Lecture 4: Probabilistic models 2

Helen Yannakoudakis and Oana Cocarascu

Department of Informatics
King's College London

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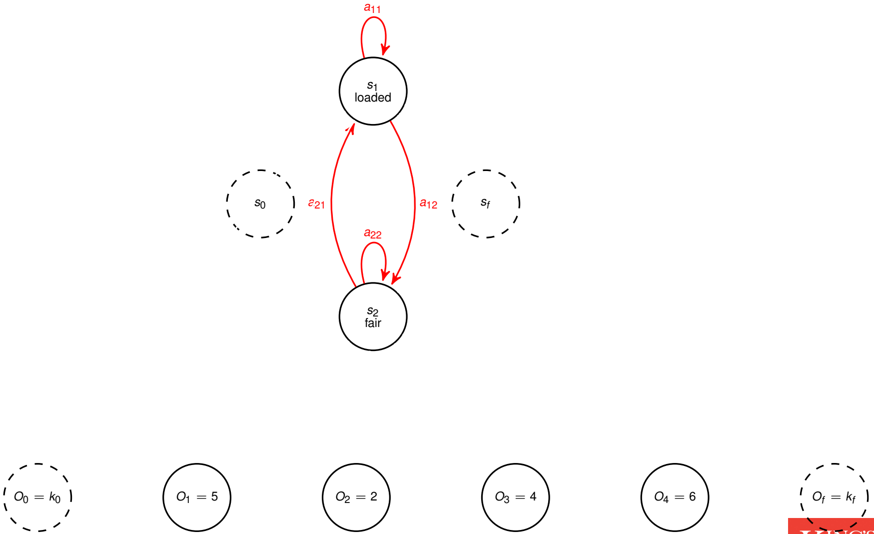
The dice HMM

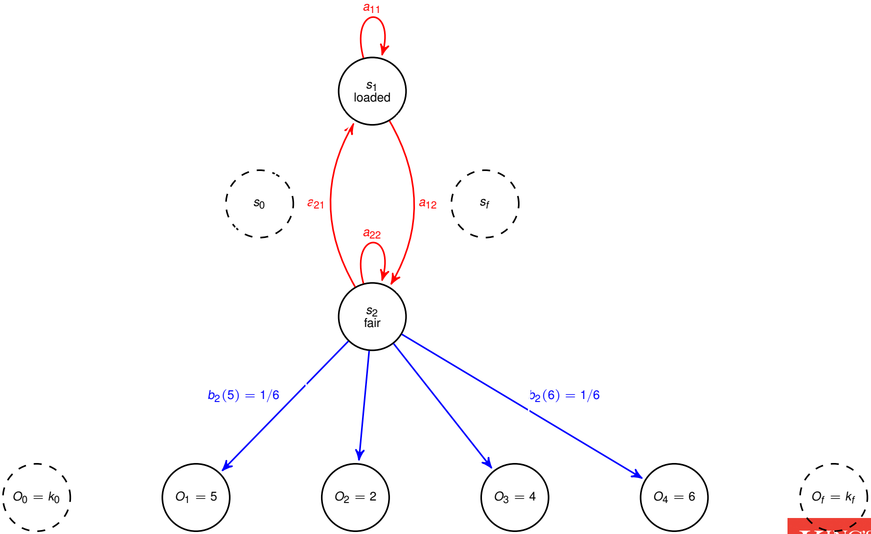
- Imagine a fraudulous croupier in a casino where customers bet on dice outcomes.
- She has two dice – a fair one and a loaded one.
- The fair one has the normal distribution of outcomes – $P(O) = \frac{1}{6}$ for each number 1 to 6.
- The loaded one has a different distribution.
- She secretly switches between the two dice.
- You don't know which dice is currently in use. You can only observe the numbers that are thrown.

The dice HMM



- There are two states (fair and loaded), and two special states (start s_0 and end s_f).
- Distribution of observations differs between the states.

