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Quality Management 444
Gehaltebestuur 444

Week 3-4: Model Selection and Data Analysis

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Agenda

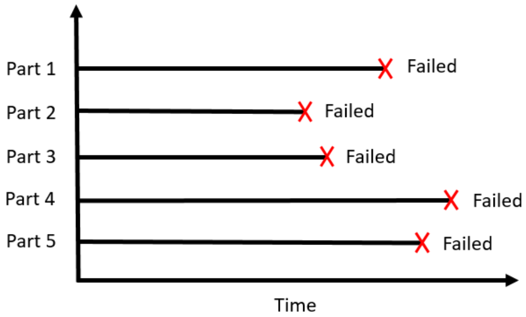
- Introduction to Reliability Engineering
- Reliability Modelling
- Component Importance
- Data Analysis:
  - Functions and terminology
  - Failure data
  - Failure timelines
    - Importance of chronological data
  - Selecting an appropriate model
  - Laplace trend test
  - Non-repairable systems - Weibull
  - Repairable systems – NHPP
- Availability

Failure Data

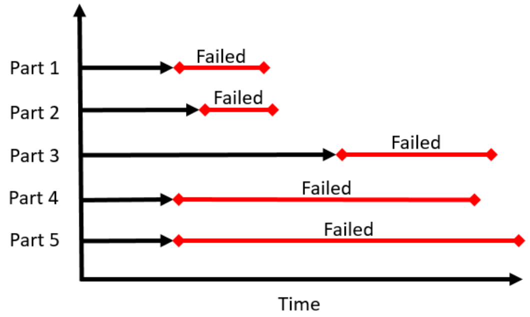
- Difficult to obtain
- Sources:
  - Computerised Maintenance Management System (CMMS)
  - Process, SCADA data
- Quality important
- Assumed that data is representative of the population of interest, if not bad estimations will be the result

