Airbreathing Propulsion HW 2

Ben Rupe

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Problem 1:

Simple turbojet engine operating with the following conditions:

$r_c = 8.0,\ T_{03} = 1200\ K,\ \dot{m} = 15\ kg/s,\ C_a = 260\ m/s,\ \eta_{\infty c} = 0.87$
$\eta_{\infty t} = 0.87,\ \eta_i = 0.95,\ \eta_j = 0.95,\ \eta_m = 0.99,\ \Delta P_b = 0.06,\ \eta_b = 0.97$

At an altitude of $h = 7,000\ m$, the ISA table in chapter 3 gives:

$P_a = 0.4111\ bar\ = 41.11\ kPa,\ T_a = 242.7\ K,\ \rho_a = 0.5901\ kg/m^3$

Find: $A_n,\ F,\ SFC$

Inlet

$T_{01}^{'} - T_a = \eta_i \frac{C_a^2}{2C_{pa}} = 0.95\frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K} = 32.09\ K$
$T_{01} = \eta_i(T_{01}^{'} - T_a) + T_a = 0.95(32.09\ K) + 242.7\ K = 273.19\ K$
$P_{01} = P_a(1 + \eta_i \frac{C_a^2}{2C_{pa}})^{\frac{\gamma}{\gamma-1}} = 0.4111\ bar(1 + 0.95\frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K})^{\frac{1.4}{0.4}}$
$P_{01} = 0.6349\ bar$

Compressor

$T_{02} - T_{01} = T_{01}[r_c^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} - 1] = 273.19\ K[8.0^{\frac{1.4-1}{1.4*0.87}} - 1] = 267.62\ K$
$P_{02} = r_c * P_{01} = 8.0 * 0.6349\ bar = 5.0792\ bar$
$T_{02} = T_{01} + (T_{02} - T_{01}) = 273.19\ K + (267.63\ K) = 540.81\ K$

Combustor

$P_{03} = P_{01} * r_c * (1 - \Delta P_b) = 0.6349\ bar * 8.0 * (0.94) = 4.7744\ bar$
$f_{actual} = \frac{C_{pg}T_{03} - C_{pa}T_{02}}{\eta_b(Q_f - C_{pg}T_{03})} = \frac{1.148*1200 - 1.0005*540.81}{(43100 - 1.148*1200)} = 0.0207$

Turbine

$T_{03} - T_{04} = \frac{1}{\eta_m}\frac{C_{pa}}{C_{pg}}(T_{02} - T_{01}) = \frac{1}{0.99}\frac{1.0005}{1.148}(267.62) = 235.59\ K$
$T_{04} = T_{03} - (T_{03} - T_{04}) = 1200\ K - 235.59\ K = 964.41\ K$
$T_{03} - T_{04} = T_{03}[1 - r_t^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}] \gg 235.59\ K = 1200\ K[1 - r_t^{\frac{0.87(0.333)}{0.333}}]$

Solving for r_t using Symbolab gives:
$r_t = 0.3659$
$P_{04} = P_{03} * r_t = 4.7744\ bar * (0.3659) = 1.7470\ bar$

Nozzle

$NPR = \frac{P_{04}}{P_a} = \frac{1.7470\ bar}{0.4111\ bar} = 4.2496$ check for choke condition
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{0.95}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.9198$
$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked $(M_5 = 1)$
$P_5 = P_c = P_{04} * \frac{1}{(\frac{P_0}{P})_{crit}} = 1.7470\ bar * \frac{1}{1.9189} = 0.9104\ bar$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$
$T_5 = T_c = T_{04} * \frac{1}{(\frac{T_0}{T})_{crit}} = 964.41\ K * \frac{1}{1.1665} = 836.76K$
$C_5 = M_5 * \sqrt{\gamma_g * R * T_5} = 1 * \sqrt{1.333 * 287 * 826.76} = 562.4m/s$
$\dot{m}_a = \rho_5 * C_5 * A_5$, solved for $A_5 = \frac{\dot{m}_a}{\rho_5 * C_5} = \frac{15\ kg/s}{0.384\ kg/m^3 * 562.4\ m/s}$
$\boxed{A_5 = 0.0695\ m^2}$
$F = \dot{m}_a(C_5 - C_a) + A_5(P_5 - P_a) = 15(562.4 - 260) + 0.0695(91040 - 41110)$
$\boxed{F = 7.985\ kN}$
$F_s = \frac{F}{\dot{m}_a} = 532.3\ Ns/kg$
$SFC = \frac{f}{F_s} = \frac{0.0207*3600}{532.3}$
$\boxed{SFC = 0.1400\ kg/hN}$

