



Uncertainty Quantification in Wind Power Forecasting

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Introduction

Integration of renewable resources into the urban power grid is a challenge due to uncertainties in power production. We focus on wind power. Reliable wind power production forecasting is crucial to:

- ▶ **Optimization of the price of electricity** for different users such as electric utilities, Transmission system operator (TSOs), Electricity Service providers (ESPs), Independent power producers (IPPs), and energy traders.
- ▶ **Allocation of energy reserves** such as water levels in dams or oil and gas reserves.
- ▶ **Operation scheduling** of conventional power plants.
- ▶ **Maintenance planning** such as that of power plants components and transmission lines.

Current State of Affairs

Wind power forecasts can be generally categorized as follows:

- ▶ physical models
- ▶ statistical methods
- ▶ artificial intelligence methods
- ▶ other hybrid approaches

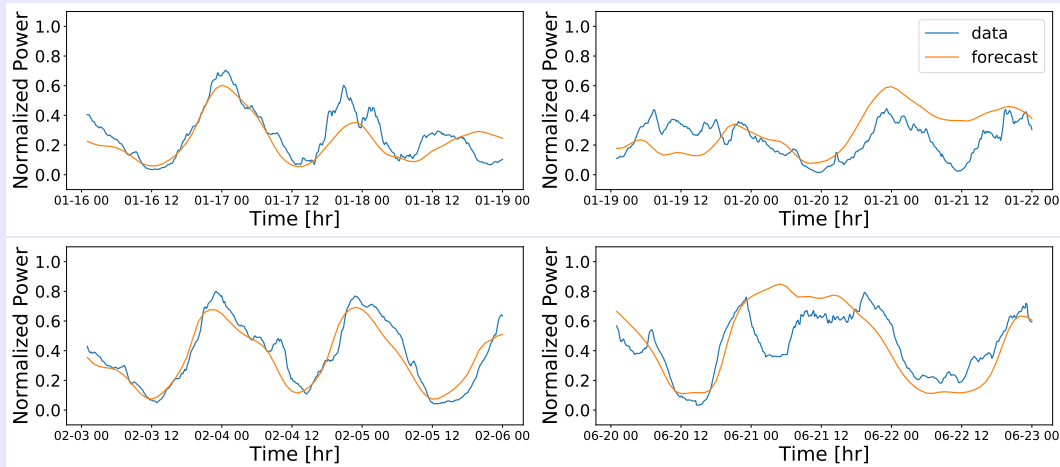
The output of such methods is usually a **deterministic forecast**.

Occasionally probabilistic forecasts are produced through uncertainty propagation or through forecast ensembles.

However, there is a lacking in **data driven stochastic forecasts** based on the real-world performance of forecasting models.

Data

This is a year long data set from Uruguay based on **1000 72-hour long paths with observations recorded every 10 min (~ half a million data points)** recorded in 2018.



Data Skewness

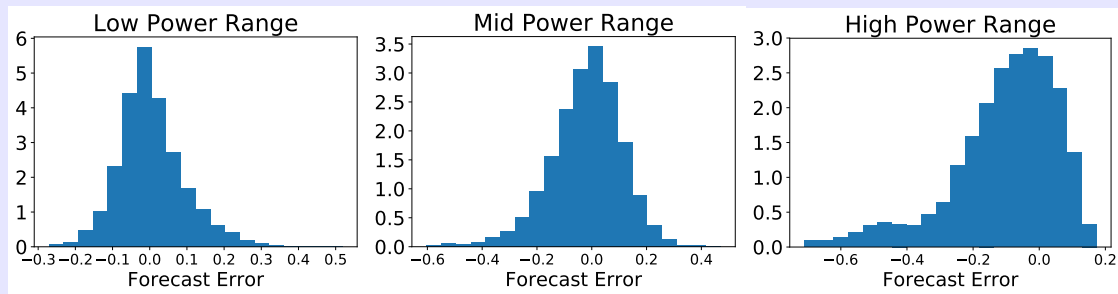


Figure 1: We see that forecast errors exhibit **skewness** near the boundaries (i.e. low and high power production regimes.)

