On the Uncertainty of Wind Power Generation continuous report

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Base Model

Moments approach with a Beta proxy. The SDE is given by

$$
\begin{aligned}
dV_t &= -\theta_t V_t \; dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} \; dW_t \\
V_0 &= v_0
\end{aligned} \tag{1}
$$

and the moments by,

$$
\frac{dm_1(t)}{dt} = -m_1(t)\theta_t \implies m_1(t_2) = V_{t_1} e^{-\int_{t_1}^{t_2} \theta_t \; dt}
$$

$$
\begin{aligned}
\frac{dm_2(t)}{dt} &= -m_2(t)\theta(1+\alpha) + \alpha\theta m_1(t)((1 - p_t - p_t^2)) + 2 \\
&\implies m_2(t_2) = v_{t_1}^2 e^{-(1+\alpha)\int_{t_1}^{t_2} \theta_t \; dt} \\
&\qquad + \alpha \int_{t_1}^{t_2} \left(\theta_s m_1(s)(1 - p_s - p_s^2) + 2 \right) e^{-(1+\alpha)\int_{t_1}^{t_2 - s} \theta_u \; du} \; ds
\end{aligned}
$$

solving for the moments numerically

We discretize and integrate numerically step-by-step using Euler,

$$
\begin{aligned}
m_{1,i+1} &= v_i \; e^{-\theta_i \Delta t} \\
m_{2,i+1} &= \frac{v_i^2 + 2\Delta t v_i \alpha \theta_i p_i (1-p_i)(1-2p_i) \alpha \theta_i p_i^2 (1-p_i)^2}{1 + 2\Delta t(\theta_i + \alpha\theta_i p_i(1-p_i))}
\end{aligned}
$$

Proposed Mini-Batch Stochastic Gradient Descent

We have the following beta log-likelihood,

$$\ell(\theta_0, \alpha | \{V_{i,j}\}) = \sum_{j=1}^{M} \sum_{i=1}^{N} (s_1 - 1) \log \left(\frac{V_{j,i+1} + 1}{2} \right) + (s_2 - 1) \log \left(1 - \frac{V_{j,i+1} + 1}{2} \right) - \log (2B(s_1, s_2))$$

where $B(\cdot, \cdot)$ is the Beta function and

$$s_1 = -\frac{(m_1 + 1)(m_1^2 + m_2 - m_1^2 - 1)}{2(m_2 - m_1^2)} \tag{2}$$

$$s_2 = \frac{(m_1 - 1)(m_1^2 + m_2 - m_1^2 - 1)}{2(m_2 - m_1^2)} \tag{3}$$

10 min data interval results

We optimize the likelihood using Nelder-Mead Method which is a derivative-free method for non-linear objectives.

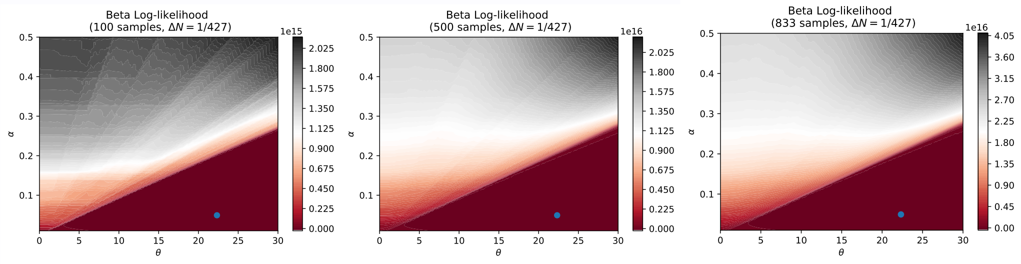

Figure 1: Likelihood evaluation with different number of sample paths. Optimal point $(\theta_0^*, \alpha^*) = (22.33, 0.049)$ shown in blue obtained using Nelder-Mead optimization method.

