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Quality Management 444
Gehaltebestuur 444

Week 2: Reliability Modelling, Component Importance & Data Analysis Functions and Terminology

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Agenda

- Introduction to Reliability Engineering
- Reliability Modelling
- Component Importance
- Data Analysis:
  - Functions and terminology
  - Failure data
  - Failure timelines
    - Importance of chronological data
  - Selecting an appropriate model
  - Laplace trend test
  - Non-repairable systems - Weibull
  - Repairable systems – NHPP
- Availability

Reliability Modelling Techniques

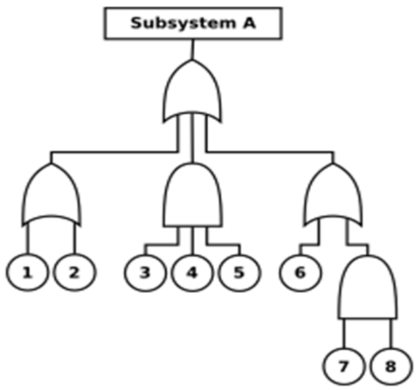
Two generally used symbolic analytical techniques, used for analysis of system reliability and characteristics

Reliability Block Diagrams (RBD)

- Work in success space – consider success combinations
- Include time-varying distributions for success
- May include repair distributions
- Generally, more difficult to convert to FTA

Fault Tree Analysis (FTA)

- Work in failure space – considers failure combinations
- Traditionally used to analyse fixed probabilities
- Can be easily converted to RBD



Reliability Block Diagram (RBD) Variables

• P	Probability
• A_i	Event related to component i
• n	Number of items installed in a system
• R	Reliability (probability of system survival)
• F	Unreliability (probability of system failure)
• k	Number of items required to survive the system in a k-out-of-n configuration
• I	Component Importance (Birnbaum's measure)
• λ	Failure rate

Most Common Configurations

- Series
  - A system that is functioning if and only if all of its n components are functioning is called a series system
  - Diagram:

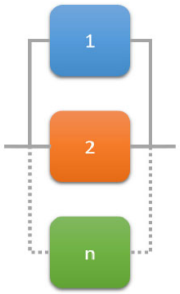
  - Function (refer to the course notes for the derivation)

$$R_S = \prod_{i=1}^{w} R_i$$

  - In short, R_s of a simple series is equal to the product of the reliabilities of the individual components

Most Common Configurations

- Active (Parallel) Redundancy
  - A system that is functioning if at least one of its *n* components is functioning is called an *active parallel* system
  - Diagram

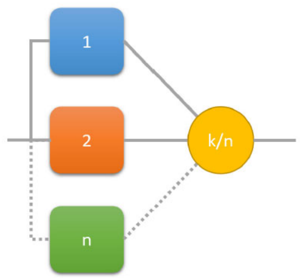


  - Function (refer to the course notes for the derivation)

$$R_S = 1 - \prod_{i=1}^{w} (1 - R_i)$$

Most Common Configurations

- k-out-of-n Redundancy
  - Cases where *k* items out of a total of *n* items are required to survive to ensure system survival
  - Diagram



  - Function (refer to the course notes for the derivation)
    - For n identical components ($R_1=R_2=\dots R_n$):

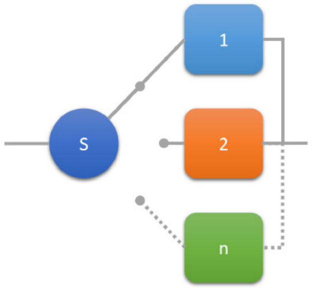
$$R_S = \sum_{j=k}^{n} \binom{n}{j} R_i^j \,(1-R_i)^{n-j}, where \binom{n}{j} = \frac{n!}{j!(n-j)!}$$

    - For n non-identical components ($R_1 \neq R_2 \neq \dots R_n$):

$$R_S = \left[\sum_{l}\left(\prod_{i} R_i \prod_{j} F_j\right)\right] + \prod_{y=1}^{n} R_y$$

Most Common Configurations

- Passive (Standby) Redundancy
  - Case where one component does not operate continuously, but is only activated when the primary component fails
  - Modelling considerably more complex because the time for which the standby component is required to operate is a variable
  - Consider only the special case of:
    - constant failure rate (CFR), $\lambda = \frac{1}{MTBF}$, and
    - identical components
  - Diagram



Most Common Configurations

- Passive (Standby) Redundancy
  - Function (refer to the course notes for the derivation)
    - For $n=2$ identical components ($R_1=R_2$):

$$R_S = (1 + \lambda t)e^{-\lambda t}, where\ \lambda = \frac{1}{MTBF}$$

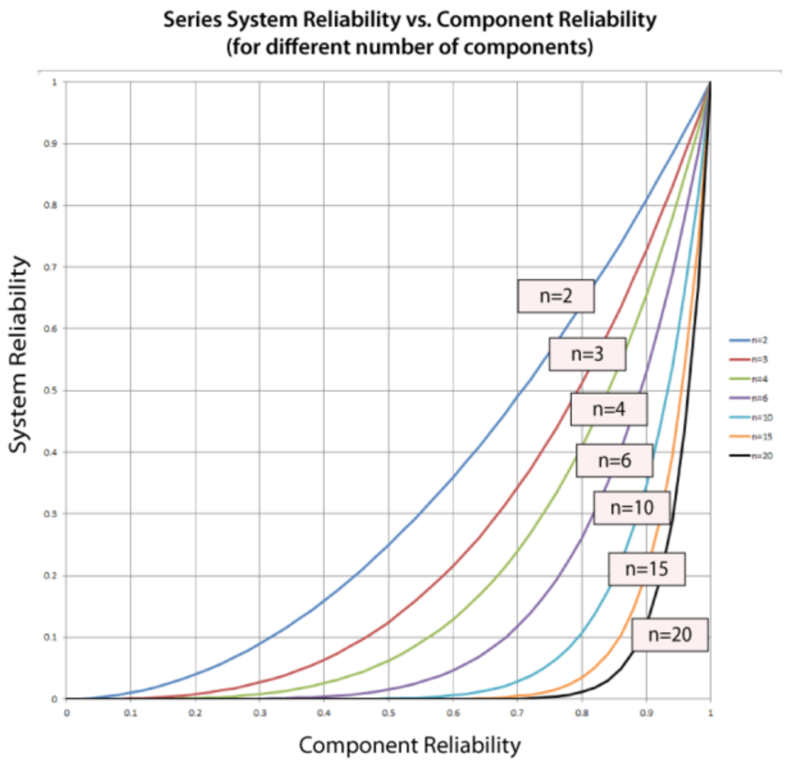
    - For n identical components ($R_1=R_2 = \ldots R_n$):

