Final

Time Series Analysis ECON - 6376

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1. Introduction

The datasets used in this assignment will explore the realm of time series data using economic indicators from the Federal Reserve Economic Data (FRED). FRED is a trusted source of economic data maintained by the Federal Reserve Bank of St. Louis, providing insights into the U.S. economy. We will analyze four key datasets: the Consumer Price Index for New Vehicles and all items, along with the Producer Price Index for Finished Consumer Foods and Finished Goods. These datasets provide valuable insights into consumer behavior, price dynamics, and production costs. We will apply time series analysis techniques to uncover patterns and trends within these datasets.

Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average (CUUR0000SETA01): This series is provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Consumer Price Index (CPI). The Consumer Price Index is a widely used economic indicator that measures the average change over time in the prices paid by urban consumers for a market basket of consumer goods and services. This dataset tracks changes in the prices of newly manufactured vehicles, including cars, trucks, and other motor vehicles. It reflects the price movements of these new vehicles in urban areas across the United States. This data is not seasonally adjusted.

Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (CPIAUCSL): This series is a fundamental economic indicator provided by the U.S. Bureau of Labor Statistics (BLS). This series measures the average change in prices over time of goods and services purchased by households in urban areas of the United States. The series is based on prices of food, clothing, shelter, fuels, transportation fares, charges for doctors' and dentists' services, drugs, and other goods and services that people buy for day-to-day living. This data is seasonally adjusted.

Producer Price Index by Commodity: Final Demand: Finished Consumer Foods (WPSFD4111): This series is provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Producer Price Index (PPI). This dataset is focused on tracking the changes in the prices of finished consumer food products at the production or wholesale level. This dataset specifically monitors the price changes for finished consumer food products. These products include a wide range of items that are ready for consumption by consumers, such as packaged food, beverages, and other consumable goods. This dataset considers final demand, which means it tracks prices for goods that are ready for sale to consumers, not intermediate goods used in the production process. This data is seasonally adjusted.

Producer Price Index by Commodity: Final Demand: Finished Goods (WPSFD49207): This series is provided by the U.S. Bureau of Labor Statistics (BLS) as part of the Producer Price Index (PPI). This dataset focuses on tracking the changes in prices for finished goods at the production or wholesale level. It encompasses a wide range of finished goods, including manufactured products that are ready for distribution and sale. These can range from electronics, appliances, and clothing to machinery, vehicles, and various consumer products. Changes in the prices of finished goods can provide insights into the overall direction of economic activity. This data is seasonally adjusted.

1.1 Midterm Summary

The Order of Integration for each series is:

- CPI for All Urban Consumers: New Vehicles in U.S. City Average (CUUR0000SETA01): **I(1)**
- CPI for All Urban Consumers: All Items in the U.S. City Average (CPIAUCSL): **I(1)**
- PPI by Commodity: Final Demand: Finished Consumer Foods (WPSFD4111): **I(1)**
- PPI by Commodity: Final Demand: Finished Goods (WPSFD49207): **I(1)**

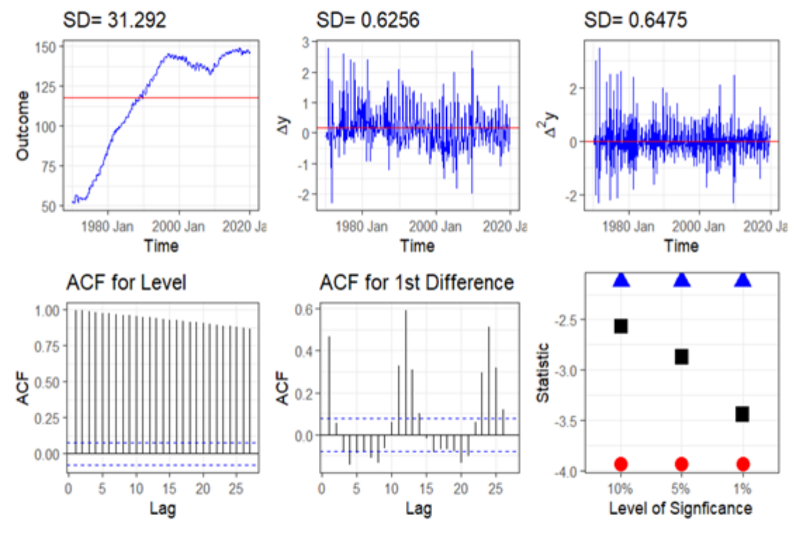

Figure 1. CPI (New Vehicles)

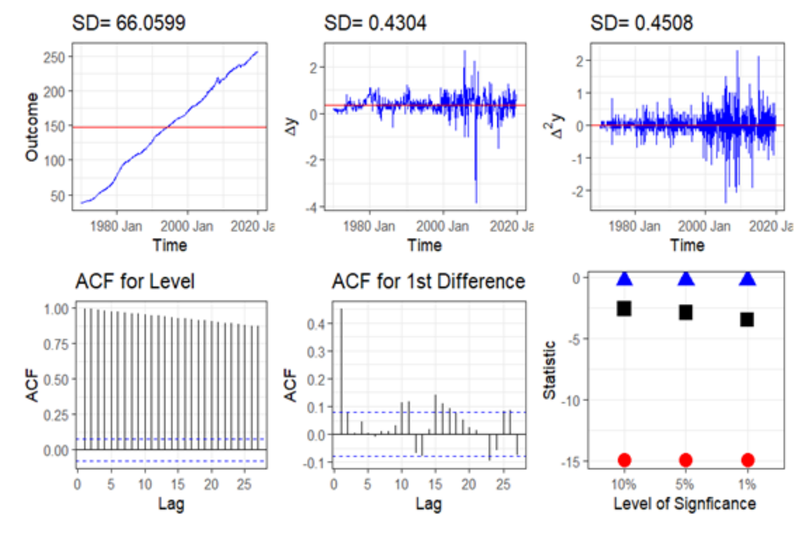

Figure 2. CPI (All Items)

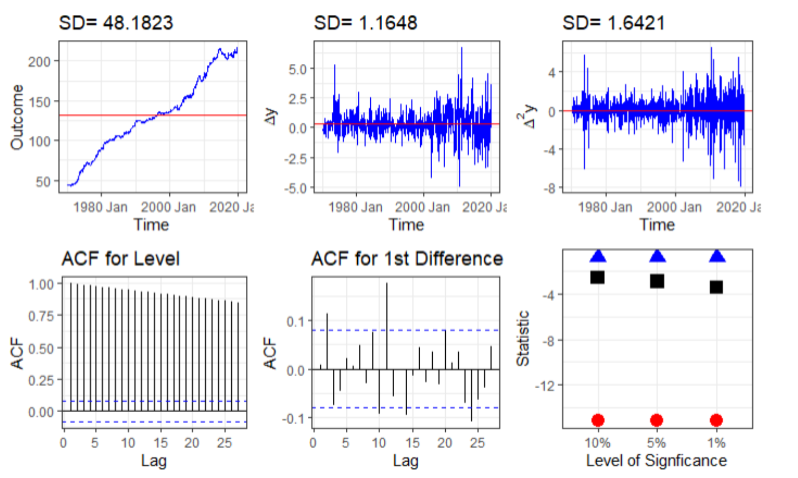

Figure 3. PPI (Finished Consumer Foods)

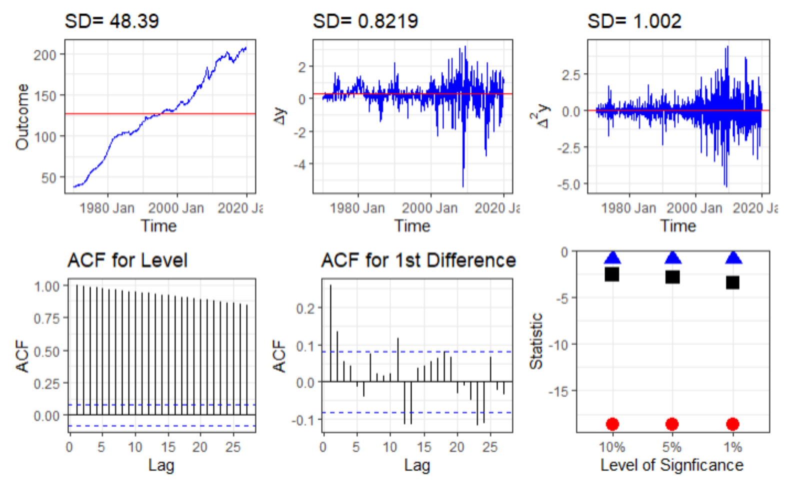

Figure 4: PPI(Finished Consumer Goods)

