Uncertainty Quantification in Wind Power Forecasting

Waleed Alhaddad* Ahmed Kebaier‡ Raúl Tempone*†

*CEMSE Division, KAUST, Saudi Arabia
‡Université Paris 13, Sorbonne Paris Cité, LAGA, CNRS (UMR 7539) , France
†Alexander von Humboldt Professor, RWTH Aachen University, Germany

October 4th, 2019

Introduction

Integration of renewable resources into the urban power grid is a challenge due to uncertainties in power production. We focus on wind power. Reliable wind power production forecasting is crucial to:

- **Optimization of the price of electricity** for different users such as electric utilities, Transmission system operator (TSOs), Electricity Service providers (ESPs), Independent power producers (IPPs), and energy traders.
- **Allocation of energy reserves** such as water levels in dams or oil and gas reserves.
- **Operation scheduling** of conventional power plants.
- **Maintenance planning** such as that of power plants components and transmission lines.

Current State of Affairs

Wind power forecasts can be generally categorized as follows:

- physical models
- statistical methods
- artificial intelligence methods
- other hybrid approaches

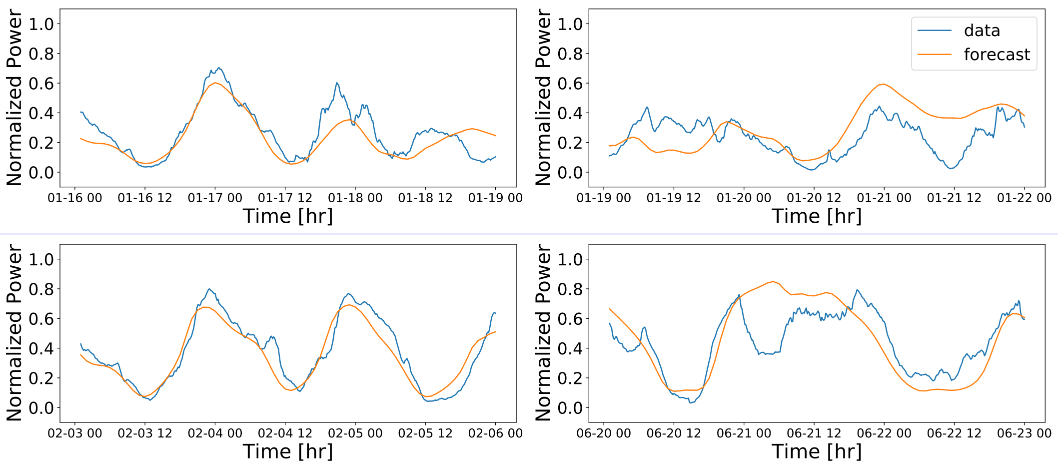
The output of such methods is usually a deterministic forecast.

Occasionally probabilistic forecasts are produced through uncertainty propagation or through forecast ensembles.

However, there is a lacking in data driven stochastic forecasts based on the real-world performance of forecasting models.

Data

This is a year long data set from Uruguay based on **1000 72-hour long paths with observations recorded every 10 min** (~ **half a million data points**) recorded in 2018.



Data Skewness

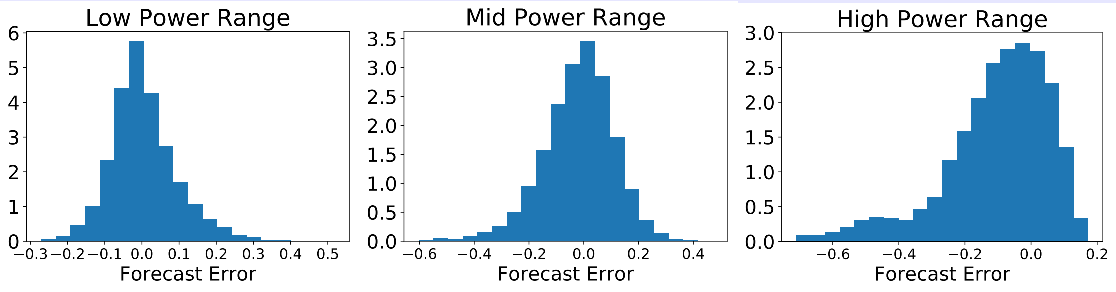


Figure 1: We see that forecast errors exhibit skewness near the boundries (i.e. low and high power production regimes.)

