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② 17 a) $V(s) = Z(s) \cdot I(s)$ "Lei das malhas"

$$\begin{bmatrix} \Sigma V_1(s) \\ \Sigma V_2(s) \end{bmatrix} = \begin{bmatrix} \Sigma Z_1(s) & - \Sigma Z_{21}(s) \\ - \Sigma Z_{12}(s) & \Sigma Z_2(s) \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + 1s & -1s \\ -1s & 1s+2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+s & -s \\ -s & s+2 \end{vmatrix} = s^2 + 3s + 2 - s^2 = 3s+2 \rightarrow I_2 = \begin{vmatrix} 1+s & V_1(s) \\ -s & 0 \end{vmatrix}$$

$$= \frac{s}{3s+2} \cdot V_1(s)$$ "Regra de Cramer"

$$V_0(s) = Z_0(s) \cdot I_2(s) \rightarrow V_0(s) = 2 \cdot \frac{s}{3s+2} V_1(s) \rightarrow \frac{V_0(s)}{V_1(s)} = \frac{2s}{3s+2}$$

b) $$V(s) = Z(s) \cdot I(s) \rightarrow \begin{bmatrix} V_1(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1 & -1 \\ -1 & 1+1s+\frac{1}{2s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -1 & 1+s+\frac{1}{2s} \end{vmatrix} = 2 + 2s + \frac{1}{s} - 1 = 2s + \frac{1}{s} + 1 \rightarrow I_2 = \begin{vmatrix} 2 & V_1(s) \\ -1 & 0 \end{vmatrix} = \frac{V_1(s)}{2s + \frac{1}{s} + 1}$$

$$V_0(s) = Z_0(s) \cdot I_s(s) \rightarrow V_0(s) = \frac{1}{2s} \cdot \frac{V_1(s)}{2s + \frac{1}{s} + 1} \rightarrow \frac{V_0(s)}{V_1(s)} = \frac{1}{4s^2 + 2s + 2}$$

18) a) $$V(s) = Z(s) \cdot I(s) \rightarrow \begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+2 & -2 \\ -2 & 2+2+2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2s+2 & -2 \\ -2 & 4+2s \end{vmatrix} = 4s^2 + 12 + 8 - 4 = 4s^2 + 12s + 4 \rightarrow I_2 = \frac{\begin{vmatrix} 2s+2 & V(s) \\ -2 & 0 \end{vmatrix}}{4s^2 + 12s + 4} = \frac{V(s)}{2s^2 + 6s + 2}$$

$$V_1(s) = Z_1(s) + I_2 = 2s \cdot \frac{V(s)}{2s^2 + 6s + 2} \rightarrow \frac{V_1(s)}{V(s)} = \frac{s}{s^2 + 3s + 1}$$

b) $$V(s) = Z(s) \cdot I(s) \rightarrow \begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2 + 2/s + 2 + \frac{2}{s} & -2 - \frac{2}{s} \\ -2 - \frac{2}{s} & 2 + \frac{2}{s} + 2 + 2s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4 + 4/s & -2 - 2/s \\ -2 - \frac{2}{s} & 4 + 4/s \end{vmatrix} = 16 + 32/s + 16/s^2 - 4 - 4/s - 4/s^2 = 12 + \frac{28}{s} + \frac{12}{s^2}$$

$$\rightarrow I_2 = \begin{vmatrix} 4 + 4/s & V(s) \\ -2 - \frac{2}{s} & 0 \end{vmatrix}$$

$$V_1(s) = \frac{2s\cdot(s^2+s)\,V(s)}{6s^2+16s+6} \rightarrow \frac{V_1(s)}{V(s)} = \frac{s^3+s^2}{3s^2+7s+3}$$

19) a) $$\begin{bmatrix} V_1(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -1 \\ -1 & 1+3s+\frac{2}{s} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\Delta} = I_2 = \frac{s\cdot V_i(s)}{6s^3+5s^2+4s+2}$$

b) $$\begin{bmatrix} V_1(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} s+s/(s^2+1) & -\frac{s}{(s^2+1)} & -s \\ -s/(s^2+1) & s/(s^2+1)+1+1/s & -1 \\ -s & -1 & s+s+1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_2 = \frac{(s^3+2s^2+2s)\,V_i}{(s^4+2s^3+3s+2)}$$

$$V_o(s) = \frac{1}{s}\cdot I_2 \rightarrow \frac{V_o(s)}{V_i(s)} = \frac{s^2+2s+2}{s^4+2s^3+3s^2+3s+2}$$

