interactions. While the PPI is an important indicator for the mid-stream industry, the CPI primarily reflects changes in the prices of products and services consumed by urban people. The work empirically elucidates the long-term cointegration relationship between CPI and PPI, exposing their persistent connection, using Vector Error Correction (VEC) modeling. **[8]**

1.4 COMPREHENSIVE ECONOMIC ASSESSMENT

The Consumer Price Index (CPI) and the Producer Price Index (PPI) are two interconnected economic indicators, each from a different perspective. The CPI measures the average change in prices paid by consumers for goods and services, while the PPI measures the average change in prices received by producers, wholesalers, and distributors. Both indices are essential for economic analysis and decision-making. The CPI is a crucial measure of inflation, while the PPI can serve as an early warning indicator for changes in consumer prices. Together, the CPI and PPI provide a comprehensive view of inflation, cost pressures, and supply chain dynamics, informing policy decisions and economic assessments.

	cpi_vehicles	cpi_all_items	ppi_finished_foods	ppi_finished_goods
cpi_vehicles	1.0000000	0.9153674	0.8833064	0.9056674
cpi_all_items	0.9153674	1.0000000	0.9868980	0.9921468
ppi_finished_foods	0.8833064	0.9868980	1.0000000	0.9957670
ppi_finished_goods	0.9056674	0.9921468	0.9957670	1.0000000

Figure 1: Correlation values for 4 time series

1.5 DATA ANALYSIS

Table 1: Summary Statistics - Values

Time Series	*CUUR0000SETA01*	*CPIAUCSL*	*WPSFD4111*	*WPSFD49207*
Min	*51.90*	*37.90*	*43.0*	*39.0*
Max	*148.99*	*258.62*	*216.3*	*208.5*
Median	*134.83*	*150.30*	*127.7*	*126.8*
Nulls	*0*	*0*	*0*	*0*
Mean	*117.55*	*147.54*	*131.2*	*126.8*
1st & 3rd Quartiles	*97.58 & 142.71*	*97.38 & 207.33*	*100.1 & 166.4*	*100.1 & 166.0*

Table 2: Summary Statistics - First Difference

Time Series	*CUUR0000SETA01*	*CPIAUCSL*	*WPSFD4111*	*WPSFD49207*
Min	*-2.30*	*-3.84*	*-4.90*	*-5.40*
Max	*2.80*	*2.70*	*6.70*	*3.20*
Median	*0.10*	*0.37*	*0.20*	*0.30*
Nulls	*1*	*1*	*1*	*1*
Mean	*0.15*	*0.36*	*0.28*	*0.28*
1st & 3rd Quartile	*-2.0 & 0.48*	*0.20 & 0.59*	*-0.30 & 0.90*	*0.00 & 0.65*

***** CUUR0000SETA01, CPIAUCSL, WPSFD4111, WPSFD49207 - Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Producer Price Index by Commodity: Final Demand: Finished Consumer Foods, Producer Price Index by Commodity: Final Demand: Finished Goods.***

Table 3: Summary Statistics - Second Difference

Time Series	***CUUR0000S ETA01***	***CPIAUCSL***	***WPSFD4111***	***WPSFD49207***
Min	*-2.31*	*-2.40*	*-7.80*	*-5.20*
Max	*3.50*	*2.29*	*6.50*	*4.40*
Median	*-0.006*	*0*	*0*	*0*
Nulls	*2*	*2*	*2*	*2*
Mean	*0.0009*	*0.001*	*0.0003*	*0.002*
1st & 3rd Quartile	*-0.30 & 0.27*	*-0.20 & 2.0*	*-0.80 & 0.90*	*-0.40 & 0.40*

Table 4: Summary Statistics - Date

Time Series	***CUUR0000S ETA01***	***CPIAUCSL***	***WPSFD4111***	***WPSFD49207***
Min Value	*1970-01-01*	*1970-01-01*	*1970-01-01*	*1970-01-01*
Max Value	*2019-12-01*	*2019-12-01*	*2019-12-01*	*2019-12-01*

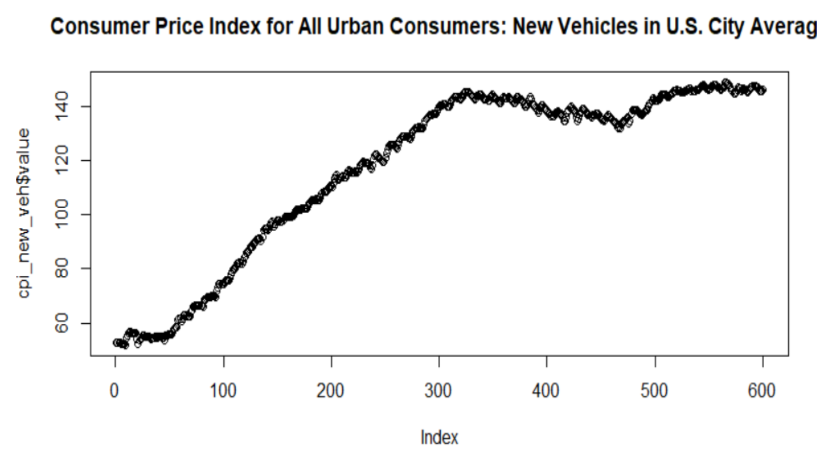

Figure 2. Consumer Price Index (New Vehicles)

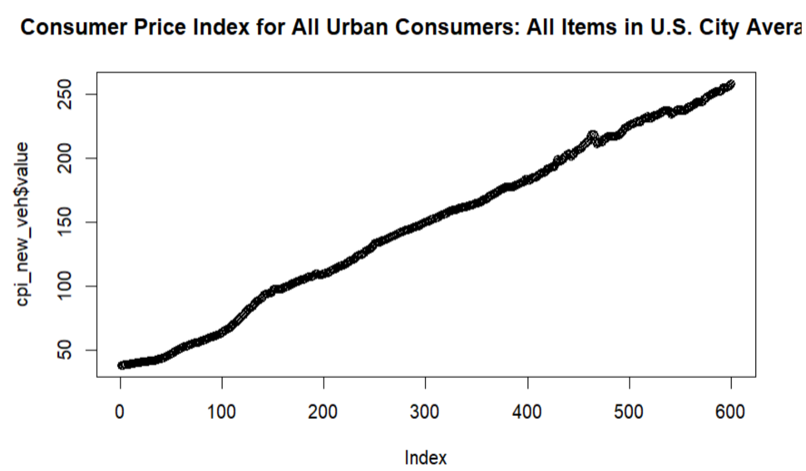

Figure 3. Consumer Price Index (All Items)

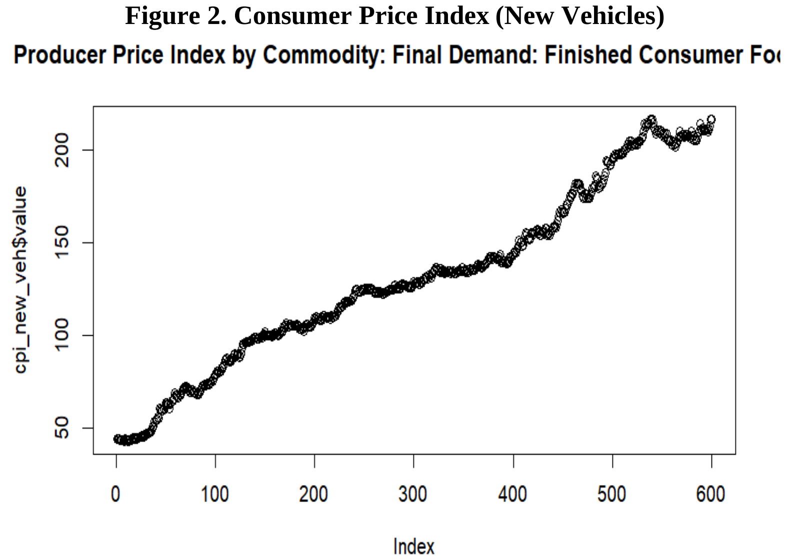

Figure 4. Produce Price Index (Finished Consumer Foods)