Failure Data

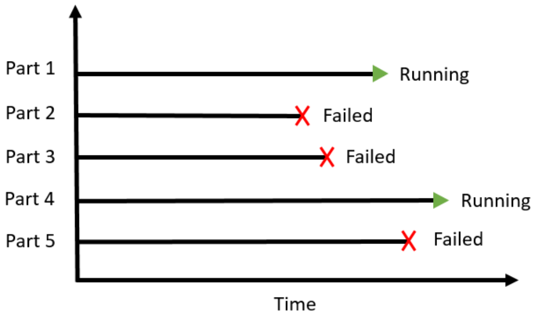
- Complete data



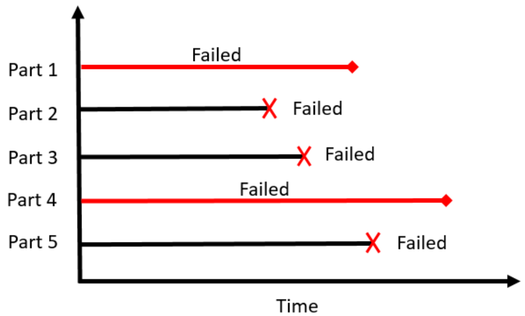
- Interval censored data



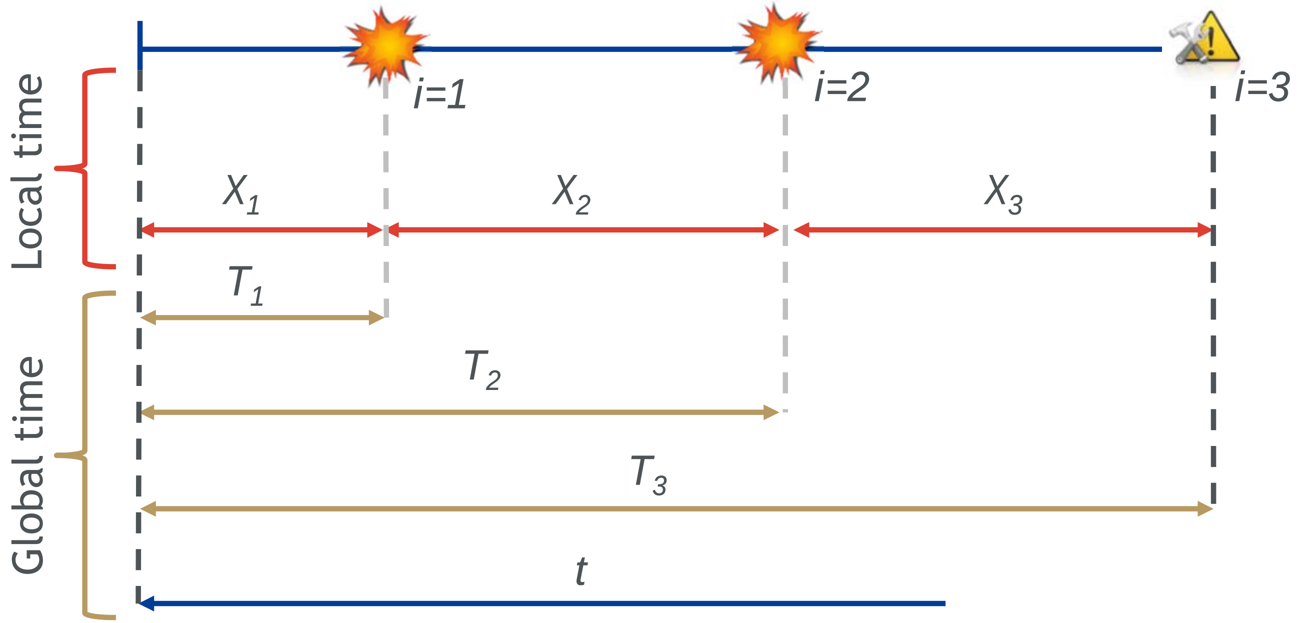
- Right censored data



- Left censored data

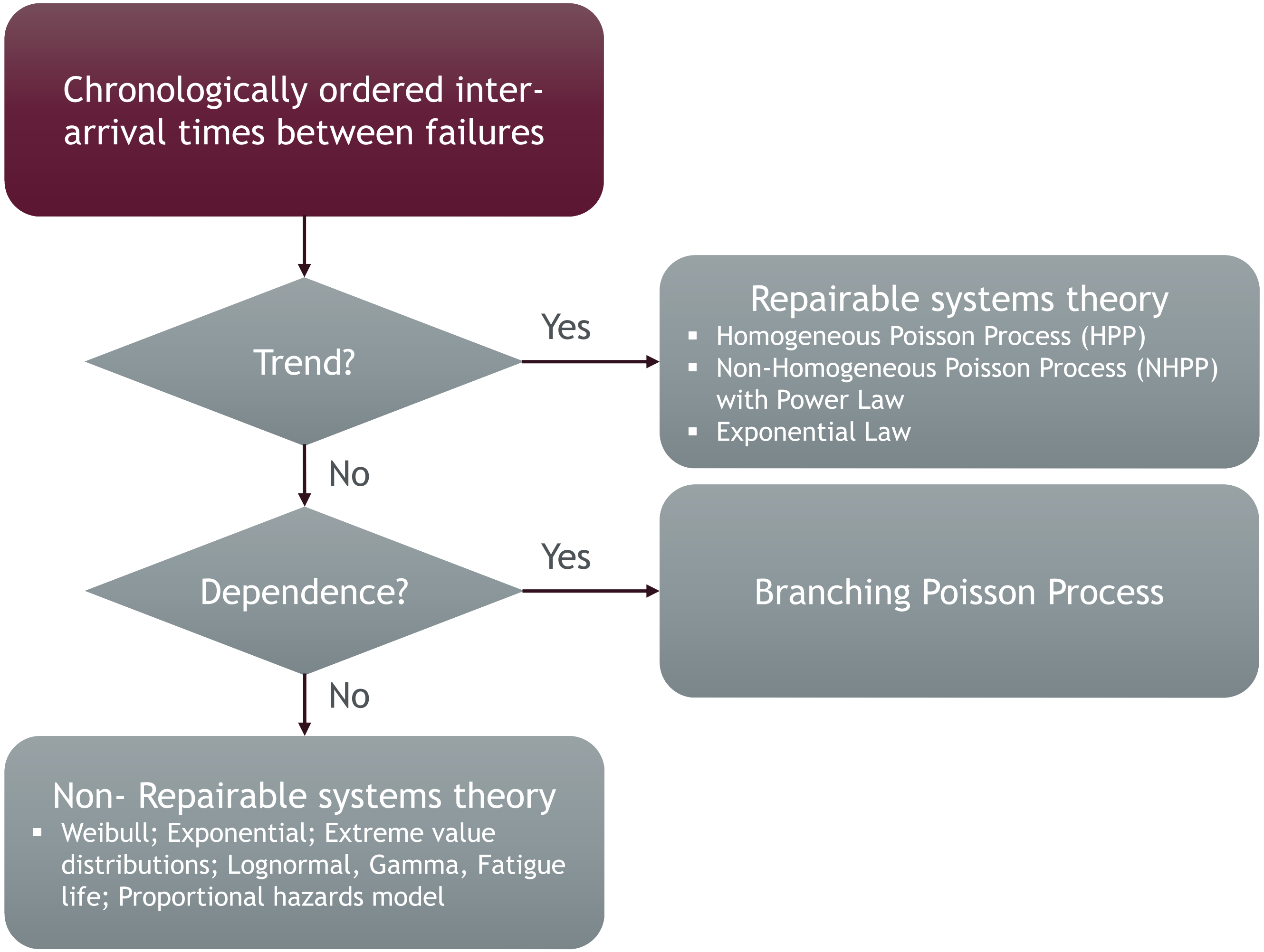


Failure Timelines



- x_i = time elapsed since most recent failure
- X_i = Inter-arrival time
- T_i = Arrival time
- C_i = Auxiliary variable recorded with X_i to indicate whether X_i was a real failure or a truncated failure observation (suspension). Use C_i=0 for a suspension and C_i=1 for a failure.

Selecting an Appropriate Model



Source: Ascher and Feingold (1984)

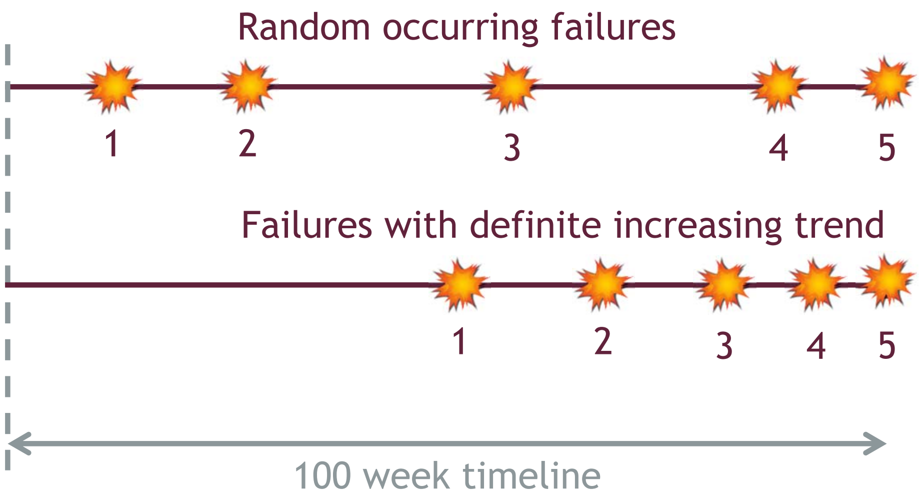
Events Table

- Generate events table with chronologically ordered interarrival times
- Select time scale and be consistent

i	X_i	C_i	T_i
1	64	1	64
2	43	1	107
3	17	0	124
4	21	1	145
5	94	1	239
6	48	0	287
7	3	1	290
8	13	1	303

Importance of Chronological Data

$$\text{MTBF} = \frac{Total\ Lifespan}{Total\ \#\ failures}$$



$$\text{MTBF} = \frac{100}{5} = 20$$

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Selecting an Appropriate Model