10 min data interval results

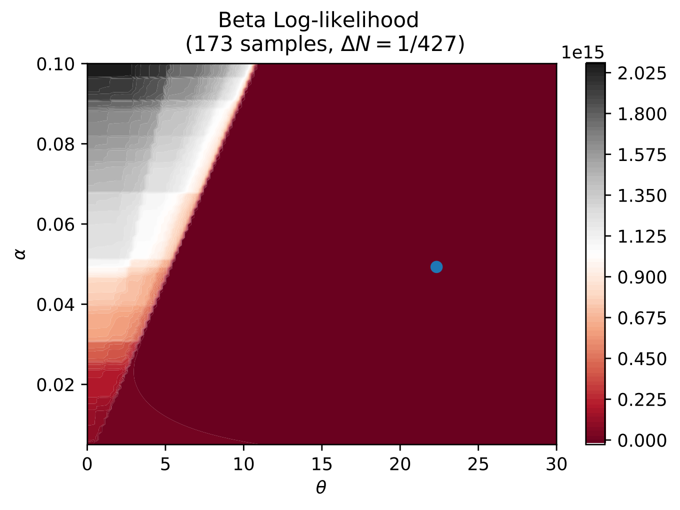

Figure 2: Likelihood evaluation with 173 paths. Optimal point $(\theta_0^*, \alpha^*) = (22.33, 0.049)$ shown in blue obtained using Nelder-Mead optimization method.

10 min data interval results

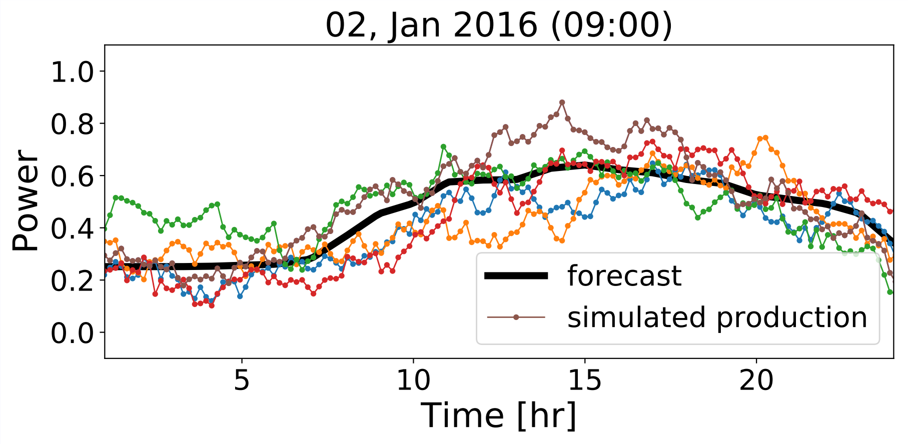

Figure 3: Example 24hr simulation paths with the optimal parameters $(\theta_0^*, \alpha^*) = (22.33, 0.049)$

10 min data interval results

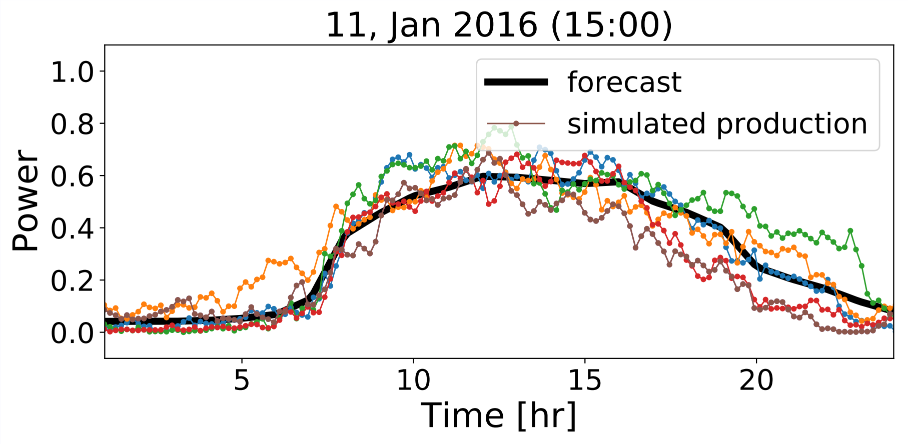

Figure 4: Example 24hr simulation paths with the optimal parameters $(\theta_0^*, \alpha^*) = (22.33, 0.049)$