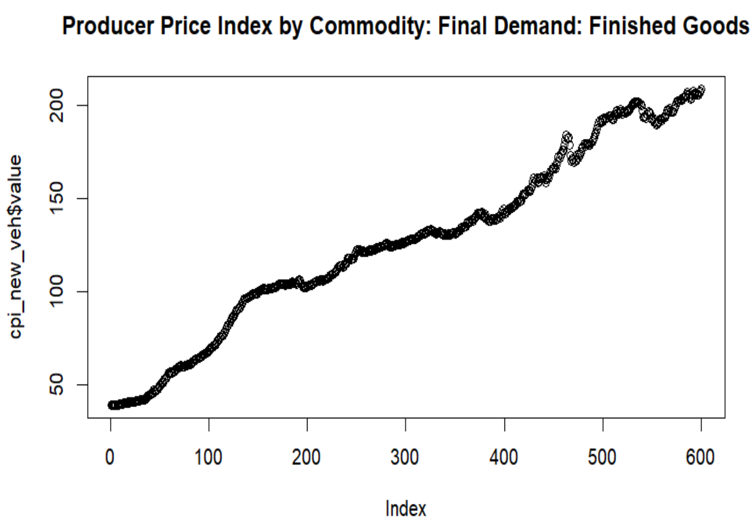

Figure 5. Produce Price Index (Finished Consumer Goods)

II. STATIONARY TIME SERIES

2.1 Strict Stationary vs Weak Stationary Time Series

Strict Stationary: A fundamental concept in time series analysis is strict stationarity, which refers to a crucial aspect of a stochastic process. Strict stationarity in the context of time series data implies that the statistical properties of the data, such as moments and joint distributions, stay stable and do not change with time.
In time series modeling and analysis, strict stationarity is a crucial presumption since it permits the use of a variety of statistical methods and guarantees the stability of model parameters throughout time. It's crucial to remember that strict stationarity is a demanding requirement that isn't always reached by actual data. In some situations, modeling and analysis may be better suited to weak stationarity assumptions.

Weak Stationary: An important concept in time series analysis is weak stationarity, which describes how a stochastic process behaves over time. Weak stationarity, as opposed to strict stationarity, loosens the requirement that a time series' statistical characteristics remain constant.
A series is set to be weak stationary if it follows:

(i) $E(y_t) = \mu \ \forall \ t$, **Constant Mean**
(ii) $E(y_t - \mu)^2 = \sigma^2 \ \forall \ t$, **Constant Variance**
(iii) $E(y_t - \mu)(y_{t-1} - \mu) = \gamma(k) \ \forall \ t$, **Constant Covariance**

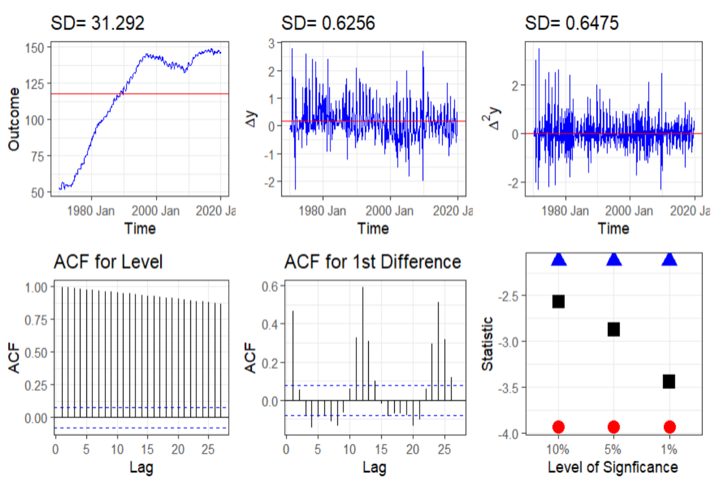

Figure 6. Consumer Price Index (New Vehicles)

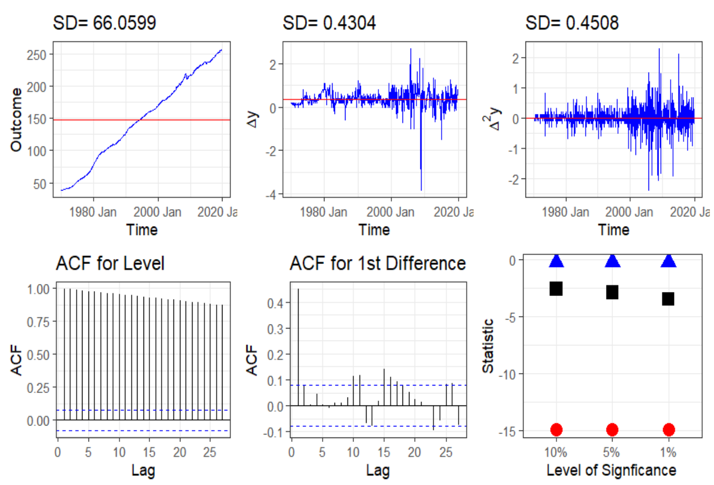

Figure 7. Consumer Price Index (All Items)

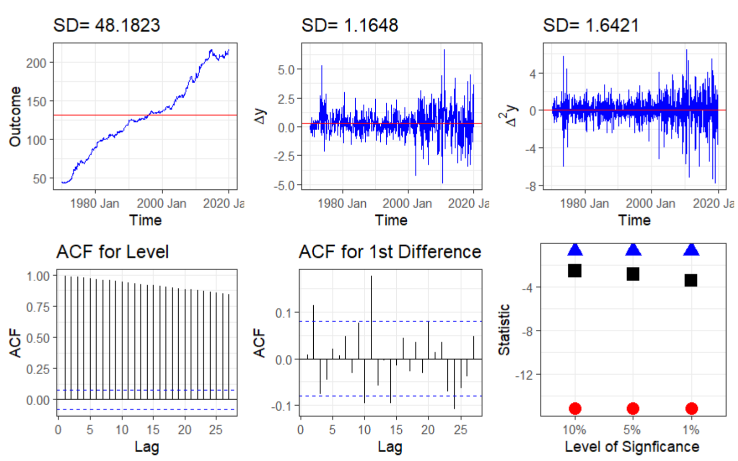

Figure 8. Produce Price Index (Finished Consumer Foods)

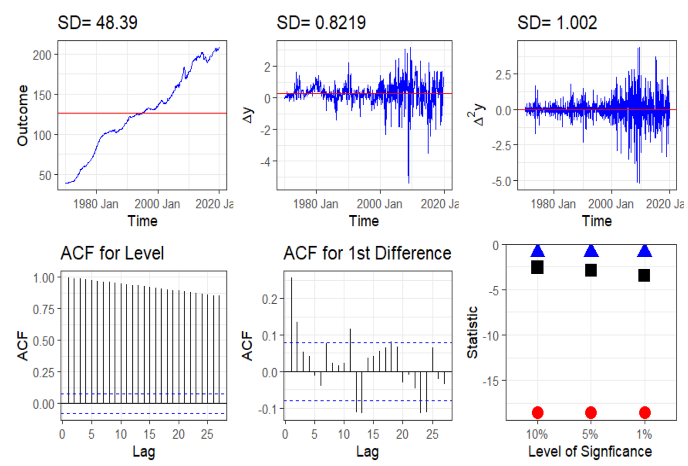

Figure 9. Produce Price Index (Finished Consumer Goods)

Evidence for stationarity in

i) Consumer Price Index (New Vehicles) (Refer Figure 6) – **First Difference stationary.**

- **Visual Evidence:** Upon visual inspection of the time series data, it is evident that the time series looks like it has a trend, but upon further inspection even after removing the trend the time series shows that it is not stationary, and hence we can confirm that is not trend stationary, and only differencing can make this stationary.

- **SD values:** By looking at the intord plot, we can notice that the level series has a SD value of 31.29, and the first difference SD value is 0.62, which is lower than half of the level series and hence we can say that it is stationary at the **first difference.**
- **ACF plot:** Upon inspection on the ACF plot, we notice that the level series is retaining a lot of memory and it does not decay with time, on the other hand, when we look at the first difference's ACF plot, it is noticed that the first difference does not retain a lot of memory and it decays faster as compared to the level series.
- **Augmented-Dickey Fuller Test:** With the help of the ADF test results from the intord function it is very clear that the series is not stationary at the levels as the test results values are above critical values, but when taking the first difference we have the test results below the critical values. Hence, this shows that the time series is stationary at the first difference.