Best fitting Univariate Model:

Table 1: Best fitting univariate models from midterm

Time Series	Best Model	AIC values
CUUR0000SETA01	ARIMA(6,1,7)	886.59
CPIAUCSL	ARIMA(4,1,0)	606.87
WPSFD4111	ARIMA(5,1,5)	1844.52
WPSFD49207	ARIMA(7,1,7)	1396.86

Finding the best fitting models is essential for producing accurate predictions, according to our midterm time series analysis. We have identified the best models (from Table 1) for each series based on the Akaike Information Criterion (AIC) values shown in the table. With an AIC value of 886.59, the ARIMA(6,1,7) model is the most favorable for the variable CUUR0000SETA01, demonstrating its capacity to achieve a balance between model complexity and goodness of fit. In the same way, the ARIMA(4,1,0) model with the lowest AIC value of 606.87 is the model of choice for the CPIAUCSL series, demonstrating its effectiveness in identifying the underlying patterns in the data. Furthermore, the WPSFD4111 and WPSFD49207 series exhibit optimal performance with the ARIMA(5,1,5) and ARIMA(7,1,7) models, respectively, based on their associated AIC values of 1844.52 and 1396.86. These results, grounded in AIC minimization principles, guide our selection of the best models for each time series.

Best Predicting Model:

Table 2: Best predicting models compared with granger bates model

Time Series	**Preferred Model RMSE Value**	**Granger Bates RMSE**
CUUR0000SETA01	0.89	0.94
CPIAUCSL	1.51	1.6
WPSFD4111	2.82	2.69
WPSFD49207	0.7	0.71

In analyzing the Root Mean Squared Error (RMSE) values from table 2, it is evident that our chosen preferred model outperforms the Granger Bates model for the CUUR0000SETA01, CPIAUCSL, and WPSFD49207 series. The lower RMSE values associated with the preferred model in these cases indicate its superior predictive ability compared to the Granger Bates alternative. However, for the WPSFD4111 series, our analysis reveals a deviation, with the Granger Bates model exhibiting a lower RMSE value than the preferred model. In this specific instance, the Granger-Bates model is deemed the better predicting model for WPSFD4111.

2. Cointegration

If two or more series are individually integrated (in the time series sense) but some linear combination of them has a lower order of integration, then the series is said to be cointegrated. Cointegration refers to the situation where the variables have a long-run equilibrium relationship. Such variables move together over time and can be said to have a similar wavelength or share a common trend. Equilibrium and long-run relationships hold a special significance in Economic theory.

Let's say we have two series x_t and y_t, generally, it is true that:

- If both x_t and y_t are I(d) series then the linear combination:

  $$z_t = x_t – ay_t, \text{ will also be I(d)}$$

- The linear combination is of the lower order of integration, and it constrains how far away the two series can drift apart

We can say that the two series are co-integrated if:

- The two series are of the same order of integration and the order of integration is greater than 0.
- A linear combination of the two series produce a lower order of integration.

Test for Cointegration:
Johansen Test:
The Johansen test is used to test cointegrating relationships between several non-stationary time series data. Compared to the Engle-Granger test, the Johansen test allows for more than one cointegrating relationship. However, it is subject to asymptotic properties (large sample size) since a small sample size would produce unreliable results. Using the test to find the cointegration of several time series avoids the issues created when errors are carried forward to the next step.

The model selection criteria chosen here is the Akaike Information Criterion (AIC), as in this case AIC is preferred over the Bayesian Information Criterion (BIC). In order to find a model that effectively captures the data while punishing unduly complicated models, AIC strikes a balance between the trade-off between model fit and model complexity. This is achieved by taking into account both the goodness of fit and the total number of model parameters. The AIC can be calculated by

$$AIC = -2(LL) + 2(K+1)$$

where 'k' is the model's parameter count and 'log-likelihood' (LL) indicates how well the model fits the data. By using AIC as the model selection criterion, we seek to identify a model that achieves the best possible balance between explaining the variation in the data and preserving simplicity.

Before proceeding with the Johansen test, we need to find the optimal lag for the Johansen test. In order to do that I used VARselect. VARselect automates the selection of the optimal lag order (p) in Vector Autoregression (VAR) models. It uses information criteria to recommend a lag order that balances model fit and complexity, streamlining the process of accurate model selection for multivariate time series analysis.

Table 3: Lags from VARselect

AIC	HQ	SC	FPE
3	3	2	3

Next, I started to proceed with the Johansen test, with **3 lags** (Using AIC selection criteria from Table 3) and modeled it with the seasonality factor, as our "**Consumer Price Index for All Urban Consumers: New Vehicles in U.S. City Average"** series is **not seasonally adjusted**. The Johansen test result is as follows:

Table 4: Johansen test results

	Test	10pct	5pct	1pct
r <= 3	0.11	6.50	8.18	11.65
r <= 2	14.56	12.91	14.90	19.19
r <= 1	26.96	18.90	21.07	25.75
r = 0	49.44	24.78	27.14	32.14

We applied the Johansen test to assess the existence of cointegration relationships among variables, focusing on the critical values at 1%, 5%, and 10% significance levels. Initially, when examining Table 4 the test value (49.44) at r = 0, the test statistic value exceeds all the critical values, and thereby rejecting the null hypothesis suggesting that there is a presence of cointegration. Then by examining the test value (26.96) at r <= 1, the test statistic is greater than all the critical values, and hence we can move on to the next step which is examining the r <= 2 level. The test value (14.56) for r <= 2 is lower than the critical values at the 1% significance level. This indicates that we have **2 cointegrating relationships** in the dataset. The test value (0.11) for r <= 3 is lower than all the critical values (10%, 5%, and 1% significance levels). This suggests the presence of **3 cointegrating relationships** within the variables under consideration.
In summary, the Johansen cointegration test results reveal that our dataset exhibits significant cointegrating relationships. Specifically, at the 1% significance level, we can confidently assert the presence of 2 cointegrating relationships (r <= 2), and furthermore, there are indications of 3 cointegrating relationships when considering a more relaxed constraint (r <= 3). These findings provide valuable insights into the long-term equilibrium relationships among the variables in our analysis.

3. Endogenous and Exogenous Variables

In econometrics, an **endogenous variable** is one that is explained within the statistical model. The values of endogenous variables are determined by the relationships specified in the model and are influenced by other variables within the system. These variables are often the primary focus of analysis as researchers seek to understand how changes in endogenous variables respond to alterations in other variables. Econometric models aim to capture the intricate relationships among endogenous variables and uncover the underlying dynamics governing their behavior.

Conversely, an **exogenous variable** in econometrics is treated as independent or external to the model. These variables are not explained within the model but are assumed to influence the behavior of endogenous variables. Exogenous variables are typically considered predetermined or controlled by external factors and are not influenced by other variables within the model.

In this case, all the variables will need equations of their own as **all of these variables are endogenous**. When examining economic trends, the Producer Price Index for Finished Goods (WPSFD49207), the Producer Price Index for Finished Consumer Foods (WPSFD4111), the Consumer Price Index for New Vehicles (CUUR0000SETA01), and the

Producer Price Index for All Urban Consumers (CPIAUCSL) can all be regarded as endogenous variables. This indicates that they are not determined externally, but rather are determined to some part within the economic system itself.

The PPIs, for instance, monitor domestic producer prices, which are impacted by domestic supply and demand dynamics. In the same way, consumer retail prices are tracked by the CPIs and are influenced by several elements within the overall economy. Changes in these price indices may be caused by shifts in production costs, pricing power of the company, consumer income and spending patterns, advancements in automobile technology, and other factors.

These variables should be viewed as jointly determined by, or endogenous to, the system rather than only being determined by external forces since they indicate prices and inflation rates that are produced as part of the operation of markets and the macroeconomy. It is essential to examine the relationships that exist between these indicators and other economic variables in order to comprehend inflationary trends and processes.