The dice HMM



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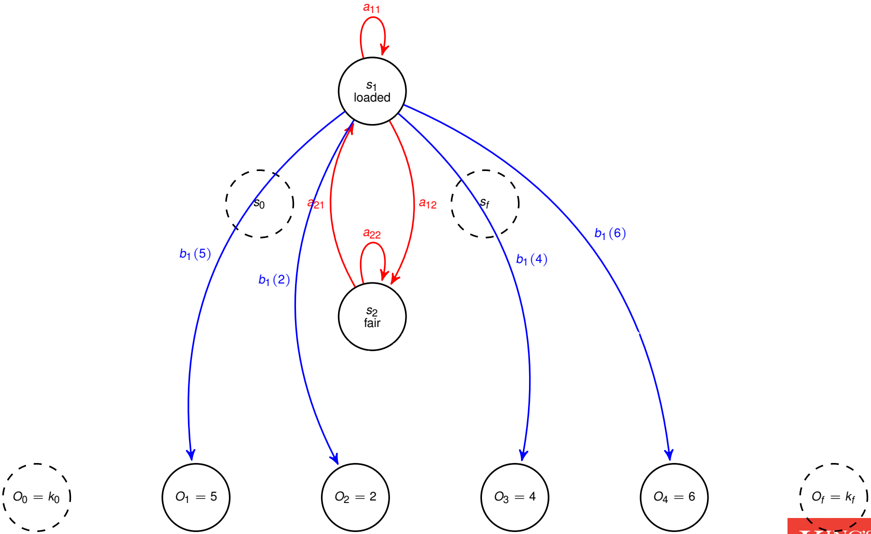

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- Distribution of observations differs between the states.

The dice HMM

- Labelled HMM learning, i.e. it has
  - Input: dual tape of state and observation (dice outcome) sequences X and O.

	F	F	F	F	L	L	L	F	F	F	F	L	L	L	L	F	F	F	L	L	
	1	3	4	5	6	6	5	1	2	3	1	4	3	5	4	1	2	6	1	2	

  - Output: HMM parameters A, B.

The dice HMM

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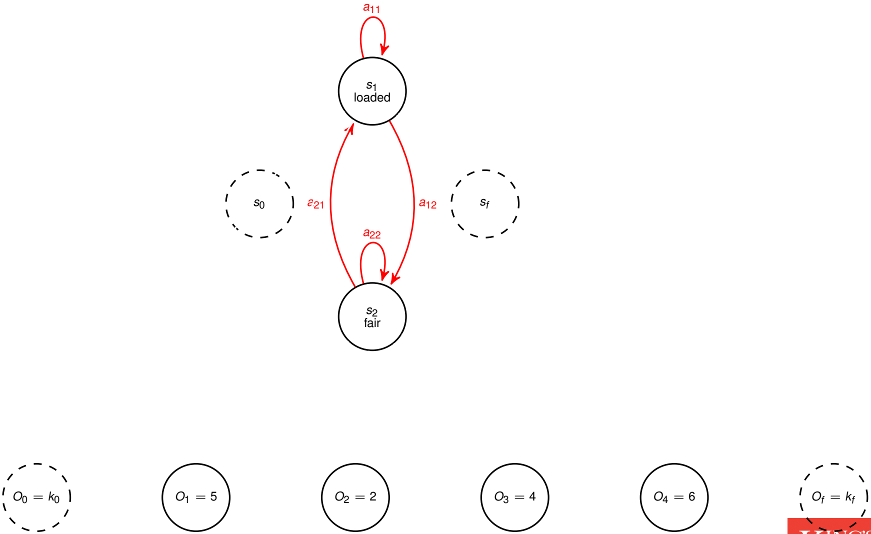
  - Input: dual tape of state and observation (dice outcome) sequences X and O.

(s_0)	F	F	F	F	L	L	L	F	F	F	F	L	L	L	L	F	F	F	L	L	(s_f)
(k_0)	1	3	4	5	6	6	5	1	2	3	1	4	3	5	4	1	2	6	1	2	(k_f)

  - Output: HMM parameters A, B.

