EM example

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EM example

Expectation Maximisation coin-flipping example (see under Additional Reading material).

Coin flipping experiment

- We have a pair of coins A and B.
- Coin A will land on heads with probability θ_A and tails with $1 - \theta_A$.
- Coin B will land on heads with probability θ_B and tails with $1 - \theta_B$.
- How can we estimate $\theta = (\theta_A, \theta_B)$?

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		Coin A	Coin B
B	H T T T H H T H T H		5 H, 5 T
A	H H H H T H H H H H	9 H, 1 T	
A	H T H H H H H T H H	8 H, 2 T	
B	H T H T T T H H T T		4 H, 6 T
A	T H H H T H H H T H	7 H, 3 T	
		24 H, 6 T	9 H, 11 T

5 sets, 10 tosses per set

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$$\hat{\theta}_A = \frac{24}{24+6} = 0.80$$

$$\hat{\theta}_B = \frac{9}{9+11} = 0.45$$

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H T T T H H T H T H

H H H H T H H H H H

H T H H H H H T H H

H T H T T T H H T T

T H H H T H H H T H

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Coin flipping experiment – EM steps

- Start with random guesses for the parameters theta.
- E step: Using these, estimate a probability distribution over the coins for each set of tosses (your hidden variables).
- M step: Based on the above, now use maximum likelihood to estimate new parameters theta.
- Alternate between the two steps until convergence.

Coin flipping experiment – EM