Model

Our goals are to produce a stochastic forecast of wind power production forecasting errors while:

- Capturing the dynamics of the forecast error process.
- Capturing the skew nature of forecast errors.
- Being forecasting-technology agnostic. Thus, compatible with past and future forecasting-technology.
- Learning from historical power production data.

Model

We propose to model wind power forecasts errors using parametric stochastic differential equations (SDEs) whose solution defines a stochastic process. This resultant stochastic process describes the time evolution dynamics of wind power forecasting errors.

$$\begin{aligned} dX_t &= a(X_t; p_t, \dot{p}_t, \boldsymbol{\theta})dt + b(X_t; p_t, \boldsymbol{\theta})dW_t \quad t > 0 \\ X_0 &= x_0 \end{aligned} \tag{1}$$

- $a(\cdot; p_t, \dot{p}_t, \boldsymbol{\theta}) : [0,1] \to \mathbb{R}$ a drift function.
- $b(\cdot; p_t, \boldsymbol{\theta}) : [0,1] \to \mathbb{R}$ a diffusion function.
- $\boldsymbol{\theta}$: a vector of parameters.
- p_t time-dependent scalar value and $\dot{p}_t$ is its time derivative at time t. (in our case p_t is a deterministic forecast).
- W_t: Standard Wiener random process in $\mathbb{R}$.

Question: How do we choose an appropriate drift and diffusion functions?

How do we choose an appropriate drift and diffusion functions?

Let $\boldsymbol{\theta} = (\theta_0, \alpha)$

1. We want the process to follow the wind forecast, thus we choose a drift term that is mean reverting and tracks the derivative of the deterministic forecast p_t, which is an input to our model.
$$a(x; p_t, \boldsymbol{\theta}) = \dot{p}_t - \theta_t (x - p_t) \tag{2}$$
where $\theta_t > 0$ is a time-dependent parameter that controls the speed of reversion.
2. We want a diffusion term that vanishes at the boundaries to prevent the process from escaping the region $[0, 1]$.
$$b(x; p_t, \boldsymbol{\theta}) = \sqrt{2\theta_t \alpha x (1 - x)} \tag{3}$$
where $\alpha > 0$ is a constant parameter that controls the path variability.
To further ensure that the process does not escape the region $[0, 1]$, the mean reversion parameter has to be selected according to the following rule,
$$\theta_t = \max\left(\theta_0 \ , \ \frac{|\dot{p}_t|}{\min(p_t, 1 - p_t)}\right), \quad \theta_0 > 0 \tag{4}$$

Model

Thus, our SDE becomes

$$\begin{aligned} dX_t &= \dot{p}_t \; dt - \theta_t (X_t - p_t) \; dt + \sqrt{2\theta_t \alpha X_t (1 - X_t)} \; dW_t \quad t > 0 \\ X_0 &= x_0 \end{aligned} \tag{5}$$

To avoid differentiation of the forecast p_t and simplify, we apply a change of variables

$$V_t = X_t - p_t$$

The model becomes,

$$\begin{aligned} dV_t &= -\theta_t V_t \; dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} \; dW_t \\ V_0 &= v_0 \end{aligned} \tag{6}$$

Note that this model is Markovian.

Model

Since V_t defined by our SDE is Markovian, the likelihood function can be written as a product of transition densities. Consider a set of M paths with N observations each, $V^{M,N} = \{V_{t_1^{M,N}}, V_{t_2^{M,N}}, \dots, V_{t_N^{M,N}}\}$ observed in intervals of Δ_N.

$$\mathcal{L}(\boldsymbol{\theta}; V) = \prod_{j=1}^{M} \prod_{i=1}^{N} \rho(V_{j,i+1} | V_{j,i}, \boldsymbol{\theta}) \rho(V_{j,0}) \tag{7}$$

The transition densities can be exactly obtained by solving the following parametric Fokker-Planck equation,

$$\begin{aligned} \frac{\partial f}{\partial t}(y, t | x, s, \theta_0, \alpha) = & -\frac{\partial}{\partial y}(a(y; p_t, \dot{p}_t, \theta_0) f(y, t | x, s, \theta_0, \alpha)) \\ & + \frac{1}{2} \frac{\partial^2}{\partial y^2}(b^2(y; p_t, \theta_0, \alpha) f(y, t | x, s, \theta_0, \alpha)) \quad t < s \end{aligned} \tag{8}$$

This is a parametric PDE which is computationally expensive to solve and optimize for every transition.