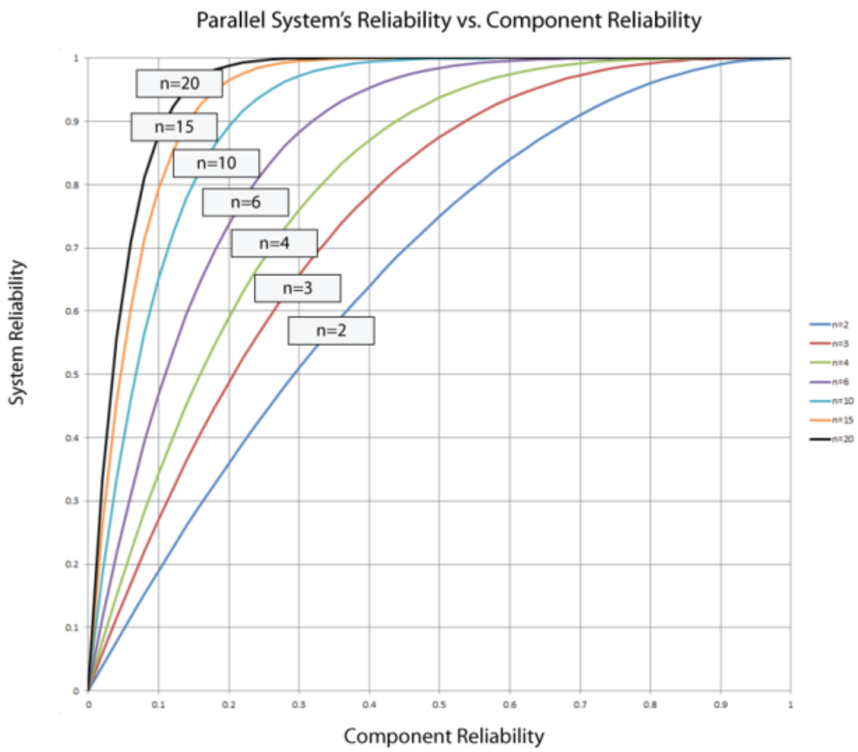
$$R_S = e^{-\lambda t}\sum_{i=0}^{n-1}\frac{(\lambda t)^i}{i!}$$

    - For a k-out-of-n system with $n=k$ identical components ($R_1=R_2 = \ldots R_n$):

$$R_S = e^{-k\lambda t}\sum_{i=0}^{n-k}\frac{(k\lambda t)^i}{i!}$$

Comparison of Series and Parallel Configurations





Other RBD Configurations

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- Series
- Active (parallel) redundancy
- k-out-of-n redundancy
- Standby redundancy
- Combined (series and parallel)
- Load sharing
- Inherited subsystems
- Multi blocks
- Mirrored blocks

Covered in this course

Refer to http://reliawiki.org/ for more information

Examples: Series

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- Three subsystems are reliability-wise in series and forms a system. Subsystem 1 has a reliability of 99.5%, subsystem 2 of 98.7% and subsystem 3 of 97.3% for a mission of 100 hours. What is the overall reliability of the system for a 100-hour mission?

$$R_S = \prod_{i=1}^{w} R_i$$

$$= R_1 \cdot R_2 \cdot R_3$$

$$= 0.995 \cdot 0.987 \cdot 0.973$$

$$= 0.9555$$

Examples: Series

- Given a series system with five components, and a system mission reliability target of at least 0.99, determine the target component reliability levels, such that all components' targets are equal.

$$R_S = \prod_{i=1}^{w} R_i$$

$$= R_i^5 = 0.99$$

$$\Rightarrow R_i = \sqrt[5]{0.99}$$

$$= 0.998$$

Examples: Active (Parallel) Redundancy

- Three subsystems are reliability-wise in parallel and make up a system. Subsystem 1 has a reliability of 99.5%, subsystem 2 has a reliability of 98.7% and subsystem 3 has a reliability of 97.3% for a mission of 100 hours. What is the overall reliability of the system for a 100-hr mission?

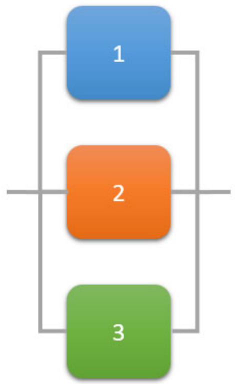
$$R_S = 1 - \prod_{i=1}^{n} (1 - R_i)$$

$$= 1 - (1 - R_1)(1 - R_2)(1 - R_3)$$

$$= 1 - (1 - 0.995)(1 - 0.987) \cdot (1 - 0.973)$$

$$= 1 - 0.00000175$$

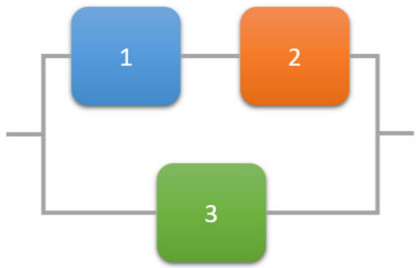
$$= 0.999998245$$



Examples: Combination

- Consider a system with three components. Units 1 and 2 are connected in series and Unit 3 is connected in parallel with the first two unit

$$R_{1,2} = R_1 \cdot R_2$$
$$= 0.995 \cdot 0.987$$
$$= 0.98207$$



$$R_S = 1 - (1 - R_{1,2})(1 - R_3)$$
$$= 1 - (1 - 0.98207)(1 - 0.973)$$
$$= 0.99952$$

Examples: k-out-of-n

- Consider a system of 6 pumps of which at least 4 must function properly for system success. Each pump has an 85% reliability for the mission duration. What is the probability of success of the system for the same mission duration?

$$R_s(k,n,R) = \sum_{r=k}^{n} \binom{n}{r} R_i^j R_i^j (1-R_i)^{n-j},$$

for $k = 4, n = 6$

$$= \sum_{r=4}^{6} \binom{6}{r} 0.85^r (1-0.85)^{6-r}$$

$$= \binom{6}{4} 0.85^4 (1-0.85)^2 + \binom{6}{5} 0.85^5 (1-0.85)^1 +$$

$$\binom{6}{6} 0.85^6 (1-0.85)^0$$

$$= 0.1762 + 0.3993 + 0.3771$$

$$= 95.26\%$$

Examples: k-out-of-n

- Three hard drives in a computer system are configured reliability-wise in parallel. At least two of them must function in order for the computer to work properly. Each hard drive is of the same size and speed, but they are made by different manufacturers and have different reliabilities. The reliability of HD #1 is 0.9, HD #2 is 0.88 and HD #3 is 0.85, all at the same mission time.