20) $$V_1 = \frac{(3s^2+2)\,V_i}{(2s+1)(3s^2+2)+2s^2}$$

$$I_2 = \frac{V_o}{V_i} = \frac{3s^2}{6s^3+5s^2+4s+2}$$

b) $$V_3 = \frac{(s^2+2s+2)\,V_i}{(s^4+2s+3s^2+3s+2)} \rightarrow I_3 = \frac{V_3}{1/s} = sV_3$$

$$V_o = \frac{1}{s}\cdot I_3 = V_3 = \frac{V_o}{V_i} = \frac{s^2+2s+2}{(s^4+2s^3+3s^2+3s+2)}$$

26) $[F] = [Z_m] \cdot [x]$

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \cdot s^2 + 5s + 2 & -5s \\ -5s & 10s^2 + 7s \end{bmatrix} \begin{bmatrix} X_2(s) \\ X_1(s) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5s+2 & -5s \\ -5s & 10s^2+7s \end{vmatrix} = 50s^3 + 55s^2 + 14s - 25s^2$$
$$= 50s^3 + 30s^2 + 14s$$

$$X2 = \frac{\begin{vmatrix} F(s) & -5s \\ 0 & 10s^2 + 7s \end{vmatrix}}{\Delta} = \frac{(10s^2 + 7s)}{50s^3 + 30s^2 + 14s} \rightarrow \frac{X_2(s)}{F(s)} = \frac{10s^2 + 7}{50s^3 + 30s^2 + 14s}$$

27) $$\begin{bmatrix} 0 \\ F(s) \end{bmatrix} = \begin{bmatrix} s^2 + 6s + 9 & -3s - 5 \\ -3s - 5 & 2s + 5s + 5 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} s^2 + 6s + 9 & -3s - 5 \\ -3s - 5 & 2s^2 + 5s + 5 \end{vmatrix} = 2s^4 + 17s^3 + 44s^2 + 45s + 20$$

$$\frac{X_1(s)}{F(s)} = \frac{-(3s+5)}{2s^4 + 17s^3 + 44s^2 + 45s + 20}$$

28) $$\begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 4s^2 & -2s & 0 \\ -2s & 4s^2 + 4s + 6 & -6 \\ 0 & -6 & 4s^2 + 2s + 6 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix}$$

$$\frac{X_3(s)}{F(s)} = \frac{3}{8s^4 + 12s^3 + 26s^2 + 18s + 36}$$

32) $[T(s)] = [Z_m(s)]\,[\theta(s)]$

$$\begin{bmatrix} T(s) \\ 0 \end{bmatrix} = \begin{bmatrix} s^2+2s+1 & -s-1 \\ -s-1 & 2s+1 \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+2s}$$

33) $(J_e s^2 + D_e s)\cdot\theta_3 = T_1 \cdot \frac{N_2 N_4}{N_1 N_3} \rightarrow \frac{\theta_3(s)}{T_1(s)} = \frac{N_2 N_4}{N_1 N_3} \cdot \frac{1}{(J_e s^2 + D_e s)}$

$/ \; J_e = J_1 \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 + (J_2 + J_3)\left(\frac{N_4}{N_3}\right)^2 + \binom{J_4}{J_5}$

$$D_e = D_1 \left(\frac{N_2 N_4}{N_1 N_3}\right)^2 + (D_2 + D_3)\left(\frac{N_4}{N_3}\right)^2 + (D_4 + D_5)$$

34) $-\; (J_e s^2 + D_e s + K_e)\,\theta_2 = T_1 \; N_2/N_1$

$\Big\{ \left[J_1\left(\frac{N_2}{N_1}\right)^2 + J_2 + J_3\left(\frac{N_3}{N_4}\right)^2\right]s^2 + \Big[D_1\left(\frac{N_2}{N_1}\right)^2 + D_2 + D_3$

$(N^3/N_4)^2\Big]s$

$+ \left[K\,(N_3/N_4)^2\right]\theta_2$

$$\frac{\theta_2(s)}{T_1(s)} = \frac{3}{(20s^2+13s+4)}$$