Aufgabe 16

Montag, 17. Juli 2023 13:52

a) ges: $\frac{\partial p}{\partial x}$ damit $Q \stackrel{!}{=} 0$

Bestimmung von Q:

$$Q = \int_A u(A)\, dA = t \int_0^b u(y)\, dy \stackrel{!}{=} 0 \qquad (1)$$

Herleitung $u(y)$:

Aus Impulsbilanz in x-Richtung:

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} = 0 \qquad \int y$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1 \qquad \int y$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2 \qquad (2)$$

Bestimmung der Integrationskonstanten C_1 und C_2 über Randwerte:

$$u(y=0) = 0 \xrightarrow{\text{in (2) einsetzen}} C_2 = 0$$

$$u(y=b) = V_0 \xrightarrow{\text{- - - -}} \frac{1}{2\mu}\frac{\partial p}{\partial x} b^2 + C_1 b = V_0$$

$$C_1 = \frac{V_0}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x} b$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + \left(\frac{V_0}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x} b\right) y$$

$$Q = t\int_0^b u(y)\, dy = t\left[\frac{1}{6\mu}\frac{\partial p}{\partial x} y^3 + \frac{V_0}{2b} y^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} b y^2\right]\Bigg|_0^b$$

$$= t\left[\underbrace{\frac{1}{6\mu}\frac{\partial p}{\partial x} b^3} + \frac{V_0}{2} b - \underbrace{\frac{1}{4\mu}\frac{\partial p}{\partial x} b^3}\right]$$

$$= t\left[-\frac{1}{12\mu}\frac{\partial p}{\partial x} b^3 + \frac{V_0}{2} b\right] \stackrel{!}{=} 0$$

$$\frac{\partial p}{\partial x} = \frac{6\mu V_0}{b^2} = 30000\ \frac{Pa}{m}$$

b) ges: $u(y)$ und $\tau(y)$

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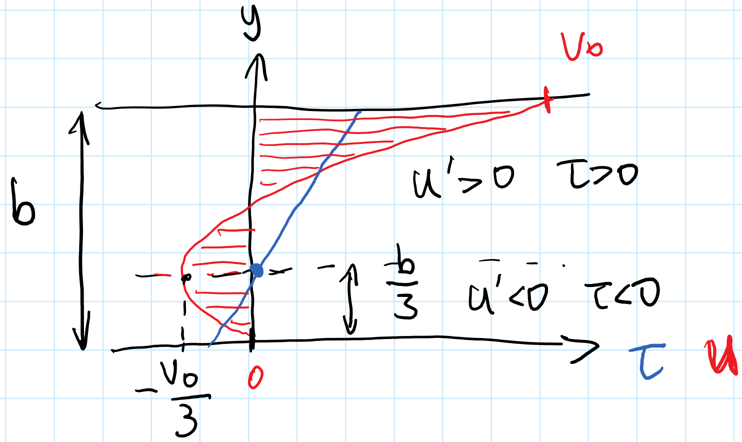
$$\tau = \mu \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x}\left(y - \frac{b}{2}\right) + \frac{\mu V_0}{b}$$

Extremewerte von $u(y)$ bei $\tau(y) = 0$:



$$\frac{\partial p}{\partial x}\left(y^* - \frac{b}{2}\right) + \frac{\mu V_0}{b} \stackrel{!}{=} 0$$

$$y^* = \frac{b}{3}$$

$$u(y^*) = -\frac{V_0}{3}$$