3.1 VECM Model and their Equations

A Vector Error Correction Model (VECM) is a type of multivariate time series model that can be used when the variables are cointegrated. This combines the advantages of a VAR model, which captures the short-run dynamics of the variables, and an ECM model, which incorporates the long-run equilibrium relationship among the variables. A VECM model can be derived from a VAR model by imposing cointegration restrictions on the parameters. This reduces the number of parameters to be estimated and improves the efficiency of the estimation.

A VECM model can be written as

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p} \Phi_i \Delta X_{t-1} + CD_t + \varepsilon_t$$

Our analysis began with the Johansen test, which revealed that our model has at least **two cointegrating relationships**. This is a crucial finding as it indicates that there are long-term equilibrium relationships among the variables in our system.
To determine the optimal lag length for our model, we utilized the VARselect function. This function employs the Akaike Information Criterion (AIC) to select the most appropriate lag length. The AIC balances the trade-off between the goodness-of-fit of the model and the complexity of the model (in terms of the number of parameters). The optimal lag length was found to be 3.

One of the variables in our model, CUUR0000SETA01, is not seasonally adjusted. To account for potential seasonal effects, we incorporated this information into the Johansen test.

Upon obtaining the results from the Johansen test, we proceeded to construct our Vector Error Correction Model (VECM). The Johansen test results were passed to the VECM(vec2var), resulting in the formation of three distinct models:

Model 1: This model was constructed with a lag length of 3, as determined by the Johansen test. The cointegrating rank, denoted as r, was set to 2. This means that there are two long-run relationships among the variables in the model.
Model 2: Similar to Model 1, this model also used a lag length of 3. However, the cointegrating rank was increased to 3, indicating the presence of three cointegrating relationships.
Model 3: For this model, we experimented with a shorter lag length of 2 while maintaining a cointegrating rank of 2.

Table 5: VECM model 1 output

Coefficient matrix of lagged endogenous variables:				
A1				
	cpi_new_veh_train.l1	cpi_all_us_train.l1	ppi_cf_train.l1	ppi_fg_train.l1
cpi_new_veh_train.l1	1.342828231	-0.08436176	-0.013815928	0.12960725
cpi_all_us_train.l1	0.025955485	1.34364111	-0.005879012	0.10844205
ppi_cf_train.l1	-0.098108464	0.05982667	0.947448708	0.06269145
ppi_fg_train.l1	0.007979632	0.52031666	-0.026030259	1.05522942
A2:				
	cpi_new_veh_train.l2	cpi_all_us_train.l2	ppi_cf_train.l2	ppi_fg_train.l2
cpi_new_veh_train	-0.48620803	0.04606267	0.008810268	-0.13891992
cpi_all_us_train	-0.01465622	-0.51235044	0.017526950	-0.09017976
ppi_cf_train	0.22742519	-0.12305687	0.080991375	0.23948249
ppi_fg_train	0.09711303	-0.66973911	0.033005296	0.01157130
A3:				
	cpi_new_veh_train.l3	cpi_all_us_train.l3	ppi_cf_train.l3	ppi_fg_train.l3
cpi_new_veh_train	0.144753122	0.02614289	0.010057107	0.01849086
cpi_all_us_train	-0.009047844	0.16627928	-0.008947839	-0.01877369
ppi_cf_train	-0.138114766	0.07380776	-0.039336377	-0.30113701
ppi_fg_train	-0.111007200	0.15771457	-0.014676906	-0.06715392

Model 1 VECM Equation:

$$X_t = \begin{bmatrix} \text{CPI new vehicles} \\ \text{CPI all items} \\ \text{PPI consumer foods} \\ \text{PPI finished goods} \end{bmatrix} \qquad \phi_i = \begin{bmatrix} \phi_{11}^{P} & \cdots & \phi_{14}^{P} \\ \vdots & & \vdots \\ \phi_{41}^{P} & \cdots & \phi_{44}^{P} \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$CD_t =$

Coefficient matrix of deterministic regressor(s).

	constant	sd1	sd2	sd3	sd4	sd5	sd6	sd7	sd8	sd9	sd10	sd11
cpi_new_veh_train	-0.05243173	-0.007645085	-0.33206810	-0.37519453	-0.2571951	-0.30622548	-0.4302494	-0.51188678	-0.598504948	-0.40126416	0.53265486	0.26000907
cpi_all_us_train	0.06338444	0.174161917	0.06476417	0.06064013	0.1248522	0.07316776	0.1739963	0.07756209	0.164523803	0.17039516	0.01364277	-0.02386801
ppi_cf_train	0.95686792	0.007586037	-0.09640508	0.20004030	-0.1586086	-0.36018780	-0.3080667	-0.08604441	0.009279152	0.05287828	0.18024513	0.17253870
ppi_fg_train	0.62660086	0.311435127	0.10585290	0.18822130	0.2562562	0.18467534	0.2515558	0.05094574	0.301310705	0.31276269	0.21967730	0.18744336

Since, it would result in large number of equations, we can substitute the above matrices and table 5 into the equation below

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p} \Phi_i \Delta X_{t-1} + CD_t + \varepsilon_t$$

Table 6: VECM model 2 output

Coefficient matrix of lagged endogenous variables:				
A1				
	cpi_new_veh_train.l1	cpi_all_us_train.l1	ppi_cf_train.l1	ppi_fg_train.l1
cpi_new_veh_train.l1	1.342821908	-0.08434696	-0.013772538	0.12956954
cpi_all_us_train.l1	0.025752172	1.34411702	-0.004483832	0.10722974
ppi_cf_train.l1	-0.094542090	0.05147853	0.922975440	0.08395688
ppi_fg_train.l1	0.007530012	0.52136913	-0.02294486	1.05254845
A2:				
	cpi_new_veh_train.l2	cpi_all_us_train.l2	ppi_cf_train.l2	ppi_fg_train.l2
cpi_new_veh_train	-0.48620040	0.04613715	0.008816299	-0.13893405
cpi_all_us_train	-0.01441105	-0.50995532	0.017720868	-0.09063432
ppi_cf_train	0.22312467	-0.16507054	0.077589803	0.24745594
ppi_fg_train	0.09765521	-0.66444236	0.033434139	0.01056608
A3:				
	cpi_new_veh_train.l3	cpi_all_us_train.l3	ppi_cf_train.l3	ppi_fg_train.l3
cpi_new_veh_train	0.144758523	0.02606276	0.010070686	0.01846296
cpi_all_us_train	-0.008874184	0.16370271	-0.008511224	-0.01967086
ppi_cf_train	-0.141160993	0.11900407	-0.046995183	-0.28539947
ppi_fg_train	-0.110623156	0.15201658	-0.013711344	-0.06913799

Model 2 VECM Equation:

$$X_t = \begin{bmatrix} \text{CPI new vehicles} \\ \text{CPI all items} \\ \text{PPI consumer foods} \\ \text{PPI finished goods} \end{bmatrix} \qquad \phi_i = \begin{bmatrix} \phi^P_{11} & \cdots & \phi^P_{14} \\ \vdots & & \vdots \\ \phi^P_{41} & \cdots & \phi^P_{44} \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$CD_t =$

```
Coefficient matrix of deterministic regressor(s).

                   constant          sd1         sd2         sd3        sd4         sd5        sd6         sd7         sd8         sd9       sd10        sd11
cpi_new_veh_train -0.05276056 -0.007658744 -0.33209486 -0.37524074 -0.2572493 -0.30626788 -0.4302824 -0.51190195 -0.59852268 -0.40127199 0.53264661  0.26000224
cpi_all_us_train   0.05281093  0.173722709  0.06390382  0.05915423  0.1231103  0.07180458  0.1729341  0.07707413  0.16395369  0.17014317 0.01337749 -0.02408763
ppi_cf_train       1.14234094  0.015290307 -0.08131336  0.22610490 -0.1280534 -0.33627586 -0.2894349 -0.07748492  0.01927965  0.05729848 0.18489851  0.17639106
ppi_fg_train       0.60321789  0.310463834  0.10395025  0.18493528  0.2524041  0.18166071  0.2492069  0.04986662  0.30004992  0.31220543 0.21909064  0.18695768
```