Moment Matching

We propose a proxy transition density. We match the moments of our SDE model with that of the proxy density. Using Itô formula, we arrive at the following iterative ODEs.

$$\frac{d\mathbb{E}[V_t^k]}{dt} = -k\theta_t \mathbb{E}[V_t^k] + \frac{k(k-1)}{2}\mathbb{E}[V_t^{k-2} b(V_t^k; \theta_t, \alpha)] \tag{9}$$

For $t \in [t_{n-1}, t]$, the first two moments are given by

$$\begin{aligned} \frac{dm_1(t)}{dt} &= -m_1(t)\theta_t \\ \frac{dm_2(t)}{dt} &= -2m_2(t)\theta_t(1+\alpha) + 2\alpha\theta_t m_1(t)(1-2p_t) + 2\alpha\theta_t p_t(1-p_t) \end{aligned} \tag{10}$$

with initial conditions, $m_1(t_{n-1}) = v_{n-1}$ and $m_2(t_{n-1}) = v_{n-1}^2$.
A suitable candidate is a Beta transition density as it is compactly supported and can morph into symmetric and asymmetric shapes.

Algorithm

Execute the following until an accuracy threshold is met:

1. initialize.
2. optimize the log-likelihood function.
   2.1 For every evaluation of the log-liklihood function:
      2.1.1 Sample a mini-batch of transitions randomly with their associated forecast and parameters.
      2.1.2 Solve the ODE system for every transition to obtain the moments.
      2.1.3 Match the resulting moments with the parameters of the chosen proxy distribution for every transition.
3. go to step 1 (i.e. re-initialize the optimization with the most recent result).

In the above:

- Choose your favorite deterministic optimization algorithm.
- Choose your favorite integrator to solve the ODE system.

Inference Results

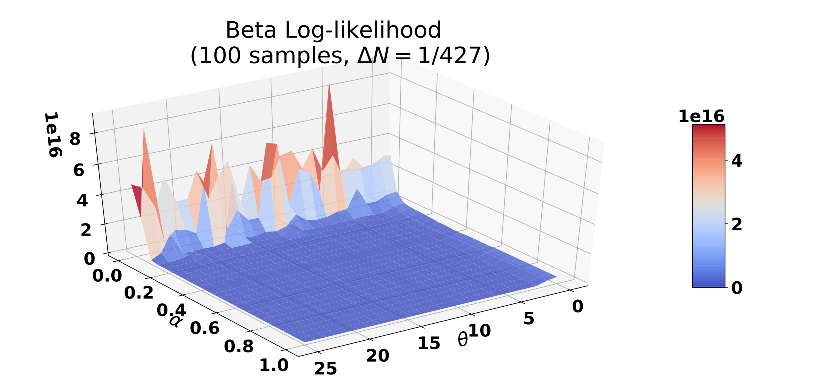


Figure 2: 3-D view of the inverted beta log-likelihood function of 100 sample paths. That is a total of $\sim 42,700$ data points