All the evidence states that the time series is **first difference stationary**.

ii) Consumer Price Index (All Items) (Refer Figure 7) **- First Difference stationary.**

- **Visual Evidence:** Upon visual inspection of the time series data, it is evident that the time series looks like it has a trend, but upon further inspection even after removing the trend the time series shows that it is not stationary, and hence we can confirm that is not trend stationary, and only differencing can make this stationary.
- **SD values:** By looking at the intord plot, we can notice that the level series has a SD value of 66.05, and the first difference SD value is 0.43, which is lower than half of the level series and hence we can say that it is stationary at the **first difference.**
- **ACF plot:** Upon inspection on the ACF plot, we notice that the level series is retaining a lot of memory and it does not decay with time, on the other hand, when we look at the first difference's ACF plot, it is noticed that the first difference does not retain a lot of memory and it decays faster as compared to the level series.
- **Augmented-Dickey Fuller Test:** With the help of the ADF test results from the intord function it is very clear that the series is not stationary at the levels as the test results values are above critical values, but when taking the first difference we have the test results below the critical values. Hence, this shows that the time series is stationary at the first difference.

All the evidence states that the time series is **first difference stationary**.

iii) Produce Price Index (Finished Consumer Foods) (Refer Figure 8) **- First Difference stationary.**

- **Visual Evidence:** Upon visual inspection of the time series data, it is evident that the time series looks like it has a trend, but upon further inspection even after removing the trend the time series shows that it is not stationary, and hence we can confirm that is not trend stationary, and only differencing can make this stationary.
- **SD values:** By looking at the intord plot, we can notice that the level series has a SD value of 48.13, and the first difference SD value is 1.16, which is lower than half of the level series and hence we can say that it is stationary at the **first difference.**
- **ACF plot:** Upon inspection on the ACF plot, we notice that the level series is retaining a lot of memory and it does not decay with time, on the other hand, when we look at the first difference's ACF plot, it is noticed that the first difference does not retain a lot of memory and it decays faster as compared to the level series.
- **Augmented-Dickey Fuller Test:** With the help of the ADF test results from the intord function it is very clear that the series is not stationary at the levels as the test results values are above critical values, but when taking the first difference we have the test results below the critical values. Hence, this shows that the time series is stationary at the first difference.

All the evidence states that the time series is **first difference stationary**.

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iv) Figure 6. Produce Price Index (Finished Consumer Goods) (Refer Figure 9) **- First Difference stationary.**

- **Visual Evidence:** Upon visual inspection of the time series data, it is evident that the time series looks like it has a trend, but upon further inspection even after removing the trend the time series shows that it is not stationary, and hence we can confirm that is not trend stationary, and only differencing can make this stationary.

- **SD values:** By looking at the intord plot, we can notice that the level series has a SD value of 48.39, and the first difference SD value is 0.82, which is lower than half of the level series and hence we can say that it is stationary at the **first difference.**
- **ACF plot:** Upon inspection on the ACF plot, we notice that the level series is retaining a lot of memory and it does not decay with time, on the other hand, when we look at the first difference's ACF plot, it is noticed that the first difference does not retain a lot of memory and it decays faster as compared to the level series.
- **Augmented-Dickey Fuller Test:** With the help of the ADF test results from the intord function it is very clear that the series is not stationary at the levels as the test results values are above critical values, but when taking the first difference we have the test results below the critical values. Hence, this shows that the time series is stationary at the first difference.

  All the evidence states that the time series is **first difference stationary**.

III. MODEL SELECTION CRITERION

The model selection criteria chosen here is ***Akaike Information Criterion (AIC),*** as in this case AIC is preferred over the Bayesian Information Criterion (BIC). In order to find a model that effectively captures the data while punishing unduly complicated models, AIC strikes a balance between the trade-off between *model fit and model complexity*. This is achieved by taking into account both the goodness of fit and the total number of model parameters. The AIC can be calculated by

$$AIC = -2(LL) + 2(K+1) \qquad \textbf{----Equation 1}$$

where 'k' is the model's parameter count and 'log-likelihood' (LL) indicates how well the model fits the data. By using AIC as the model selection criterion, we seek to identify a model that achieves the best possible balance between explaining the variation in the data and preserving simplicity.

The AIC also allows for a wider variety of statistical models and is more adaptable. AIC is easily adapted to a variety of modeling approaches, including time series analysis, linear regression, and non-linear models, among others. Due to its adaptability, AIC is a popular choice among researchers that work with a variety of datasets and modeling techniques. BIC, on the other hand, could not be as universally applicable and might prove to be limiting in complex modeling settings. As a result, our choice of AIC as the model selection criterion is robust and contextually suitable. Our analysis, which uses a diverse dataset and an exploratory research methodology, is in line with the nature of AIC.

IV. TWO SERIES SELECTION

For this analysis, I will focus on two economic indicators, specifically, *CUUR0000SETA01* (**Consumer Price Index – New Vehicles**), and CPIAUCSL (**Consumer Price Index – All Items**), obtained from the Federal Reserve Economic Data (FRED). These indicators are essential for understanding inflation, reflecting the average change in prices of a basket of goods and services purchased by urban consumers over time.

IV.I Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average

From Figure 6, we can find that the series has an upward trend, but when removing the trend, we also noticed that it didn't not make the series stationary. Other evidence with respect to the SD values, ACF plots and ADF tests also showed clear evidence that we need to use the first difference series for our analysis.

Total Observations: 600 = (594 observations Training Set) + 6 observation Test set

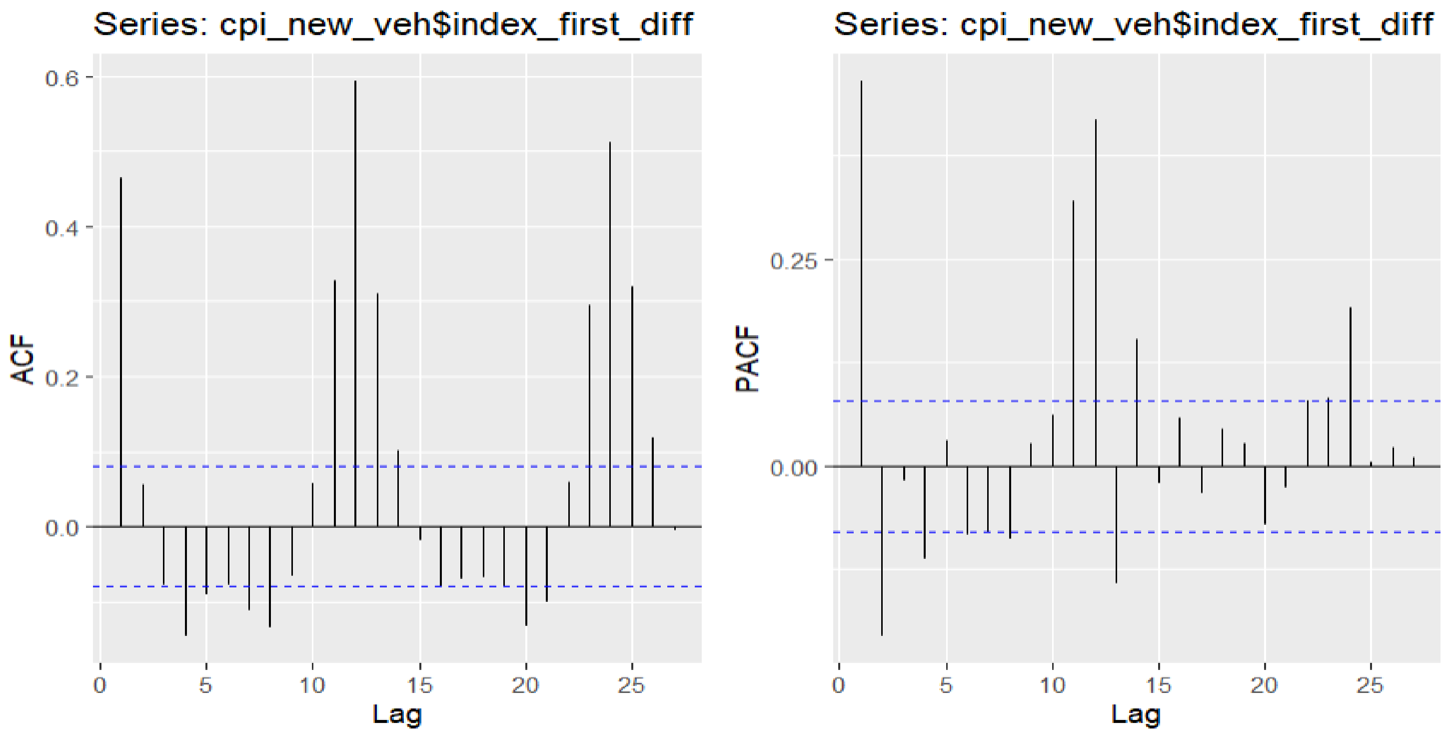

Figure 10: ACF & PACF graph for the Consumer Price Index – All vehicles- First Difference

From the above plot (Refer Figure 10), we can see that the PACF plots decay and the ACF plots decay is almost the same. So, from the plot we can derive the P, D, Q values from both the PACF and ACF plots.