Since, it would result in large number of equations, we can substitute the above matrices and table 6 into the equation below

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p} \Phi_i \Delta X_{t-1} + CD_t + \varepsilon_t$$

Table 7: VECM model 3 output

Coefficient matrix of lagged endogenous variables:				
A1				
	cpi_new_veh_train.l1	cpi_all_us_train.l1	ppi_cf_train.l1	ppi_fg_train.l1
cpi_new_veh_train.l1	1.29858007	-0.1364797	-0.01710642	0.1403100
cpi_all_us_train.l1	0.02724457	1.2708150	-0.01003845	0.1204399
ppi_cf_train.l1	-0.02941187	0.4218694	0.95334213	-0.0276647
ppi_fg_train.l1	0.04862811	0.5010570	-0.03130007	1.0619410
A2:				
	cpi_new_veh_train.l2	cpi_all_us_train.l2	ppi_cf_train.l2	ppi_fg_train.l2
cpi_new_veh_train	-0.29726543	0.1254638	0.01983975	-0.13004282
cpi_all_us_train	-0.02468253	-0.2728256	0.01491389	-0.12387465

ppi_cf_train	0.01392912	-0.4113542	0.01723280	0.05039155
ppi_fg_train	-0.05432769	-0.4945763	0.02040743	-0.05671907

Model 3 VECM Equation:

$$X_t = \begin{bmatrix} \text{CPI new vehicles} \\ \text{CPI all items} \\ \text{PPI consumer foods} \\ \text{PPI finished goods} \end{bmatrix} \quad \phi_i = \begin{bmatrix} \phi^P_{11} & \cdots & \phi^P_{14} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \phi^P_{41} & \cdots & \phi^P_{44} \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$CD_t =$

```
Coefficient matrix of deterministic regressor(s).

                      constant          sd1         sd2         sd3        sd4         sd5        sd6         sd7        sd8        sd9         sd10        sd11
cpi_new_veh_train -0.06394775 -0.024182141 -0.29127586 -0.33440080 -0.1776716 -0.20268528 -0.3417392 -0.42542505 -0.5037867 -0.2918676 0.6668209385  0.40380227
cpi_all_us_train   0.02311099  0.186936123  0.07573709  0.04729207  0.1066965  0.06819285  0.1577705  0.06864162  0.1414804  0.1565396 0.0006534193 -0.05075082
ppi_cf_train       1.40200950  0.006996445 -0.20184738  0.16873074 -0.2518108 -0.45692807 -0.4107479 -0.22424481 -0.1230570 -0.1155959 0.0200120437  0.01405791
ppi_fg_train       0.56271619  0.337760933  0.08571075  0.15190617  0.1895351  0.11451265  0.1791621 -0.01595776  0.2167949  0.2231223 0.1168680142  0.05872014
```

Since, it would result in large number of equations, we can substitute the above matrices and table 7 into the equation below

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p} \Phi_i \Delta X_{t-1} + CD_t + \varepsilon_t$$

4. Model Forecasts and Comparision

4.1 Forecast point estimates for VECM models

Table 8: Point forecast estimate for Model 1

Point Forecast Estimate	**cpi_new_veh_train**	**cpi_all_us_train**	**ppi_cf_train**	**ppi_fg_train**
1	147.1039	253.8003	210.1406	204.1888
2	146.9813	254.2010	210.5341	204.6526
3	146.8508	254.5378	210.6203	204.9790
4	146.5867	254.9544	210.7727	205.3545
5	146.2015	255.3077	211.1189	205.5691

6	145.6844	255.6900	211.5467	205.9667
7	145.3586	256.1115	211.9907	206.3890
8	146.0465	256.3941	212.5886	206.7380
9	146.7807	256.6041	213.0929	207.0013
10	147.1274	256.8399	213.5403	207.1629

Table 9: Point forecast estimate for Model 2

Point Forecast Estimate	**cpi_new_veh_train**	**cpi_all_us_train**	**ppi_cf_train**	**ppi_fg_train**
1	147.1039	253.7998	210.1499	204.1876
2	146.9810	254.2004	210.5380	204.6514
3	146.8505	254.5365	210.6383	204.9763
4	146.5861	254.9513	210.8138	205.3478
5	146.2001	255.3022	211.1833	205.5575
6	145.6820	255.6822	211.6323	205.9501
7	145.3554	256.1017	212.0952	206.3673
8	146.0427	256.3825	212.7102	206.7115
9	146.7764	256.5909	213.2299	206.9701
10	147.1229	256.8253	213.6914	207.1271

Table 10: Point forecast estimate for Model 3

Point Forecast Estimate	**cpi_new_veh_train**	**cpi_all_us_train**	**ppi_cf_train**	**ppi_fg_train**
1	147.1147	253.7830	210.7543	204.2227
2	147.0327	254.2021	211.0182	204.6499
3	146.9372	254.5663	211.0803	204.9901
4	146.6979	254.9951	211.1853	205.3703
5	146.3275	255.3509	211.5163	205.5858
6	145.8217	255.7318	211.9294	205.9789
7	145.5062	256.1507	212.3604	206.3947
8	146.2043	256.4303	212.9445	206.7362
9	146.9483	256.6370	213.4351	206.9917
10	147.3046	256.8693	213.8690	207.1450