The dice HMM



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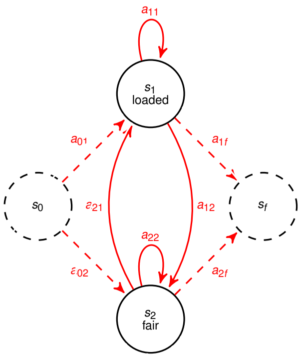
The dice HMM



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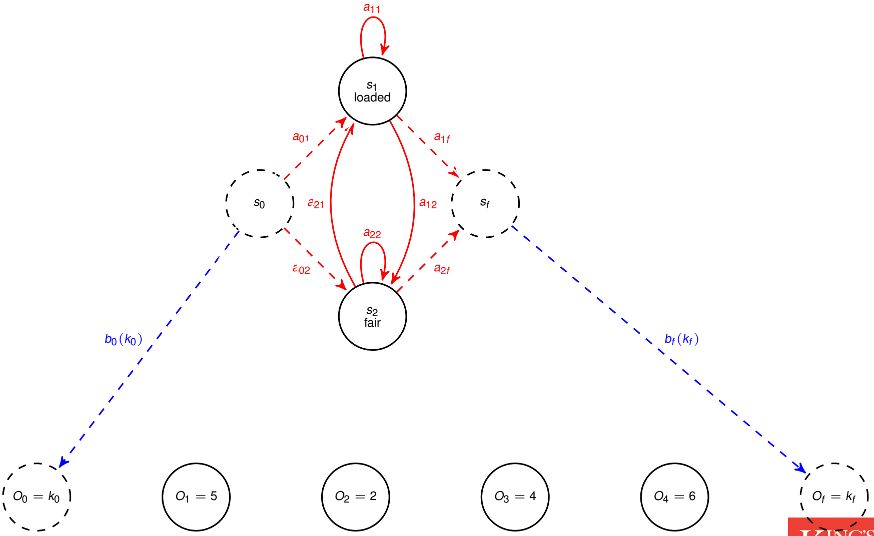

- There are two states (fair and loaded), and two special states (start s_0 and end s_f).
- Distribution of observations differs between the states.

Parameter estimation of HMM parameters A, B

(s_0)	F	F	F	F	L	L	L	F	F	F	F	L	L	L	L	F	F	F	L	L	(s_f)
(k_0)	1	3	4	5	6	6	5	1	2	3	1	4	3	5	4	1	2	6	1	2	(k_f)

- Transition matrix A consists of transition probabilities a_{ij}

$$a_{ij} = P(X_{t+1} = s_j | X_t = s_i) \sim \frac{count_{trans}(X_t = s_i, X_{t+1} = s_j)}{count_{trans}(X_t = s_i)}$$

- Emission matrix B consists of emission probabilities $b_i(k_j)$

$$b_i(k_j) = P(O_t = k_j | X_t = s_i) \sim \frac{count_{emission}(O_t = k_j, X_t = s_i)}{count_{emission}(X_t = s_i)}$$

Emission probabilities	k0	1	2	3	4	5	6	kf
F	0.00	0.36	0.18	0.18	0.09	0.09	0.09	0.00
L	0.00	0.11	0.11	0.11	0.22	0.22	0.22	0.00
s0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
sf	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Transition probabilities	to F	to L	to s0	to sf
From s0	1.00	0.00	0.00	0.00
From F	0.73	0.27	0.00	0.00
From L	0.22	0.67	0.00	0.11
From sf	0.00	0.00	0.00	0.00

Parameter estimation of HMM parameters A, B

(s_0)	F	F	F	F	L	L	L	F	F	F	F	L	L	L	L	F	F	F	L	L	(s_f)
(k_0)	1	3	4	5	6	6	5	1	2	3	1	4	3	5	4	1	2	6	1	2	(k_f)

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- Emission matrix B consists of emission probabilities $b_i(k_j)$

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Parameter estimation of HMM parameters A, B

(s_0)	F	F	F	F	L	L	L	F	F	F	F	L	L	L	L	F	F	F	L	L	(s_f)
(k_0)	1	3	4	5	6	6	5	1	2	3	1	4	3	5	4	1	2	6	1	2	(k_f)

- Transition matrix A consists of transition probabilities a_{ij}

$$a_{ij} = P(X_{t+1} = s_j | X_t = s_i) \sim \frac{count_{trans}(X_t = F, X_{t+1} = L)}{count_{trans}(X_t = F)}$$

- Emission matrix B consists of emission probabilities $b_i(k_j)$

$$b_i(k_j) = P(O_t = k_j | X_t = s_i) \sim \frac{count_{emission}(O_t = 6, X_t = L)}{count_{emission}(X_t = L)}$$