Problem 2:

At takeoff with the following conditions:

$$P_a = 1.1\ bar,\ T_a = 288\ K,\ P_{07} = 3.8\ bar,\ T_{07} = 1000\ K,\ \dot{m} = 23\ kg/s$$

A: Find A_9 required and F produced assuming isentropic convergent nozzle

$NPR = \frac{P_{07}}{P_a} = \frac{3.8\ bar}{1.01\ bar} = 3.76$

Assuming nozzle efficiency $\eta_j = 1$

$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.852$
$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked ($M_9 = 1$)
$P_9 = P_c = P_{07} * \frac{1}{(\frac{P_0}{P})_{crit}} = 3.8\ bar * \frac{1}{1.852} = 2.052\ bar$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$
$T_9 = T_c = T_{07} * \frac{1}{(\frac{T_0}{T})_{crit}} = 1000\ K * \frac{1}{1.1665} = 857.27K$
$C_9 = M_9 * \sqrt{\gamma_g * R * T_9} = 1 * \sqrt{1.333 * 287 * 857.27} = 572.0m/s$
$\rho_9 = \frac{P_9}{R*T_9} = \frac{205.2/kPa}{0.287\ kJ/kgK*857.27\ K} = 0.834\ kg/m^3$
$\dot{m} = \rho_9 * C_9 * A_9$, solved for $A_9 = \frac{\dot{m}}{\rho_9 * C_9} = \frac{23\ kg/s}{0.834\ kg/m^3*572.0\ m/s}$
$\boxed{A_5 = 0.0482\ m^2}$
$F = \dot{m}_a(C_9 - C_a) + A_9(P_9 - P_a) = 23(572.0 - 0) + 0.0482(205200 - 101000)$
$\boxed{F = 18.18\ kN}$

B: Find A_9 required and F produced assuming isentropic C-D nozzle

Since isentropic: $P_{09} = P_{07}$

Since fully expanded: $P_9 = P_7,\ T_9 = T_7$

$\frac{P_09}{P_9} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} = 3.762 = (1 + 0.1665M^2)^{4.003}$

Solving for M gives:

$M = 1.536$
$C_9 = M_9 * \sqrt{\gamma * R * T} = M_9 * \sqrt{1.333 * 287\ kJ/kgK * 288\ K} = 509.85\ m/s$
$\rho_9 = \frac{P_9}{R*T_9} = \frac{101\ kPa}{0.287\ kJ/kgK*288\ K} = 1.222\ kg/m^3$
$A_9 = \frac{\dot{m}}{\rho_9 * C_9} = \frac{23\ kg/s}{1.222\ kg/m^3*509.85\ m/s}$
$\boxed{A_9 = 0.0369\ m^2}$
$F = \dot{m}_a(C_9 - C_a) + A_9(P_9 - P_a) = 23(509.85 - 0) + 0.0369(101000 - 101000)$
$\boxed{F = 11.73\ kN}$

C: Comment on which nozzle you would pick as a design engineer:

The increased thrust generated by the convergent nozzle makes it the more desirable option in this case. Because the C-D nozzle fully expands the gas in this aircraft, the pressure thrust term becomes zero, and thus it produces less thrust.