4.2 Forecast Plots for Model 1

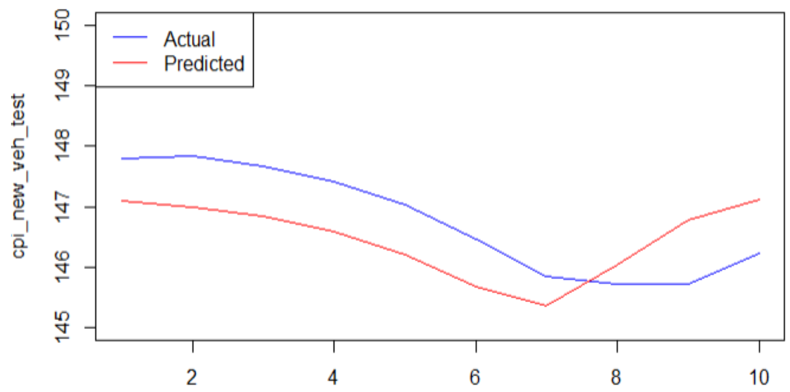

Figure 5: Model 1 Forecast for cpi_new_veh

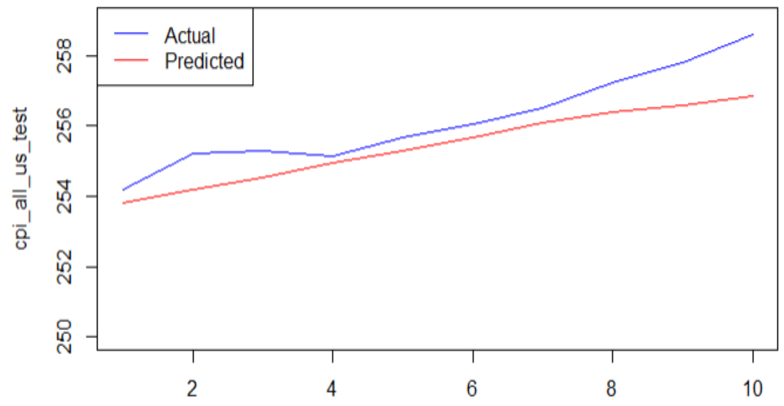

Figure 6: Model 1 Forecast for cpi_all_us

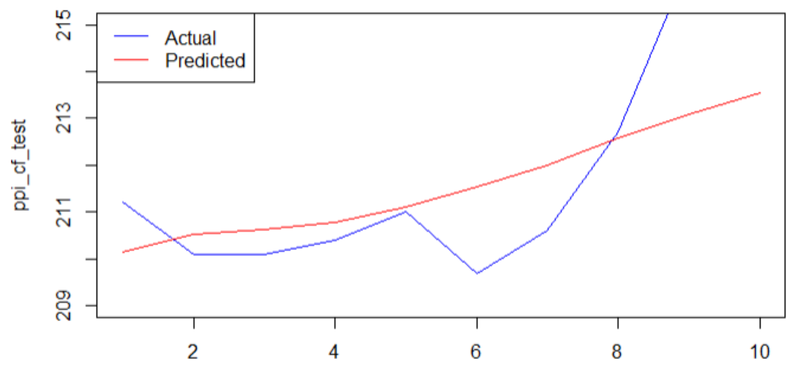

Figure 7: Model 1 Forecast for ppi_cf

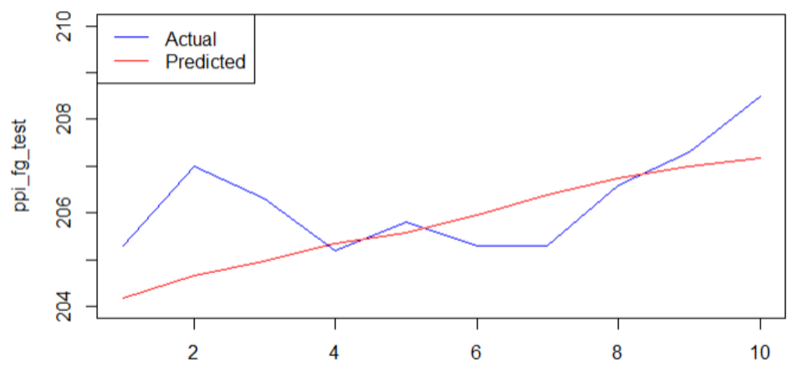

Figure 8: Model 1 Forecast for ppi_fg

4.3 Forecast Plots for Model 2

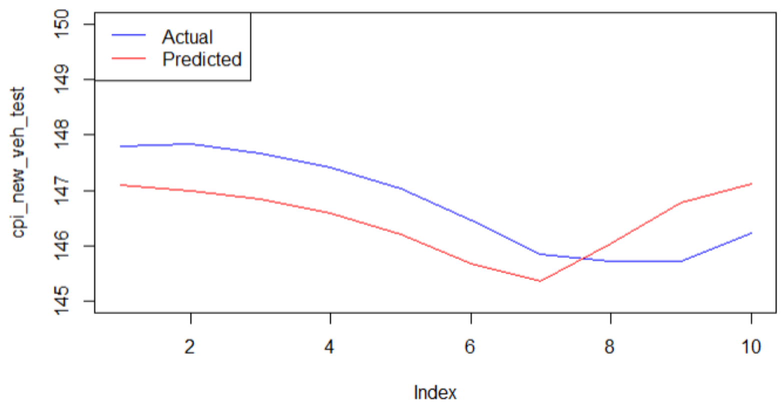

Figure 9: Model 2 Forecast for cpi_new_veh

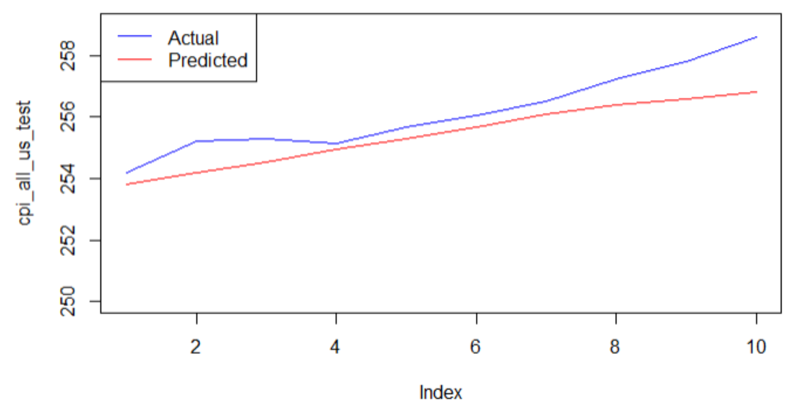

Figure 10: Model 2 Forecast for cpi_all_us

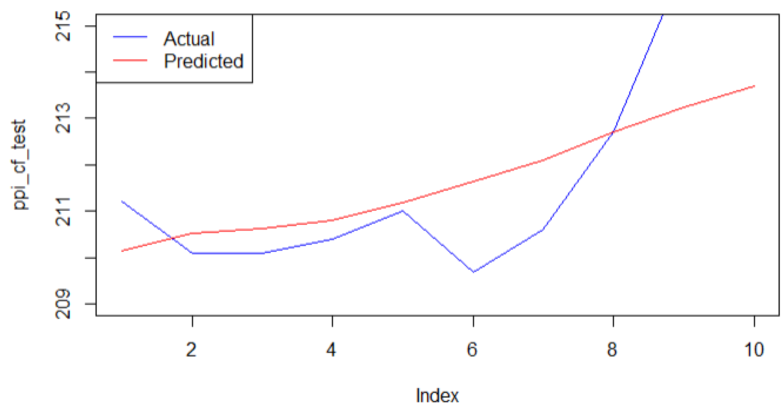

Figure 11: Model 2 Forecast for ppi_cf

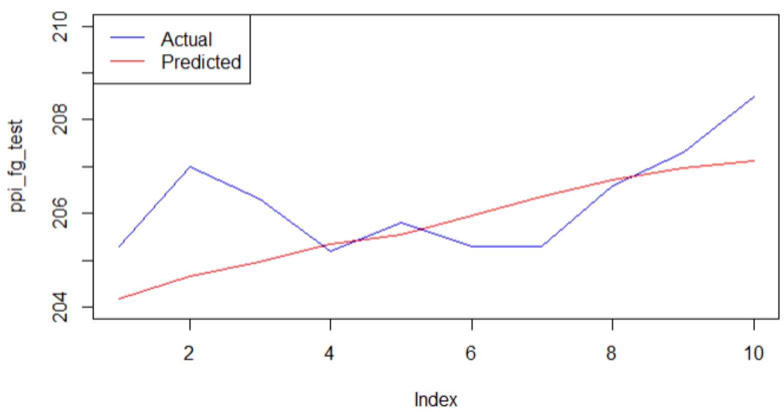

Figure 12: Model 2 Forecast for ppi_fg

4.4 Forecast Plots for Model 3

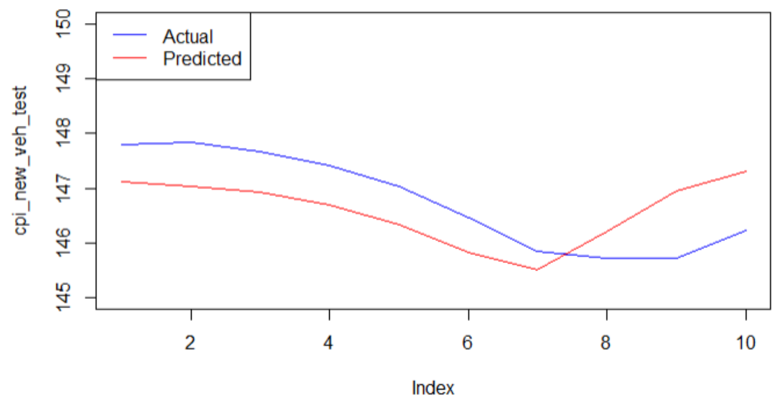

Figure 13: Model 3 Forecast for cpi_new_veh

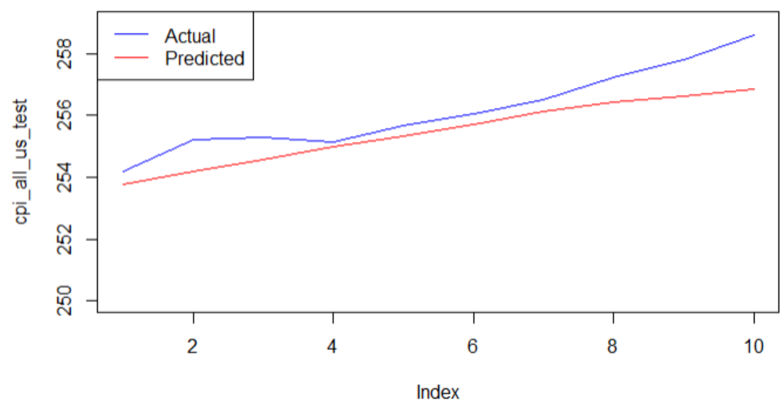

Figure 14:Model 3 Forecast for cpi_all_us

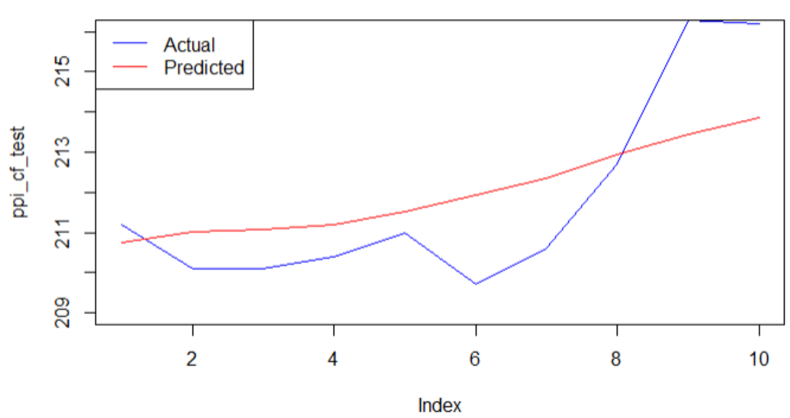

Figure 15: Model 3 Forecast for ppi_cf

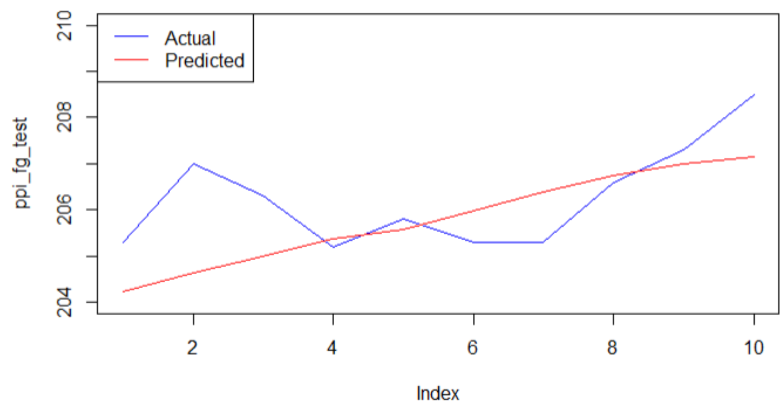

Figure 16: Model 3 Forecast for ppi_fg

Mean Squared Error (MSE):

Mean Squared Error (MSE) is a metric commonly used to evaluate the accuracy of predictions in regression and forecasting. It measures the average squared difference between predicted and actual values, providing a comprehensive assessment of the model's performance. The squaring of errors amplifies the impact of larger deviations, making MSE sensitive to outliers. It's particularly valuable in applications where precise estimation of extreme errors is crucial, such as financial modeling, as it prioritizes minimizing the impact of large deviations from actual values.

$$MSE = \frac{1}{N} \cdot \sum_{i=1}^{N} (x_i - m_i)^2$$

Mean Absolute Error (MAE):

Mean Absolute Error (MAE) is another metric used to assess prediction accuracy. Unlike MSE, MAE calculates the average absolute differences between predicted and actual values, without squaring the errors. This makes MAE less sensitive to outliers, as each error contributes proportionally to the overall evaluation. MAE provides a straightforward measure of average prediction accuracy, making it easier to interpret than MSE. It's often favored in situations where extreme errors are less critical, and the emphasis is on capturing the overall magnitude of prediction deviations.

$$MAE = \frac{1}{N} \cdot \sum_{i=1}^{N} |x_i - m_i|$$

Why MSE?

To assess the predictive accuracy of Vector Error Correction Models (VECMs), it is essential to choose a suitable metric for comparison. Mean Squared Error (MSE) shows up as a well-rounded statistic in this situation, capturing the dynamics of the entire system as well as the performance of individual series. By calculating the average squared discrepancies between expected and actual values, MSE offers a thorough evaluation and is skilled at assessing forecast accuracy across all observations.

Several important factors support the choice of Mean Squared Error (MSE) over Mean Absolute Error (MAE) as the comparison metric for assessing Vector Error Correction Models (VECMs). Because of the squared term, MSE penalizes greater prediction mistakes more severely, making it a more sensitive way to assess model performance—especially when extreme values are particularly concerning. In financial or economic forecasting, where outliers or big departures from actual values might have major ramifications, this trait is very important. MSE is also differentiable, which makes it easier to apply in optimization techniques for model estimation.

Table 11: MSE for the models

Models	cpi_new_veh_train	cpi_all_us_train	ppi_cf_train	ppi_fg_train
Model 1	0.61176	0.75413	2.44503	1.20939
Model 2	0.61092	0.76680	2.34767	1.21539
Model 3	0.60611	0.71652	2.46556	1.20828

Overall Model 1 loss: 5.02031

Overall Model 2 loss: 4.94078

Overall Model 3 loss: 4.99647