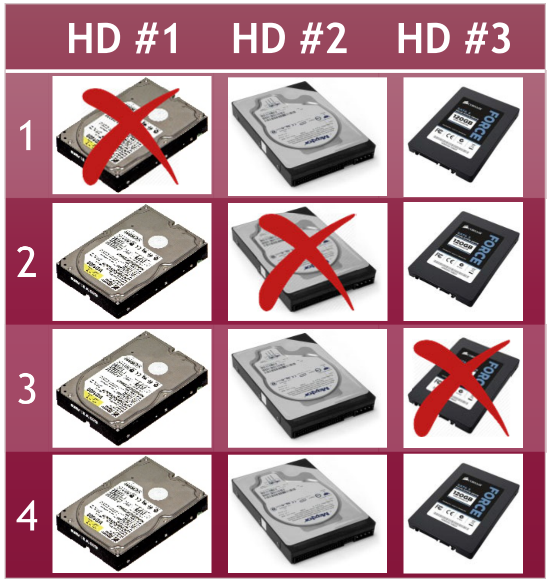
$$R_s = \left[\sum_l \left(\prod_i R_i \prod_j F_j\right)\right] + \prod_{y=1}^{n} R_y$$

$$= R_2R_3F_1 + R_1R_3F_2 + R_1R_2F_3 + \prod_{i=1}^{3} R_i$$

$$= 0.88 \cdot 0.85 \cdot 0.1 + 0.9 \cdot 0.85 \cdot 0.12 +$$

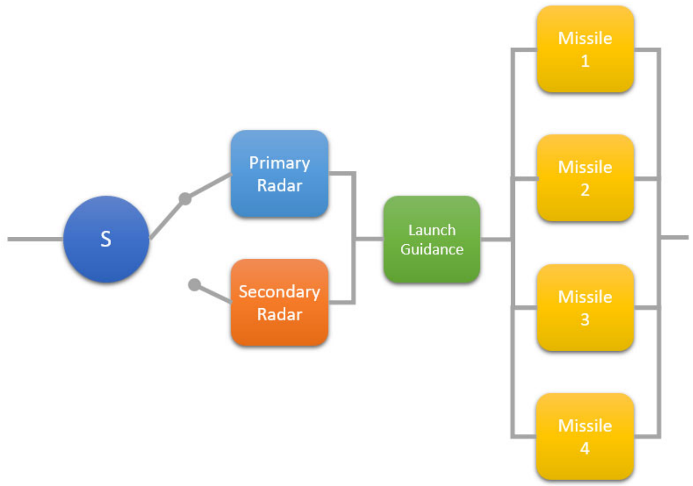
$$0.9 \cdot 0.88 \cdot 0.15 + 0.9 \cdot 0.88 \cdot 0.85$$

$$= 0.9586$$



Examples: Passive (Standby)

A missile system consists of two warning radars, a launch guidance system and the missiles. The radars are arranged so that either can give warning if the other fails, in a standby redundant configuration with perfect switching. Four missiles are available for firing and the system is considered to be reliable if three out of the four missiles can be fired and guided.



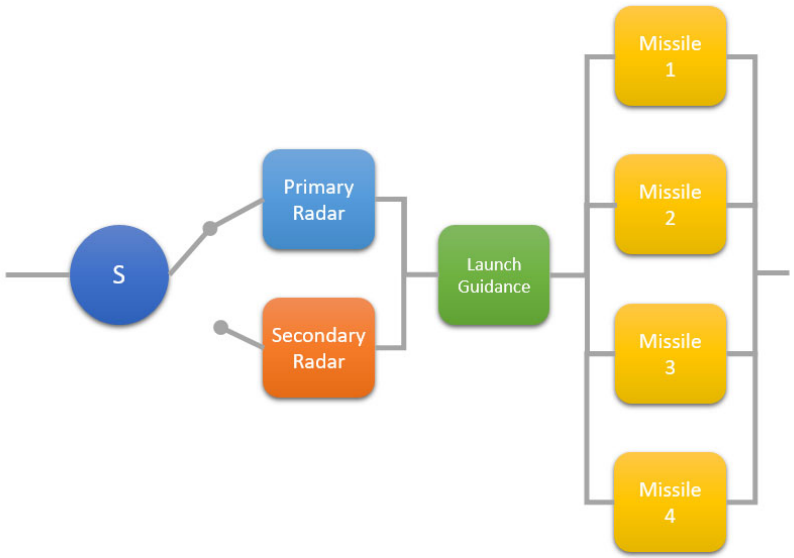
Over a 24hr period the reliability of each missile is 0.9 and for the launch guidance system it is 0.9685. The MTBF of both the primary and secondary radar systems is 1000 hours and 750 hours for the launch guidance system. Determine the reliability of the system over 24hr.

Examples: Passive (Standby)

$$R_R = (1+\lambda t)e^{-\lambda t}$$
$$= \left(1 + \frac{1}{1000}\cdot 24\right) e^{-\frac{1}{1000}\cdot 24}$$
$$= 0.9997$$

$$R_M = \sum_{r=k}^{n} \binom{n}{r} R_i^j R_i^j (1-R_i)^{n-j}$$
$$= \binom{4}{3} 0.9^3 (1-0.9)^1 + \binom{4}{4} 0.9^4 (1-0.9)^0 = 0.9477$$

$$R_S = R_R R_{LG} R_M$$
$$= 0.9997 \cdot 0.9685 \cdot 0.9477$$
$$= 0.9176$$



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- Component Importance
- Data Analysis:
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  - Failure data
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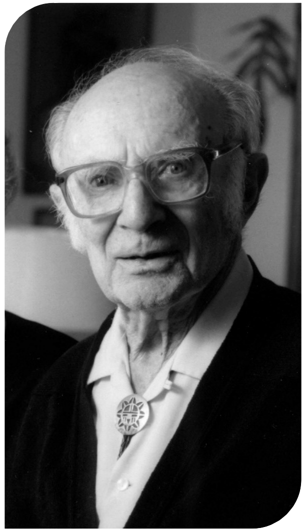
Component Importance

- Identify the most important component in a system
- Objective:
  - To identify components for improving system design
  - To allocate inspection and maintenance resources to most important components
- Birnbaum's measure:

$$I_i = \frac{\partial R_S}{\partial R_i}$$

  - Partial differentiation of R_S with respect to the component's reliability R_i
  - The larger I_i the more important the component
  - If I_i is large a small change in R_i will result in equally large change in R_S



