Uncertainty of Power Generation Forecasts

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Reader's Guide

List of changes in this iteration:

- Extrapolate one hour backwards to circumvent the big error of the initial point.
- Implemented the new parameterized drift term to prevent the process from leaving the permissible range.

Next steps:

- Find some metric to measure how good is the model
- Compute the confidence intervals in another way.

Note :

- Green slides: possible future extensions
- Red slides: notes to be removed

Base Model

Let V_t be the deviation of the wind power generation forecasts from actual wind power generation. That is, V_t models the errors or uncertainty of a given set of wind power generation forecasts.
We propose a forecast-error model based on the following parameterized stochastic differential equation (SDE),

$$\begin{aligned} dV_t &= a(V_t;\boldsymbol{\theta})dt + b(V_t;\boldsymbol{\theta})dW_t \quad t > 0 \\ V_0 &= v_0 \end{aligned} \tag{1}$$

- $a(V_t;\boldsymbol{\theta})$: Drift function
- $b(V_t;\boldsymbol{\theta})$: Diffusion function
- $\boldsymbol{\theta}$: a vector of parameters
- dW_t: Standard Wiener random process.

Physical Restriction

It is necessary to keep the power generation forecast plus its errors inside the range $[0,1]$ where we have normalized against the power generating capacity of the power plant. To ensure that, we choose our model to have zero diffusion near the boundaries. Hence, we formulate the following diffusion term,

$$b(V_t;\boldsymbol{\theta}) = \sqrt{2\theta\alpha p_t(1-p_t)X_t(1-X_t)} = \sqrt{2\theta\alpha p_t(1-p_t)(V_t+p_t)(1-V_t-p_t)}$$

It remains to control the drift term. As a first requirement, we need the drift term to be mean reverting to the power generation forecast. Thus, we formulate the following drift,

$$a(V_t;\boldsymbol{\theta}) = -\theta(X_t-p_t) = -\theta V_t \tag{2}$$

However, this does not track the rate of change of the forecast $\dot{p}$ which may results in X_t exiting the interval $[0,1]$ when the forecast is a small δ distance away from the boundaries.

Physical Restriction - Drift Near the Boundaries

To see the issue, we switch to the point of view of the forecast prediction process X_t. In this point of view, we have shown in our results that it is necessary to include derivative tracking in order to follow the power production forecast accurately.The derivative tracking model is given by,

$$\begin{aligned} dX_t &= \hat{a}(X_t; \boldsymbol{\theta})dt + \hat{b}(X_t; \boldsymbol{\theta})dW_t \quad t > 0 \\ X_0 &= x_0 \end{aligned} \tag{3}$$

where

$$\begin{aligned} \hat{a}(X_t; \boldsymbol{\theta}) &= \dot{p}_t + a(X_t; \boldsymbol{\theta}) = \dot{p}_t - \theta\left(X_t - p_t\right) \\ \hat{b}(X_t; \boldsymbol{\theta}) &= b(X_t; \boldsymbol{\theta}) = \sqrt{2\theta\alpha p_t(1-p_t)X_t(1-X_t)} \end{aligned} \tag{4}$$

It is clear in this point of view to see that term $\dot{p}_t$ is not controlled to maintain that X_t stays inside the range $[0, 1]$.

Physical Restriction - Drift Near the Boundaries

We have that the first moment of the process X_t is given by the following ODE,

$$\dot{m}_1(t) = \frac{d\mathbb{E}[X_t]}{dt} = \dot{p}_t - \theta\mathbb{E}[X_t] + \theta p_t = \dot{p}_t - \theta m(t) + \theta p_t \tag{5}$$

Re-arranging,

$$\frac{\dot{m}_1(t)}{\theta} + m(t) = \frac{\dot{p}_t}{\theta} + p_t \tag{6}$$

Note that by setting $\hat{a}(X_t;\boldsymbol{\theta}) = 0$, we find that $\frac{\dot{p}_t}{\theta} + p_t$ is also a line of zero drift or a line of mean stationarity Thus, must have that,

$$0 \leq \frac{\dot{p}_t}{\theta} + p_t \leq 1 \tag{7}$$

to keep the process X_t inside the interval $[0,1]$. The condition can be rewritten as,

$$\frac{-|\dot{p}_t|}{p_t} \leq \theta \leq \frac{|\dot{p}_t|}{1-p_t} \tag{8}$$

Physical Restriction - Drift Near the Boundaries

To enforce the condition,

$$0 \leq \frac{\dot{p}_t}{\theta} + p_t \leq 1 \tag{9}$$

We define an adjusted drift $\theta = \theta_0 f(p_t, \dot{p}_t)$ which satisfies the condition when

$$\theta = \theta_0 f(p_t, \dot{p}_t) \leq \frac{|\dot{p}_t|}{\min(p_t, 1 - p_t)} \tag{10}$$

Thus we choose θ such that,

$$\theta = \max\left(\theta_0 \ , \ \frac{|\dot{p}_t|}{\min(p_t, 1 - p_t)}\right) \tag{11}$$

Base Model

We combine the previous results to obtain the following SDE model,

$$\begin{aligned} dV_t &= a(V_t;\boldsymbol{\theta})dt + b(V_t;\boldsymbol{\theta})dW_t \quad t > 0 \\ V_0 &= v_0 \end{aligned} \tag{12}$$

- $a(V_t;\boldsymbol{\theta})$: Drift function
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- $\boldsymbol{\theta}$: a vector of parameters
- dW_t: Standard Wiener random process.

In order to respect the physical restrictions discussed above, we prescribe the following specifications

$$\begin{aligned} a(V_t;\boldsymbol{\theta}) &= -\theta V_t \\ b(V_t;\boldsymbol{\theta}) &= \sqrt{2\theta\alpha p_t(1-p_t)(V_t+p_t)(1-V_t-p_t)} \end{aligned} \tag{13}$$

with

$$\theta = \max\left(\theta_0\ ,\ \frac{|\dot{p}_t|}{\min(p_t, 1-p_t)}\right) \tag{14}$$

Where

- p_t: Numerical wind power generation forecast.
- $\theta > 0$: Mean reversion parameter.
- $\alpha > 0$: Variability parameter.

Implementation

- We linearly interpolate the forecast and data. Note that higher-order of interpolation cause the forecast and data to escape the interval $[0, 1]$.
- Extrapolate one hour backwards and take that as the new initial point.
- We find the first integer κ such that the line of mean stationarity $\frac{\dot{p}_t}{\theta} + p_t$ is inside the range $[0, 1]$.
- Simulate each forecast using the modified $\theta = \theta_0 f(p_t)$ with its path specific constant κ.
- Obtain the empirical confidence intervals of each forecast.

Results

See attached two PDF files, the file 6hr.pdf covers the first six hours while 72hr.pdf covers 72 hours.

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