Usa corrector de Ingles!

Initial Guess

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March 13, 2020

Quadratic mean minimization

We consider the transition $\Delta V_i = V_{i+1} - V_i$ with $\Delta t = t_{i+1} - t_i$. $(V_{i+1}|V_i)$ is a random variable which mean can be approximated by the solution of the system

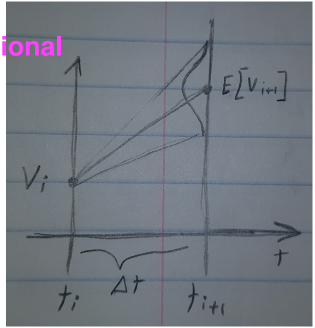
conditional

dE[V] dt = -theta_t E[V] dt

Text

$$\begin{cases} \mathrm{d}V & = -\theta_t V \,\mathrm{d}t \\ V(t_i) & = V_i, \end{cases}$$

evaluated in t_{i+1} (i.e., $V(t_{i+1})$). Then, the random variable $(V_{i+1} - V(t_{i+1}))$ has approximately zero mean.



E[V](t_i+1)

If we assume that $\theta_t = \theta_0$ for all $t \in [t_i, t_{i+1}]$, then $V(t_{i+1}) = V_i e^{-\theta_0 \Delta t}$. If we have a total of n transitions, then:

We write the regression problem for the conditional mean with L^2 loss function, namely

$$\theta_0^* \approx \arg\min_{\theta_0} \left[\sum_{i=1}^{n} \left(V_{i+1} - V_i e^{-\theta_0 \Delta t} \right)^2 \right]. \tag{1}$$

Quadratic mean minimization

We take the first order approximation w.r.t. θ_0

$$e^{-\theta_0 \Delta t} = 1 - \theta_0 \Delta t + \mathscr{O}(\theta_0^2),$$

and introduce it in equation (1). We get

$$\theta_0^* \approx \arg\min_{\theta_0} \underbrace{\left[\sum_{i=1}^{n} \left(V_{i+1} - V_i (1 - \theta_0 \Delta t) \right)^2 \right]}_{:= f(\theta_0)}. \tag{2}$$

As $f(\theta_0)$ is convex in θ_0, solving (2) (finding θ_0^*) is equivalent to solving $\frac{\partial f}{\partial \theta_0}(\theta_0^*) = 0$.

Quadratic mean minimization

$$
\begin{aligned}
\frac{\partial f}{\partial \theta_0} &= \sum_{i=1}^{n} 2(-V_i)(-\Delta t)(V_{i+1} - V_i(1-\theta_0 \Delta t)) \\
&= \sum_{i=1}^{n} 2V_i \Delta t (V_{i+1} - V_i(1-\theta_0 \Delta t)) \\
&= \sum_{i=1}^{n} 2V_{i+1} V_i \Delta t - 2V_i^2 \Delta t + 2V_i^2 \Delta t^2 \theta_0.
\end{aligned}
$$

Then, θ_0^* satisfies

$$
\theta_0^* \approx \frac{\sum_{i=1}^{n} V_i \Delta t (V_i - V_{i+1})}{\sum_{i=1}^{n} (V_i \Delta t)^2}. \tag{3}
$$

Notice that θ_0^* has dimension $\mathbf{time}^{-1}$.

Quadratic variation

One more time, we approximate the SDE by its E-M scheme. In particular, we approximate the Itô quadratic variation with the discrete one:

- Itô process quadratic variation: $[V]_t = \int_0^t \sigma_s^2 \, \mathrm{d}s$.
- Discrete process quadratic variation: $[V]_t = \sum_{0<s\leq t} (\Delta V_s)^2$.

Then, considering Δt the time between measurements, we approximate:

$$\theta_0^* \alpha^* \approx \frac{\sum_{i=1}^n (\Delta V_i)^2}{2\Delta t \sum_{i=1}^n (V_i + p_i)(1 - V_i - p_i)}. \tag{4}$$

Initial parameters

We first approximate $\theta_0^* \alpha^*$ from (4). Then, using all the data, assuming $\theta_t = \theta_0$ and equation (3), we can approximate θ_0^*. However, as we have $\theta_0^* \alpha^*$, we can check for which days the assumption $\theta_t = \theta_0$ was wrong, and remove that days from all the data.

We repeat the process until the assumption is correct.

Boundedness of θ_t

We have that

$$\theta_t = \max\left(\theta_0, \frac{\alpha\theta_0 + |2\dot{p}_t|}{2\min(1-p_t, p_t)}\right).$$

Then, if $p_t \in [\delta, 1-\delta]$ for some $0 < \delta < \frac{1}{2}$ and all $t \in [0, T]$, then $\theta_t < M(\delta) < \infty$ for all $t \in [0, T]$. So, using the corrected forecast

$$p_t^\delta = \begin{cases} \delta & \text{if} \quad p_t < \delta \\ p_t & \text{if} \quad \delta \leq p_t < 1-\delta \\ 1-\delta & \text{if} \quad p_t > 1-\delta, \end{cases}$$

we guaranty the boundedness of θ_t for all $t \in [0, T]$.