



**Universidade Federal de Uberlândia**

**Faculdade de Engenharia Elétrica**

**Sistemas de Controle**

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**Uberlândia**

$$2. a) C(s) = \frac{5}{s(s+5)} = \frac{1}{s} - \frac{1}{s+5} \Rightarrow c(t) = 1 - e^{-5t}$$

$$T = \frac{1}{5}, T_r = \frac{2,2}{5} = 0,44 \quad T_s = \frac{4}{5} = 0,8$$

$$b) C(s) = \frac{20}{s(s+20)} = \frac{1}{s} - \frac{1}{s+20} \Rightarrow c(t) = 1 - e^{-20t}$$

$$T = \frac{1}{20}, T_r = \frac{2,2}{20} = 0,11 \quad T_s = \frac{4}{20} = 0,2$$

8.	Polos	Zeros	$c(t)$
a)	-2		$A + B e^{-2t}$
b)	-3, -6		$A + B e^{-3t} + C e^{-6t}$
c)	-10, -20	-7	$A + B e^{-10t} + C e^{-20t}$
d)	$-3 \pm j3\sqrt{15}$ , $(-3 - j3\sqrt{15})$		$A + B e^{-3t} \cos(3\sqrt{15}t + \phi)$
e)	$j3$ , $-j3$	-2	$A + B \cos(3t + \phi)$
f)	-10, -10	-5	$A + B e^{-10t} + C t e^{-10t}$

→

- a) primeira ordem  
b) superamortecido  
c) superamortecido  
d) subamortecido  
e) não amortecido  
f) criticamente amortecido

10.  $G(s) = C(sI - A)^{-1} B$

$$A = \begin{bmatrix} 8 & -4 & 1 \\ -3 & 2 & 0 \\ 5 & 17 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -4 \\ -3 \\ 4 \end{bmatrix} \quad C = [2 \ 8 \ -3]$$

$$C(sI - A)^{-1} = \frac{1}{s^3 - s^2 - 31s + 67} \begin{bmatrix} (s-2)(s+9) & -(4s+29) & (s-2) \\ -(3s+29) & (s^2+s-77) & -3 \\ 5s-31 & 17s-76 & (s^2-10s+4) \end{bmatrix}$$

$$G(s) = \frac{-49s^2 + 291s + 1814}{s^3 - s^2 - 31s + 67}$$

Polos: 9,683  
0,7349  
-9,4179

$$12. V_c(s) \left( \frac{1}{R_2} + \frac{1}{Ls} + Cs \right) + \frac{V_c(s) - V(s)}{R_1} = 0$$

$$\frac{V_c(s)}{V(s)} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{Ls} + Cs} = \frac{10s}{s^2 + 20s + 500}$$

Polos:  $-10 \pm j20$        $V_c(t) = A e^{-10t} \cos(20t + \phi)$

$$19. F(s) = (Ms^2 + fvs + ks)X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fvs + ks} = \frac{1}{s^2 + s + 5}$$

$$\begin{aligned} X(s) &= \frac{1}{s(s^2 + s + 5)} = \frac{1/5}{s} - \frac{1/5s + 1/5}{(s + 1/2)^2 + 19/4} \\ &= \frac{1/5 (s + 1/2) + \frac{1}{5\sqrt{19}} \frac{\sqrt{19}}{2}}{(s + 1/2)^2 + \frac{19}{4}} \end{aligned}$$

$$x(t) = \frac{1}{5} - \frac{1}{5} e^{-0,5t} \left( \cos \frac{\sqrt{19}}{2} t + \frac{1}{\sqrt{19}} \sin \frac{\sqrt{19}}{2} t \right)$$

$$\begin{aligned} 26. T(s) &= (1,07s^2 + 1,53s)\theta_1(s) - 1,53\theta_2(s) \\ 0 &= 1,53\theta_1(s) + (1,53s + 1,92)\theta_2(s) \end{aligned}$$

$$\theta_2(s) = \frac{\begin{vmatrix} 1,07s^2 + 1,53s & T(s) \\ -1,53s & 0 \end{vmatrix}}{\Delta} = \frac{0,935T(s)}{s^2 + 1,25s + 1,99}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{0,935}{s^2 + 1,25s + 1,99}$$

$$\omega_n = 1,34 ; 2\zeta\omega_n = 1,25 ; \zeta = 0,467 ; \%OS = 13,1$$

$$T_s = 6,40 ; T_p = 2,66$$

20.

$\omega_m^2$	$\xi$	$\omega_m$	$T_s$	$T_p$	%OS
a) $16 \pi^2$	0,375	4	$\frac{4}{\xi \omega_m} = 2,667$	0,8472 s	28,06%
b) $0,04 \pi^2$	0,05	0,2	400 s	15,732	85,45%
c) $1,05 \cdot 10^7 \pi^2$	0,247	3240	0,005 s	0,001 s	44,92%

 $T_n$ 

a) 0,356 s

b) 5,26 s

c)  $3,88 \cdot 10^{-4}$  s

$$23. a) \xi = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0,56 ; \omega_m = \frac{4}{\xi T_s} = 11,92$$

$$\text{Poles: } -\xi \omega_m \pm j \omega_m \sqrt{1-\xi^2} = -6,67 \pm j 9,88$$

$$b) \xi = 0,591 ; \omega_m = \frac{\pi}{T_p \sqrt{1-\xi^2}} = 0,779$$

$$\text{Poles: } -0,4605 \pm j 0,6283$$

$$c) \xi \omega_m = \frac{4}{T_s} = 0,571 ; \omega_m \sqrt{1-\xi^2} = \frac{\pi}{T_p} = 1,047$$

$$\text{Poles: } 0,571 \pm j 1,047$$

$$25. a) F(s) = (5s^2 + 5s + 28)X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1/5}{s^2 + s + 28/5}$$

$$b) \omega_n^2 = 28/5 ; 2\zeta\omega_n = 1 \quad \zeta = 0,211 ; \omega_n = 2,37$$

$$T_s = \frac{4}{\zeta\omega_n} = 8,01s ; T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1,36s$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100 = 50,7\% ; \omega_n T_n = (1,76\zeta^2 - 0,417\zeta^4 + 1,039\zeta^6 + 1)$$

$$T_n = 0,519s$$