Bayesian games

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Bayesian games - intuitive definition

So far we have assumed that each player knows the payoff of other players. Ex. Prisoners dilemma. We didn't know whether our colleague will cooperate with police, but we knew payoffs for all possibilities.

Closer to real life are games with incomplete information (uncertainty about payoffs) called Bayesian Games.

Bayesian games - intuitive definition

In games with incomplete information we have
- Players
  - One player can have multiple types each having different payoffs
  - Player has probability assigned for each type of his opponent
  - Player 2 knows that player 1 has beliefs.
    Player 1 knows that player 2 knows that player 1 has beliefs.
    Player 2 knows that player 1 knows that player 2 knows that player 1 has beliefs.
    ... and so on
- Actions
- Payoffs

So in short, the difference is these types and beliefs about these types.

Bayesian games - intuitive definition

Bayesian game eliminates infinite loops in situations where players try to predict each other's thoughts. For instance, a player might think, "If I expect *player B* to take a certain action, then *player B* will predict that I expect this action, so I need to predict *player B's* prediction", and so on. Bayesian games simplify this by assigning probability weights to each outcome.

Complete information

	Star Wars	Titanic
Star Wars	(2, 1)	(0, 0)
Titanic	(0, 0)	(2, 1)

Incomplete information

p=0.75

	Star Wars	Titanic
Star Wars	(2, 1)	(0, 0)
Titanic	(0, 0)	(2, 1)

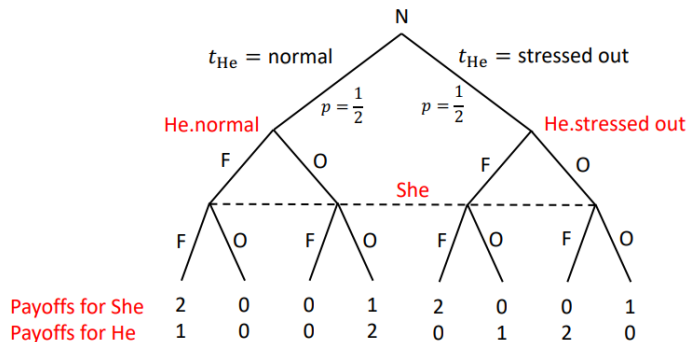*Woman wishes to meet with man*

p=0.25

	Star Wars	Titanic
Star Wars	(2, 0)	(0, 2)
Titanic	(0, 1)	(1, 0)

Woman wishes to avoid man

Watch out! Here she prefers football and he prefers opera.

Suspect is criminal with prob p

Sheriff's action

	Shoot	Not
Suspect's action Shoot	(0, 0)	(2, -2)
Not	(-2, -1)	(-1, 1)

Suspect is civilian with prob 1-p

Sheriff's action

	Shoot	Not
Suspect's action Shoot	(-3, -1)	(-1, -2)
Not	(-2, -1)	(0, 0)

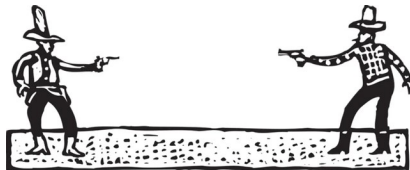**Suspect is criminal with prob p**

Sheriff's action

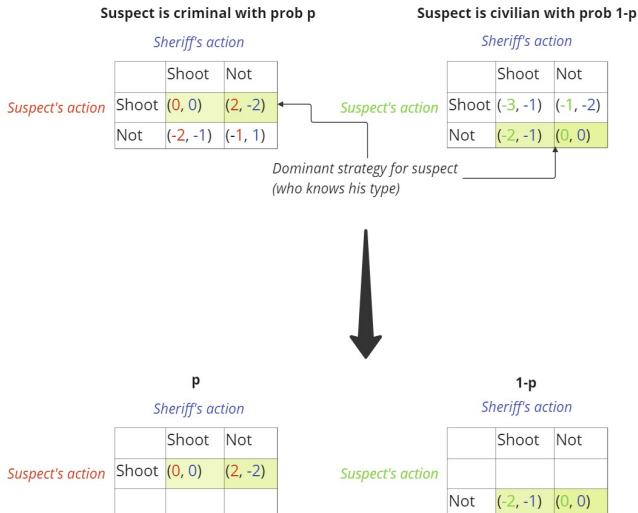|	Shoot	Not
Shoot	(0, 0)	(2, -2)
Not	(-2, -1)	(-1, 1)

Suspect's action

Suspect is civilian with prob 1-p

Sheriff's action

	Shoot	Not
Shoot	(-3, -1)	(-1, -2)
Not	(-2, -1)	(0, 0)

Suspect's action

Dominant strategy for suspect (who knows his type)

p

Sheriff's action

	Shoot	Not
Shoot	(0, 0)	(2, -2)

Suspect's action

1-p

Sheriff's action

	Shoot	Not
Not	(-2, -1)	(0, 0)

Suspect's action

	p				1-p	
	Sheriff's action				*Sheriff's action*	

	Shoot	Not
Shoot	(0, 0)	(2, -2)

Suspect's action

	Shoot	Not
Not	(-2, -1)	(0, 0)

Suspect's action

Expected payoff for sheriff if he shoots:

$$0 \cdot p + (-1)(1 - p) = p - 1$$

and if he does not shoot:

$$-2 \cdot p + 0 \cdot (1 - p) = -2p$$

Therefore, sheriff should shoot only when

$$p - 1 > -2p \qquad p > \frac{1}{3}$$

Mathematical definition

A Bayesian game is defined by (N, A, T, p, u), where:

- N - **Set of players**

- $a_i \in A$ - **Actions:** The set of actions available to Player i.

- $t_i \in T$ - **Types:** The set of types for player i. Captures the private information a player can have.

- u - **Payoff functions:** Assign a payoff to a player given their type and the action profile.

- p - **Types probabilities:** Where $p(t) = p(t_1, \ldots, t_N)$ is the probability that Player 1 has type t_1 and Player N has type t_N.

- $N = \{\text{Suspect, Sheriff}\}$
- $A_{\text{Suspect}} = \{\text{Shoot, Not}\}$, $A_{\text{Sheriff}} = \{\text{Shoot, Not}\}$
- $T_{\text{Suspect}} = \{\text{Criminal, Civilian}\}$, $T_{\text{Sheriff}} = \{\text{Default}\}$
- $p_{\text{Criminal}} = p$, $p_{\text{Civilian}} = (1 - p)$
- Payoffs u are the tables we have seen before

- She and He have an arbitrary number of types each.
- There are more than two players.
- Players choose sequentially. The player playing second can observe the action, but not the type of the player playing first.

- There are two cars:
  - A high-quality car worth $100,000, sold for $100,000.
  - A defective car with hidden flaws worth $50,000, also sold for $100,000.
- The buyer does not know which car has hidden flaws, so they take this into account and negotiate a price in the middle: $75,000.
- Since sellers of high-quality cars cannot sell them for lower prices than the cars are worth, they leave the market. Only low-quality cars can be sold for lower prices.
- As a result, the average quality and price in the market decrease.
- This cycle repeats until buyers only want cars for free.

In Poland, nearly twice as many used cars are sold as new ones, so there are, of course, ways to deal with this problem, such as warranties.

Resources

- https://en.wikipedia.org/wiki/Bayesian_game
- https://www.ehu.eus/iaguirre/Chapter%201.%20Bayesian%20Games%20in%20Normal%20Form.pdf