Zygmunt William Birnbaum

Component Importance

- Recapping some differential calculus
  - Rule 1: If c is a constant, then:

$$\frac{\partial}{\partial x}(c) = 0$$

  - Rule 2: If n is a integer, then

$$\frac{\partial}{\partial x}(x^n) = nx^{n-1}$$

- For more differentiation rules visit: https://en.wikipedia.org/wiki/Differentiation_rules

Examples: Series

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- Consider a simple series system where R1=0.98 and R2=0.96. What is the component importance of both components?

$$R_S = \prod_{i=1}^{w} R_i = R_1 \cdot R_2$$

- Rule 2: If n is a integer, then

$$\frac{\partial}{\partial x}(x^n) = nx^{n-1}$$

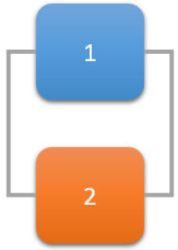
$$I_1 = \frac{\partial R_S}{\partial R_1}(R_1 R_2) = 1 \cdot R_1^{1-1} R_2 = R_2 = 0.96$$

$$I_2 = \frac{\partial R_S}{\partial R_2}(R_1 R_2) = 1 \cdot R_2^{1-1} R_1 = R_1 = 0.98$$

Examples: Active (Parallel) Redundancy

- Consider a simple parallel system where R1=0.98 and R2=0.96. What is the component importance of both components?

$$R_S = 1 - \prod_{i=1}^{n} (1 - R_i)$$
$$= 1 - (1 - R_1)(1 - R_2)$$
$$= 1 - (1 - R_2 - R_1 + R_1 R_2)$$
$$= \underline{R_2} + \underline{R_1} - \underline{R_1 R_2}$$



- Rule 1: If c is a constant, then:

$$\frac{\partial}{\partial x}(c) = 0$$

- Rule 2: If n is a integer, then

$$\frac{\partial}{\partial x}(x^n) = n x^{n-1}$$

$$I_1 = \frac{\partial R_S}{\partial R_1} = \underline{0} + \underline{1 \cdot R_1^{1-1}} - \underline{1 \cdot R_1^{1-1} R_2} = 1 - R_2 = 0.04$$

$$I_2 = \frac{\partial R_S}{\partial R_2} = \underline{1 \cdot R_2^{1-1}} + \underline{0} - \underline{1 \cdot R_2^{1-1} R_1} = 1 - R_1 = 0.02$$

Examples: k-out-of-n

- Consider a 2-out-of-3 system, where R1=0.98, R2=0.96 and R3=0.94. Which component has the highest importance?

$$R_S = R_1R_2(1-R_3) + R_1R_3(1-R_2) + R_2R_3(1-R_1) + R_1R_2R_3$$
$$= R_1R_2 + R_1R_3 + R_2R_3 - 2R_1R_2R_3$$

$$I_1 = \frac{\partial R_S}{\partial R_1} = R_2 + R_3 - 2R_2R_3 = \underline{0.0952}$$

$$I_2 = \frac{\partial R_S}{\partial R_2} = R_1 + R_3 - 2R_1R_3 = 0.0776$$

$$I_3 = \frac{\partial R_S}{\partial R_3} = R_1 + R_2 - 2R_1R_2 = 0.0584$$

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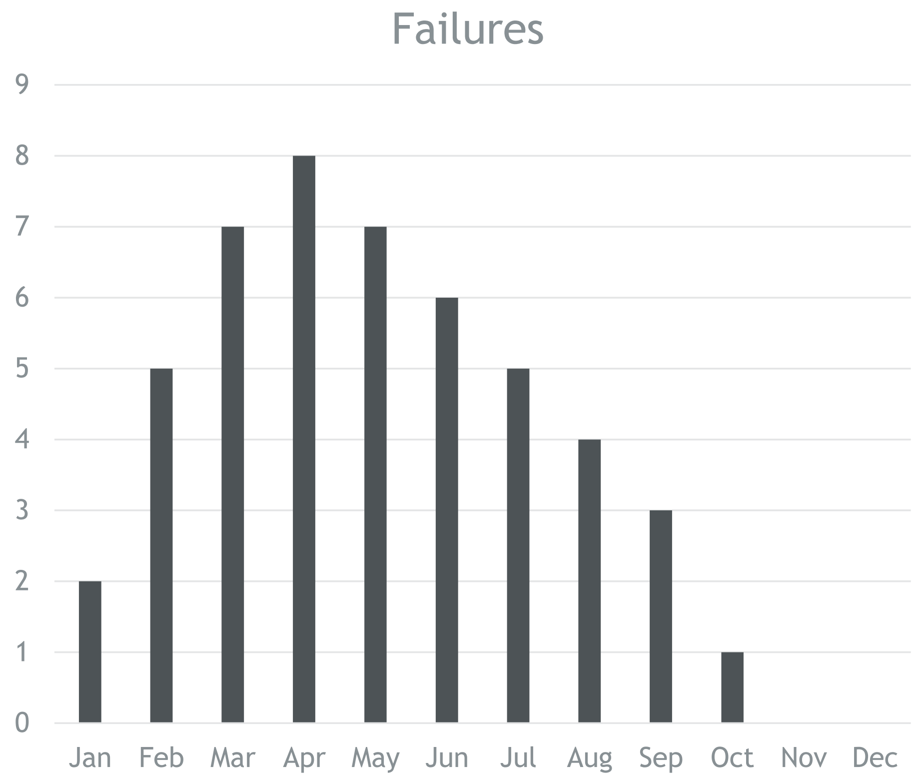
Relative Frequency Diagram

- 48 pumps are installed in January
- All have failed by November

Month	Failures
Jan	2
Feb	5
Mar	7
Apr	8
May	7
Jun	6
Jul	5
Aug	4
Sep	3
Oct	1
Nov	0
Dec	0

