On the Uncertainty of Wind Power Generation

Waleed Alhaddad Raul Tempone Ahmad kebaier

September 22, 2019

Introduction

Integration of renewable resources into the urban power grid is a challenge due to uncertainties in power production. We focus on wind power. Reliable wind power production forecasting is crucial to:

- **Optimization of the price of electricity** for different users such as electric utilities, Transmission system operator (TSOs), Electricity Service providers (ESPs), Independent power producers (IPPs), and energy traders.
- **Allocation of energy reserves** such as water levels in dams or oil and gas reserves.
- **Operation scheduling** of conventional power plants.
- **Maintenance planning** such as that of power plants components and transmission lines.

Status quo

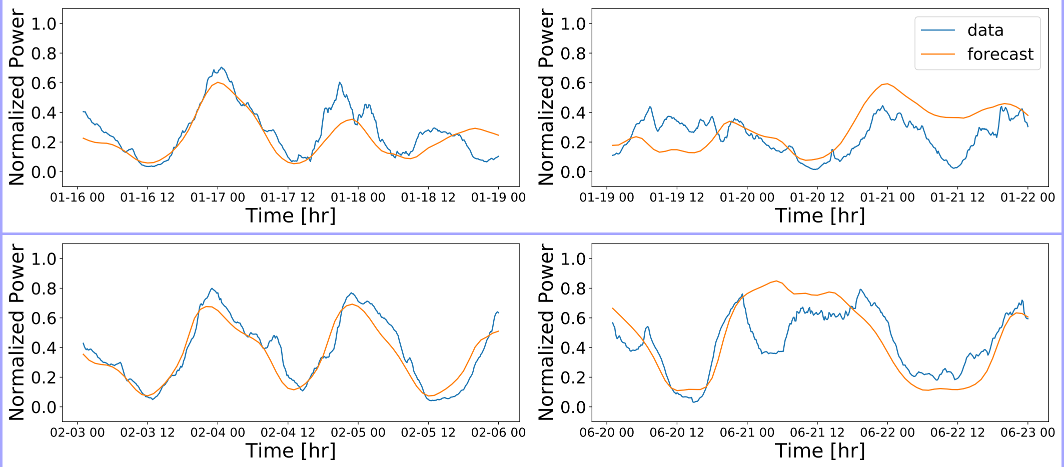
Wind power forecasts can be generally categorized as follows:

- physical models
- statistical methods
- artificial intelligence methods
- spatial correlation methods
- persistence models
- other hybrid approaches

The output of such methods is usually a **deterministic forecast**. Occasionally probabilistic forecasts are produced through uncertainty propagation in the data, parameters or through forecast ensembles. However, little has been done in terms of producing **data driven probabilistic forecasts** based on real-world performance of forecasting models.

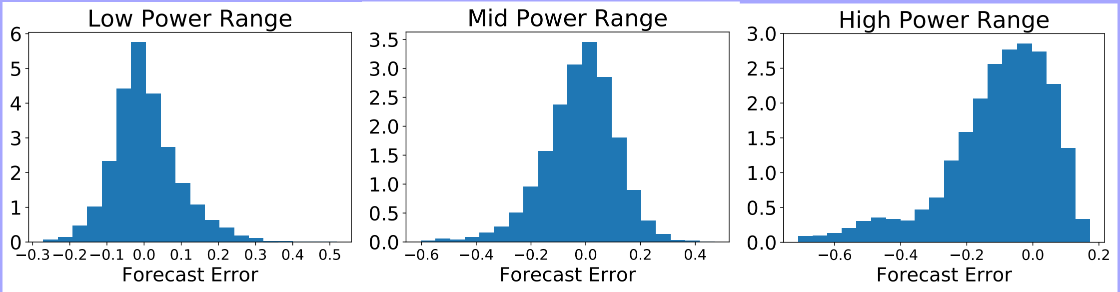
Data

This is data from Uruguay based on **10 minute observation interval and 1000 paths**.



Have a better data set ? send it our way.

Data Skewness



Model

We wish to:

- Generate a probabilistic forecast from a deterministic forecast.
- Capture the dynamics and correlation structure.
- Capture the skew nature of the process.
- Be forecasting technology agnostic. Thus, compatible with future forecasting technology.
- Learn from historical power production data.