Emission probabilities	k0	1	2	3	4	5	6	kf
F	0.00	0.36	0.18	0.18	0.09	0.09	0.09	0.00
L	0.00	0.11	0.11	0.11	0.22	0.22	0.22	0.00
s0	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
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Fundamental tasks with HMMs

- Problem 1 (Labelled Learning)
  - Given a parallel observation and state sequence O and X, learn the HMM parameters A and B. → looked at this
- Problem 2 (Unlabelled Learning)
  - Given an observation sequence O (and only the set of emitting states S_e), learn the HMM parameters A and B.
- Problem 3 (Likelihood)
  - Given an HMM $\mu = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\mu)$.
- Problem 4 (Decoding)
  - Given an observation sequence O and an HMM $\mu = (A, B)$, discover the best hidden state sequence X. → now to this

Viterbi algorithm, initialisation

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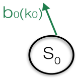
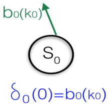
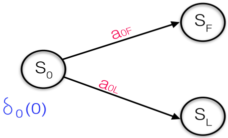


Viterbi algorithm, initialisation

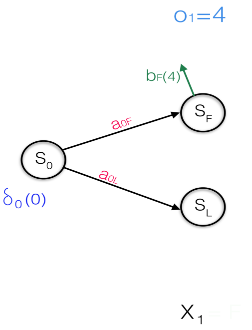


Viterbi algorithm, main step

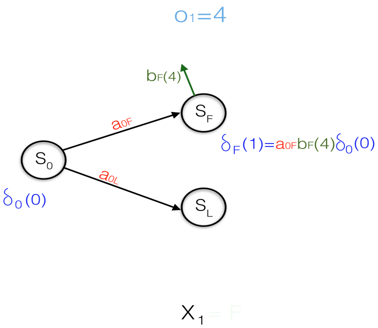


X_1

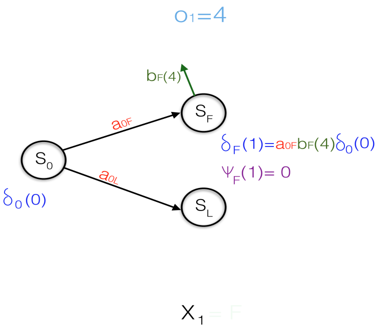
Viterbi algorithm, main step: observation is 4



Viterbi algorithm, main step: observation is 4

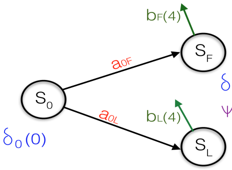


Viterbi algorithm, main step: observation is 4



Viterbi algorithm, main step: observation is 4

$o_1 = 4$

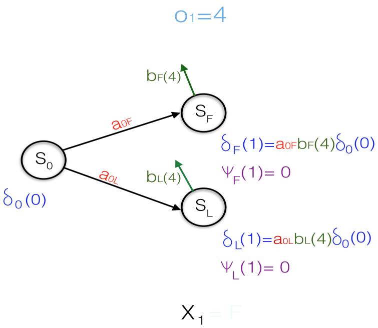


$\delta_F(1) = a_{0F} b_F(4) \delta_0(0)$

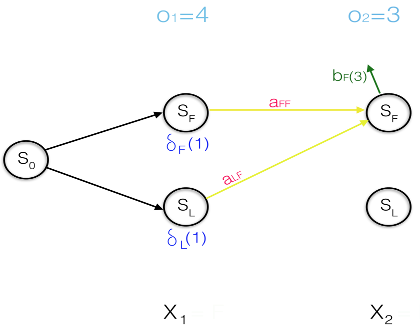
$\Psi_F(1) = 0$

X_1

Viterbi algorithm, main step: observation is 4



Viterbi algorithm, main step: observation is 3



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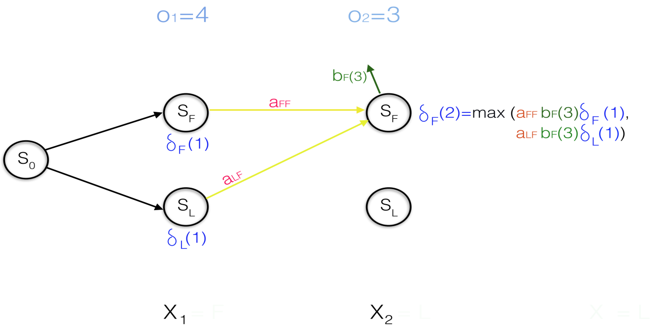