Table 5: Consumer Price Index -All vehicles - P, D, Q values

MODEL	P	D	Q
Model 1	6	1	7
Model 2	4	1	7
Model 3	6	1	5

Table 6: Consumer Price Index-All vehicles – ARIMA Models

	Model 1		Model 2		Model 3	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Φ1	-0.0933	0.1656	0.4253	0.2219	0.7720	0.1845
Φ2	0.3549	0.0667	0.9315	0.1711	0.1503	0.2493
Φ3	-0.0725	0.0736	-0.7275	0.1530	0.6894	0.1275
Φ4	0.162	0.073	-0.3339	0.2131	-0.7551	0.1873
Φ5	-0.6454	0.0488	-	-	-0.1879	0.2474
Φ6	-0.4256	0.1383	-	-	0.3020	0.1201
Φ7	-	-	-	-	-	-
θ1	0.6349	0.1706	0.1152	0.2209	-0.2617	0.1763
θ2	-0.2711	0.0827	-1.1422	0.0684	-0.5126	0.1505
θ3	-0.1242	0.0704	0.2397	0.2375	-0.9098	0.0251
θ4	-0.2472	0.0496	0.6172	0.1698	0.2471	0.1676

θ5	0.6336	0.0524	0.2080	0.1068	0.5717	0.1429
θ6	0.8722	0.1391	0.0996	0.0629	-	-
θ7	0.1002	0.1233	-0.0966	0.0485	-	-
Num. Obs	594					
AIC	886.59		903.8		903.35	
AICc	887.31		904.33		903.89	
BIC	947.98		956.42		955.97	

SERIAL CORRELATION

Serial correlation occurs in a time series when a variable and a lagged version of itself (for instance a variable at times T and at T-1) are observed to be correlated with one another over periods of time. Repeating patterns often show serial correlation when the level of a variable affects its future level.

Ljung-Box test

The Ljung-Box test is employed to determine whether serial correlation or autocorrelation exists in time series data. Serial correlation is the relationship between a series of observations, where the value at one point in time is influenced by measurements made earlier. If these relationships exist beyond what would be predicted by chance, the test determines if they do. The Ljung-Box test results for the 3 models are:

Table 7: Consumer Price Index Models - ARIMA Models

Model	X-Squared value	p-value	Significance
Model 1	1.3365	0.2476	**Not Significant**
Model 2	1.4944	0.2215	**Not Significant**
Model 3	0.00061	0.9802	**Not Significant**

Based on the Ljung-Box test we can see that none of the above models show serial correlations, as the p-values of the test are above the significance level and hence we can conclude that there is not serial correlation.

Final Model Conclusion for Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average

Due to the fact that our data is not seasonally adjusted, a thorough investigation of seasonality was conducted before we built our time series model. As seasonality is often an important consideration in time series modeling, our careful examination of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots revealed a lack of obvious patterns or significant spikes, which allowed us to reasonably conclude that seasonality was not a significant factor in our dataset.

Subsequently, the primary objective was to find the relevant P, D, Q values for the ARIMA Framework, so that we can have a better predicting model. We use the ARMA model as we can see from the ACF and PACF graphs that decay is nearly identical. The models were then created after determining the values of P, D, and Q from the ACF and PACF plots.

AIC is established as the principal model selection criterion, given its ability to strike a balance between goodness of fit and model complexity. Model 1 surpassed all others, by having a lower AIC value. Following the model selection process, we ran the Ljung-Box test to see if the selected models had serial correlation. All models underwent the Ljung-Box test. None of the models in our data showed serial correlation, confirming the strength and suitability of our modeling efforts. **Model 1** met the requirements of the AIC selection and the lack of serial correlation, and thus emerged as the most appropriate and efficient model.

IV.II Consumer Price Index for All Urban Consumers: All Items in U.S. City Average

From Figure 7, we can find that the series has an upward trend, but when removing the trend, we also noticed that it didn't not make the series stationary. Other evidence with respect to the SD values, ACF plots and ADF tests also showed clear evidence that we need to use the first difference series for our analysis.

Total Observations: 600 = (594 observations Training Set) + 6 observation Test set

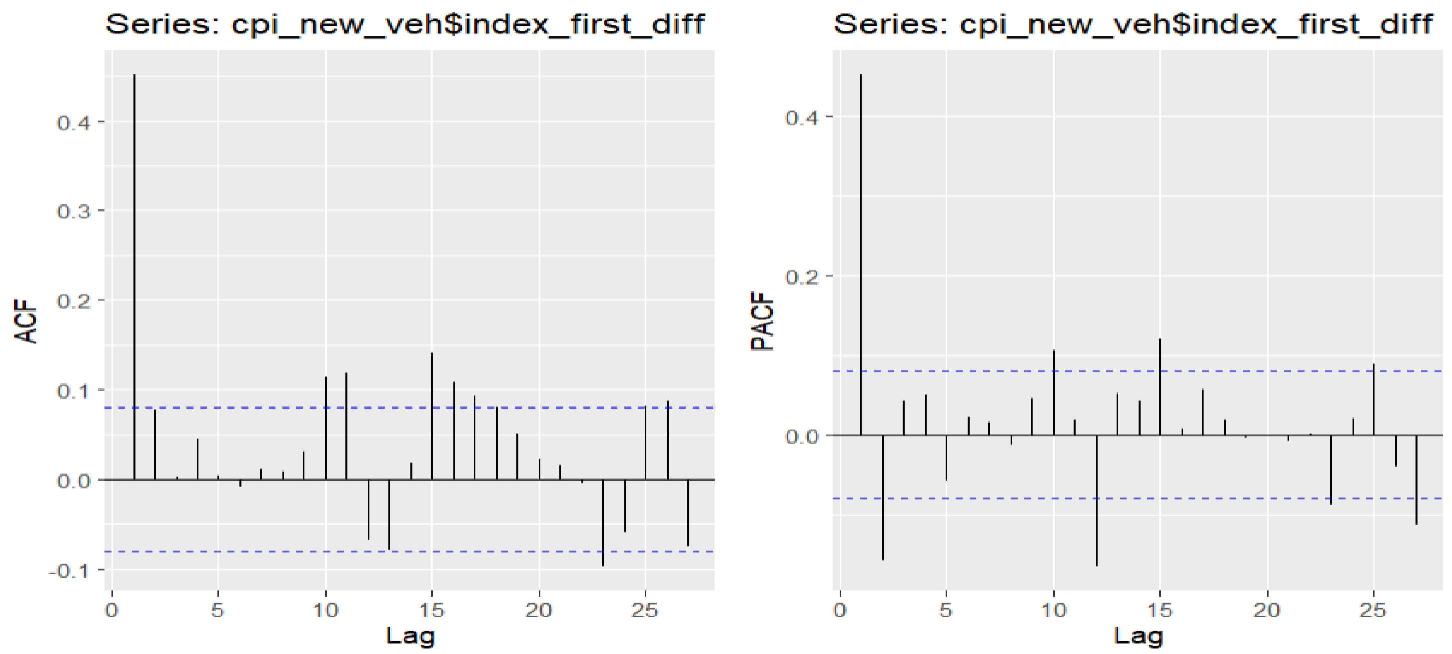

Figure 11: ACF & PACF graph for the Consumer Price Index – All Items- First Difference

From the above plot (Refer Figure 11), we can see that PACF decays faster as compared to ACF. So, from the plot we can derive the P value from the PACF graph, and we are using an AR model. We have also considered an ARMA model to see the performance of the model.

Table 8: Consumer Price Index – All Items- P, D, Q values

MODEL	P	D	Q
Model 1	5	1	0
Model 2	3	1	2
Model 3	4	1	0

Table 9: Consumer Price Index- All items - ARIMA Models

	Model 1		Model 2		Model 3	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Φ1	0.6406	0.0410	0.4403	0.0327	0.6491	0.0404
Φ2	-0.1140	0.0484	-0.8036	0.0143	-0.1103	0.0483
Φ3	0.0794	0.0486	0.6949	0.0296	0.0741	0.0484
Φ4	0.1414	0.0484	-	-	0.1730	0.0405
Φ5	0.0487	0.0411	-	-	-	-
Φ6	-	-	-	-	-	-
Φ7	-	-	-	-	-	-

θ1	-	-	0.2499	0.0226	-	-
θ2	-	-	0.9589	0.0207	-	-
Num. Obs	594					
AIC	607.47		624.27		606.87	
AICc	607.61		624.42		606.97	
BIC	633.78		650.58		628.8	

SERIAL CORRELATION

Serial correlation occurs in a time series when a variable and a lagged version of itself (for instance a variable at times T and at T-1) are observed to be correlated with one another over periods of time. Repeating patterns often show serial correlation when the level of a variable affects its future level.