Model

We propose to model wind power **forecasts errors using parametric stochastic differential equations (SDEs)** whose solution defines a stochastic process. This resultant stochastic process describes the time evolution dynamics of wind power forecast errors.

$$\begin{aligned} dX_t &= a(X_t;\boldsymbol{\theta})dt + b(X_t;\boldsymbol{\theta})dW_t \quad t > 0 \\ X_0 &= X_0 \end{aligned} \tag{1}$$

- $a(\cdot;\boldsymbol{\theta}) : [0,1] \to \mathbb{R}$ a drift function.
- $b(\cdot;\boldsymbol{\theta}) : [0,1] \to \mathbb{R}$ a diffusion function.
- $\boldsymbol{\theta}$: a vector of parameters.
- W_t: Standard Wiener random process in $\mathbb{R}$.

Question: How do we choose an appropriate drift and diffusion functions ?

Model

Answer:

1. We want the process to follow the wind forecast, thus we choose a drift term that is mean reverting and tracks the derivative of the deterministic forecast p which is an input to our model.
$$a(X_t; \boldsymbol{\theta}) = \dot{p}\ dt - \theta_t (X_t - p_t) \tag{2}$$
where θ_t is a time-dependent paramter that controls the speed of reversion.
2. We want a diffusion term that vanishes at the boundries to prevent the process from escaping the region $[0, 1]$.
$$b(\cdot; \boldsymbol{\theta}) = \sqrt{2\theta_t \alpha x(1-x)} \tag{3}$$
where α is a constant prameter that controls the path variability.
To further ensure that the process does not escape the region $[0, 1]$, the mean reversion parameter has to be selected according to the following rule,
$$\theta_t = \max\left(\theta_0\ ,\ \frac{|\dot{p}_t|}{\min(p_t, 1-p_t)}\right) \tag{4}$$

Model

Thus, our SDE becomes

$$
\begin{aligned}
dX_t &= \dot{p}_t \; dt - \theta_t(X_t - p_t) \; dt + \sqrt{2\theta_t \alpha x(1-x)} \; dW_t \quad t > 0 \\
X_0 &= x_0 \\
\theta_t &= \max\left(\theta_0 \; , \; \frac{|\dot{p}_t|}{\min(p_t, 1-p_t)}\right)
\end{aligned} \tag{5}
$$

To avoid differentiation of the forecast p_t and simplify, we apply a change of variables

$$V_t = X_t - p_t$$

The model becomes,

$$
\begin{aligned}
dV_t &= -\theta_t V_t \; dt + \sqrt{2\theta_t \alpha (V_t + p_t)(1 - V_t - p_t)} \; dW_t \\
V_0 &= v_0 \\
\theta_t &= \max\left(\theta_0 \; , \; \frac{|\dot{p}_t|}{\min(p_t, 1-p_t)}\right)
\end{aligned} \tag{6}
$$

Note that this model is Markovian.

Model

Since V_t defined by the SDE in (6) is Markovian, the likelihood function can be written as product of transition densities.

$$\mathcal{L}(\boldsymbol{\theta}; V) = \prod_{j=1}^{M} \prod_{i=1}^{N} \rho(V_{j,i+1} | V_{j,i}, \boldsymbol{\theta}) \rho(V_{j,0}) \tag{7}$$

The transition densities can be exactly obtained by solving the following parametric Fokker-Planck equation,

$$\begin{aligned} \frac{\partial f}{\partial t}(y, t | x, s, \theta_t, \alpha) = & -\frac{\partial}{\partial y}(a(y; \dot{p}_t, p_t, \theta_t) f(y, t | x, s, \theta_t, \alpha)) \\ & + \frac{1}{2} \frac{\partial^2}{\partial y^2}(b(y; \theta_t, \alpha) f(y, t | x, s, \theta_t, \alpha)) \quad t < s \end{aligned} \tag{8}$$

This is a parametric PDE which computationally prohibitive to solve for every transition.

Moment Matching

Instead of solving for exact transition densities by the Fokker-Planck, we propose a **proxy transition denisty**. A suitable candidate is a Beta transition density as it can morph into symmetric and asymetric shapes.

$$\frac{d\mathbb{E}[V_t^k]}{dt} = -k\theta_t \mathbb{E}[V_t^k] + \frac{k(k-1)}{2}\mathbb{E}[V_t^{k-2} b(y;\theta_t,\alpha)] \tag{9}$$