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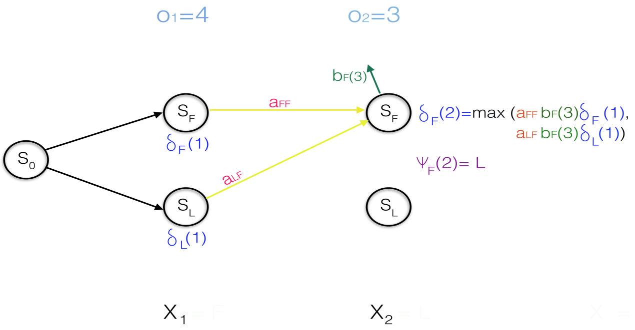

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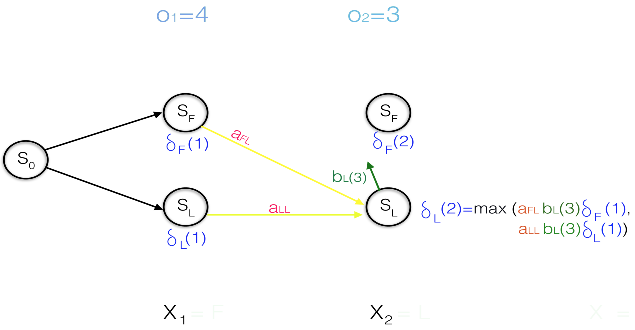

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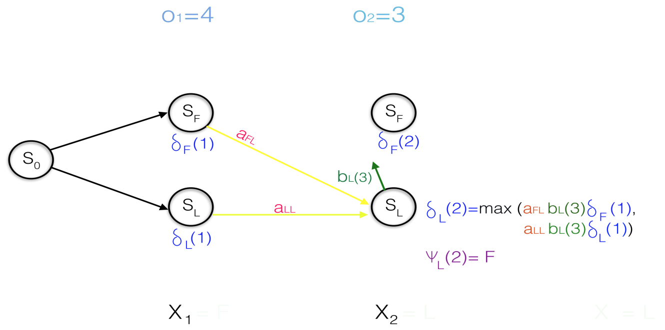
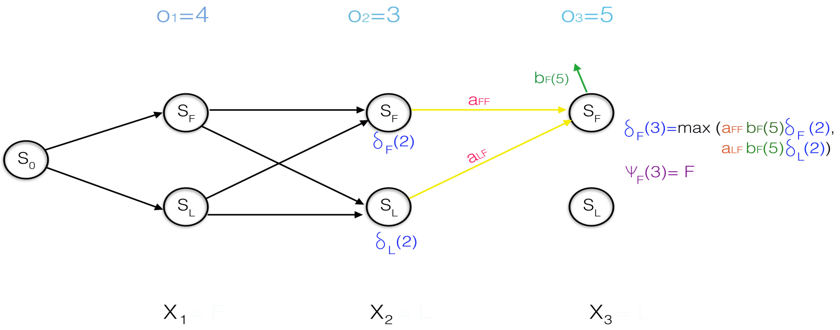
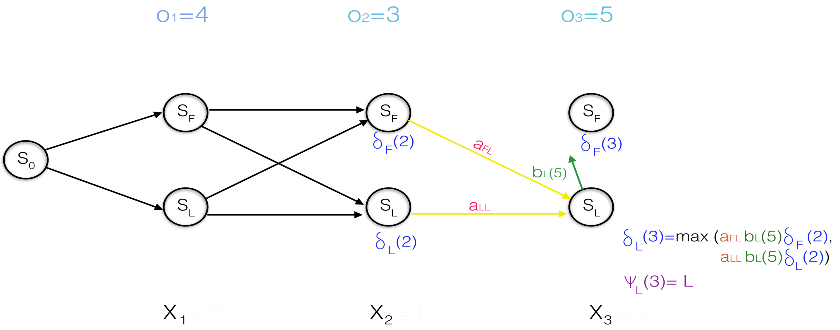