Inference Results

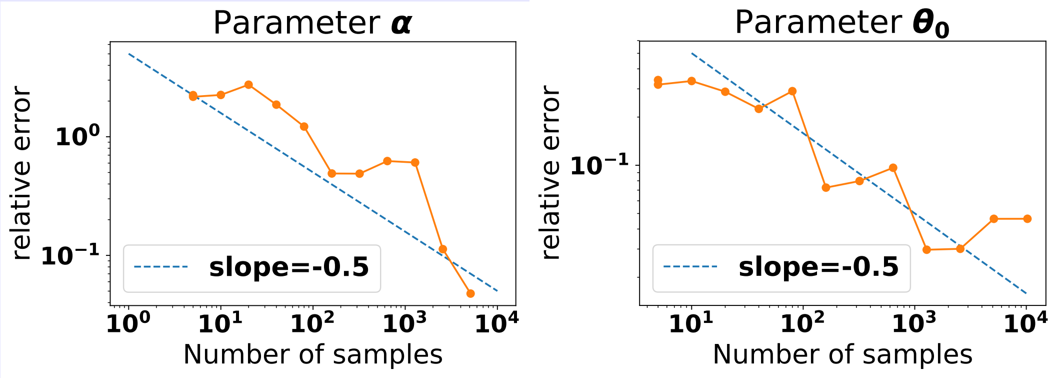


Figure 3: We have self-convergence of our algorithm at a rate that matches the convergence rate of Monte Carlo.

Future Wind Power Simulation

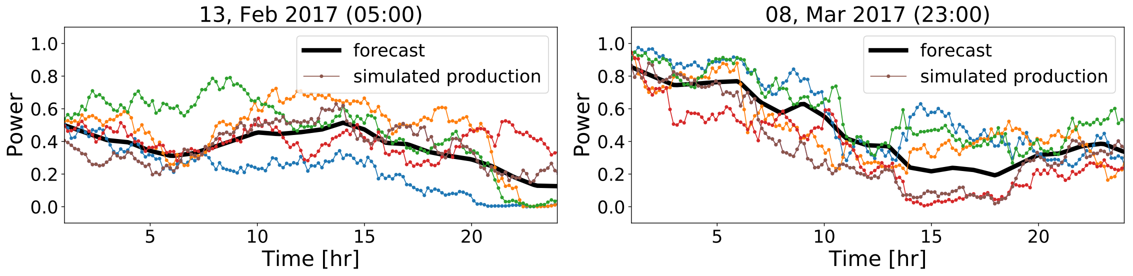


Figure 4: We simulate five possible future wind power production paths using the obtained optimal parameters $(\theta_0, \alpha) = (12, 0.3)$

Confidence Bands

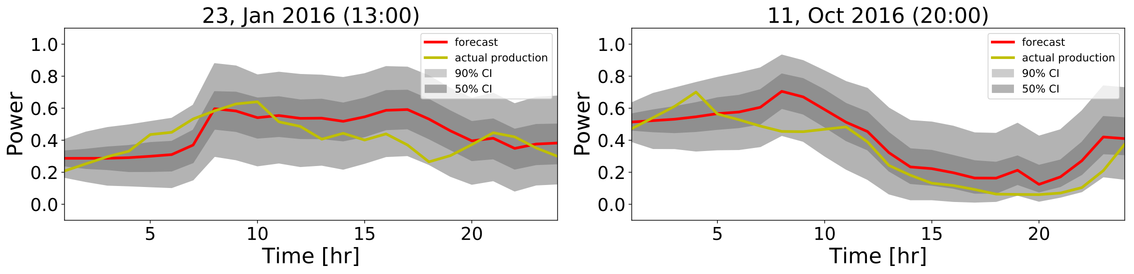


Figure 5: We obtain confidence intervals for future wind power production using the obtained optimal parameters $(\theta_0, \alpha) = (12, 0.3)$. Actual production plotted in retrospect.

State-Independent Diffusion Formulation

The model we have demonstrated previously is state-dependent diffusion formulation, that is the diffusion of the SDE depends on the state.

$$\begin{aligned} dV_t &= -\theta_t V_t \; dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} \; dW_t \\ V_0 &= v_0 \end{aligned} \tag{11}$$

Why are we interested in a state-independent diffusion formulation? Because it's more tractable and numerically stable.
We apply a **Lamperti transform** to obtain the following state-independent diffusion SDE,

$$\begin{aligned} dZ_t &= \frac{-\theta_t(1 + \sin(Z_t) - 2p_t) + \alpha \theta_t \sin(Z_t)}{\cos(Z_t)} \; dt + \sqrt{2\alpha\theta_t} dW_t \\ Z_0 &= z_0 \end{aligned} \tag{12}$$

where $Z_t = \arcsin\left(\frac{1}{2}(V_t + p_t) - 1\right)$.