Upon analyzing the MSE losses for individual series in Table X for different models, it becomes evident that Model 3 outperforms the other models, exhibiting the lowest individual loss for cpi_new_veh_train, cpi_all_us_train, and ppi_fg_train. However, it is noteworthy that Model 3 incurs a higher loss for ppi_cf_train. As a result, we conclude that Model 3 might not be the best option based only on individual losses. In order to arrive at a better informed judgment, we calculated the aggregate loss while taking into account the predictive power of all the series combined. Even if Model 2's losses for individual series aren't the lowest, its overall performance is clearly better than the other models. From this, we can say that **Model 2** is our best model.

5. Innovation accounting

5.1 Impulse Response Functions:

Impulse Response Functions or IRFs are used to study the effects of shocks or impulses in a VAR or VECM system. It traces out one unit or one standard deviation shock to an endogenous variable and its effects on all the endogenous variables in a VAR or VECM, keeping all other variables and shocks constant.

Impulse Response Functions are the most important application of VAR and VECM models. They are useful in understanding the dynamic behavior of variables in a system. Furthermore, IRFs are important tools in predicting the effects of impulses/shocks and policy analysis.

This is a general equation of an IRF in a Vector Autoregressive model. The IRF of a VAR model shows the dynamic effects of a one-standard-deviation shock to one variable on all of the other variables in the model.

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \epsilon_{yt-i} \\ \epsilon_{zt-i} \end{bmatrix}$$

where

- y_t and z_t are the two variables in the VAR model

- $\overline{y}$ and $\overline{z}$ are the mean values of y_t and z_t, respectively
- Φ_{jk} (0) are the impact multipliers
- ϵ_{yt-i} and ϵ_{zt-i} are the shock terms for variables y_t and z_t , respectively

The set of coefficients $\{\phi_{11}(i), \phi_{12}(i), \phi_{21}(i), \phi_{22}(i)\}$ are the impulse response functions.

Now let's see the difference between an orthogonal impulse response function and one that isn't orthogonal.
In **Orthogonal IRFs**, these functions isolate the independent effects of shocks to individual variables in a system. This is achieved by applying a technique called orthogonalization, which removes any inherent correlations between the shocks. Each orthogonal IRF shows the isolated response of one variable to a shock in another variable, holding all other shocks constant. This makes it easier to understand the specific causal effect of each shock on the system.
For example, In Orthogonal IRFs, It's like throwing each ball individually, observing the impact on the target, and then erasing the target before throwing the next ball. This allows you to see the isolated effect of each shock.

In **Non-orthogonal IRFs,** these functions capture the combined effects of all shocks to the system, including any correlations between them. They do not isolate individual shocks. Non-orthogonal IRFs show the total response of a variable to all shocks, making it difficult to disentangle the specific contribution of each individual shock.
For example, In Non-orthogonal IRFs, It's like throwing all three balls at the same time and observing the combined impact on the target. While you can see the overall result, it's difficult to pinpoint the specific contribution of each individual ball.

By using IRFs, we can say that IRFs act like X-rays for systems, revealing the hidden impact of shocks and their intricate dance over time. By studying their shape and timing, we can isolate individual effects, understand how shocks ripple through the system, and even predict future behavior, empowering us to make informed decisions in economics, finance, ecology, and beyond.

5.2 Forecast Error Variance Decomposition:

In econometrics and other applications of multivariate time series analysis, a variance decomposition or forecast error variance decomposition (FEVD) is used to aid in the interpretation of a vector autoregression (VAR) model once it has been fitted. The variance decomposition indicates the amount of information each variable contributes to the other variables in the autoregression. It determines how much of the forecast error variance of each of the variables can be explained by exogenous shocks to the other variables.

