Airbreathing Propulsion HW 2

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Problem 1:

Simple turbojet engine operating with the following conditions:

$r_c = 8.0,\ T_{03} = 1200\ K,\ \dot{m} = 15\ kg/s,\ C_a = 260\ m/s,\ \eta_{\infty c} = 0.87$
$\eta_{\infty t} = 0.87,\ \eta_i = 0.95,\ \eta_j = 0.95,\ \eta_m = 0.99,\ \Delta P_b = 0.06,\ \eta_b = 0.97$

At an altitude of $h = 7,000\ m$, the ISA table in chapter 3 gives:

$P_a = 0.4111\ bar\ = 41.11\ kPa,\ T_a = 242.7\ K,\ \rho_a = 0.5901\ kg/m^3$

Find: $A_n,\ F,\ SFC$

Inlet

$T_{01}^{'} - T_a = \eta_i \frac{C_a^2}{2C_{pa}} = 0.95 \frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K} = 32.09\ K$
$T_{01} = \eta_i(T_{01}^{'} - T_a) + T_a = 0.95(32.09\ K) + 242.7\ K = 273.19\ K$
$P_{01} = P_a(1 + \eta_i \frac{C_a^2}{2C_{pa}})^{\frac{\gamma}{\gamma-1}} = 0.4111\ bar(1 + 0.95 \frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K})^{\frac{1.4}{0.4}}$
$P_{01} = 0.6349\ bar$

Compressor

$T_{02} - T_{01} = T_{01}[r_c^{\frac{\gamma-1}{\gamma \eta_{\infty c}}} - 1] = 273.19\ K[8.0^{\frac{1.4-1}{1.4*0.87}-1}] = 267.62\ K$
$P_{02} = r_c * P_{01} = 8.0 * 0.6349\ bar = 5.0792\ bar$
$T_{02} = T_{01} + (T_{02} - T_{01}) = 273.19\ K + (267.63\ K) = 540.81\ K$

Combustor

$P_{03} = P_{01} * r_c * (1 - \Delta P_b) = 0.6349\ bar * 8.0 * (0.94) = 4.7744\ bar$
$f_{actual} = \frac{C_{pg}T_{03} - C_{pa}T_{02}}{\eta_b(Q_f - C_{pg}T_{03})} = \frac{1.148*1200 - 1.0005*540.81}{(43100 - 1.148*1200)} = 0.0207$

Turbine

$T_{03} - T_{04} = \frac{1}{\eta_m}\frac{C_{pa}}{C_{pg}}(T_{02} - T_{01}) = \frac{1}{0.99}\frac{1.0005}{1.148}(267.62) = 235.59\ K$
$T_{04} = T_{03} - (T_{03} - T_{04}) = 1200\ K - 235.59\ K = 964.41\ K$
$T_{03} - T_{04} = T_{03}[1 - r_t^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}] \gg 235.59\ K = 1200\ K[1 - r_t^{\frac{0.87(0.333)}{0.333}}]$

Solving for r_t using Symbolab gives:
$r_t = 0.3659$
$P_{04} = P_{03} * r_t = 4.7744\ bar * (0.3659) = 1.7470\ bar$

Nozzle

$NPR = \frac{P_{04}}{P_a} = \frac{1.7470\ bar}{0.4111\ bar} = 4.2496$ check for choke condition
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{0.95}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.9198$
$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked $(M_5 = 1)$
$P_5 = P_c = P_{04} * \frac{1}{(\frac{P_0}{P})_{crit}} = 1.7470\ bar * \frac{1}{1.9189} = 0.9104\ bar$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$
$T_5 = T_c = T_{04} * \frac{1}{(\frac{T_0}{T})_{crit}} = 964.41\ K * \frac{1}{1.1665} = 836.76K$
$C_5 = M_5 * \sqrt{\gamma_g * R * T_5} = 1 * \sqrt{1.333 * 287 * 826.76} = 562.4m/s$
$\dot{m}_a = \rho_5 * C_5 * A_5$, solved for $A_5 = \frac{\dot{m}_a}{\rho_5 * C_5} = \frac{15\ kg/s}{0.384\ kg/m^3 * 562.4\ m/s}$
$\boxed{A_5 = 0.0695\ m^2}$
$F = \dot{m}_a(C_5 - C_a) + A_5(P_5 - P_a) = 15(562.4 - 260) + 0.0695(91040 - 41110)$
$\boxed{F = 7.985\ kN}$
$F_s = \frac{F}{\dot{m}_a} = 532.3\ Ns/kg$
$SFC = \frac{f}{F_s} = \frac{0.0207*3600}{532.3}$
$\boxed{SFC = 0.1400\ kg/hN}$

Problem 2:

At takeoff with the following conditions:

$P_a = 1.1\ bar,\ T_a = 288\ K,\ P_{07} = 3.8\ bar,\ T_{07} = 1000\ K,\ \dot{m} = 23\ kg/s$

A: Find A_9 required and F produced assuming isentropic convergent nozzle

$NPR = \frac{P_{07}}{P_a} = \frac{3.8\ bar}{1.01\ bar} = 3.76$

Assuming nozzle efficiency $\eta_j = 1$

$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.852$

$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked ($M_9 = 1$)

$P_9 = P_c = P_{07} * \frac{1}{(\frac{P_0}{P})_{crit}} = 3.8\ bar * \frac{1}{1.852} = 2.052\ bar$

$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma - 1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$

$T_9 = T_c = T_{07} * \frac{1}{(\frac{T_0}{T})_{crit}} = 1000\ K * \frac{1}{1.1665} = 857.27K$

$C_9 = M_9 * \sqrt{\gamma_g * R * T_9} = 1 * \sqrt{1.333 * 287 * 857.27} = 572.0m/s$

$\rho_9 = \frac{P_9}{R * T_9} = \frac{205.2/kPa}{0.287\ kJ/kgK * 857.27\ K} = 0.834\ kg/m^3$

$\dot{m} = \rho_9 * C_9 * A_9$, solved for $A_9 = \frac{\dot{m}}{\rho_9 * C_9} = \frac{23\ kg/s}{0.834\ kg/m^3 * 572.0\ m/s}$

$\boxed{A_5 = 0.0482\ m^2}$

$F = \dot{m}_a(C_9 - C_a) + A_9(P_9 - P_a) = 23(572.0 - 0) + 0.0482(205200 - 101000)$

$\boxed{F = 18.18\ kN}$