Aufgabe 18

Montag, 17. Juli 2023 13:52

a) ges: $u(\frac{\partial p}{\partial x})$, $\tau(\frac{\partial p}{\partial x})$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \quad \int y$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1 \quad \int y$$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$

oben: $u_B = \frac{1}{2\mu_B}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$

$$\tau_B = \frac{\partial p}{\partial x} y + \mu_B C_1$$

unten: $u_A = \frac{1}{2\mu_A}\frac{\partial p}{\partial x} y^2 + C_3 y + C_4$

$$\tau_A = \frac{\partial p}{\partial x} y + \mu_A C_3$$

4 Unbekannte $\rightarrow$ 4 Randbedingungen !

$u_B(b)=0 \rightsquigarrow \frac{1}{2\mu_B}\frac{\partial p}{\partial x} b^2 + \frac{\mu_A C_3}{\mu_B} b + C_2 = 0$ ①

$u_A(-b)=0 \rightsquigarrow \frac{1}{2\mu_A}\frac{\partial p}{\partial x} b^2 + C_3(-b) + C_2 = 0$ ②

$u_B(0) = u_A(0) \rightsquigarrow C_2 = C_4$

$\tau_B(0) = \tau_A(0) \rightsquigarrow \mu_B C_1 = \mu_A C_3 \Rightarrow C_1 = \frac{\mu_A C_3}{\mu_B}$ ③

① − ② $\quad \frac{\partial p}{\partial x} b^2 \frac{1}{2}\left(\frac{1}{\mu_B} - \frac{1}{\mu_A}\right) + b C_3\left(\frac{\mu_A}{\mu_B} + 1\right) = 0$

$$C_3 = \frac{\partial p}{\partial x}\frac{b}{2}\frac{\mu_B - \mu_A}{(\mu_B + \mu_A)\mu_A}$$

in ③ einsetzen: $C_1 = \frac{\partial p}{\partial x}\frac{b}{2}\frac{\mu_B - \mu_A}{(\mu_B + \mu_A)\mu_B}$

in ② einsetzen: $C_2 = -b^2\frac{\partial p}{\partial x}\frac{1}{\mu_B + \mu_A} = C_4$

$\tau_{AB} = \frac{\partial p}{\partial x} y + \frac{\partial p}{\partial x}\frac{b}{2}\frac{\mu_B - \mu_A}{\mu_B + \mu_A}$

$$u_B = \frac{1}{2\mu_B}\frac{\partial p}{\partial x}\left(y^2 + b\,\frac{\mu_B-\mu_A}{\mu_B+\mu_A}\,y\right) - b^2\frac{\partial p}{\partial x}\frac{1}{\mu_B+\mu_A}$$

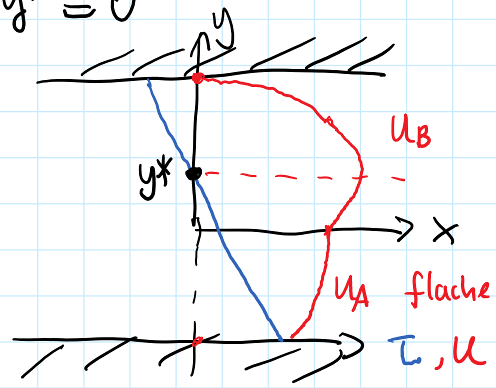
$$u_A = \frac{1}{2\mu_A}\frac{\partial p}{\partial x}\left(y^2 + b\,\frac{\mu_B-\mu_A}{\mu_B+\mu_A}\,y\right) - b^2\frac{\partial p}{\partial x}\frac{1}{\mu_B+\mu_A}$$

b)

u_B, u_A, τ_{AB}

$$\tau_{AB} = \frac{\partial p}{\partial x}y + \frac{\partial p}{\partial x}\frac{b}{2}\frac{\mu_B-\mu_A}{\mu_B+\mu_A}$$

$\tau_{y^*} = 0$



$$\tau_{AB}(y^*) = 0$$

$$\frac{\partial p}{\partial x}y^* + \frac{\partial p}{\partial x}\frac{b}{2}\frac{\mu_B-\mu_A}{\mu_B+\mu_A} = 0$$

$$y^* = \frac{b}{2}\frac{\mu_A-\mu_B}{\mu_B+\mu_A} > 0 \quad \text{mit } \mu_A > \mu_B$$