FEVD helps pinpoint the main drivers of forecasting errors, highlighting which variables need more attention for better predictions. By analyzing how error contributions change over time, you can gain insights into the interactions and feedback loops within the system, leading to more accurate forecasts. Knowing which shocks have the biggest impact can inform policy decisions, allowing you to focus on mitigating the most disruptive ones.

One-step ahead prediction of a var can be written as:

$$E_t x_{t+1} = A_0 + A_1 x_t \qquad \text{where } E_t \text{ is the conditional expectation.}$$

Forecast error is:

$$x_{t+1} - E_t x_{t+1} = e_{t+1}$$

After some math we get the proportions of $\sigma_y(n)^2$ (which is the Forecast variation for Y) due to the shocks in ε_{yt} and ε_{zt}.

$$\frac{\sigma_x^2[\phi_{11}(0)^2 + \ldots + \phi_{11}(n-1)^2]}{\sigma_y(n)^2}$$

This equation tells us what proportion of movements in a sequence are from own shocks versus other variables.
If shocks to ϵ_{zt} explain none of the forecast error variances of y_t, then we can say y_t is exogenous to z_t.

In simple terms, the forecast error variance, which is the squared difference between the expected and actual values, is first broken down into its component parts. Imagine it as dividing the pie into individual pieces. We employ a sophisticated method called orthogonalization to prevent duplicate counting the impacts of shocks that may be interrelated. This guarantees that every slice accurately captures the distinct contribution of a particular shock. After giving the pie slice sizes, we finally compute the portion of the overall error variance that can be attributed to each shock.

6. Orthogonal Impulse Response Function plots for the best model

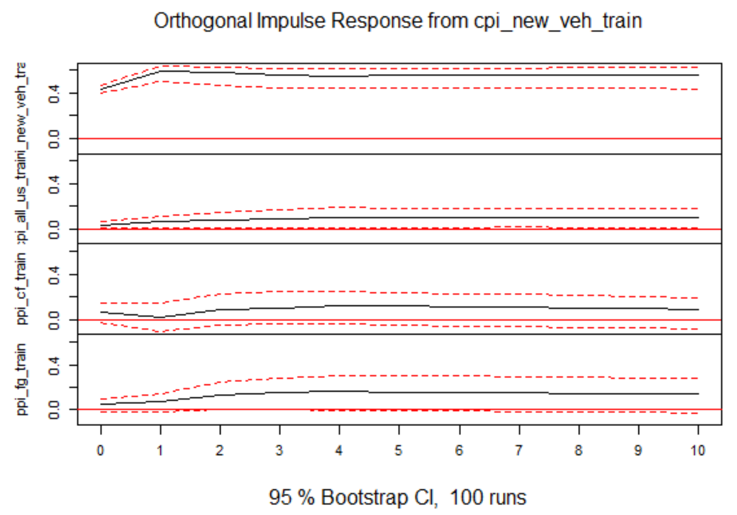

Figure 17: Model 2s IRF function when cpi_new_veh is shocked

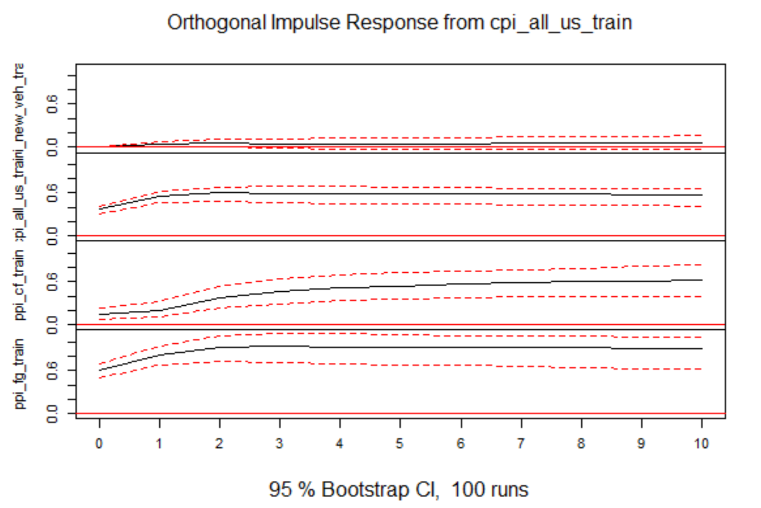

Figure 18: Model 2s IRF function when cpi_all_us is shocked

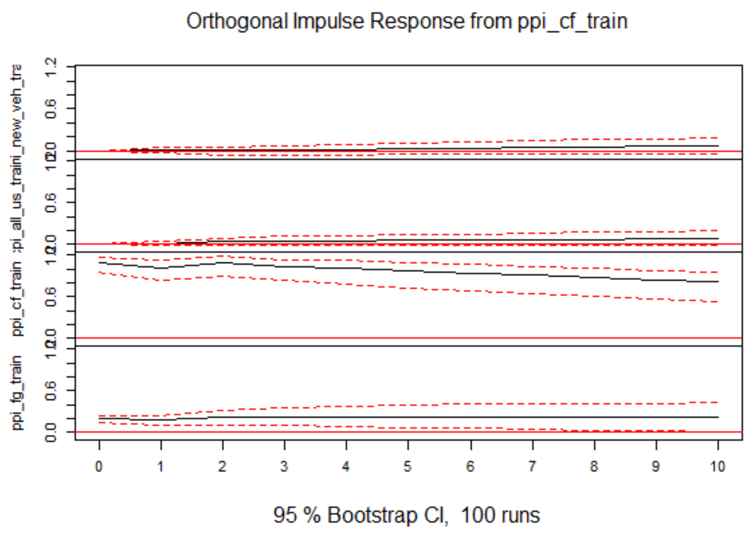

Figure 19: Model 2s IRF function when ppi_cf is shocked

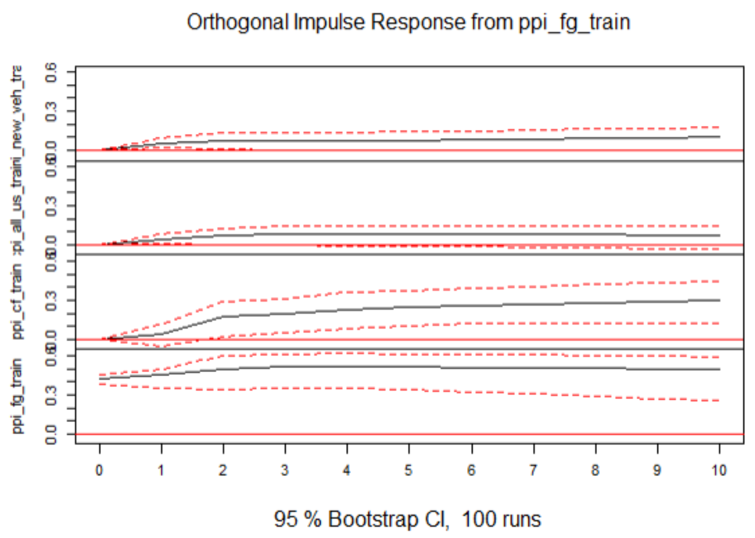

Figure 20: Model 2s IRF function when ppi_fg is shocked

Based on the previous analysis, we can see that our overall best predicting model is **Model 2**. So we plot the IRF function for that model, and based on the output shown in Fig 17, we can infer that one unit shock in **cpi_new_veh_train** causes:

- **cpi_new_veh_train**: an initial increase in the variable during the first time period, stabilizing at 0.5 thereafter.
- **cpi_all_us_train:** Demonstrates no discernible change, highlighting its relative insensitivity to the specified shock.
- **ppi_cf_train:** Experiences an initial drop to 0 at time period 1, followed by a slight increase and subsequent stabilization in later periods.
- **ppi_fg_train:** Shows a gradual increase until the second time period, followed by stabilization.

A one-unit shock in **cpi_all_us_train** (refer to fig.18) causes:

- **cpi_new_veh_train**: Demonstrates no discernible change, highlighting its relative insensitivity to the specified shock.
- **cpi_all_us_train:** an initial increase in the variable during the first time period until 0.6, stabilizing at 0.5 thereafter

- **ppi_cf_train:** Experiences a slight increase and subsequent stabilization in later periods.
- **ppi_fg_train:** Seems to be affected due to the shock, as we can see it starts at 0.6 and stabilizes in the later period.

