Lab 1 Report

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1 Task 1

1.1 (a) find the coefficients for the standard form

Given the points as the code below

```
x = [1; 2; 3];
y = [2; 3; 5];
```

I create this function to find the coefficients:

```
function coeffs = compute_poly_coeffs(x, y)
    % Construct the Vandermonde matrix
    A = vander(x);
    A = fliplr(A);
    % Solve for the coefficients
    coeffs = A \ y;
end
```

And I got the coefficients:

```
Computed polynomial coefficients:
    2.0000    -0.5000    0.5000
```

Figure 1: Polynomial coefficients

1.2 (b)evaluate the inter-polating polynomial at a given point

According the given points in (a), we choose the evaluation point u = 1.5

```
% Evaluate polynomial at a given point u
u = 1.5;  % Example evaluation point
p_value = polyval(flip(coeffs), u);
```

And we get the following:
$P(1.500000) = 2.375000$

1.3 (c) Plot the interpolating polynomial;

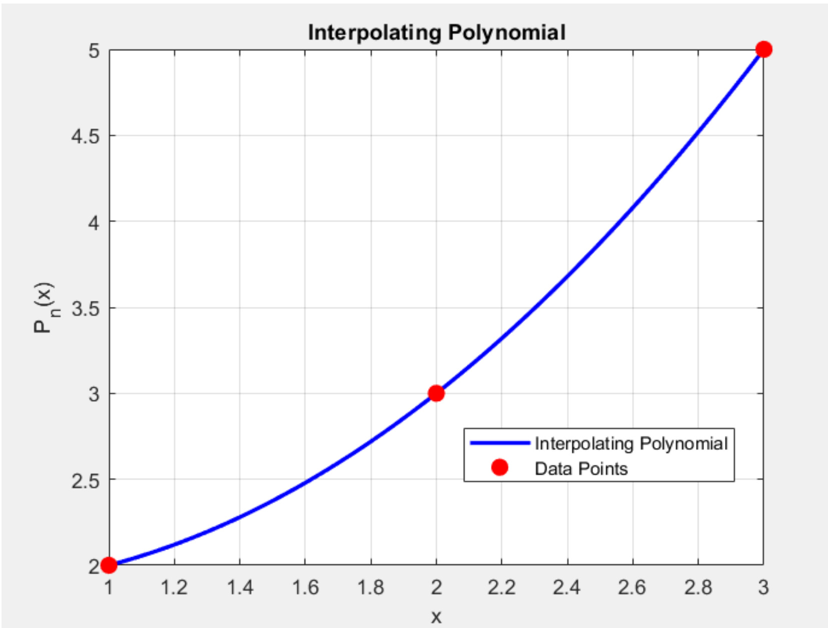

Figure 2: interpolating

2 Task 2

2.1 vander

Help construct the Vandermonde matrix with coefficients.

2.2 fliplr

Help flip the martix from left to right.

2.3 polyval

Help find the point's value in polynomial with coefficients.

3 Task 3

3.1 (a)Write down the de nition of the standard form of the interpolating polynomials of degree n

The interpolating polynomial of degree n in standard form is given by:

$$P_n(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

where the coefficients $a_n, a_{n-1}, \ldots, a_0$ are determined such that the polynomial passes through the given data points:

$$P_n(x_i) = y_i, \quad \text{for } i = 0, 1, 2, \ldots, n.$$

3.2 (b) describe how to construct it.

To find $P_n(x)$, we substitute the given data points into the polynomial equation:

$$\begin{cases} a_0 + a_1x_0 + a_2x_0^2 + \cdots + a_nx_0^n = y_0 \\ a_0 + a_1x_1 + a_2x_1^2 + \cdots + a_nx_1^n = y_1 \\ \vdots \\ a_0 + a_1x_n + a_2x_n^2 + \cdots + a_nx_n^n = y_n \end{cases}$$

which can be written in matrix form as:

$$A \cdot c = b$$

where A is the Vandermonde matrix:

$$A = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \dots & x_m^n \end{bmatrix}$$

The coefficient vector c is:

$$c = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}$$

and the right-hand side vector b is:

$$b = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}$$

4 Task 4

4.1 Existence

The standard form of the interpolating polynomial is given by:

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

To find its coefficients, we set up the system:

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \dots & x_m^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}$$

If the Vandermonde matrix A is invertible, then the system has a unique solution, ensuring the existence of $P_n(x)$.

4.2 Uniqueness

Suppose that there exist two different polynomials $P_n(x)$ and $Q_n(x)$ that interpolate the same points. Define your difference:

$$R_n(x) = P_n(x) - Q_n(x)$$

Since both $P_n(x)$ and $Q_n(x)$ satisfy $P_n(x_i) = Q_n(x_i) = y_i$, we have:

$$R_n(x_i) = 0, \quad \text{for } i = 0, 1, \dots, n$$

Since $R_n(x)$ is a polynomial of n-th degree with $n+1$ distinct roots, it must be identically zero:

$$R_n(x) \equiv 0$$

Thus, $P_n(x) = Q_n(x)$, proving uniqueness.