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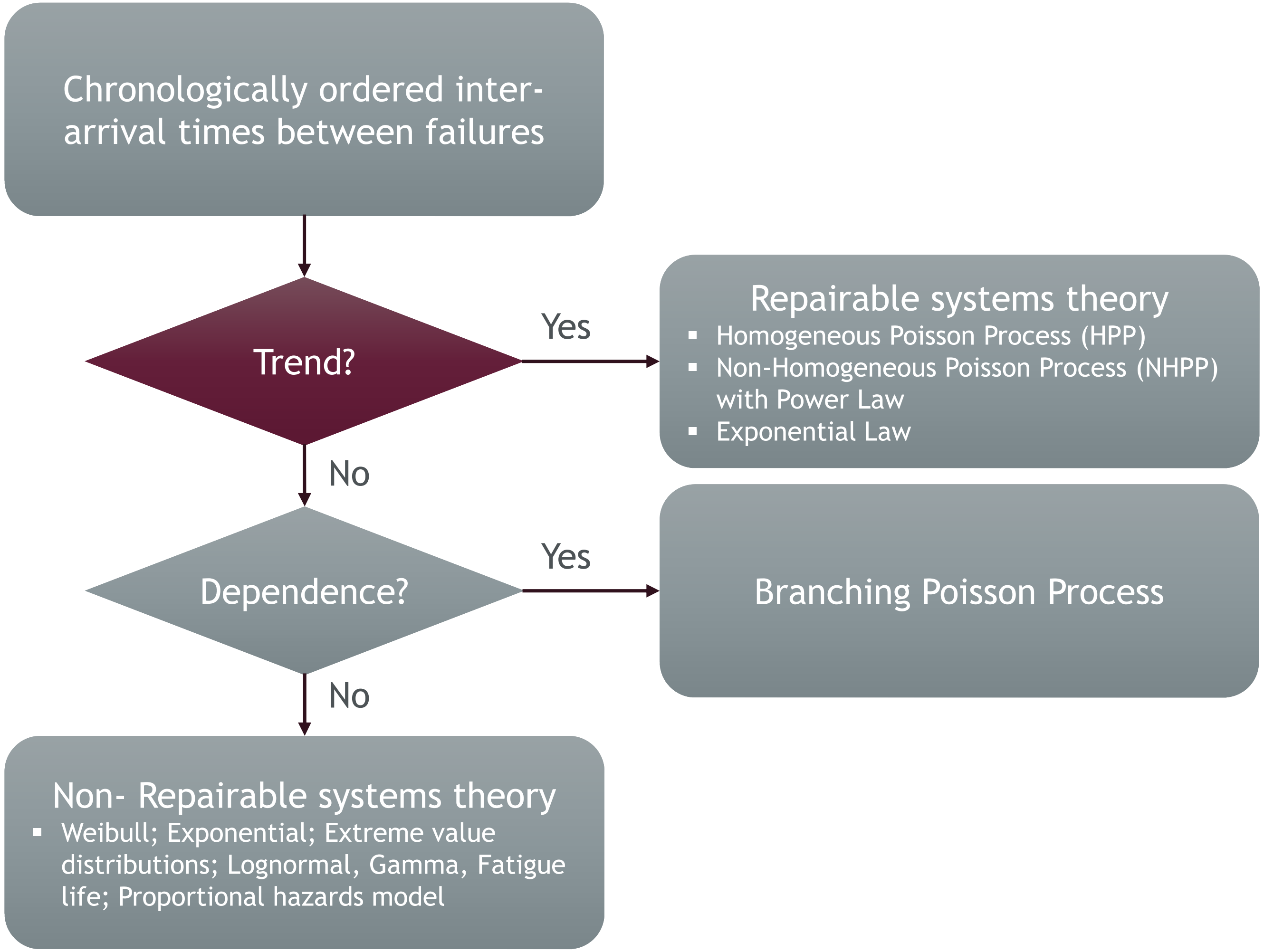
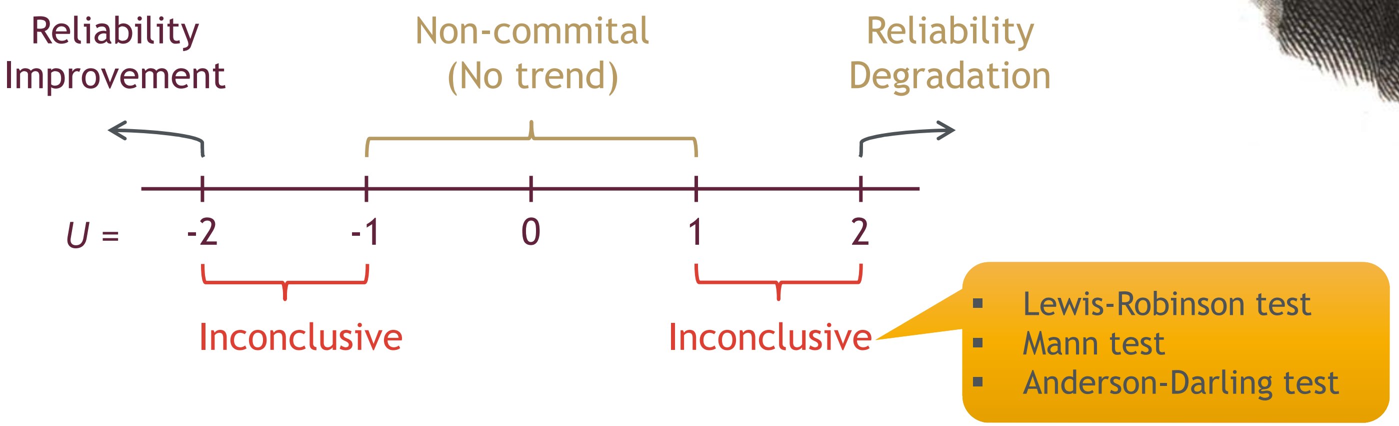


Source: Ascher and Feingold (1984)

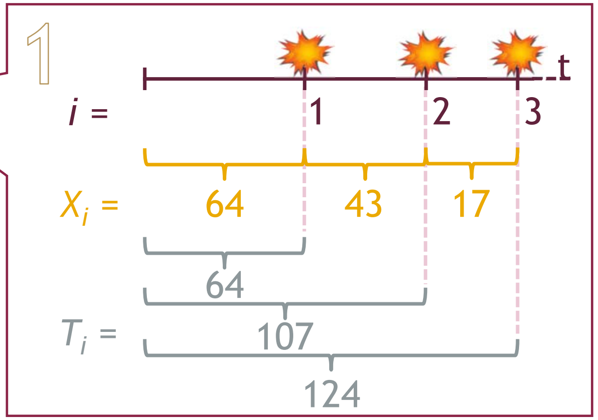
Laplace Trend Test

$$U = \frac{\frac{\sum_{i=1}^{r-1} T_i}{r-1} - \frac{T_r}{2}}{T_r\sqrt{\frac{1}{12(r-1)}}}$$