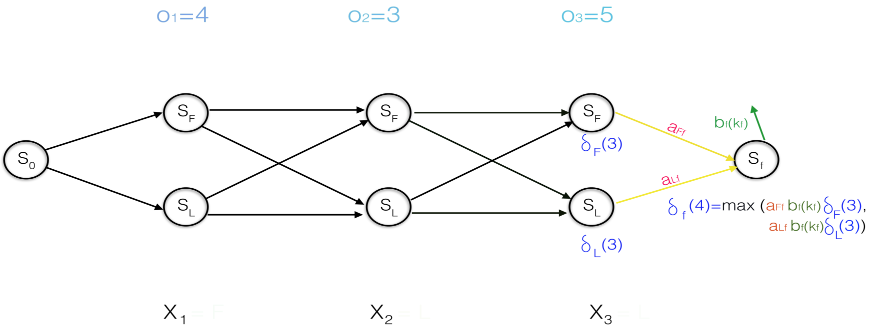
Viterbi algorithm, main step: observation is 5



Viterbi algorithm, main step: observation is 5



Viterbi algorithm, termination



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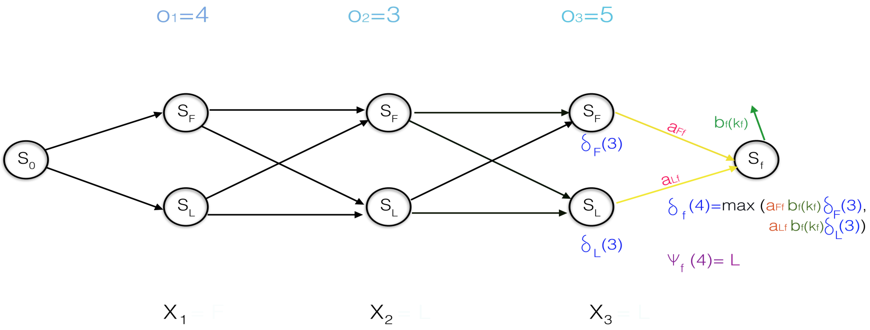
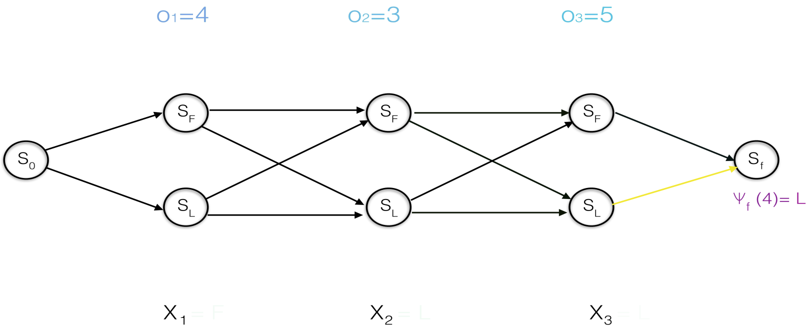


Viterbi algorithm, backtracing



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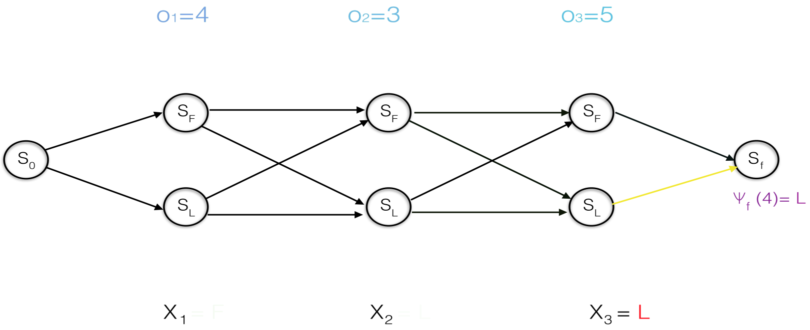