4.3 Determinant of the Vandermonde Matrix

The determinant of the Vandermonde matrix is given by:

$$\det(A) = \prod_{0 \leq i < j \leq n} (x_j - x_i)$$

Since all x_i are distinct, we have $\det(A) \neq 0$, ensuring invertibility of A and thus the uniqueness of $P_n(x)$.

5 Task 5

Evaluation of $P_2(x)$ at different points:

$$P_2(0.50) = 6.4167$$
$$P_2(0.75) = 9.0833$$
$$P_2(1.20) = 1.9133$$

6 Task 6

The condition number of a matrix A is given by:

$$\kappa(A) = \|A\| \cdot \|A^{-1}\|$$

The error norm is defined as:

$$\text{Error} = \|A \cdot c - y\|$$

where A is the Vandermonde matrix, c is the computed polynomial coefficient vector, and y is the vector of function values. The condition number and the 2-norm of error are

```
Condition number of Vandermonde matrix:
   85.4871


Error norm:
   3.0767e-15
```

Figure 3: condition number and error

6.1 (a)

To construct the interpolating polynomial $P_n(x)$, we find its coefficients by solving the linear system:

$$A \cdot c = y$$

where A is the Vandermonde matrix:

$$A = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \dots & x_m^n \end{bmatrix}$$

6.1.1 MATLAB Implementation

The following MATLAB script computes the polynomial coefficients:

```
x = [1/3; 1/4; 1];
y = [2; -1; 7];

A = vander(x);

A = fliplr(A);

coeffs = A \ y;

disp(coeffs');
```

```
Polynomial coefficients (from lowest to highest order):
  -13.1667    58.1667   -38.0000
```

Figure 4: Polynomial coefficients

6.2 (b)

```
u = 0.5;

p_value = polyval(flip(coeffs), u);

fprintf('P_n(%.2f) = %.4f\n', u, p_value);
```

$P_n(0.50) = 6.4167$

6.3 Task 7

The following MATLAB is subroutine(function)

```
    function p_value = polyinterp_sta(x, y, u)
    A = vander(x);
    A = fliplr(A);

    coeffs = A \ y;

    p_value = polyval(flip(coeffs), u);
end
```

As you can see, it is familiar to the function that I describe in the first of the report.
The difference is we have to print the value of polynomial
However, we still have the same solution in task6(b)
$P_n(0.50) = 6.4167$

6.4 Task 8

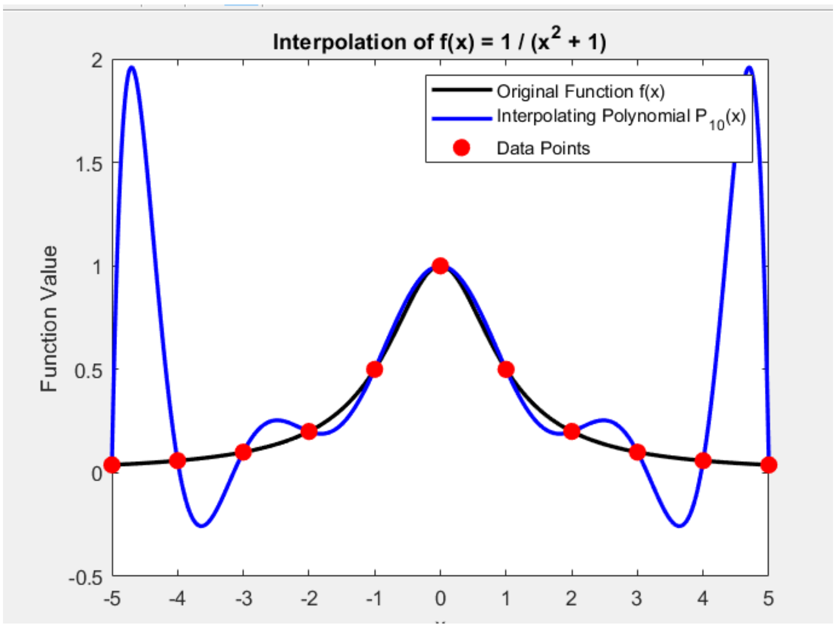

Figure 5: Interpolation

6.5 Task 9

```
Condition number of Vandermonde matrix: 3.0484e+07
Error norm ||A * c - y||_2: 4.9570e-14
```

Figure 6: condition number and error

As the image we get in Task 8, we can see that the condition number is unstable as we see.