

## Tutorial 04

(Version 2.0; exercises from Ann Copestake)

1. This question is a variant of the fraudulent croupier scenario discussed in the lecture. In this version, the croupier has three six-sided dice: a fair (F) dice, and two loaded ones (L1 and L2). The croupier again secretly switches between dice. You have some training data sequences, of different lengths, with the observed dice rolls paired with the type of dice.
  - (a) Assuming a first-order HMM, show what the two HMM probability matrices would look like for this scenario, explaining the notation you use.
  - (b) Draw an HMM model and illustrate the different types of probabilities involved (you do not have to label all the arcs).
  - (c) Suppose the croupier is acting according to certain rules which determine when the dice is switched. For each of the following rules, describe the effect in terms of the parameters of the HMM, and discuss whether the behaviour would be modelled by an HMM.
    - i. The croupier never switches directly from F to L2.
    - ii. The croupier knows in advance how many dice throws the sequence will contain and makes sure that the dice is always F on the last roll.
    - iii. The croupier never rolls L2 more than twice in a row.
    - iv. The croupier always switches dice after rolling a 6.
2. Viterbi is an algorithm that allows you to process the input observation in time that is linear to the observation sequence. With a first order HMM, we keep N (number of states) maximum probabilities per observation at each step.
  - (a) How many states do we need to keep for an N order HMM?
  - (b) What are the implications for the asymptotic complexity of Viterbi?

### Version list

- Version 1.0, February 4th 2020.
- Version 2.0, January 31st 2021.