Laplace Trend Test: Example

Events (i)	Inter-arrival times of failures (X_i)	Arrival times of failures (T_i)
1	64	64
2	43	107 (=64+43)
3	17	124 (=107+17)
4	21	145
5	94	239
6	48	287
7	3	290
8	13	303
9	96	399
10	91	490
11	16	506
12	63	569
13	38	607
14 (= *r-1*)	69	676
15 (= r)	50	726 (= T_r)



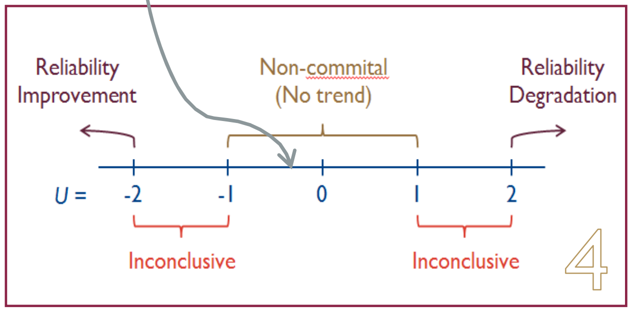
2

$\sum = 4806$

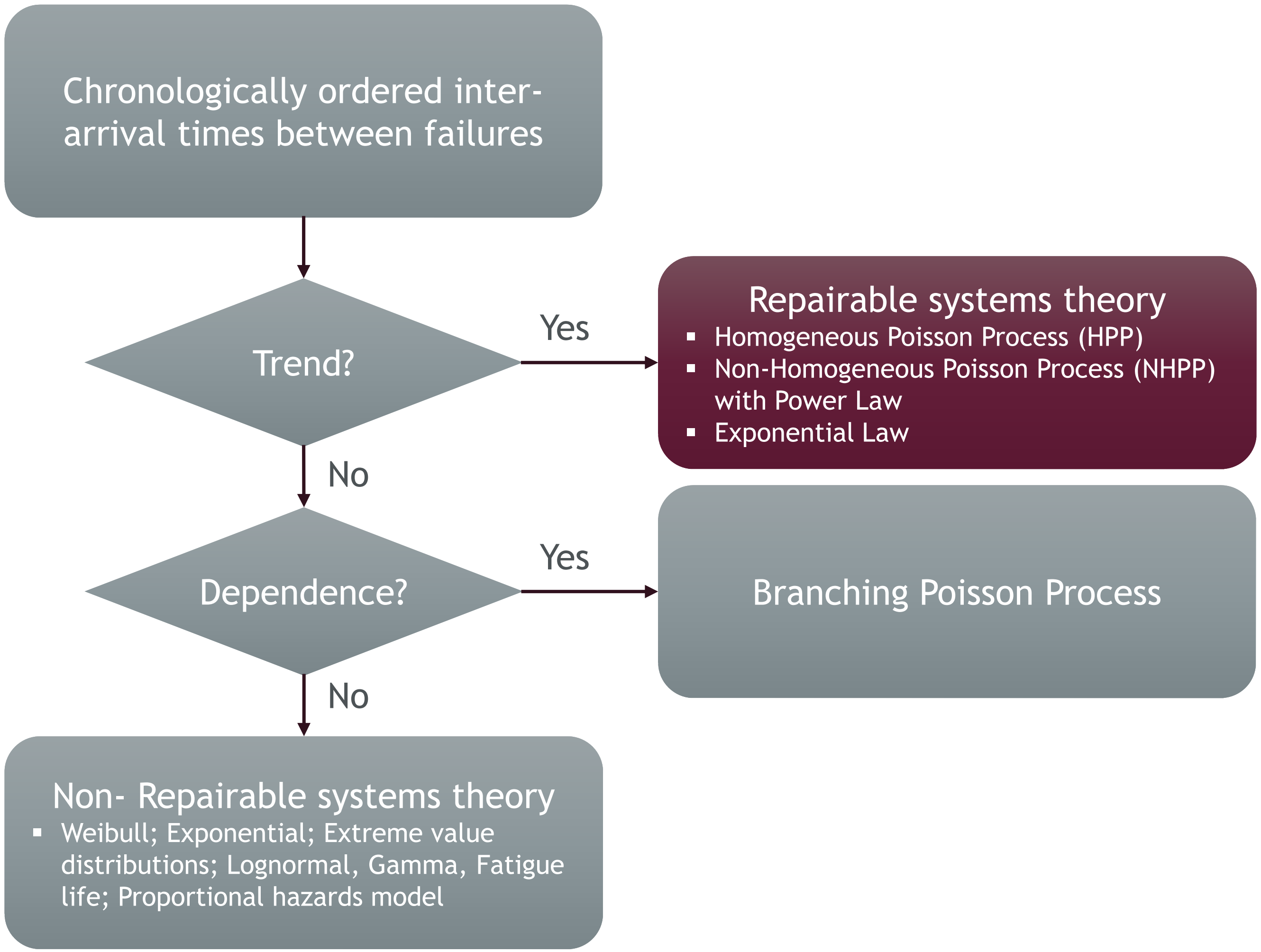
3

$$U = \frac{\frac{\sum_{i=1}^{r-1} T_i}{r-1} - \frac{T_r}{2}}{T_r\sqrt{\frac{1}{12(r-1)}}} = \frac{\frac{4806}{14} - \frac{726}{2}}{726\sqrt{\frac{1}{12(15-1)}}}$$