Model

Our goals are to produce a stochastic forecast of wind power production forecasting errors while:

- ▶ Capturing the dynamics of the forecast error process.
- ▶ Capturing the skew nature of forecast errors.
- ▶ Being forecasting-technology agnostic. Thus, compatible with past and future forecasting-technology.
- ▶ Learning from historical power production data.

Model

We propose to model wind power forecasts errors using **parametric stochastic differential equations (SDEs)** whose solution defines a stochastic process. This resultant stochastic process describes the time evolution dynamics of wind power forecasting errors.

$$\begin{aligned}dX_t &= a(X_t; p_t, \dot{p}_t, \boldsymbol{\theta})dt + b(X_t; p_t, \boldsymbol{\theta})dW_t \quad t > 0 \\ X_0 &= x_0\end{aligned}\tag{1}$$

- ▶ $a(\cdot; p_t, \dot{p}_t, \boldsymbol{\theta}) : [0, 1] \rightarrow \mathbb{R}$ a drift function.
- ▶ $b(\cdot; p_t, \boldsymbol{\theta}) : [0, 1] \rightarrow \mathbb{R}$ a diffusion function.
- ▶ $\boldsymbol{\theta}$: a vector of parameters.
- ▶ p_t time-dependent scalar value and \dot{p}_t is its time derivative at time t . (in our case p_t is a deterministic forecast).
- ▶ W_t : Standard Wiener random process in \mathbb{R} .

Question: How do we choose an appropriate drift and diffusion functions?

How do we choose an appropriate drift and diffusion functions?

Let $\theta = (\theta_0, \alpha)$

1. We want the process to follow the wind forecast, thus we choose a drift term that is mean reverting and tracks the derivative of the deterministic forecast p_t , which is an input to our model.

$$a(x; p_t, \theta) = \dot{p}_t - \theta_t(x - p_t) \quad (2)$$

where $\theta_t > 0$ is a time-dependent parameter that controls the speed of reversion.

2. We want a diffusion term that vanishes at the boundaries to prevent the process from escaping the region $[0, 1]$.

$$b(x; p_t, \theta) = \sqrt{2\theta_t \alpha x(1-x)} \quad (3)$$

where $\alpha > 0$ is a constant parameter that controls the path variability.

To further ensure that the process does not escape the region $[0, 1]$, the mean reversion parameter has to be selected according to the following rule,

$$\theta_t = \max\left(\theta_0, \frac{|\dot{p}_t|}{\min(p_t, 1-p_t)}\right), \quad \theta_0 > 0 \quad (4)$$

Model

Thus, our SDE becomes

$$\begin{aligned}dX_t &= \dot{p}_t dt - \theta_t(X_t - p_t) dt + \sqrt{2\theta_t \alpha X_t(1 - X_t)} dW_t \quad t > 0 \\X_0 &= x_0\end{aligned}\tag{5}$$

To avoid differentiation of the forecast p_t and simplify, we apply a change of variables

$$V_t = X_t - p_t$$

The model becomes,

$$\begin{aligned}dV_t &= -\theta_t V_t dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} dW_t \\V_0 &= v_0\end{aligned}\tag{6}$$

Note that this model is **Markovian**.