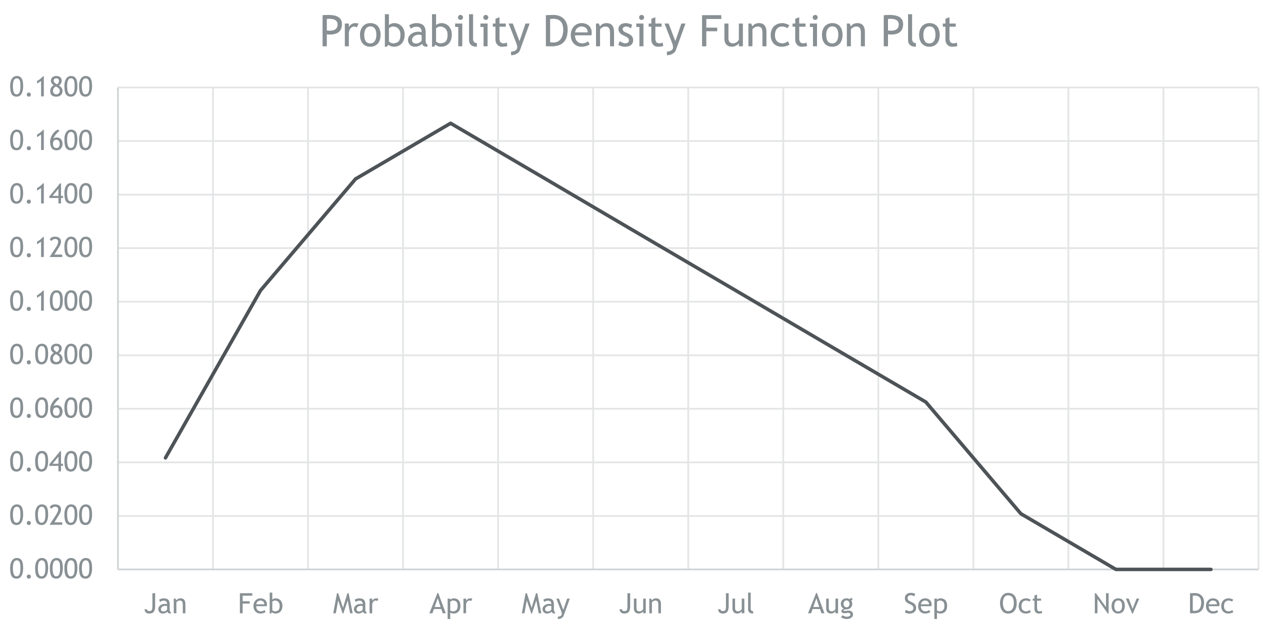


Probability Density Function (PDF)

- Probability of failure occurring at **any specific time**
- PDF gives probability of failure occurring during the following time unit
- $\mathrm{f}(t) = \frac{1}{N} \cdot \frac{\Delta n}{\Delta t}$
  - $\Delta n =$ number of failures in the time interval $[\mathrm{t}, \mathrm{t} + \Delta n]$
  - $\Delta t =$ length of time interval
  - $N =$ original population

Probability Density Function (PDF)

- The area underneath the function's plot represents the cumulative probability that pumps would have failed by a specific time
- The total area underneath the plot adds to 1 or 100%

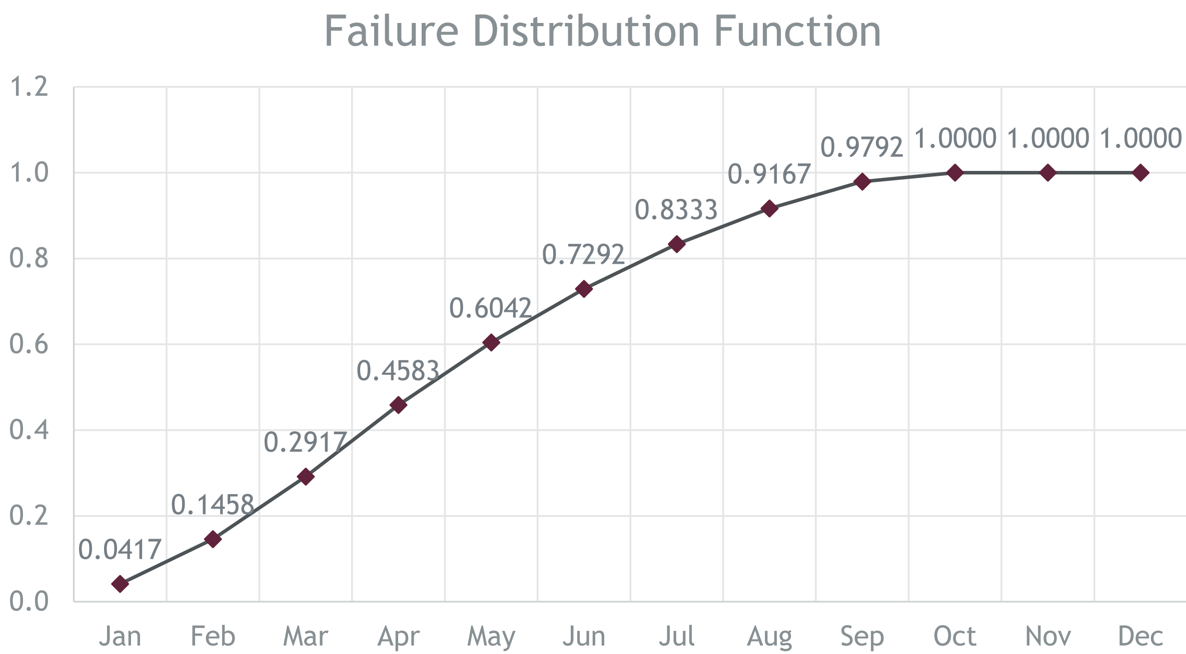


Failure Distribution Function

- Cumulative probability density (CPD)
- Probability that failure has occurred **<u>on or before a specific time</u>**
- $\mathrm{F}(t) = \frac{\sum n_i}{N}$
  - $\sum n_i$ = number of failures up to time, t
  - N = original population

Failure Distribution Function

- 45.8% (22/48) pumps have failed by April
- 83% of pumps have failed by mid-July
- 100% by November