Problem 3:

A high-bypass turbofan designed for $M_\infty = 0.85$, with characteristics

$\eta_\infty = 0.90,\ \eta_i =\ 0.95, BPR = 6.2\ FRP = 1.55\ r_o = 34$
$T_{04} = 1350\ K, \dot{m} = 220\ kg/s, \Delta P_b = 0.06$
assume $\eta_m = 1,\ \eta_j = 1$ since not given

At an altitude of $h = 11,000\ m$, the ISA table in chapter 3 gives:

$P_a = 0.227\ bar,\ T_a = 216.9\ K,\ a_a = 295.2\ m/s$

Fan

$r_c = \frac{r_o}{FPR} = \frac{34}{1.55} = 21.94$
$T_{01} = T_a(1 + \frac{\gamma-1}{2} * M_\infty^2) = 216.8\ K(1 + \frac{1.4-1}{2} * 0.85^2) = 248.13\ K$
$\frac{T_{02}}{T_{01}} = (FPR)^{\frac{\gamma-1}{\gamma \eta_{infty}}} = 1.55^{0.3175} = 1.149$
$T_{02} = T_{01}\frac{T_{02}}{T_{01}} = 248.13 * 1.149 = 285.1\ K$
$P_{01} = P_a(1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} = 0.3561\ bar$
$P_{02} = FPR * P_{01} = 1.55(0.3561) = 0.5520\ bar$
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.4}{2.4})]^{\frac{1.4}{0.4}}} = 1.8929$

check for choked condition

$\frac{P_{02}}{P_a} = \frac{0.5520}{0.227} = 2.4317$
$2.4317 > 1.8929$

flow is choked: $M_8 = 1$

Fan Nozzle

$P_8 = P_c = P_{02} * \frac{1}{(\frac{P_0}{P})_{crit}} = 0.5520 * \frac{1}{1.8929} = 0.2916\ bar$
$C_a = M_\infty * a_a = 0.85 * 295.2 = 250.92\ m/s$
$\dot{m}_c = \frac{\dot{m}BPR}{BPR+1} = \frac{220*6.2}{7.2} = 189.44\ kg/s$
$T_{08} = T_{02} = 285.1\ K$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.4-1}{2}(1)] = 1.2$
$T_8 = T_c = T_{08} * \frac{1}{(\frac{T_0}{T})_{crit}} = 285.1\frac{1}{1.2} = 237.58\ K$

$C_8 = M_8\sqrt{\gamma RT_8} = 1\sqrt{1.4 * 287 * 237.58} = 308.97\ m/s$
$\rho_8 = \frac{P_8}{R*T_8} = \frac{29160\ Pa}{287\ J/kgK*237.58\ K} = 0.4277\ kg/m^3$
$\dot{m}_c = \rho_8 * C_8 * A_8$, solved for $A_8 = \frac{\dot{m}_c}{\rho_8*C_8} = \frac{189.44\ kg/s}{0.4277\ kg/m^3*308.97\ m/s}$
$A_8 = 1.434\ m^3$
$F_c = \dot{m}_c(C_8 - C_a) + A_8(P_8 - P_a) = 189.44(308.97 - 250.92) + 1.424(29160 - 22700)\ kN$
$F_c = 20.26\ kN$

HPC

$T_{03} = T_{02}(r_c)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 285.1(21.94)^{0.3175} = 760.05\ K$
$P_{03} = P_{02} * r_c = 0.5520 * 21.94 = 12.1101\ bar$
$T_{03} - T_{02} = 760.05 - 285.1 = 474.96\ K$

Combustor

$P_{04} = P_{03}(1 - \Delta P_b) = 12.1101(0.94) = 11.3835\ bar$
$T_{04} = 1350\ K$

HPT

$\dot{m}_h = \dot{m} - \dot{m}_c = 220 - 189.44 = 30.56\ kg/s$

Work balance with HPC:

$\eta_m C_{pg}(T_{04} - T_{05}) = C_{pa}(T_{03} - T_{02})$ gives
$T_{04} - T_{05} = \frac{C_{pa}}{\eta_m C_{pg}}(T_{03} - T_{02}) = \frac{1.0005}{1*1.148}(474.95) = 413.92\ K$
$T_{05} = T_{04} - (T_{04} - T_{05}) = 1350 - 413.92 = 936.08\ K$
$\frac{P_{05}}{P_{04}} = (\frac{T_{05}}{T_{04}})^{\frac{\gamma}{\eta_\infty(\gamma-1)}} = (\frac{936.08}{1350})^{1/0.2248} = 0.1964$
$P_{05} = P_{04}(\frac{P_{05}}{P_{04}}) = 11.3835 * 0.1964 = 2.2357\ bar$

LPT

Work balance with Fan:

$\dot{m}_h\eta_m C_{pg}(T_{05} - T_{06}) = \dot{m}C_{pa}(T_{02} - T_{01})$ gives
$T_{05} - T_{06} = \frac{\dot{m}}{\dot{m}_h}\frac{C_{pa}}{\eta_m C_{pg}}(T_{02} - T_{01}) = \frac{220}{30.56}\frac{1.0005}{1*1.148}(285.1 - 248.13) = 231.95\ K$
$T_{06} = T_{05} - (T_{05} - T_{06}) = 851.18 - 231.95 = 704.12\ K$
$\frac{P_{06}}{P_{05}} = (\frac{T_{06}}{T_{05}})^{\frac{\gamma}{\eta_\infty(\gamma-1)}} = (\frac{704.12}{936.08})^{1/0.2248} = 0.2818$

$P_{06} = P_{05}(\frac{P_{06}}{P_{05}}) = 2.2357 * 0.2818 = 0.6300\ bar$

Core Nozzle

Check for choked condition

$\frac{P_{06}}{P_a} = \frac{0.6300}{0.227} = 2.775$
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.8524$
$1.0969 > 1.8524$, flow is choked
$M_7 = 1$
$P_7 = P_c = P_{06} * \frac{1}{(\frac{P_0}{P})_{crit}} = 0.6300\ bar * \frac{1}{1.8524} = 0.3401\ bar$
$T_7 = T_c = T_{06} * \frac{1}{(\frac{T_0}{T})_{crit}} = 704.12\ K * \frac{1}{1.1665} = 603.62K$
$C_7 = M_7 * \sqrt{\gamma_g * R * T_7} = 1 * \sqrt{1.333 * 287 * 603.62} = 480.55m/s$
$\rho_7 = \frac{P_7}{R*T_7} = \frac{34.01/kPa}{0.287\ kJ/kgK*603.62\ K} = 0.1963\ kg/m^3$
$\dot{m}_h = \rho_7 * C_7 * A_7$, solved for $A_7 = \frac{\dot{m}_h}{\rho_7 * C_7} = \frac{30.56\ kg/s}{0.1963\ kg/m^3*480.55\ m/s}$
$A_7 = 0.323\ m^3$
$F_h = \dot{m}_h(C_7 - C_a) + A_7(P_7 - P_a) = 30.56(480.55 - 250.92) + 0.323(34010 - 22700)$
$F_h = 10.6\ kN$
$F = F_h + F_c = 10.6 + 20.26$
$\boxed{F = 30.86\ kN}$
$f_{actual} = \frac{C_{pg}T_{04} - C_{pa}T_{03}}{\eta_b(Q_f - C_{pg}T_{04})} = \frac{1.148*1350 - 1.0005*760.05}{(43100 - 1.148*1350)} = 0.0190$
$TSFC = \frac{f}{(B+1)F/\dot{m}_a} = \frac{0.0190*3600}{7.2(30860)/220} = 0.068\ kg/Nh$
$\boxed{TSFC = 0.068\ kg/Nh}$

B: Proposed changes for extended range?

By increasing bypass ratio, the thrust and fuel efficiency of the aircraft will increase. Both of these changes will lead to decreased TSFC, allowing for more range.