Model

Since V_t defined by our SDE is Markovian, the **likelihood function can be written as a product of transition densities**. Consider a set of M paths with N observations each, $V^{M,N} = \{V_{t_1^{M,N}}, V_{t_2^{M,N}}, \dots, V_{t_N^{M,N}}\}$ observed in intervals of Δ_N .

$$\mathcal{L}(\boldsymbol{\theta}; V) = \prod_{j=1}^M \prod_{i=1}^N \rho(V_{j,i+1} | V_{j,i}, \boldsymbol{\theta}) \rho(V_{j,0}) \quad (7)$$

The transition densities can be exactly obtained by solving the following parametric Fokker-Planck equation,

$$\begin{aligned} \frac{\partial f}{\partial t}(y, t | x, s, \boldsymbol{\theta}_0, \alpha) = & -\frac{\partial}{\partial y} (a(y; p_t, \dot{p}_t, \boldsymbol{\theta}_0) f(y, t | x, s, \boldsymbol{\theta}_0, \alpha)) \\ & + \frac{1}{2} \frac{\partial^2}{\partial y^2} (b^2(y; p_t, \boldsymbol{\theta}_0, \alpha) f(y, t | x, s, \boldsymbol{\theta}_0, \alpha)) \quad t < s \end{aligned} \quad (8)$$

This is a parametric PDE which is **computationally expensive to solve** and optimize for every transition.

Moment Matching

We propose a **proxy transition density**. We match the moments of our SDE model with that of the proxy density. Using Itô formula, we arrive at the following iterative ODEs.

$$\frac{d\mathbb{E}[V_t^k]}{dt} = -k\theta_t\mathbb{E}[V_t^k] + \frac{k(k-1)}{2}\mathbb{E}[V_t^{k-2}b(V_t^k;\theta_t,\alpha)] \quad (9)$$

For $t \in [t_{n-1}, t]$, the first two moments are given by

$$\begin{aligned} \frac{dm_1(t)}{dt} &= -m_1(t)\theta_t \\ \frac{dm_2(t)}{dt} &= -2m_2(t)\theta_t(1+\alpha) + 2\alpha\theta_t m_1(t)(1-2p_t) + 2\alpha\theta_t p_t(1-p_t) \end{aligned} \quad (10)$$

with initial conditions, $m_1(t_{n-1}) = v_{n-1}$ and $m_2(t_{n-1}) = v_{n-1}^2$.

A suitable candidate is a **Beta transition** density as it is compactly supported and can morph into symmetric and asymmetric shapes.

Algorithm

Execute the following until an accuracy threshold is met:

1. **initialize**.
2. **optimize** the log-likelihood function.
 - 2.1 For every evaluation of the log-likelihood function:
 - 2.1.1 **Sample** a mini-batch of transitions randomly with their associated forecast and parameters.
 - 2.1.2 **Solve** the ODE system for every transition to obtain the moments.
 - 2.1.3 **Match** the resulting moments with the parameters of the chosen proxy distribution for every transition.
3. go to step 1 (i.e. **re-initialize** the optimization with the most recent result).

In the above:

- ▶ Choose your favorite deterministic optimization algorithm.
- ▶ Choose your favorite integrator to solve the ODE system.

Inference Results

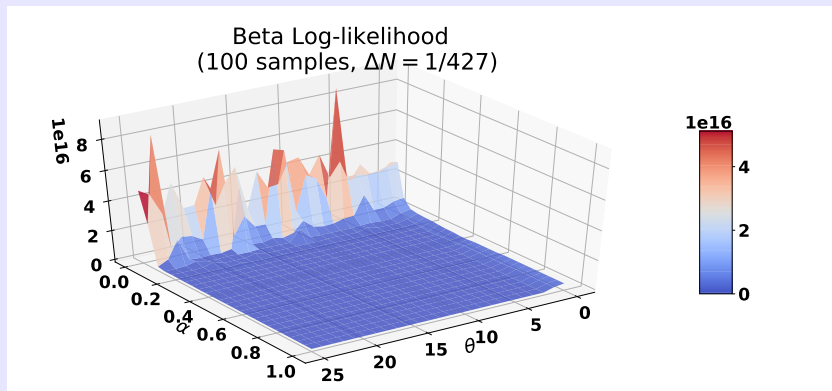


Figure 2: 3-D view of the **inverted beta log-likelihood** function of 100 sample paths. That is a total of $\sim 42,700$ data points

