Airbreathing Propulsion HW 2

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Problem 1:

Simple turbojet engine operating with the following conditions:

$r_c = 8.0,\ T_{03} = 1200\ K,\ \dot{m} = 15\ kg/s,\ C_a = 260\ m/s,\ \eta_{\infty c} = 0.87$
$\eta_{\infty t} = 0.87,\ \eta_i = 0.95,\ \eta_j = 0.95,\ \eta_m = 0.99,\ \Delta P_b = 0.06,\ \eta_b = 0.97$

At an altitude of $h = 7,000\ m$, the ISA table in chapter 3 gives:

$P_a = 0.4111\ bar\ = 41.11\ kPa,\ T_a = 242.7\ K,\ \rho_a = 0.5901\ kg/m^3$

Find: $A_n,\ F,\ SFC$

Inlet

$T_{01}^{'} - T_a = \eta_i \frac{C_a^2}{2C_{pa}} = 0.95\frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K} = 32.09\ K$
$T_{01} = \eta_i(T_{01}^{'} - T_a) + T_a = 0.95(32.09\ K) + 242.7\ K = 273.19\ K$
$P_{01} = P_a(1 + \eta_i \frac{C_a^2}{2C_{pa}})^{\frac{\gamma}{\gamma-1}} = 0.4111\ bar(1 + 0.95\frac{(260\ m/s)^2}{2*1.0005\ kJ/kg*K})^{\frac{1.4}{0.4}}$
$P_{01} = 0.6349\ bar$

Compressor

$T_{02} - T_{01} = T_{01}[r_c^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} - 1] = 273.19\ K[8.0^{\frac{1.4-1}{1.4*0.87}-1}] = 267.62\ K$
$P_{02} = r_c * P_{01} = 8.0 * 0.6349\ bar = 5.0792\ bar$
$T_{02} = T_{01} + (T_{02} - T_{01}) = 273.19\ K + (267.63\ K) = 540.81\ K$

Combustor

$P_{03} = P_{01} * r_c * (1 - \Delta P_b) = 0.6349\ bar * 8.0 * (0.94) = 4.7744\ bar$
$f_{actual} = \frac{C_{pg}T_{03} - C_{pa}T_{02}}{\eta_b(Q_f - C_{pg}T_{03})} = \frac{1.148*1200-1.0005*540.81}{(43100-1.148*1200)} = 0.0207$

Turbine

$T_{03} - T_{04} = \frac{1}{\eta_m} \frac{C_{pa}}{C_{pg}}(T_{02} - T_{01}) = \frac{1}{0.99} \frac{1.0005}{1.148}(267.62) = 235.59\ K$
$T_{04} = T_{03} - (T_{03} - T_{04}) = 1200\ K - 235.59\ K = 964.41\ K$
$T_{03} - T_{04} = T_{03}[1 - r_t^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}] \gg 235.59\ K = 1200\ K[1 - r_t^{\frac{0.87(0.333)}{0.333}}]$

Solving for r_t using Symbolab gives:
$r_t = 0.3659$
$P_{04} = P_{03} * r_t = 4.7744\ bar * (0.3659) = 1.7470\ bar$

Nozzle

$NPR = \frac{P_{04}}{P_a} = \frac{1.7470\ bar}{0.4111\ bar} = 4.2496$ check for choke condition
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{0.95}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.9198$
$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked $(M_5 = 1)$
$P_5 = P_c = P_{04} * \frac{1}{(\frac{P_0}{P})_{crit}} = 1.7470\ bar * \frac{1}{1.9189} = 0.9104\ bar$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$
$T_5 = T_c = T_{04} * \frac{1}{(\frac{T_0}{T})_{crit}} = 964.41\ K * \frac{1}{1.1665} = 836.76K$
$C_5 = M_5 * \sqrt{\gamma_g * R * T_5} = 1 * \sqrt{1.333 * 287 * 826.76} = 562.4m/s$
$\dot{m}_a = \rho_5 * C_5 * A_5$, solved for $A_5 = \frac{\dot{m}_a}{\rho_5 * C_5} = \frac{15\ kg/s}{0.384\ kg/m^3 * 562.4\ m/s}$
$\boxed{A_5 = 0.0695\ m^2}$
$F = \dot{m}_a(C_5 - C_a) + A_5(P_5 - P_a) = 15(562.4 - 260) + 0.0695(91040 - 41110)$
$\boxed{F = 7.985\ kN}$
$F_s = \frac{F}{\dot{m}_a} = 532.3\ Ns/kg$
$SFC = \frac{f}{F_s} = \frac{0.0207*3600}{532.3}$
$\boxed{SFC = 0.1400\ kg/hN}$

Problem 2:

At takeoff with the following conditions:

$$P_a = 1.1\ bar,\ T_a = 288\ K,\ P_{07} = 3.8\ bar,\ T_{07} = 1000\ K,\ \dot{m} = 23\ kg/s$$

A: Find A_9 required and F produced assuming isentropic convergent nozzle

$NPR = \frac{P_{07}}{P_a} = \frac{3.8\ bar}{1.01\ bar} = 3.76$

Assuming nozzle efficiency $\eta_j = 1$

$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.852$

$NPR > (\frac{P_0}{P})_{crit}$, meaning nozzle is choked ($M_9 = 1$)

$P_9 = P_c = P_{07} * \frac{1}{(\frac{P_0}{P})_{crit}} = 3.8\ bar * \frac{1}{1.852} = 2.052\ bar$

$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.333-1}{2}(1)] = 1.1665$

$T_9 = T_c = T_{07} * \frac{1}{(\frac{T_0}{T})_{crit}} = 1000\ K * \frac{1}{1.1665} = 857.27K$

$C_9 = M_9 * \sqrt{\gamma_g * R * T_9} = 1 * \sqrt{1.333 * 287 * 857.27} = 572.0m/s$

$\rho_9 = \frac{P_9}{R*T_9} = \frac{205.2/kPa}{0.287\ kJ/kgK*857.27\ K} = 0.834\ kg/m^3$

$\dot{m} = \rho_9 * C_9 * A_9$, solved for $A_9 = \frac{\dot{m}}{\rho_9 * C_9} = \frac{23\ kg/s}{0.834\ kg/m^3*572.0\ m/s}$

$\boxed{A_5 = 0.0482\ m^2}$

$F = \dot{m}_a(C_9 - C_a) + A_9(P_9 - P_a) = 23(572.0 - 0) + 0.0482(205200 - 101000)$

$\boxed{F = 18.18\ kN}$

B: Find A_9 required and F produced assuming isentropic C-D nozzle

Since isentropic: $P_{09} = P_{07}$

Since fully expanded: $P_9 = P_7,\ T_9 = T_7$

$\frac{P_09}{P_9} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} = 3.762 = (1 + 0.1665M^2)^{4.003}$

Solving for M gives:

$M = 1.536$

$C_9 = M_9 * \sqrt{\gamma * R * T} = M_9 * \sqrt{1.333 * 287\ kJ/kgK * 288\ K} = 509.85\ m/s$

$\rho_9 = \frac{P_9}{R*T_9} = \frac{101\ kPa}{0.287\ kJ/kgK*288\ K} = 1.222\ kg/m^3$

$A_9 = \frac{\dot{m}}{\rho_9*C_9} = \frac{23\ kg/s}{1.222\ kg/m^3*509.85\ m/s}$

$\boxed{A_9 = 0.0369\ m^2}$

$F = \dot{m}_a(C_9 - C_a) + A_9(P_9 - P_a) = 23(509.85 - 0) + 0.0369(101000 - 101000)$

$\boxed{F = 11.73\ kN}$

C: Comment on which nozzle you would pick as a design engineer:

The increased thrust generated by the convergent nozzle makes it the more desirable option in this case. Because the C-D nozzle fully expands the gas in this aircraft, the pressure thrust term becomes zero, and thus it produces less thrust.

Problem 3:

A high-bypass turbofan designed for $M_\infty = 0.85$, with characteristics

$\eta_\infty = 0.90,\ \eta_i = \ 0.95, BPR = 6.2\ FRP = 1.55\ r_o = 34$
$T_{04} = 1350\ K, \dot{m} = 220\ kg/s, \Delta P_b = 0.06$
assume $\eta_m = 1,\ \eta_j = 1$ since not given

At an altitude of $h = 11,000\ m$, the ISA table in chapter 3 gives:

$P_a = 0.227\ bar,\ T_a = 216.9\ K,\ a_a = 295.2\ m/s$

Fan

$r_c = \frac{r_o}{FPR} = \frac{34}{1.55} = 21.94$
$T_{01} = T_a(1 + \frac{\gamma-1}{2} * M_\infty^2) = 216.8\ K(1 + \frac{1.4-1}{2} * 0.85^2) = 248.13\ K$
$\frac{T_{02}}{T_{01}} = (FPR)^{\frac{\gamma-1}{\gamma\eta_{infty}}} = 1.55^{0.3175} = 1.149$
$T_{02} = T_{01}\frac{T_{02}}{T_{01}} = 248.13 * 1.149 = 285.1\ K$
$P_{01} = P_a(1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}} = 0.3561\ bar$
$P_{02} = FPR * P_{01} = 1.55(0.3561) = 0.5520\ bar$
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.4}{2.4})]^{\frac{1.4}{0.4}}} = 1.8929$