Reliability Function

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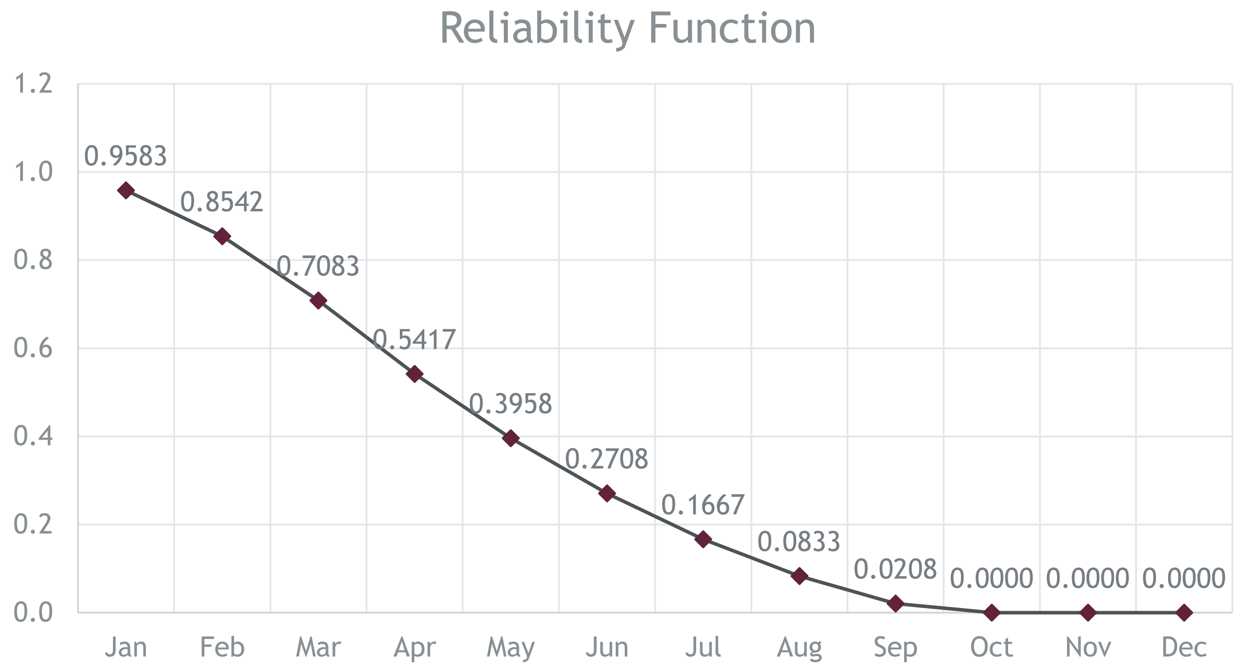
- Probability of survival **up to a specific time**
- Reliability function is the complement of the failure distribution function
- $\mathrm{R}(t) = 1 - F(t)$

Reliability Function

- Probability that a pump will last until July is 17% (1-0.83)
- What is the probability that a pump will last for 6 months?
  - 27%

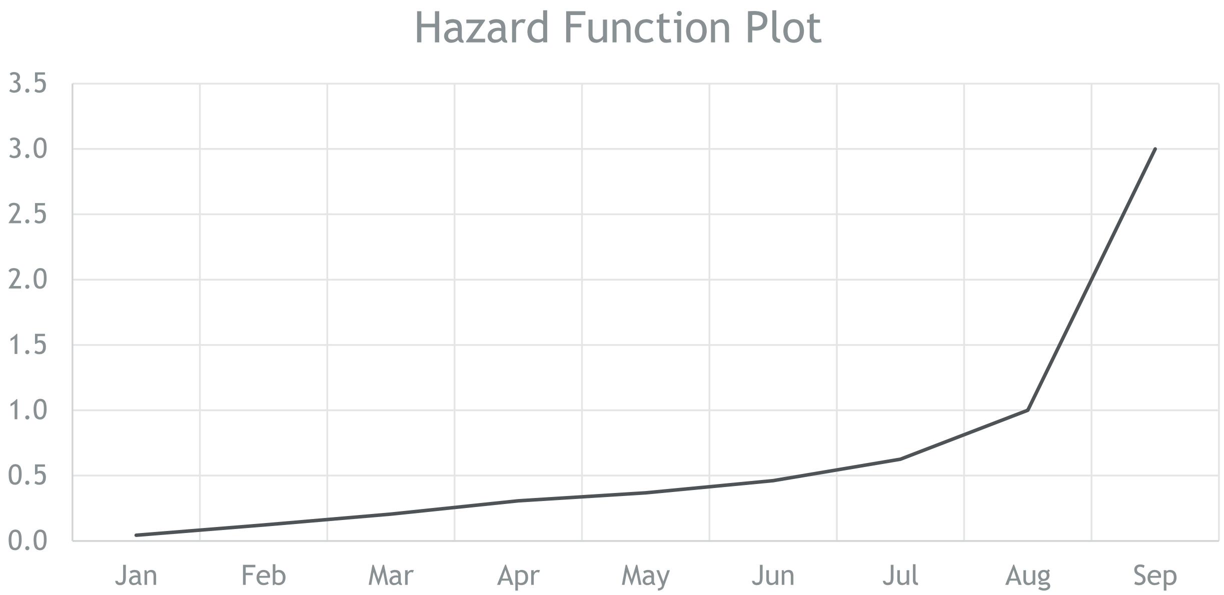


Hazard Function (Rate)

- Probability that a pump will fail at a certain age, given that it survived up to that age
- Indication of the risk of failure at a specific time
- $\mathrm{h}(t) = \frac{1}{n(t)} \cdot \frac{\Delta n}{\Delta t} = \frac{f(t)}{R(t)}$
  - $\Delta n =$ number of failures in the time interval $[\mathrm{t}, \mathrm{t} + \Delta n]$
  - $\Delta t =$ length of time interval
  - $n(t) =$ population surviving at time t

Hazard Function (Rate)

- Hazard rate can take the form of three shapes:
  - Decreasing, Increasing, Constant
- The combined shapes are referred to, as the so-called “bathtub curve”



Example

The table consists of failure observations, i, (in column 1) and operating times for 10 electronic components which failed in the same position (in column 2).

The first component failed at 8 hours. It was replaced with a second new component, which then failed after another 12 hours, and so forth.

Failure Obs., i	Operating Time
1	8
2	20
3	34
4	46
5	63
6	86
7	111
8	141
9	186
10	266

Example

Obs, i	Interval, h	Op. Time	△t	f(t)	F(t)	R(t)	H(t)
1	0≤h<8	8	8	0.0125	0	1	0.0125
2	8≤h<20	20	12	0.0083	0.1	0.9	0.0093
3	20≤h<34	34	14	0.0071	0.2	0.8	0.0089
4	34≤h<46	46	12	0.0083	0.3	0.7	0.0119
5	46≤h<63	63	17	0.0059	0.4	0.6	0.0098
6	63≤h<86	86	23	0.0043	0.5	0.5	0.0087
7	86≤h<111	111	25	0.0040	0.6	0.4	0.01
8	111≤h<141	141	30	0.0033	0.7	0.3	0.0111
9	141≤h<186	186	45	0.0022	0.8	0.2	0.0111
10	186≤h<266	266	80	0.0013	0.9	0.1	0.0125

$$f_4(t) = \frac{1}{N} \cdot \frac{\Delta n}{\Delta t} = \frac{1}{10} \cdot \frac{1}{12} = 0.0083$$

$$R_4(t) = 1 - F_4(t) = 1 - 0.3 = 0.7$$

$$F_4(t) = \frac{\sum n_i}{N} = \frac{3}{10} = 0.3$$

$$h_4(t) = \frac{1}{n(t)} \cdot \frac{\Delta n}{\Delta t} = \frac{1}{7} \cdot \frac{1}{12} = 0.0119$$