Inference Results

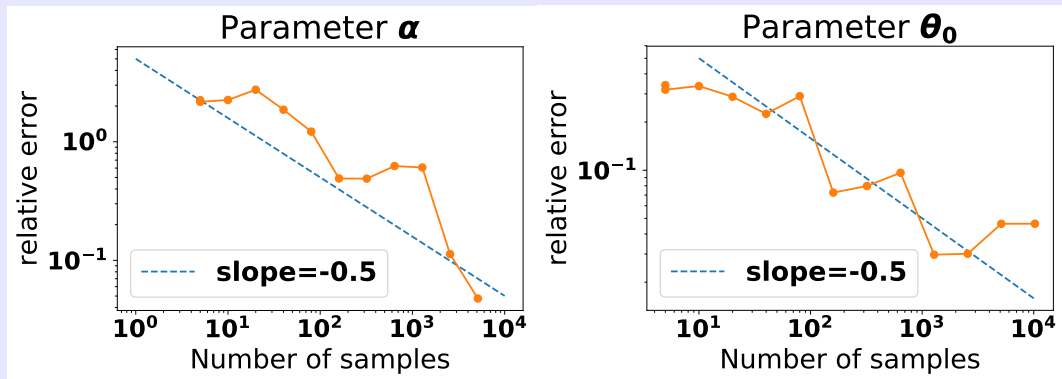


Figure 3: We have **self-convergence** of our algorithm at a rate that matches the convergence rate of Monte Carlo.

Future Wind Power Simulation

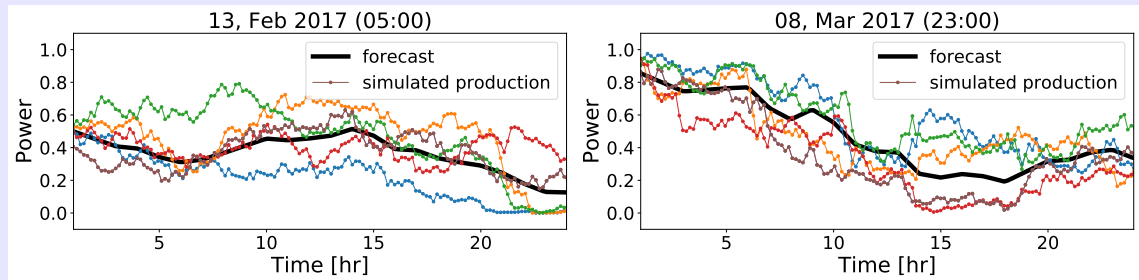


Figure 4: We simulate five possible future wind power production paths using the obtained optimal parameters $(\theta_0, \alpha) = (12, 0.3)$

Confidence Bands

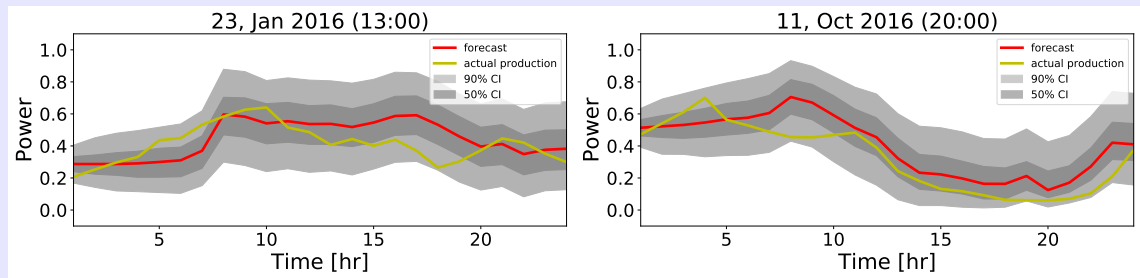


Figure 5: We obtain confidence intervals for future wind power production using the obtained optimal parameters $(\theta_0, \alpha) = (12, 0.3)$. Actual production plotted in retrospect.