$$U = -0.35$$

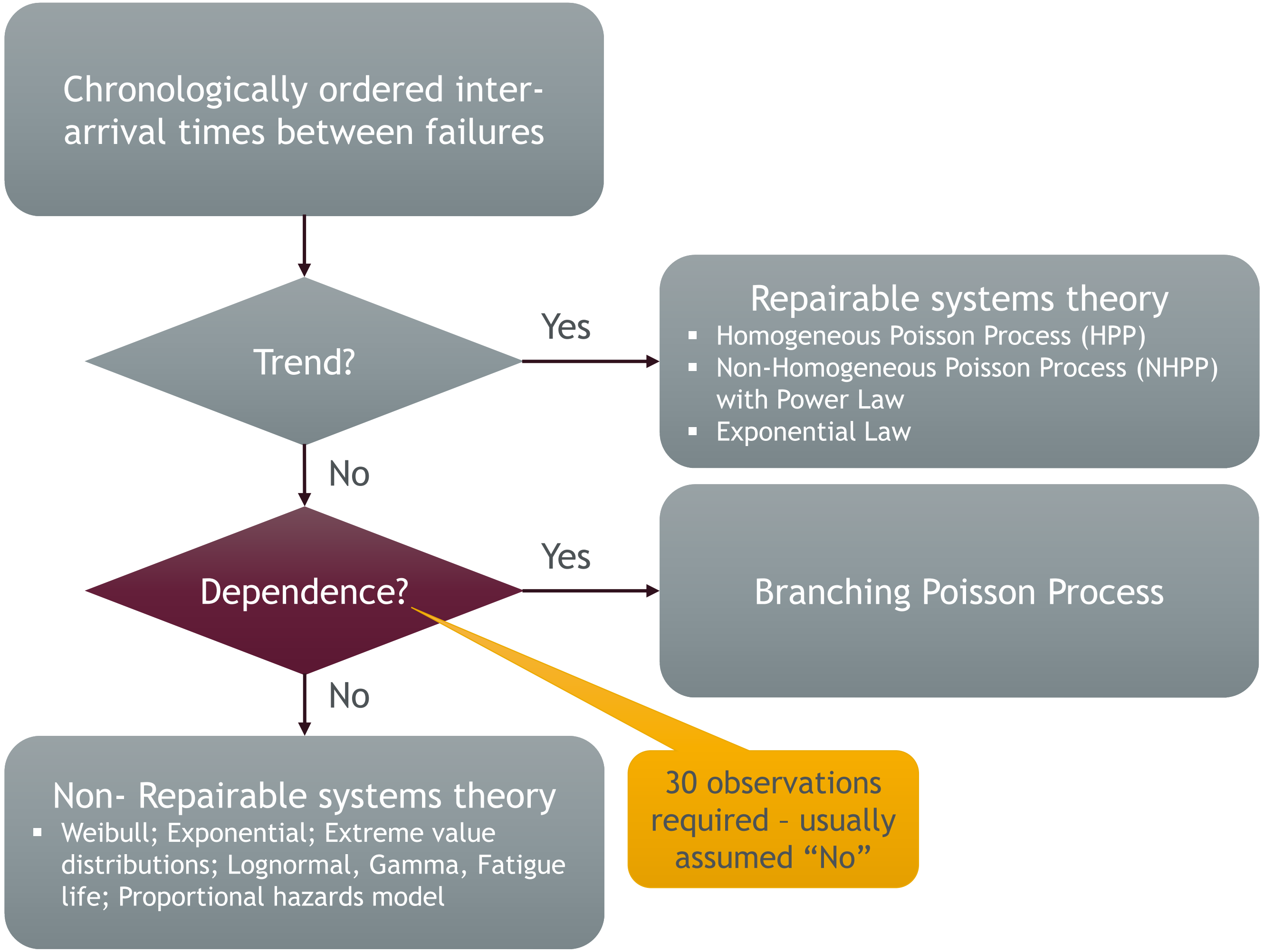


Selecting an Appropriate Model



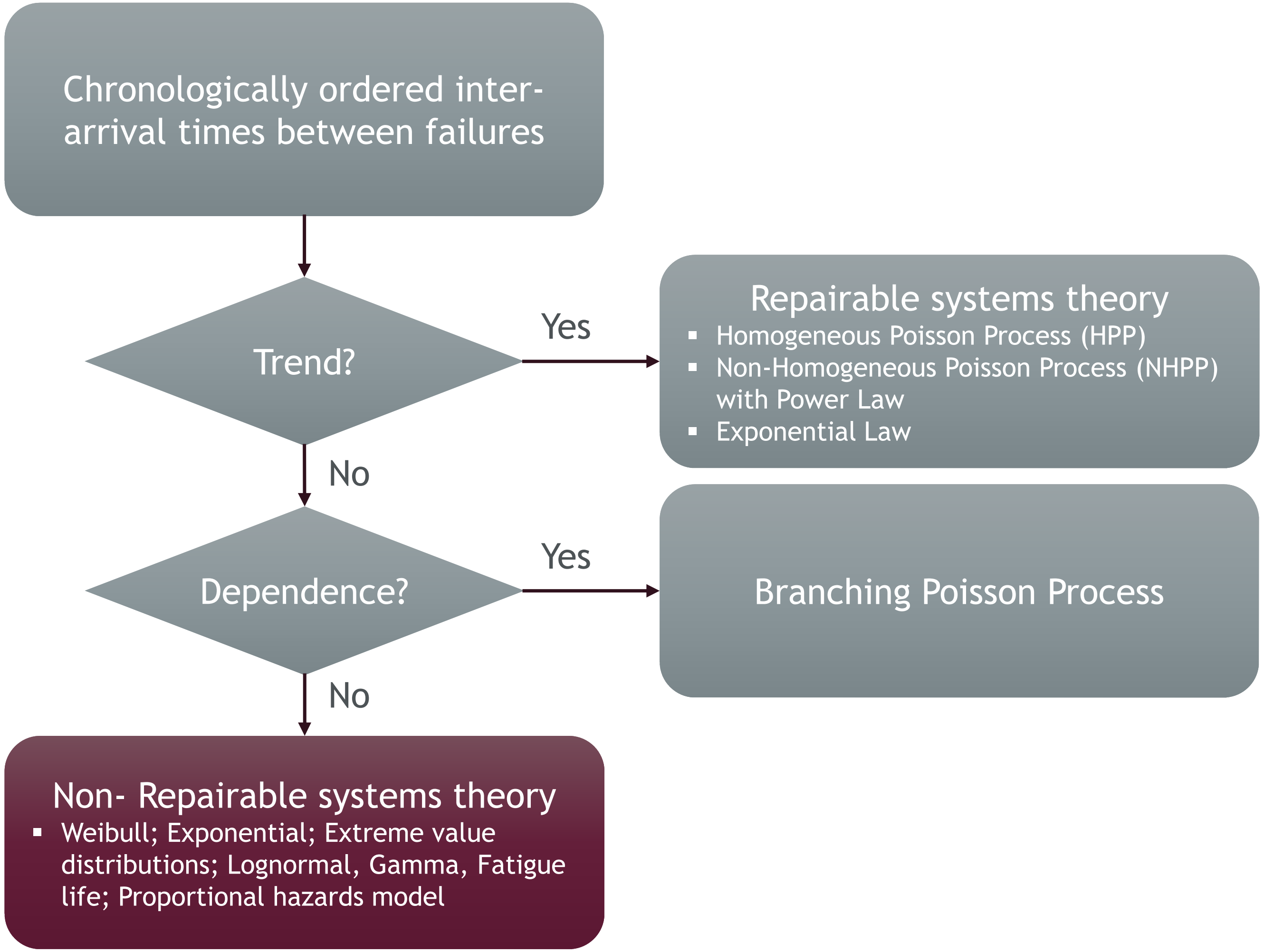
Source: Ascher and Feingold (1984)

