

EM example

Helen Yannakoudakis and Oana Cocarascu

Department of Informatics
King's College London

Expectation Maximisation coin-flipping example (see under Additional Reading material).

Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?

Coin flipping experiment






- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.

Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.
- So assuming we know which coin was used in each set of tosses:

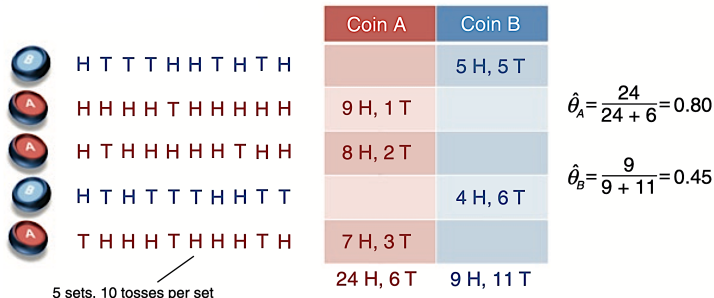
Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.
- So assuming we know which coin was used in each set of tosses:

		Coin A	Coin B
	H T T T H H T H T H		5 H, 5 T
	H H H H T H H H H H	9 H, 1 T	
	H T H H H H H T H H	8 H, 2 T	
	H T H T T T H H T T		4 H, 6 T
	T H H H T H H H T H	7 H, 3 T	
5 sets, 10 tosses per set		24 H, 6 T	9 H, 11 T

Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.
- So assuming we know which coin was used in each set of tosses:



Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.
- But what if we don't know which coin was used in each set of tosses?

Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?
- Randomly pick a coin (with equal probability) and perform 10 independent coin tosses; Repeat this 5 times, so total 50 coin tosses.
- But what if we don't know which coin was used in each set of tosses?



5 sets, 10 tosses per set

Coin flipping experiment – EM steps

- Start with random guesses for the parameters θ .
- E step: Using these, estimate a probability distribution over the coins for each set of tosses (your hidden variables).
- M step: Based on the above, now use maximum likelihood to estimate new parameters θ .
- Alternate between the two steps until convergence.

Coin flipping experiment – EM