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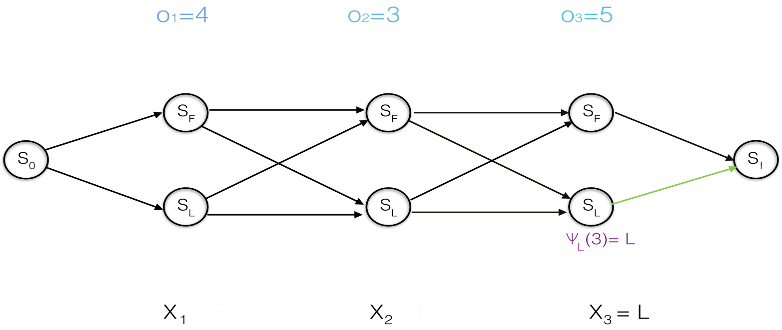

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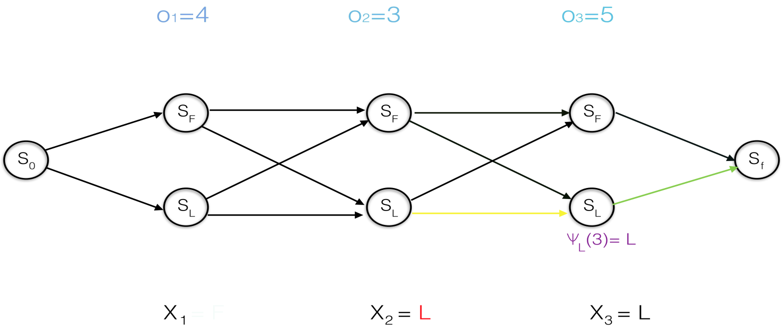

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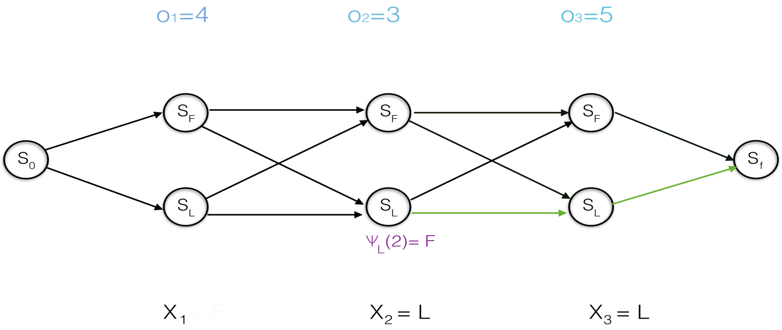

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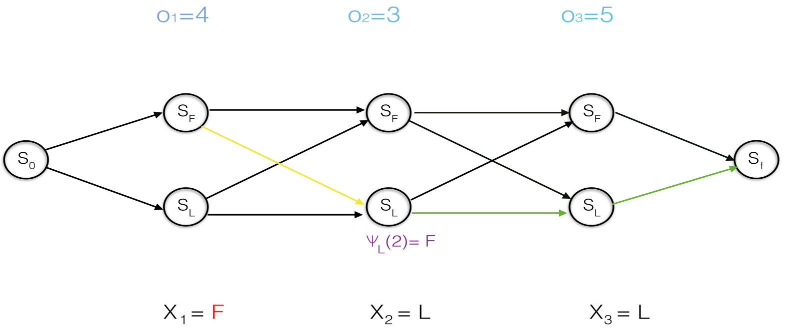

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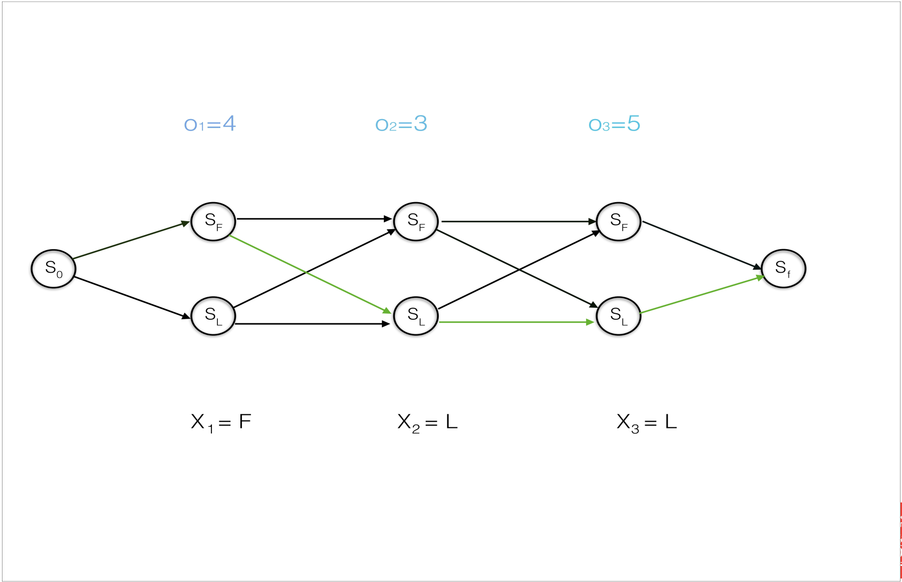