Selecting an Appropriate Model



Source: Ascher and Feingold (1984)

Selecting an Appropriate Model



Source: Ascher and Feingold (1984)

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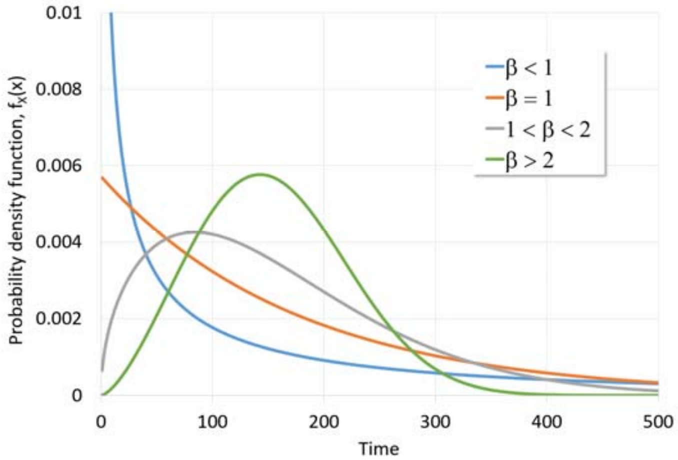
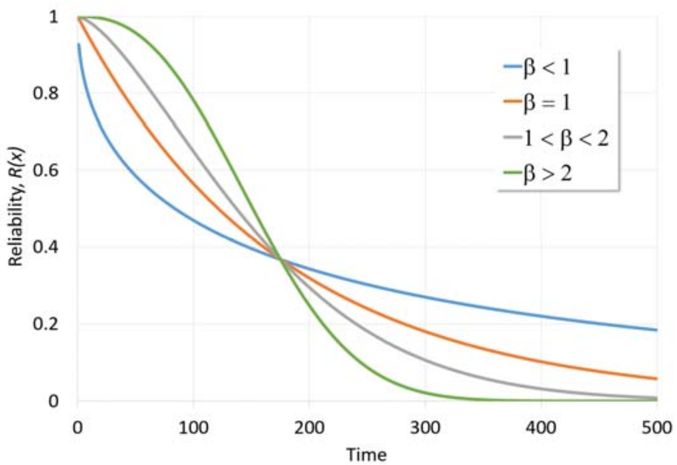
Weibull Analysis

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \cdot e^{\left(-\left(\frac{x}{\eta}\right)^{\beta}\right)}$$

β= shape parameter,
and
η= characteristic life

$$\mathrm{R}(x) = e^{\left(-\left(\frac{x}{\eta}\right)^{\beta}\right)}$$





Waloddi Weibull
Swedish Engineer
1887-1979

Weibull Analysis

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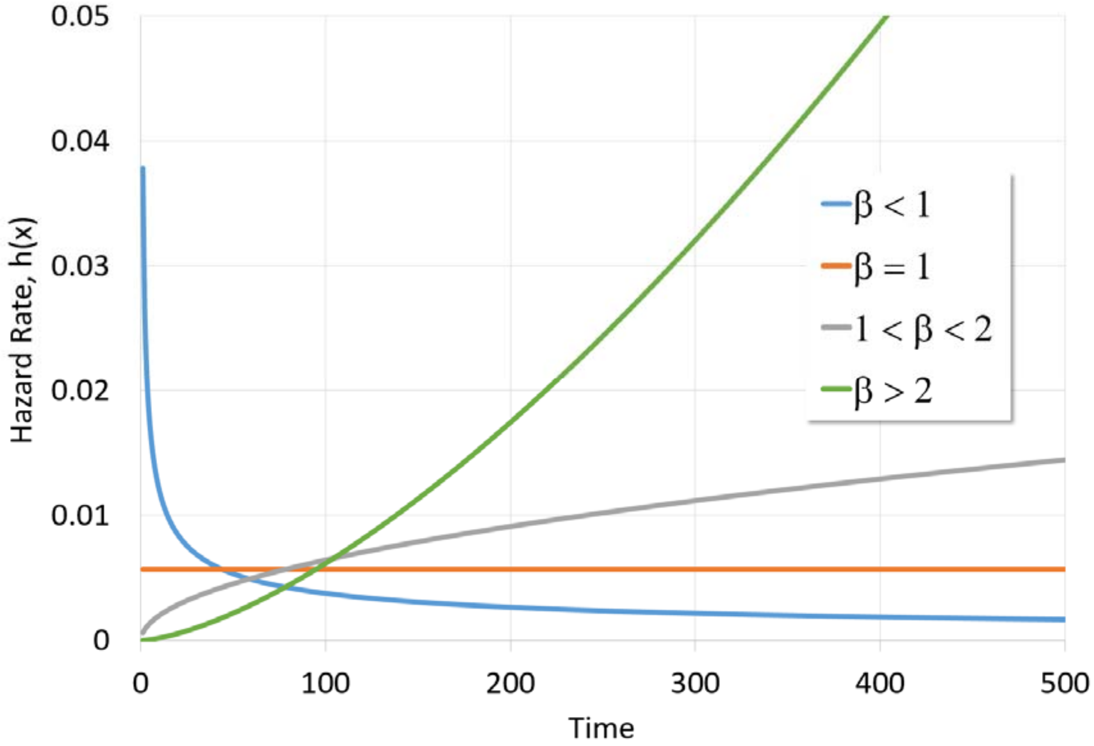
$$\mathrm{h}(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