Ljung-Box test

The Ljung-Box test is employed to determine whether serial correlation or autocorrelation exists in time series data. Serial correlation is the relationship between a series of observations, where the value at one point in time is influenced by measurements made earlier. If these relationships exist beyond what would be predicted by chance, the test determines if they do. The Ljung-Box test results for the 3 models are:

Table 10: Consumer Price Index Models - ARIMA Models

Model	X-Squared value	p-value	Significance
Model 1	1.0564	0.304	**Not Significant**
Model 2	4.0724	0.04359	**Significant**
Model 3	1.3903	0.2384	**Not Significant**

Based on the Ljung-Box test we can see the p-value for model 2 is less than the critical value of 0.05 and we can conclude that the **model 2** is significant and it has serial correlation. But the rest of the models here do not have any serial correlation.

Final Model Conclusion for Consumer Price Index for All Urban Consumers: All Items in U.S. City Average

Due to the fact that our data is seasonally adjusted, even then a thorough investigation of seasonality was conducted before we built our time series model. As seasonality is often an important consideration in time series modeling, our careful examination of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots revealed a lack of obvious patterns or significant spikes, which allowed us to reasonably conclude that seasonality was not a significant factor in our dataset.

Subsequently, the primary objective was to find the relevant P, D, Q values for the ARIMA Framework, so that we can have a better predicting model. We use both AR and ARMA model as we can see from the PACF decays faster than the ACF and we also considered the case of almost same decay of ACF and PACF. The models were then created after determining the values of P, D, and Q from the ACF and PACF plots.

AIC is established as the principal model selection criterion, given its ability to strike a balance between goodness of fit and model complexity. Model 1 surpassed all others, by having a lower AIC value. Following the model selection process, we ran the Ljung-Box test to see if the selected models had serial correlation. All models underwent the Ljung-Box test. Model 2 shows a serial correlation and hence we can discard the model, but the other models do not show any sign of serial correlation. **Model 3** met the requirements of the AIC selection and the lack of serial correlation, and thus emerged as the most appropriate and efficient model.

V. Model Forecasts and its Loss Functions

V.I Model forecast and its loss function for the Consumer Price Index – All Urban Consumers: New Vehicles in U.S. City Average

Model 1 forecast (Preferred Model):

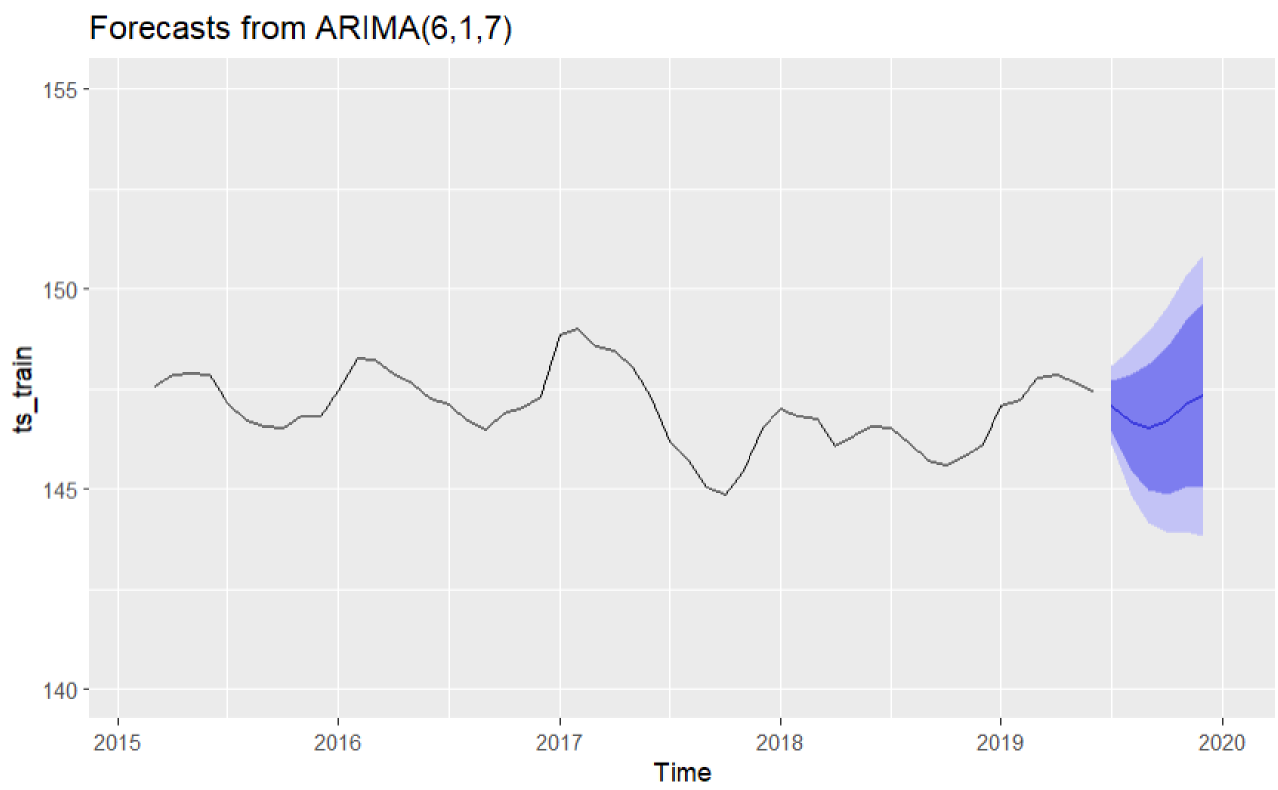

Figure 12: Test forecast for Model 1(Our preferred model)

Table 11: Model 1 point forecast values (Preferred Model):

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jul 2019	147.0808	146.4376	147.7239	146.0972	148.0644
Aug 2019	146.6595	145.4776	147.8413	144.8520	148.4670
Sept 2019	146.5389	144.9823	148.0955	144.1583	148.9195
Oct 2019	146.7205	144.8660	148.5751	143.8842	149.5568
Nov 2019	147.1209	145.0434	149.1983	143.9437	150.2981
Dec 2019	147.3439	145.0434	149.6445	143.8255	150.8623

Model 2 forecast:

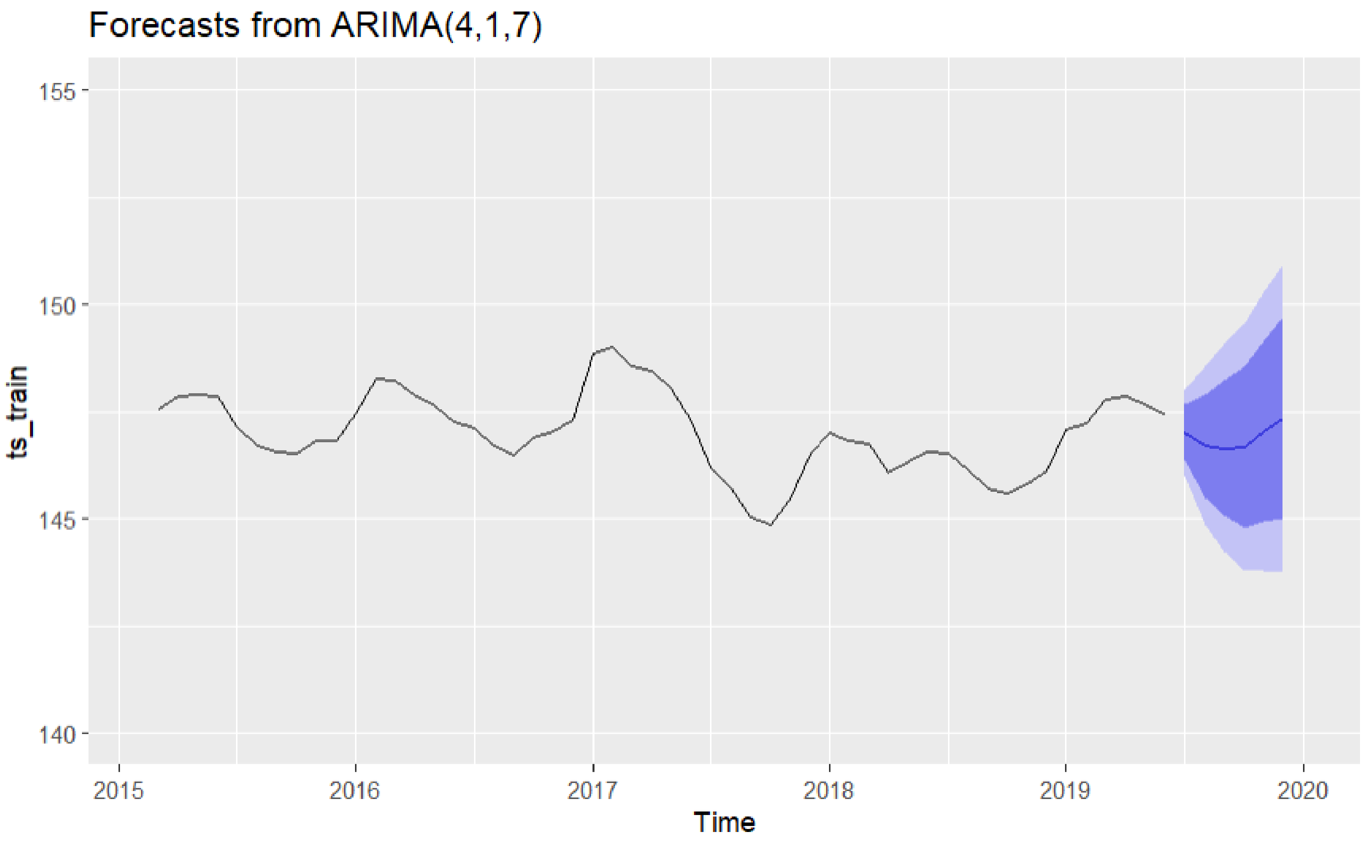

Figure 13: Test forecast for Model 2

Table 12: Model 2 point forecast values

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jul 2019	147.0104	146.3558	147.6651	146.0092	148.0116
Aug 2019	146.7032	145.5009	147.9056	144.8644	148.5421
Sept 2019	146.6455	145.0681	148.2230	144.2331	149.0580
Oct 2019	146.6778	144.7902	148.5653	143.7909	149.5646
Nov 2019	147.0392	144.9109	149.1674	143.7843	150.2940
Dec 2019	147.3450	145.0004	149.6896	143.7592	150.9307

Table 13: Loss Function Values

Model	RMSE	RMAE
Model 1(preffered)	0.885527	0.860759
Model 2	0.873575	0.861322

By looking at the forecasts values we can see that the last 6 observations (Figure 12 and 13) are being predicted by both the models, but inorder to know how good these models are predicting we need to use loss functions, the loss functions used here to compare the better predicting model is RMSE and RMAE. The two loss functions can be calculated by using the following equation:

$$MSE = \frac{1}{N} \cdot \sum_{i=1}^{N} (x_i - m_i)^2$$

$$MAE = \frac{1}{N} \cdot \sum_{i=1}^{N} |x_i - m_i|$$

The RMSE can be calculated as

$$\text{RMSE} = \sqrt{MSE} \qquad \text{---- Equation 2}$$

The RMAE can be calculated as

$$\text{RMAE} = \sqrt{MAE} \qquad \text{---- Equation 3}$$

Comparison of forecasted values using RMSE and RMAE

While Model 1 remains our top pick for this research, a closer look at the evaluation metrics yields important information. Notably, we saw that Model 1's Root Mean Square Error (RMSE)(Table 13) was greater than Model 2's. But when we concentrate on the Root Mean Absolute Error (RMAE) (Table 13), **Model 1** emerges as the better performance, showing a smaller RMAE in comparison to Model 2.

In summary, the AIC metric shows that **Model 1** is our preferred model and matches our model selection requirements. However, the selection of the loss function offers a different viewpoint. **Model 2** is the better predictive model for cases where RMSE is the main criterion for evaluation. **Model 1** is the better option when RMAE is the selected loss function, on the other hand. This finding emphasizes how important it is to match the evaluation metric chosen with the analysis's unique objectives, resulting in a thorough and contextually appropriate evaluation of model performance.

Uncertainty surrounding the forecast:

Tables 11 and 12 are useful for understanding the degree of uncertainty in our point projected values. The "Lo95" and "Hi95" columns in these tables are crucial for illuminating the range of possible mistakes and uncertainties surrounding our point estimations. The "Lo95" number in these tables denotes the lower limit of the uncertainty range. It stands for the point estimate with the least amount of error or uncertainty. The "Hi95" number, on the other hand, denotes the point estimate with the largest error or uncertainty and serves as the upper bound of the uncertainty range.

We now have an important understanding of the range of uncertainty surrounding our point estimate forecasts due to the information provided. It enables us to determine the possible range in which the true values might fall, improving our understanding of the inherent uncertainty, which is essential for making wise decisions and evaluating risk.

V.II Model forecast and its loss function for the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average

Model 3 forecast (Preferred Model):

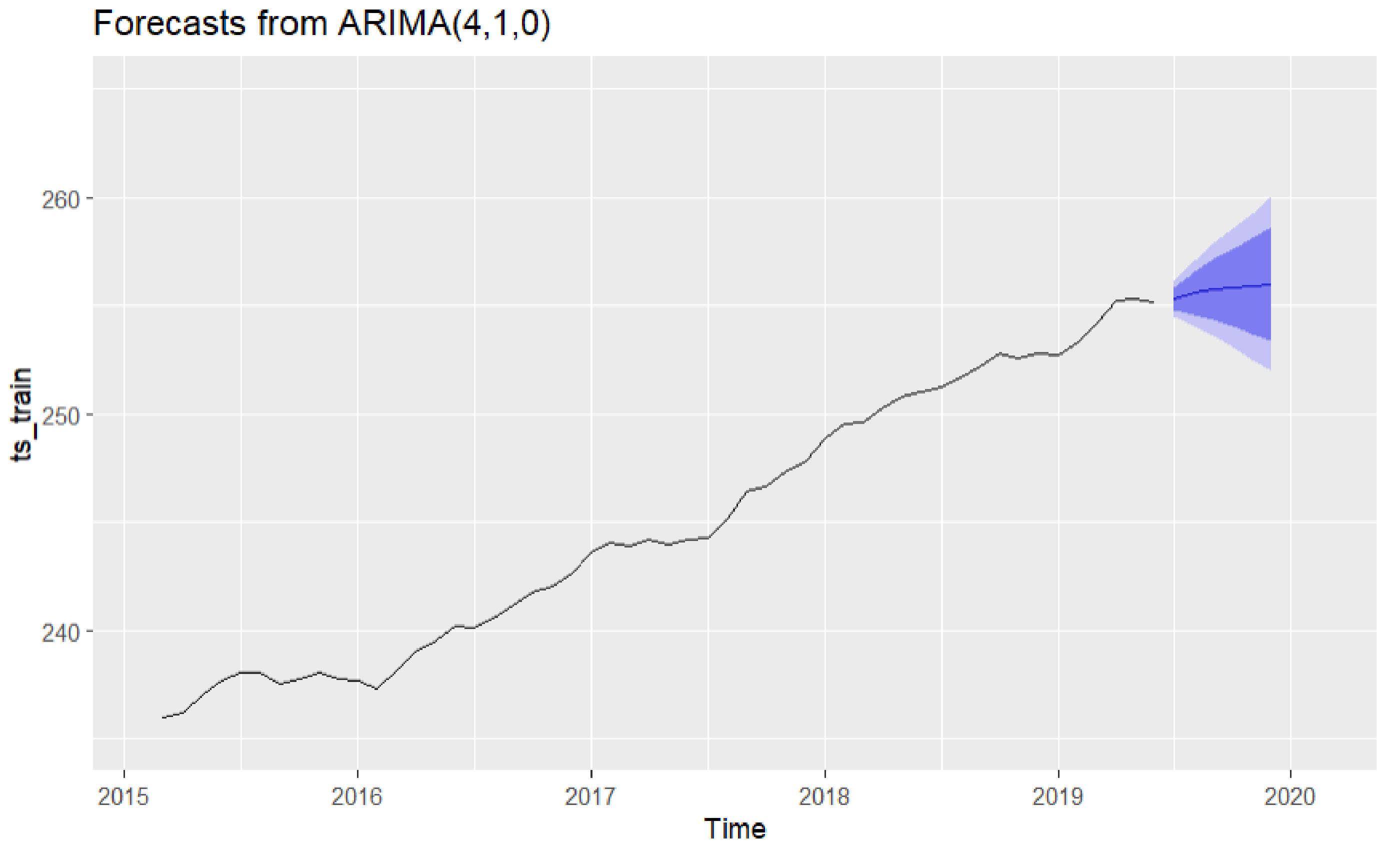

Figure 14: Test forecast for Model 3 (Preferred model)