10 min data interval results

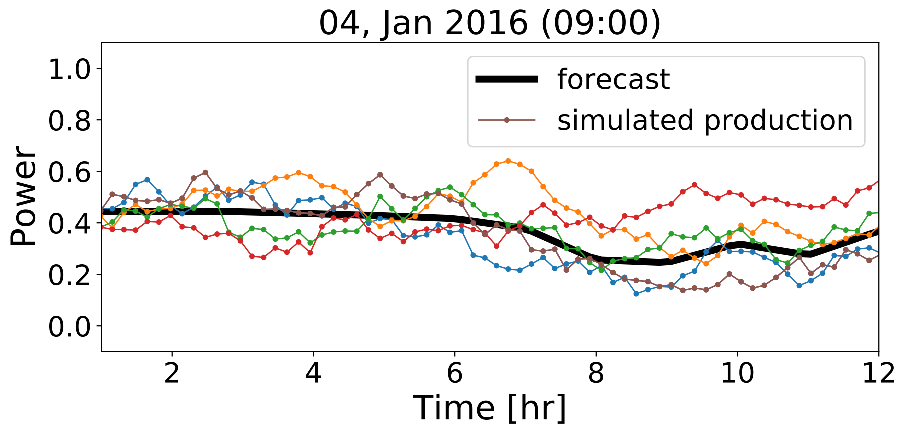

Figure 5: Example 12hr simulation paths with the optimal parameters $(\theta_0^*, \alpha^*) = (22.33, 0.049)$

10 min data interval results

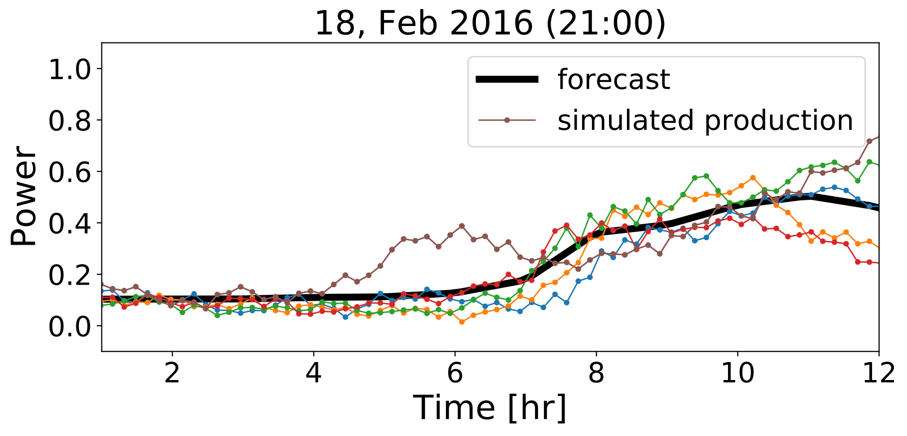

Figure 6: Example 12hr simulation paths with the optimal parameters $(\theta_0^*, \alpha^*) = (22.33, 0.049)$

Proposed Mini-Batch Stochastic Gradient Descent

Let B be the mini-batch size and η the learning rate.
We iterate from an initial guess $\Theta_0 = (\theta_0, \alpha)^T$ in the following way,

$$\Theta = \Theta - \eta \nabla \ell (\Theta; V_{1:B,1:N}) \tag{4}$$

until an accuracy threshold is reached. Note that $V_{1:B,1:N}$ is a mini-batch of size B of complete sample paths. The components of the gradient $\nabla \ell = (\frac{\partial \ell}{\partial \theta_0}, \frac{\partial \ell}{\alpha})^T$ are given by,

$$\frac{\partial \ell}{\partial \theta_0} = \frac{\partial \ell}{\partial s_1}\frac{\partial s_1}{\partial m_1}\frac{\partial m_1}{\partial \theta_t}\frac{\partial \theta_t}{\partial \theta_0} + \frac{\partial \ell}{\partial s_1}\frac{\partial s_1}{\partial m_2}\frac{\partial m_2}{\partial \theta_t}\frac{\partial \theta_t}{\partial \theta_0} + \frac{\partial \ell}{\partial s_2}\frac{\partial s_2}{\partial m_1}\frac{\partial m_1}{\partial \theta_t}\frac{\partial \theta_t}{\partial \theta_0} + \frac{\partial \ell}{\partial s_2}\frac{\partial s_2}{\partial m_2}\frac{\partial m_2}{\partial \theta_t}\frac{\partial \theta_t}{\partial \theta_0}$$

$$\frac{\partial \ell}{\partial \alpha} = \frac{\partial \ell}{\partial s_1}\frac{\partial s_1}{\partial m_2}\frac{\partial m_2}{\partial \alpha} + \frac{\partial \ell}{\partial s_2}\frac{\partial s_2}{\partial m_2}\frac{\partial m_2}{\partial \alpha}$$

Proposed Mini-Batch Stochastic Gradient Descent

We run into an issue here because the term $\frac{\partial \theta_t}{\partial \theta_0}$ is undefined as θ_t is non-differentiable and given by,

$$\theta_t = \max\left(\theta_0, \frac{|\dot{p}|}{\min(p, 1-p)}\right)$$

We can either regularize or continue to use the Nelder-Mead optimization method which is derivative-free. Is it possible to use mini-batches with Nelder-Mead?