- β<1: Decreasing $h(x)$
  - Run-to-failure
  - Condition-based Maint.
- β=1: Constant $h(x)$
  - Run-to-failure
- β>1: Increasing $h(x)$
  - Usage-based Maint.
- β>6: Time to become suspicious
  - Highly accelerated wear-out, brittle parts
- Characteristic of life, η
  - Point in time at which 63.2% of parts would have failed



Weibull Analysis

- Parameter Estimation by maximising the likelihood

$$lnL(X,\theta) = \sum_{i=1}^{m}\left[ln\frac{\beta}{\eta} + (\beta - 1)ln\frac{X_i}{\eta}\right] - \sum_{j=1}^{r}\left(\frac{X_j}{\eta}\right)^{\beta}$$

- Residual life estimation
  - Predict arrival time of next failure event
  - Determine confidence levels for the residual life estimate
- Preventive maintenance instance
  - Minimise maintenance cost over system life, considering
    - Unexpected failures at cost, C_f
    - Preventive maintenance cost, C_p
    - Preventive maintenance duration, a
    - Failure replacement duration, b

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Non-Homogenous Poisson Process (NHPP)

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- Log-linear NHPP
  - Function: $\rho_1(t) = e^{(\alpha_0+\alpha_1 t)}$
  - Expected number of failures, N, between time, t_1 and t_2:

$$E[N(t_1 \rightarrow t_2)] = \frac{1}{\alpha_1}[e^{(\alpha_0+\alpha_1 t_2)} - e^{(\alpha_0+\alpha_1 t_1)}]$$

  - MTBF from time, t_1 and t_2: $MTBF_{\rho 1}(t_1 \rightarrow t_2) = \dfrac{\alpha_1(t_2 - t_1)}{e^{(\alpha_0+\alpha_1 t_2)} - e^{(\alpha_0+\alpha_1 t_1)}}$

  - Reliability from time, t_1 and t_2: $R(t_1 \rightarrow t_2) = e^{-[\frac{e^{(\alpha_0+\alpha_1 t_2)} - e^{(\alpha_0+\alpha_1 t_1)}}{\alpha_1}]}$

Non-Homogenous Poisson Process (NHPP)

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- Power Law NHPP
  - Function: $\rho_2(t) = \lambda\delta t^{\delta-1}$
  - Expected number of failures, N, between time, t_1 and t_2:

$$E[N(t_1 \rightarrow t_2)] = \lambda({t_2}^{\delta} - {t_1}^{\delta})$$

  - MTBF from time, t_1 and t_2: $MTBF_{\rho 2}(t_1 \rightarrow t_2) = \dfrac{(t_2 - t_1)}{\lambda({t_2}^{\delta} - {t_1}^{\delta})}$
  - Reliability from time, t_1 and t_2: $R(t_1 \rightarrow t_2) = e^{-\lambda({t_2}^{\delta} - {t_1}^{\delta})}$

NHPP Parameter Estimation

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Log-linear NHPP

- Least-Squares Method
  - T_r is a failure: $\min(\hat{\alpha_0}, \hat{\alpha_1}) : \sum_{i=1}^{r} [E[N(0 \to T_i)] - N(0 \to T_i)]^2$
  - T_r is a suspension: $\min(\hat{\alpha_0}, \hat{\alpha_1}) : \sum_{i=1}^{r-1} [E[N(0 \to T_i)] - N(0 \to T_i)]^2$
- Likelihood Method
  - T_r is a failure: $l_{\rho_1}(\alpha_0, \alpha_1) = r\alpha_0 + \alpha_1 \sum_{i=1}^{r} T_i - \frac{e^{\alpha_0}\left(e^{\alpha_1 T_r} - 1\right)}{\alpha_1}$
  - T_r is a suspension: $l_{\rho_1}(\alpha_0, \alpha_1) = r\alpha_0 + \alpha_1 \sum_{i=1}^{r} T_i - \frac{e^{\alpha_0}\left(e^{\alpha_1 T_r} - 1\right)}{\alpha_1} - \frac{1}{\alpha_1} \cdot \left(e^{\alpha_0 + \alpha_1 T_r} - e^{\alpha_0 + \alpha_1 T_{r-1}}\right)$

Power Law NHPP

- Least-Squares Method
  - T_r is a failure: $\min(\hat{\lambda}, \hat{\delta}) : \sum_{i=1}^{r} [E[N(0 \to T_i)] - N(0 \to T_i)]^2$
  - T_r is a suspension: $\min(\hat{\lambda}, \hat{\delta}) : \sum_{i=1}^{r-1} [E[N(0 \to T_i)] - N(0 \to T_i)]^2$
- Likelihood Method
  - T_r is a failure: $l_{\rho_2}(\lambda, \delta) = r \ln \lambda + r \ln \delta - \lambda {T_r}^{\delta} + (\delta - 1) \sum_{i=1}^{r} \ln T_i$
  - T_r is a suspension: $l_{\rho_2}(\lambda, \delta) = r \ln \lambda + r \ln \delta - \lambda {T_r}^{\delta} + (\delta - 1) \sum_{i=1}^{r} \ln T_i - \lambda(T_2^{\delta} - T_1^{\delta})$

NHPP Analysis Outputs

- Residual life estimation
  - Predict arrival time of next failure event
  - Determine confidence levels for the residual life estimate
    - Upper confidence limit may not converge
- Preventive maintenance replacement
  - Minimise maintenance cost over system life, considering
    - Average cost of minimal repairs, C_m
    - Cost of replacement of the system, C_s
    - Number of minimal preventive maintenance repairs, N^*
    - Optimal global time, T^*

Data Analysis Conclusions

- Limitations due to assumptions
- Renewal assumption for non-repairable systems
  - Overall system deterioration influence future components, even if components are replaced.
- Minimal repair assumption of repairable systems more realistic, however:
  - Human interference not accounted for
- Shortcoming of conventional failure data analysis:
  - Inability to include diagnostic information
  - Long term nature of cost optimisation often rejected by impatient maintenance practitioners.