Data Skewness after Lamperti Transformation

As stated before, the state-independent diffusion SDE follows a Lamperti transformed process Z_t given by $Z_t = \arcsin\left(\frac{1}{2}\left(V_t + p_t\right) - 1\right)$.

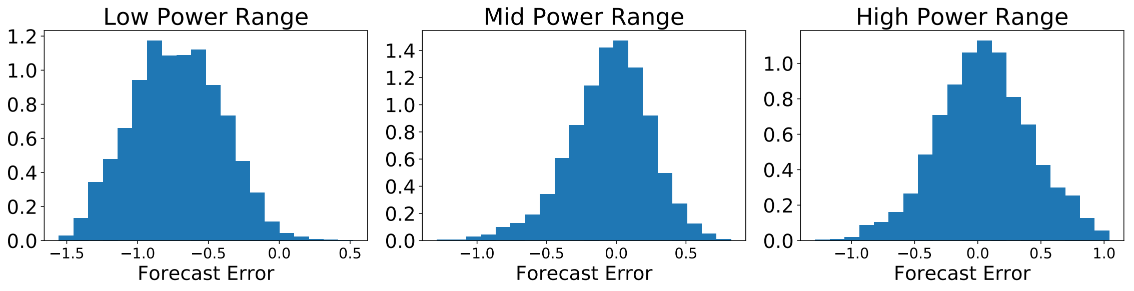


Figure 6: We observe that skewness has been greatly reduced after the Lamperti transformation. This motivates us to use a Gaussian transition density as a proxy density.

State-Independent diffusion formulation

Similarly, we try to obtain a system of ODEs to determine the centered moments of the Lamperti transformed process V_t. Due to the non-linearity in the drift, we can only approximate the centered moments by the following ODEs,

$$\begin{aligned}\frac{dm_1(t)}{dt} &= -m_1(t)\theta_t(1-\alpha) - \theta(1-2p_t) \\ \frac{dvar(t)}{dt} &= 2var(t)\theta_t(2p_t-1)\tan(m_1(t))\sec(m_1(t)) + \theta_t(\alpha-1)\sec^2(m_1(t)) + 2\theta_t\alpha\end{aligned} \tag{13}$$

with initial conditions, $m_1(t_{n-1}) = v_{n-1}$ and $var(t_{n-1}) = v_{n-1}^2 - v_{n-1}$.

These are not exact ODEs for the centered moments, however they are accurate enough for small time intervals.

Result Comparison in the Different Spaces

Formulation	parameters (θ_0, α)
Without Lamperti transform	$(12, 0.3)$
With Lamperti transform	$(12, 0.29)$

Table 1: We compare the parameters obtained in both the original and Lamperti space.

Concluding Remarks

We were able to:

- simulate future wind power production based on real data.
- obtain an analytical description of the uncertainty of wind power forecasts in the form of an SDE.
- develop a forecasting technology agnostic method.
- capture skewness of the error process and its dynamics.

References

- Alhaddad, W. , Kebaier, A. , & Tempone, R. (2019). Stochastic Wind Power Forecasting. In preparation.
- Møller, J. K., Zugno, M., & Madsen, H. (2016). Probabilistic Forecasts of Wind Power Generation by Stochastic Differential Equation Models. Journal of Forecasting, 35(3), 189-205. doi:10.1002/for.2367
- Elkantassi, S., Kalligiannaki, E., & Tempone, R. (2017). Inference And Sensitivity In Stochastic Wind Power Forecast Models. Proceedings of the 2nd International Conference on Uncertainty Quantification in Computational Sciences and Engineering (UNCECOMP 2017). doi:10.7712/120217.5377.16899
- Giebel, G., Brownsword, R., Kariniotakis, G., Denhard, M., & Draxl, C. (2011). The State-Of-The-Art in Short- Term Prediction of Wind Power: A Literature Overview, 2nd edition. ANEMOS.plus. https://doi.org/10.11581/DTU:00000017
- Chang, W.-Y. (2014) A Literature Review of Wind Forecasting Methods. Journal of Power and Energy Engineering, 2, 161-168. http://dx.doi.org/10.4236/jpee.2014.24023