State-Independent Diffusion Formulation

The model we have demonstrated previously is **state-dependent diffusion** formulation, that is the diffusion of the SDE depends on the state.

$$\begin{aligned}dV_t &= -\theta_t V_t dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} dW_t \\ V_0 &= v_0\end{aligned}\tag{11}$$

Why are we interested in a state-independent diffusion formulation? Because it's more tractable and numerically stable.

We apply a **Lamperti transform** to obtain the following **state-independent diffusion** SDE,

$$\begin{aligned}dZ_t &= \frac{-\theta_t(1 + \sin(Z_t) - 2p_t) + \alpha\theta_t \sin(Z_t)}{\cos(Z_t)} dt + \sqrt{2\alpha\theta_t} dW_t \\ Z_0 &= z_0\end{aligned}\tag{12}$$

where $Z_t = \arcsin\left(\frac{1}{2}(V_t + p_t) - 1\right)$.

Data Skewness after Lamperti Transformation

As stated before, the state-independent diffusion SDE follows a Lamperti transformed process Z_t given by $Z_t = \arcsin\left(\frac{1}{2}(V_t + p_t) - 1\right)$.

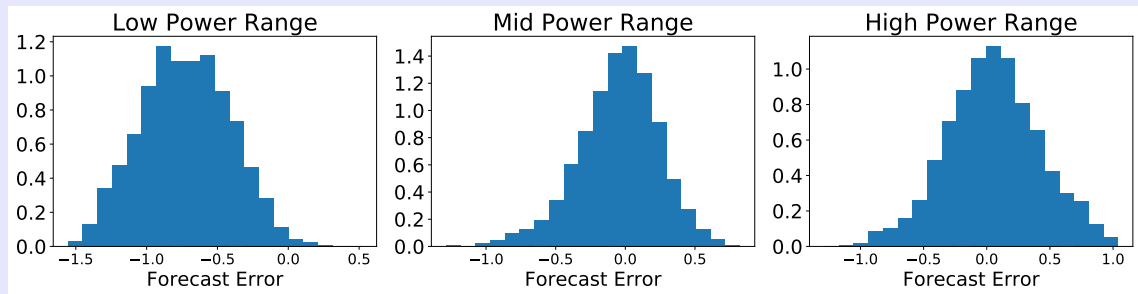


Figure 6: We observe that skewness has been greatly reduced after the Lamperti transformation. This motivates us to use a **Gaussian transition** density as a proxy density.

State-Independent diffusion formulation

Similarly, we try to obtain a system of ODEs to determine the centered moments of the Lamperti transformed process V_t . Due to the non-linearity in the drift, we can only approximate the centered moments by the following ODEs,

$$\begin{aligned}\frac{dm_1(t)}{dt} &= -m_1(t)\theta_t(1 - \alpha) - \theta(1 - 2p_t) \\ \frac{dvar(t)}{dt} &= 2var(t)\theta_t(2p_t - 1)\tan(m_1(t))\sec(m_1(t)) + \theta_t(\alpha - 1)\sec^2(m_1(t)) + 2\theta_t\alpha\end{aligned}\tag{13}$$

with initial conditions, $m_1(t_{n-1}) = v_{n-1}$ and $var(t_{n-1}) = v_{n-1}^2 - v_{n-1}$.

These are not exact ODEs for the centered moments, however they are accurate enough for small time intervals.

Result Comparison in the Different Spaces

Formulation	parameters (θ_0, α)
Without Lamperti transform	(12, 0.3)
With Lamperti transform	(12, 0.29)

Table 1: We compare the parameters obtained in both the original and Lamperti space.

Concluding Remarks

We were able to:

- ▶ simulate future wind power production based on real data.
- ▶ obtain an analytical description of the uncertainty of wind power forecasts in the form of an SDE.
- ▶ develop a forecasting technology agnostic method.
- ▶ capture skewness of the error process and its dynamics.

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