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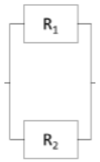
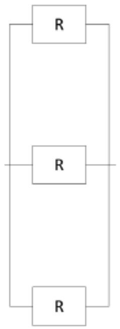
Availability

- Availability Definition
  - The probability that an item will be available when required
- *A=f(ROCOF, repair rate)*
- Various definitions
  - Inherent
  - Achieved
  - Operational
  - Steady-state
  - Instantaneous

Availability

- Steady-state Availability
  - Proportion of total time that system is available
  - For simple unit with constant ROCOF, $\lambda = \frac{1}{MTBF}$, and repair rate, $\mu = \frac{1}{MTTR}$,
  - $A = \frac{\mu}{\lambda+\mu}$
- Instantaneous Availability
  - Probability that a system will be available at time, t
  - $A(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$
  - Approaches steady-state as t becomes large

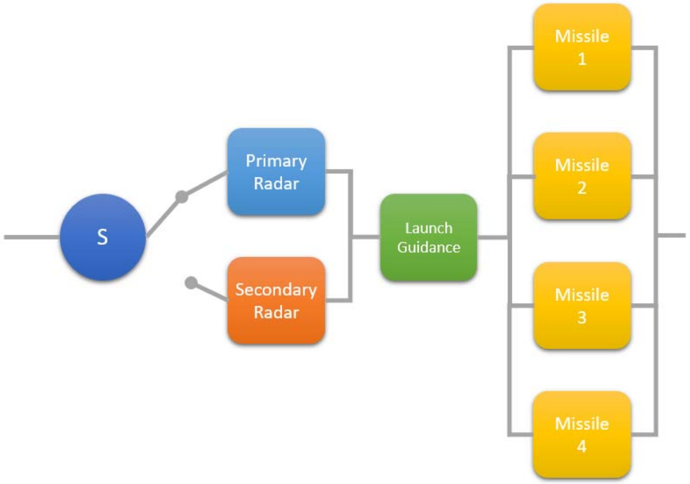
Some Steady-state Availability Functions

Reliability Configuration		Steady-state availability, A, repair rate, μ, ROCOF, λ	General steady-state availability, A, n blocks
	–	$\frac{\mu}{\lambda+\mu}$	–
	–	$\frac{\mu_1\mu_2}{\mu_1\mu_2+\mu_1\lambda_2+\mu_2\lambda_1+\lambda_1\lambda_2}$	$\prod_{i=1}^{w} \frac{\mu_i}{\lambda_i+\mu_i}$
	Active	$\frac{\mu^2+2\mu\lambda}{\mu^2+2\mu\lambda+2\lambda^2}$ [1]	$1-\prod_{i=1}^{w} \frac{\lambda_i}{\lambda_i+\mu_i}$
	Standby	$\frac{\mu^2+\mu\lambda}{\mu^2+\mu\lambda+\lambda^2}$	–
	Active 1/3	$\frac{\mu^3+3\mu^2\lambda+6\mu\lambda^2}{\mu^3+3\mu^2\lambda+6\mu\lambda^2+6\lambda^3}$	As above (active)
	Active 2/3	$\frac{\mu^3+3\mu^2\lambda}{\mu^3+3\mu^2\lambda+6\mu\lambda^2+6\lambda^3}$	$1-\frac{1}{(\lambda+\mu)^w}\sum_{i=0}^{k-1}\binom{w}{i}\mu^i\lambda^{w-i}$
	Standby 1/3	$\frac{\mu^3+\mu^2\lambda+\mu\lambda^2}{\mu^3+\mu^2\lambda+\mu\lambda^2+\lambda^3}$	–

Availability Example

- Recall the missile system example

A missile system consists of two warning radars, a launch guidance system and the missiles. The radars are arranged so that either can give warning if the other fails, in a standby redundant configuration with perfect switching. Four missiles are available for firing and the system is considered to be reliable if three out of the four missiles can be fired and guided.



Over a 24hr period the reliability of each missile is 0.9 and for the launch guidance system it is 0.9685. The MTBF of both the primary and secondary radar systems is 1000 hours and 750 hours for the launch guidance system. Determine the reliability of the system over 24hr.

Availability Example

Determine the steady-state availability of the system, excluding the missiles, if the mean repair time for all units is 2 hours.

$$\mu = \frac{1}{MTTR} = \frac{1}{2} = 0.5$$

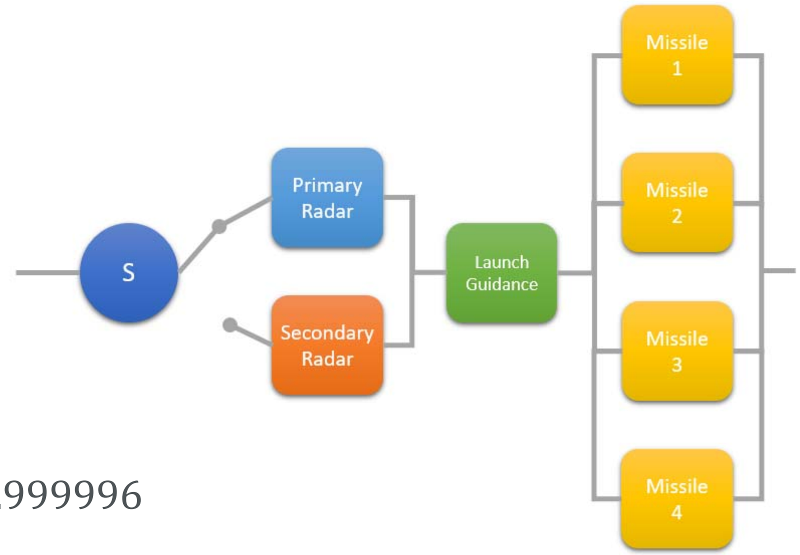
$$\lambda_{\text{LG}} = \frac{1}{750} = 0.0013$$

$$\lambda_{\text{R}} = \frac{1}{1000} = 0.001$$

$$\text{A}_{\text{R}} = \frac{\mu^2 + \mu\lambda}{\mu^2 + \mu\lambda + \lambda^2} = \frac{0.5^2 + 0.5 \cdot 0.001}{0.5^2 + 0.5 \cdot 0.001 + 0.001^2} = 0.999996$$

$$\text{A}_{\text{LG}} = \frac{\mu}{\lambda + \mu} = \frac{0.5}{0.0013 + 0.5} = 0.9974$$

$$\text{A}_{\text{S}} = \text{A}_{\text{R}}\text{A}_{LG} = 0.9974$$



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Thank you
Enkosi
Dankie

Photo by Stefan Els