A one-unit shock in **ppi_cf_train** (refer to fig.19) causes:

- **cpi_new_veh_train**: Demonstrates no discernible change, highlighting its relative insensitivity to the specified shock.
- **cpi_all_us_train:** Demonstrates no discernible change, highlighting its relative insensitivity to the specified shock.
- **ppi_cf_train:** We can note that after the initial shock, we can observe a gradual decrease in the later periods.
- **ppi_fg_train:** Demonstrates no discernible change, highlighting its relative insensitivity to the specified shock.

A one-unit shock in **ppi_fg_train** (refer fig.20) causes:

- **cpi_new_veh_train**: Demonstrates a very slight increase and then stabilizing in the later period
- **cpi_all_us_train:** Demonstrates a very slight increase and then stabilizing in the later period.
- **ppi_cf_train:** We can note that there is a sudden increase observed from time period 1 to 2 and then later there is a gradual increase.
- **ppi_fg_train:** There is a gradual increase observed from the initial shock to time period 3 and then a slight decrease can be observed in the later time period.

7. Forecast Error Variance Decomposition for the best model

Table 12: Forecast error variance decomposition table when cpi_new_veh is shocked

Time Period	cpi_new_veh	cpi_all_us	ppi_cf	ppi_fg
1	1.0000000	0.000000000	0.000000000	0.000000000
2	0.9903570	0.003751762	0.0002317018	0.005659548
3	0.9856337	0.005335517	0.0001756965	0.008855095
4	0.9834847	0.005710031	0.0002217650	0.010583505
5	0.9821545	0.005755159	0.0004031296	0.011687251
6	0.9806627	0.005870926	0.0007465736	0.012719790
7	0.9788366	0.006120831	0.0012365208	0.013806045
8	0.9766891	0.006486055	0.0018605060	0.014964323
9	0.9742638	0.006941622	0.0026054593	0.016189144
10	0.9715900	0.007473956	0.0034586095	0.017477439

Table 12 shows the proportion of the forecast error variance of each variable that can be attributed to a one unit shock in cpi_new_veh_train, over 10 time periods.
In period 1, the shock in cpi_new_veh_train explains 100% of its own forecast error variance, and 0% of the forecast error variance of the other variables. This means that the shock has no immediate impact on the other variables in the system. However, as the time periods increase, the table also reveals that the shock in cpi_new_veh_train has little impact on ppi_fg_train, explaining about 1.5% of its forecast error variance in period 10. This suggests that there is a weak relationship between the new vehicle CPI and the finished goods PPI. On the other hand, the shock in cpi_new_veh_train has the smallest impact on cpi_all_us_train, explaining only about 0.7% of its forecast error variance

in period 10.

Table 13: Forecast error variance decomposition table when cpi_all_us is shocked

Time Period	cpi_new_veh	cpi_all_us	ppi_cf	ppi_fg
1	0.008725523	0.9912745	0.0000000000	0.000000000
2	0.011298764	0.9835104	0.0006131168	0.004577741
3	0.013932328	0.9750872	0.0021883106	0.008792127
4	0.016377154	0.9686840	0.0036587352	0.011280066
5	0.018339847	0.9640239	0.0049134856	0.012722763
6	0.019776193	0.9606553	0.0059999426	0.013568606
7	0.020848529	0.9580469	0.0070103253	0.014094222
8	0.021704512	0.9558670	0.0079914167	0.014437099
9	0.022430470	0.9539370	0.0089629684	0.014669588
10	0.023070011	0.9521670	0.0099318271	0.014831156

The table 13 shows the proportion of the forecast error variance of each variable in the system that can be explained by the shock to cpi_all_us_train, for different time periods (from 1 to 10). For example, in period 1, the shock to cpi_all_us_train explains 99.13% of its own forecast error variance, 0.87% of the forecast error variance of cpi_new_veh_train, 0% of the forecast error variance of ppi_cf_train, and 0% of the forecast error variance of ppi_fg_train. This means that the shock to cpi_all_us_train has a strong impact on itself only, but a weak or negligible impact on the other variables. As the time period increases, the proportions change, reflecting the dynamic effects of the shock over time. For example, in period 10, the shock to cpi_all_us_train explains 95.22% of its own forecast error variance, 2.31% of the forecast error variance of cpi_new_veh_train, 0.99% of the forecast error variance of ppi_cf_train, and 0.014% of the forecast error variance of ppi_fg_train. This means that the shock to cpi_all_us_train has a persistent impact on itself, but a slightly increasing impact on the cpi_new_veh_train.

Table 14: Forecast error variance decomposition table when ppi_cf is shocked

Time Period	cpi_new_veh	cpi_all_us	ppi_cf	ppi_fg
1	0.003024547	0.01823874	0.9787367	0.0000000000
2	0.001777305	0.02827062	0.9693880	0.0005640599
3	0.002942291	0.05811363	0.9303273	0.0086168112
4	0.004249285	0.08489690	0.8968279	0.0140259337
5	0.005524160	0.10846964	0.8666286	0.0193775702
6	0.006344316	0.12810361	0.8416345	0.0239175767
7	0.006844944	0.14518503	0.8199873	0.0279827563
8	0.007128339	0.16064542	0.8005505	0.0316757461
9	0.007271485	0.17507166	0.7825334	0.0351234898
10	0.007318885	0.18877448	0.7655157	0.0383909718

The FEVD results (from Table 14) reveal that when ppi_cf_train is shocked, the variable itself accounts for 97.87% of the forecast error variance in period 1, indicating a strong initial impact of the shock. However, this proportion declines over time, reaching 76.55% by period 10. This suggests that the shock to ppi_cf_train dissipates gradually and other variables become more influential in explaining the forecast error variance.

The variable that is most affected by the shock to ppi_cf_train is cpi_all_us_train, which shows an increasing proportion of the forecast error variance from 1.82% in period 1 to 18.88% in period 10. This implies that there is a positive and significant long-run relationship between ppi_cf_train and cpi_all_us_train, consistent with the economic theory that producer prices influence consumer prices.

The variables cpi_new_veh_train and ppi_fg_train show relatively minor and stable effects of the shock to ppi_cf_train, with proportions ranging from 0.30% to 0.73% and 0% to 4.24%, respectively. This indicates that these variables are less sensitive to the changes in ppi_cf_train and have weaker connections with the producer price index for finished consumer foods.

Table 15: Forecast error variance decomposition table when ppi_fg is shocked

Time Period	cpi_new_veh	cpi_all_us	ppi_cf	ppi_fg
1	0.004497376	0.6176945	0.06885539	0.3089528
2	0.005392772	0.6904932	0.05005527	0.2540587
3	0.010043898	0.7065034	0.04646151	0.2369912
4	0.013336422	0.7109466	0.04501515	0.2307018
5	0.015279422	0.7123337	0.04442334	0.2279635
6	0.016358512	0.7133885	0.04403977	0.2262132
7	0.016975598	0.7143946	0.04377472	0.2248551
8	0.017327766	0.7153936	0.04357619	0.2237025
9	0.017516956	0.7163736	0.04342364	0.2226858
10	0.017598531	0.7173376	0.04330179	0.2217621

The FEVD results (From Table 15) reveal that the shock to ppi_fg_train has a significant impact on cpi_all_us_train, explaining about 62% of its forecast error variance in the first period and gradually increasing to about 72% by the tenth period. This suggests a strong positive relationship between the producer price index for finished goods and the consumer price index for all items. The shock also affects ppi_cf_train, but to a lesser extent, accounting for about 7% of its forecast error variance in the first period and slightly decreasing to about 4.3% by the tenth period. This indicates a moderate negative relationship between the producer price index for finished goods and the producer price index for finished consumer foods. The shock has a negligible impact on cpi_new_veh_train, explaining less than 2% of its forecast error variance throughout the time periods. This implies a weak or no relationship between the producer price index for finished goods and the consumer price index for new vehicles. The shock explains the largest proportion of the forecast error variance of ppi_fg_train itself, starting from about 31% in the first period and decreasing to about 22% by the tenth period.