Example

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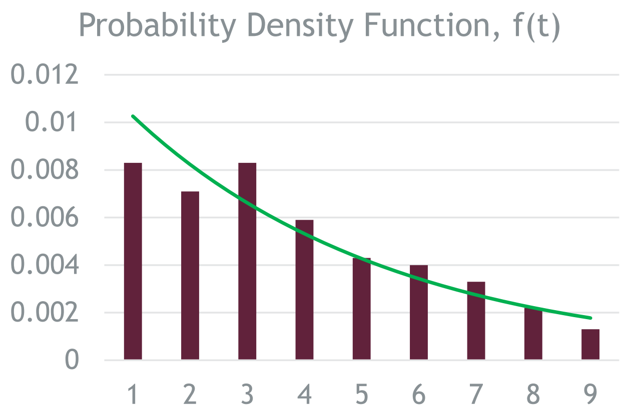

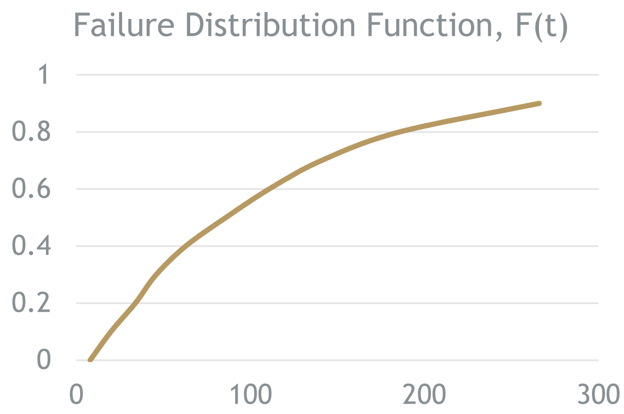

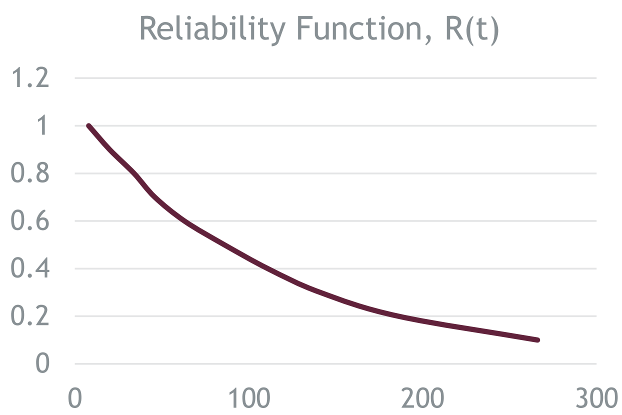

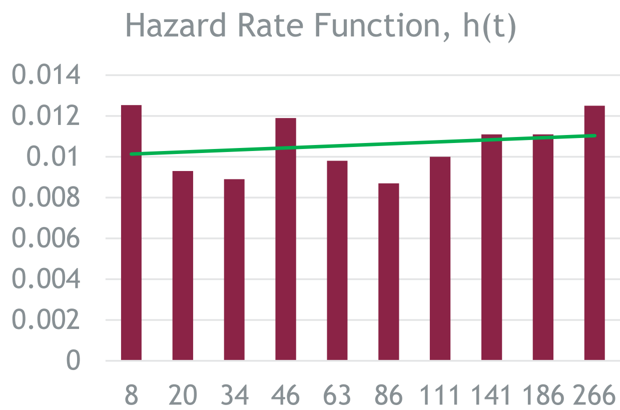

Terminology

- Part (or Component)
  - Item not subject to disassembly; discarded after its first, and only failure
- Socket
  - Space or equipment position which, at any given time, holds a part of a given type
- System
  - Collection of two or more sockets and their associated parts, interconnected to perform one or more functions.
- Non-Repairable System
  - A system which is discarded the first time that it ceases to perform to specification
- Repairable System
  - A system, which, after failing to perform at least one of its require functions, can be restored to performing all of its required functions by any method, other than replacement of the entire system.

Terminology

- Pump Network Example

Three Types of Reliability Statistics

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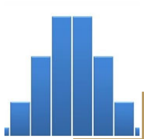
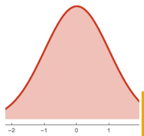


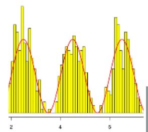
Discrete probability distributions

- Describe discrete events, e.g. light bulb works or not; pressure vessel passes or fails inspection
- Binomial- and Poisson distributions



Continuous probability distributions

- Describe events with continuous variables, e.g. time; distance travelled
- Normal-, lognormal-, exponential-, gamma-, Weibull distributions



Stochastic point processes

- Describes events where more than one failure can occur in the time continuum
- Ordinary renewal process, Non-homogeneous Poisson process, generalized renewal process

Non-Repairable Components (Systems)

- A system that is discarded after its first failure on system level
- Independent and identically distributed (IID) failure data
  - Each variable has the same probability distribution as the other
  - All variables are mutually independent
- Life data analysis
- Component assumed “as-good-as-new”

Repairable Systems

- A system that can be restored to perform its function by any method other than complete replacement after its first failure on system level
- Stochastic point process
  - Sequence of inter-dependent random events
  - Events occurring first, affect future failures
- System assumed “same-as-old”
  - Part in socket is “as-good-as-new”, but various other parts keeps operating at various ages
- System generating failure data with <u>increasing or decreasing trend</u>

Failure Rates

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- Frequency with which parts or systems failure
- Failure per time unit