Table 14: Model 3 point forecast values (Preferred Model):

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jul 2019	255.2923	254.7780	255.8066	254.5057	256.0789
Aug 2019	255.5737	254.5818	256.5656	254.0567	257.0907
Sept 2019	255.7456	254.3313	257.1598	253.5827	257.9085
Oct 2019	255.8133	254.0135	257.6132	253.0608	258.5659
Nov 2019	255.8823	253.6751	258.0894	252.5068	259.2578
Dec 2019	255.9810	253.3444	258.6175	251.9487	260.0132

Model 1 forecast:

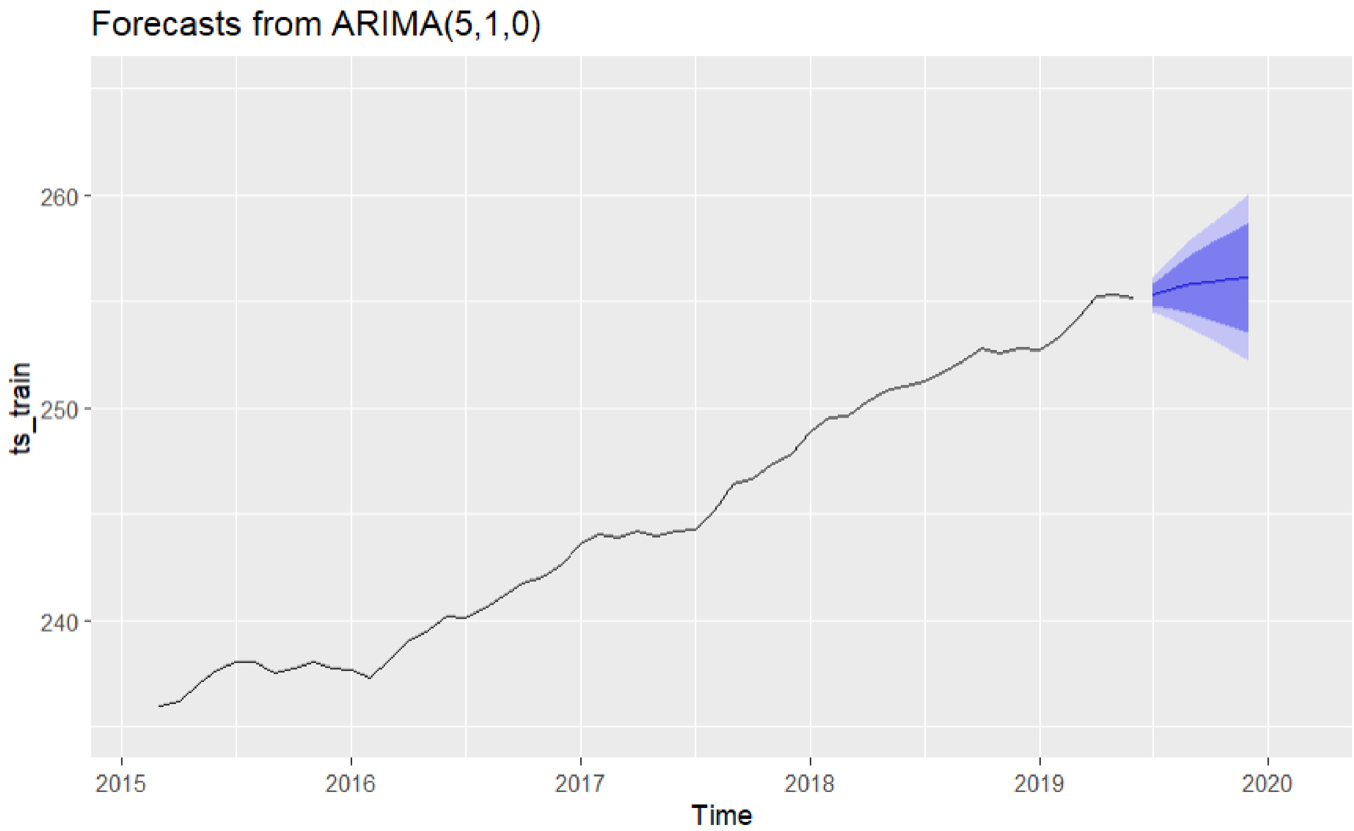

Figure 15: Test forecast for Model 1

Table 15: Model 1 point forecast values

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jul 2019	255.3001	254.7860	255.8142	254.5138	256.0864
Aug 2019	255.5973	254.6095	256.5851	254.0866	257.1081
Sept 2019	255.8216	254.4188	257.2243	253.6763	257.9668
Oct 2019	255.9279	254.1472	257.7085	253.2046	258.6511
Nov 2019	256.0076	253.8360	258.1792	252.6865	259.3287
Dec 2019	256.1133	253.5247	258.7018	252.1544	260.0721

Table 16: Loss Function Values

Model	RMSE	RMAE
Model 3(preffered)	1.50858	1.12766
Model 1	1.41749	1.09165

Comparison of forecasted values using RMSE and RMAE

By looking at the forecasts values we can see that the last 6 observations (Figure 14 and 15) are being predicted by both the models, but inorder to know how good these models are predicting we need to use loss functions, the loss functions used here to compare the better predicting model is RMSE and RMAE.

While Model 3 remains our top pick for this research, a closer look at the evaluation metrics yields important information. Notably, we saw that Model 3's Root Mean Square Error (RMSE) (Table 16) was greater than Model 1's. It is the same case when take the RMAE values as well, **Model 1** is a better predicting model when compared to Model 3.

In summary, even though our preferred model is Model 3 based on the AIC metric. When we use RMSE and RMAE loss functions, Model 1 turns out to be a better predicting model. This means that our preferred model does not always be the better predicting model.

Uncertainty surrounding the forecast:
Tables 14 and 15 are useful for understanding the degree of uncertainty in our point projected values. The "Lo95" and "Hi95" columns in these tables are crucial for illuminating the range of possible mistakes and uncertainties surrounding our point estimations. The "Lo95" number in these tables denotes the lower limit of the uncertainty range. It stands for the point estimate with the least amount of error or uncertainty. The "Hi95" number, on the other hand, denotes the point estimate with the largest error or uncertainty and serves as the upper bound of the uncertainty range.

We now have an important understanding of the range of uncertainty surrounding our point estimate forecasts due to the information provided. It enables us to determine the possible range in which the true values might fall, improving our understanding of the inherent uncertainty, which is essential for making wise decisions and evaluating risk.

VI. Forecast Combination Techniques

Granger-Bates forecast combination is a statistical technique that combines various time series forecasts by determining whether prior predictions from one model can improve those from another. To evaluate the historical performance of several forecasting models and ascertain how they affect each other's accuracy over time, it makes use of the Granger causality theory. This technique tries to produce a combined forecast that takes advantage of the capabilities of many models, leading to more precise and trustworthy forecasts by discovering causal links between forecast errors.