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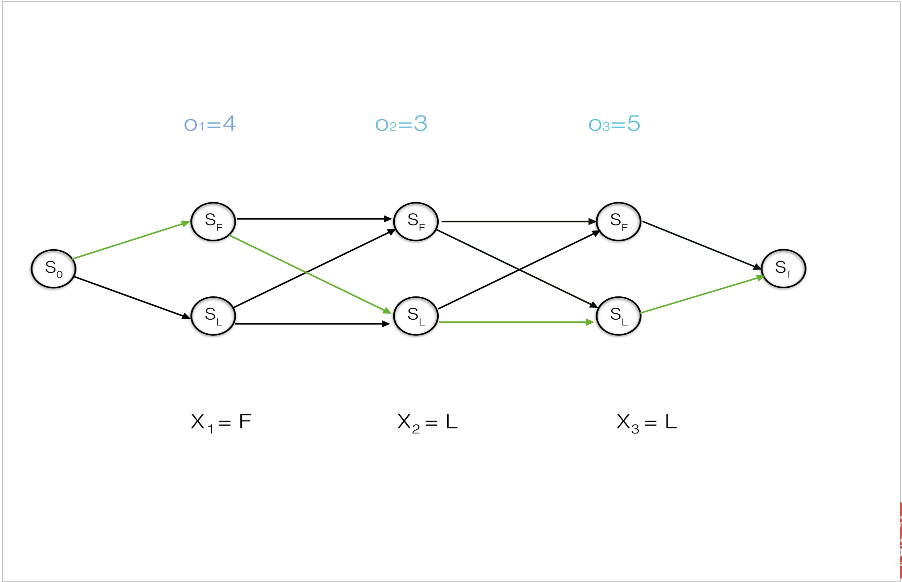

Precision and Recall

- *How would we calculate the precision and recall for a the predicted sequence? Why is it useful to know the precision for the prediction of a sequence?*

Precision and Recall

- We can measure system success using accuracy.
- But sometimes it's only one type of instances that we find interesting.
- We don't want a summary measure that averages over interesting and non-interesting instances, as accuracy does.
- In those cases, we use precision, recall and F-measure.
- These metrics are imported from the field of information retrieval, where the difference beween interesting and non-interesting examples is particularly high.
- Accuracy doesn't work well when the types of instances are unbalanced.

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Precision and Recall

Measure precision of L (P_L), recall of L (R_L) and F-measure of L (F_L).

System says:

Truth is:	F	L	Total
F	a	b	a+b
L	c	d	c+d
Total	a+c	b+d	a+b+c+d

- Precision of L: $P_L = \frac{d}{b+d} = \frac{\text{no of correctly predicted L}}{\text{number of predicted L}}$
- Recall of L: $R_L = \frac{d}{c+d} = \frac{\text{no of correctly predicted L}}{\text{true number of L}}$
- F-measure of L: $F_L = \frac{2P_L R_L}{P_L + R_L}$
- Accuracy: $A = \frac{a+d}{a+b+c+d}$

Significance testing

- *Would you mind explaining the 2-tailed test in detail? What is the meaning of ”direction” here? (Two-Tailed vs. One-Tailed Tests p73)*

Binomial Distribution $B(N, q)$

- Call the probability of a negative outcome q (here $q = 0.5$)
- Probability of observing $X = k$ negative events out of N:

$$P_q(X = k|N) = \binom{N}{k} q^k (1-q)^{N-k}$$

- At most k negative events:

$$P_q(X \leqslant k|N) = \sum_{i=0}^{k} \binom{N}{i} q^i (1-q)^{N-i}$$