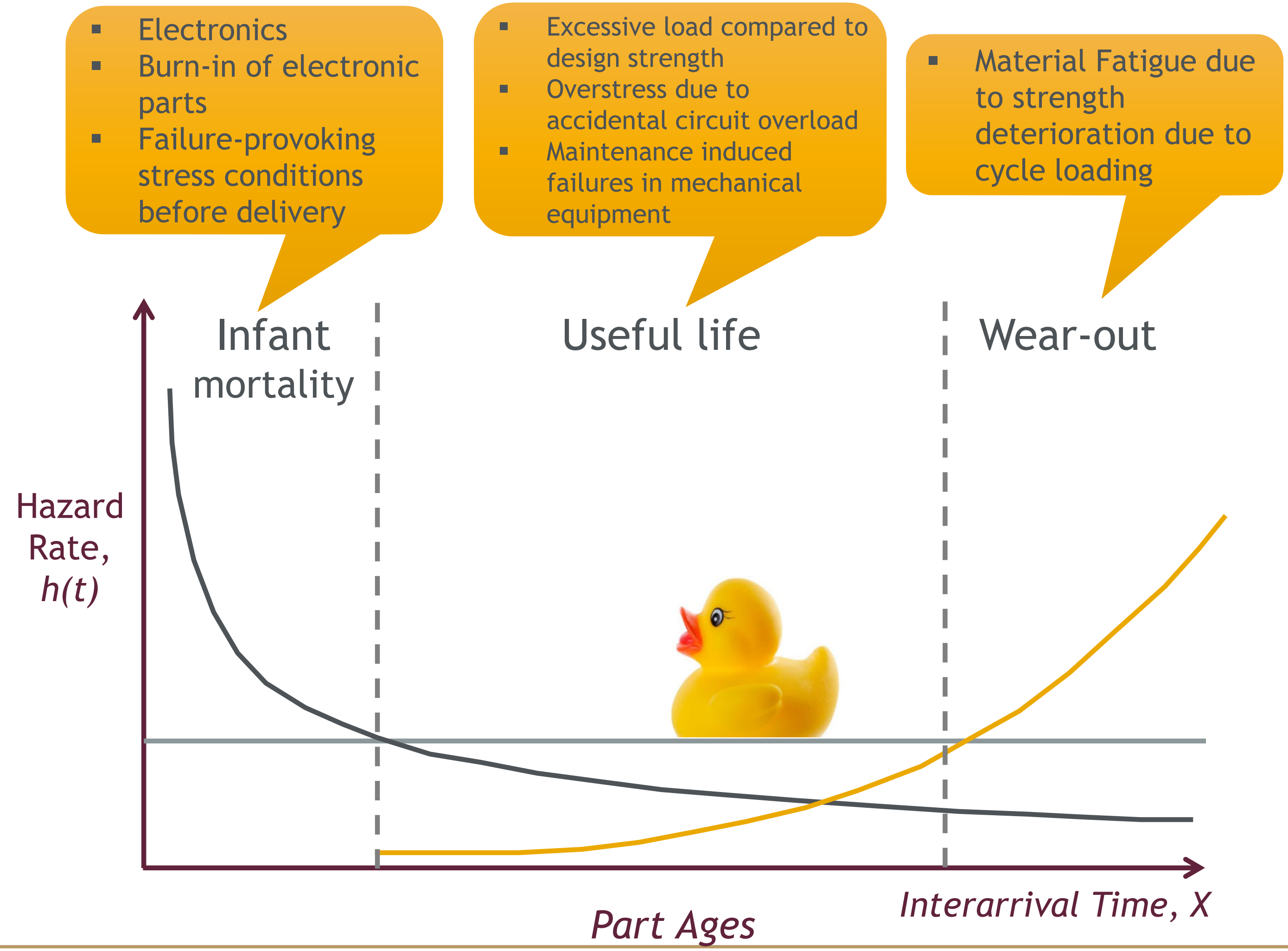
Hazard Rate

- Non-repairable parts
- Conditional probability of failure, given failure has not occurred
- Local time (interarrival times), X
- Decreasing, Constant, or Increasing

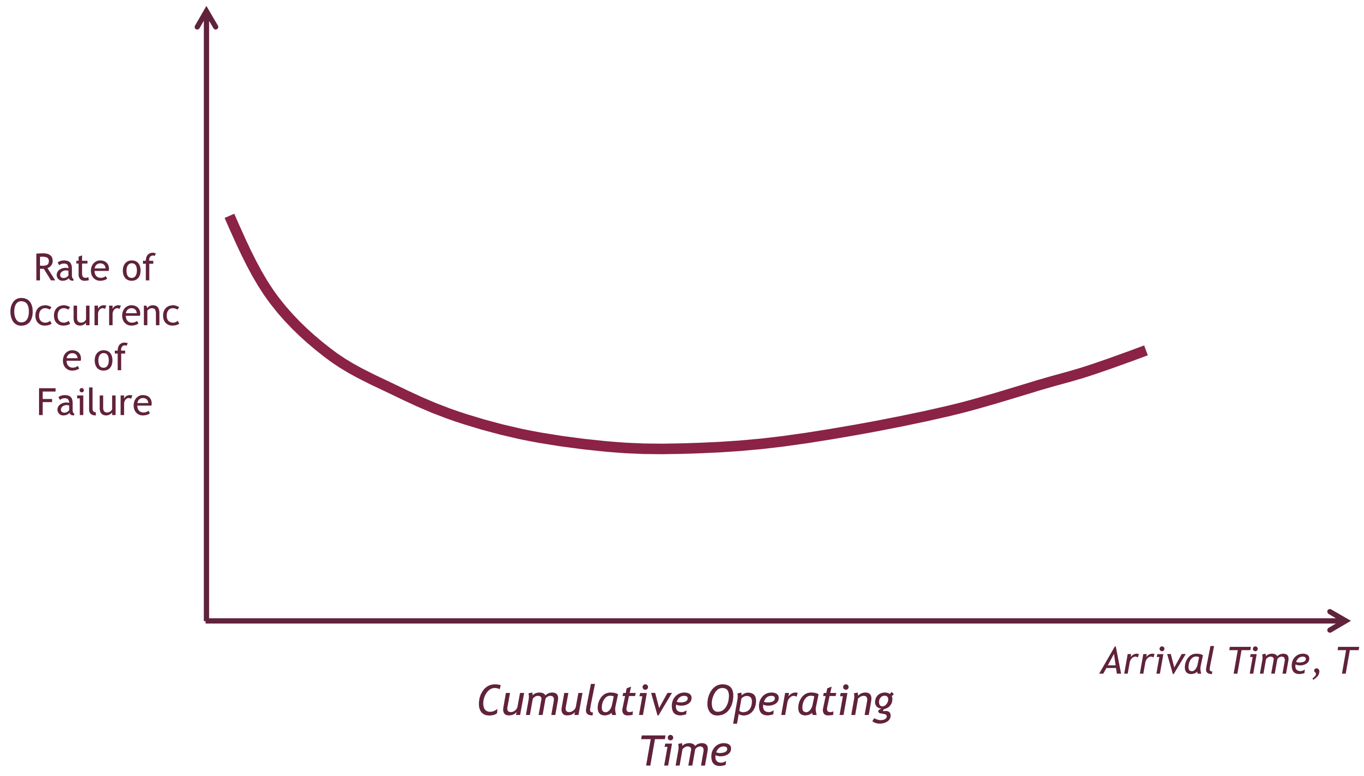
Rate of Occurrence of Failure (ROCOF)

- Repairable systems
- Unconditional probability of failure
- Global time (arrival times), T
- Decreasing, Constant, or Increasing

Bathtub Curve of the Hazard Rate



Bathtub Curve of the ROCOF



Failure Distribution Research (Nolan and Heap, 1978)