VI.I Forecast combination technique for Consumer Price Index – All Urban Consumers: New Vehicles in U.S. City Average

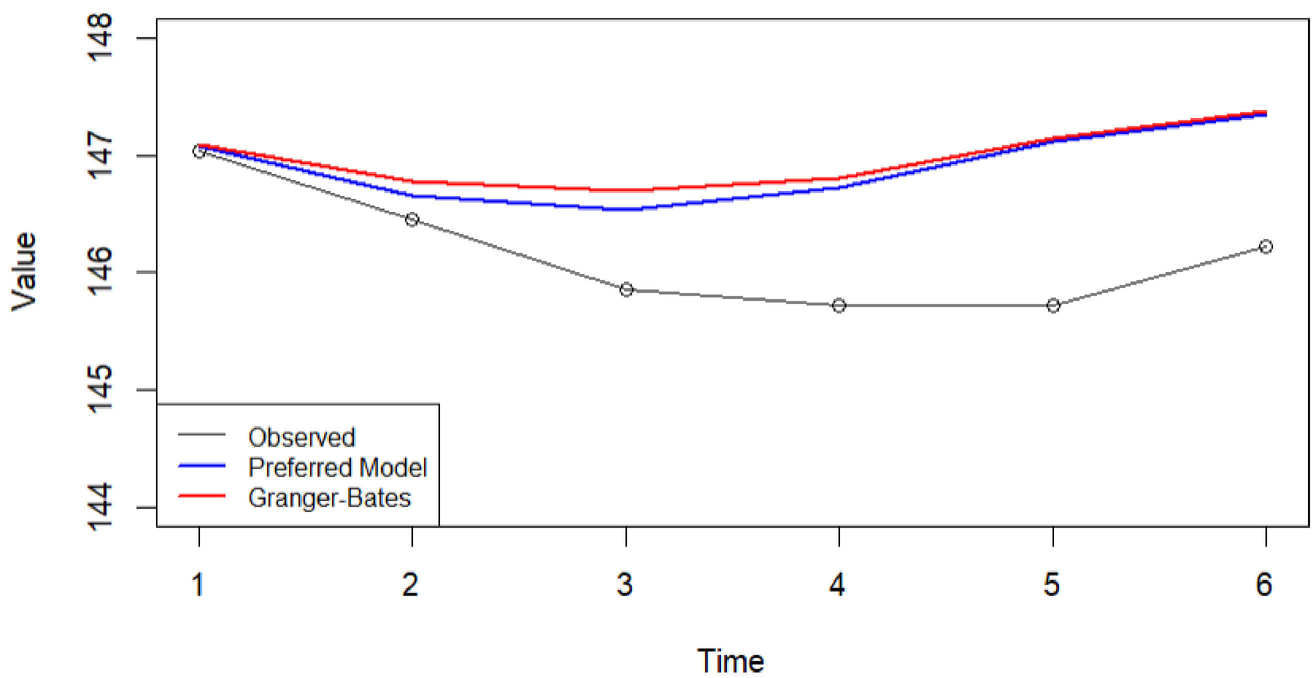

Figure 16: Forecast combination for CPI – All Urban Consumers: New vehicles

```
$Accuracy_Train
                  ME      RMSE       MAE        MPE      MAPE      ACF1 Theil's U
Test set -0.8084733 0.9383905 0.8084733 -0.5540232 0.5540232 0.5333435  2.359649

$Input_Data
$Input_Data$Actual_Train
Time Series:
Start = 1
End = 6
Frequency = 1
[1] 147.035 146.456 145.854 145.726 145.728 146.220

$Input_Data$Forecasts_Train
Time Series:
Start = 1
End = 6
Frequency = 1
  Series 1 Series 2 Series 3
1 147.0808 147.0104 147.2177
2 146.6595 146.7032 147.1316
3 146.5389 146.6455 147.0524
4 146.7205 146.6778 147.1552
5 147.1209 147.0392 147.3390
6 147.3439 147.3450 147.4712


$Weights
prediction_matrix.Series 1 prediction_matrix.Series 2 prediction_matrix.Series 3
                 0.3836832                  0.3942541                  0.2220628
```

Figure 17: Forecast combination results (Granger Bates) method

Table 17: Loss Function Values for Granger bates and preferred model.

	RMSE
Preferred Model (Model 1)	0.89
Granger-Bates Model	0.94

From the above figure 16, we can see that that was the result of the forecast combination of all the three models, our preferred model, and the actual test observations. We can see that its not the best prediction, but it can still predict closer to the actual observations. We gave each model a different amount of weight (from figure 17), with Series 2 receiving the most (about 39.43%), Series 1 coming in second (around 38.37%), and Series 3 receiving the least (roughly 22.21%). These weightings were determined by the performance and relative contributions of the models.

Based on the values from Table 17, we can see that the RMSE value for Preferred model (**Model 1**) is better than the Granger-Bates model.

VI.II Forecast combination technique for Consumer Price Index - All Urban Consumers: All Items in U.S. City Average

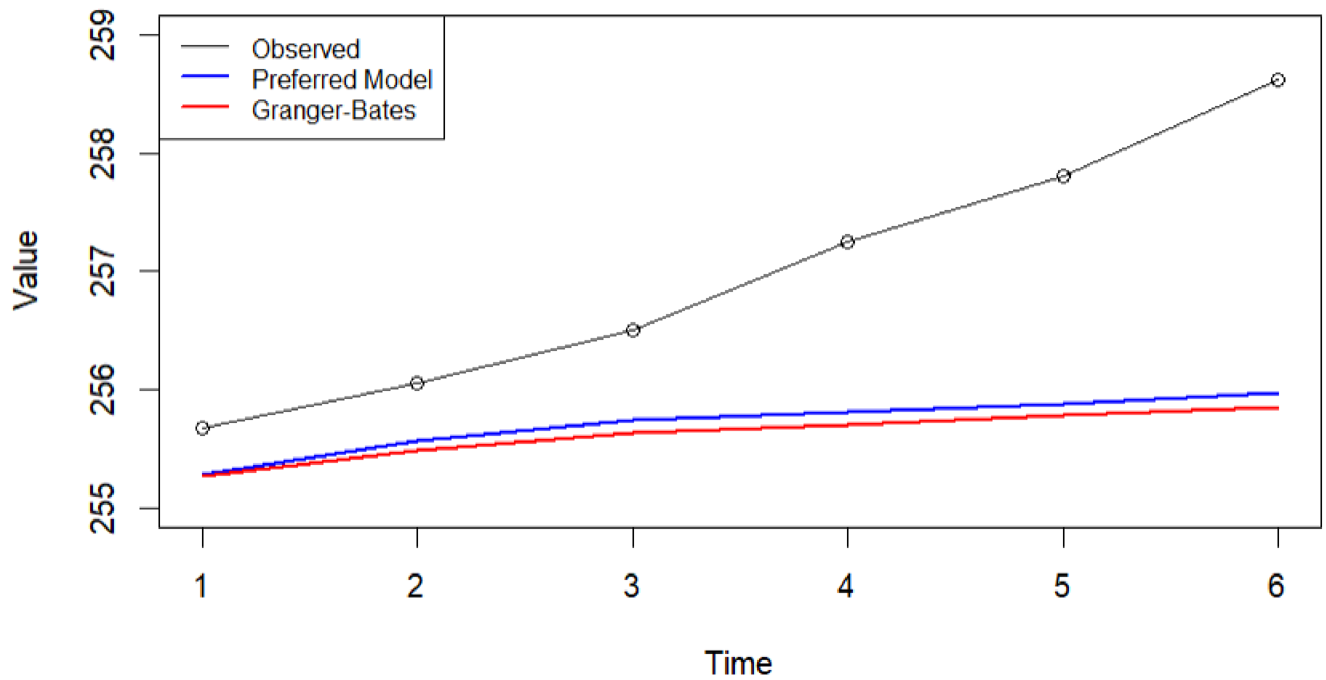

Figure 18: Forecast combination for CPI – All Items

```
$Accuracy_Train
               ME     RMSE      MAE       MPE      MAPE      ACF1 Theil's U
Test set 1.361506 1.598115 1.361506 0.5285271 0.5285271 0.4985972  2.854466

$Input_Data
$Input_Data$Actual_Train
Time Series:
Start = 1
End = 6
Frequency = 1
[1] 255.685 256.059 256.511 257.244 257.803 258.616

$Input_Data$Forecasts_Train
Time Series:
Start = 1
End = 6
Frequency = 1
  Series 1 Series 2 Series 3
1 255.3001 255.1552 255.2923
2 255.5973 255.0387 255.5737
3 255.8216 254.8994 255.7456
4 255.9279 254.9291 255.8133
5 256.0076 254.9731 255.8823
6 256.1133 254.8719 255.9810


$Weights
prediction_matrix.Series 1 prediction_matrix.Series 2 prediction_matrix.Series 3
                 0.4409179                  0.1698017                  0.3892804
```

Figure 19: Forecast combination results (Granger Bates) method

Table 18: Loss Function Values for Granger bates and preferred model.

	RMSE
Preferred Model (Model 3)	1.51
Granger-Bates Model	1.6

From the above figure 18, we can see that that was the result of the forecast combination of all the three models, our preferred model, and the actual test observations. We can see that it's not the best prediction, but it can still predict closer to the actual observations. We gave each model a different amount of weight (from figure 19), with Series 1 receiving the most (about 44.1%), Series 3 coming in second (around 38.9%), and Series 2 receiving the least (roughly 16.9%). These weightings were determined by the performance and relative contributions of the models.
Based on the values from Table 18, we can see that the RMSE value for Preferred model (**Model 3**) is better than the Granger-Bates model.

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