check for choked condition

$\frac{P_{02}}{P_a} = \frac{0.5520}{0.227} = 2.4317$
$2.4317 > 1.8929$

flow is choked: $M_8 = 1$

Fan Nozzle

$P_8 = P_c = P_{02} * \frac{1}{(\frac{P_0}{P})_{crit}} = 0.5520 * \frac{1}{1.8929} = 0.2916\ bar$
$C_a = M_\infty * a_a = 0.85 * 295.2 = 250.92\ m/s$
$\dot{m}_c = \frac{\dot{m}BPR}{BPR+1} = \frac{220*6.2}{7.2} = 189.44\ kg/s$
$T_{08} = T_{02} = 285.1\ K$
$(\frac{T_0}{T})_{crit} = [1 + \frac{\gamma-1}{2}(M)] = [1 + \frac{1.4-1}{2}(1)] = 1.2$
$T_8 = T_c = T_{08} * \frac{1}{(\frac{T_0}{T})_{crit}} = 285.1\frac{1}{1.2} = 237.58\ K$

$C_8 = M_8\sqrt{\gamma RT_8} = 1\sqrt{1.4 * 287 * 237.58} = 308.97\ m/s$
$F_c = \dot{m}_c(C_8 - C_a) = 189.44(308.97 - 250.92) = 11.00\ kN$

HPC

$T_{03} = T_{02}(r_c)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 285.1(21.94)^{0.3175} = 760.05\ K$
$P_{03} = P_{02} * r_c = 0.5520 * 21.94 = 12.1101\ bar$
$T_{03} - T_{02} = 760.05 - 285.1 = 474.96\ K$

Combustor

$P_{04} = P_{03}(1 - \Delta P_b) = 12.1101(0.94) = 11.3835\ bar$
$T_{04} = 1350\ K$

HPT

$\dot{m}_h = \dot{m} - \dot{m}_c = 220 - 189.44 = 30.56\ kg/s$

Work balance with HPC:

$\eta_m C_{pg}(T_{04} - T_{05}) = C_{pa}(T_{03} - T_{02})$ gives
$T_{04} - T_{05} = \frac{C_{pa}}{\eta_m C_{pg}}(T_{03} - T_{02}) = \frac{1.4}{1*1.333}(474.95) = 498.82\ K$
$T_{05} = T_{04} - (T_{04} - T_{05}) = 1350 - 498.82 = 851.18\ K$
$\frac{P_{05}}{P_{04}} = (\frac{T_{05}}{T_{04}})^{\frac{\gamma}{\eta_\infty(\gamma-1)}} = (\frac{851.18}{1350})^{1/0.2248} = 0.1285$
$P_{05} = P_{04}(\frac{P_{05}}{P_{04}}) = 11.3835 * 0.1285 = 1.4628\ bar$

LPT

Work balance with Fan:

$\dot{m}_h\eta_m C_{pg}(T_{05} - T_{06}) = \dot{m}C_{pa}(T_{02} - T_{01})$ gives
$T_{05} - T_{06} = \frac{\dot{m}}{\dot{m}_h}\frac{C_{pa}}{\eta_m C_{pg}}(T_{02} - T_{01}) = \frac{220}{30.56}\frac{1.4}{1*1.333}(285.1 - 248.13) = 279.52\ K$
$T_{06} = T_{05} - (T_{05} - T_{06}) = 851.18 - 279.52 = 571.66\ K$
$\frac{P_{06}}{P_{05}} = (\frac{T_{06}}{T_{05}})^{\frac{\gamma}{\eta_\infty(\gamma-1)}} = (\frac{571.66}{851.18})^{1/0.2248} = 0.1702$
$P_{06} = P_{05}(\frac{P_{06}}{P_{05}}) = 1.4628 * 0.1702 = 0.2490\ bar$

Core Nozzle

Check for choked condition

$\frac{P_{06}}{P_a} = \frac{0.2490}{0.227} = 1.0969$
$(\frac{P_0}{P})_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma-1}{\gamma+1})]^{\frac{\gamma}{\gamma-1}}} = \frac{1}{[1-\frac{1}{1}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.8524$
$1.0969 < 1.8524$, flow is not choked
$T_{07} = T_{06},\ P_7 = P_a$
$T_{07} - T_7 = \eta_j T_{07}[1 - (\frac{P_7}{P_{06}})^{\frac{\gamma-1}{\gamma}}] = 1(812.35)[1 - (\frac{1}{1.0969})^{\frac{0.333}{1.333}}] = 443.08\ K$
$T_7 = T_{07} - (T_{07} - T_7) = 812.35 - 443.08 = 369.27\ K$