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QUALITY MANAGEMENT 444

Lecture 19 (Week 10) & Tutorial 9

Chapter 20 – Accurate and reliable data and measurement systems >> Design of Experiments

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Lecture 19 - DoE

After completing the lecture and assignment you will be able to:

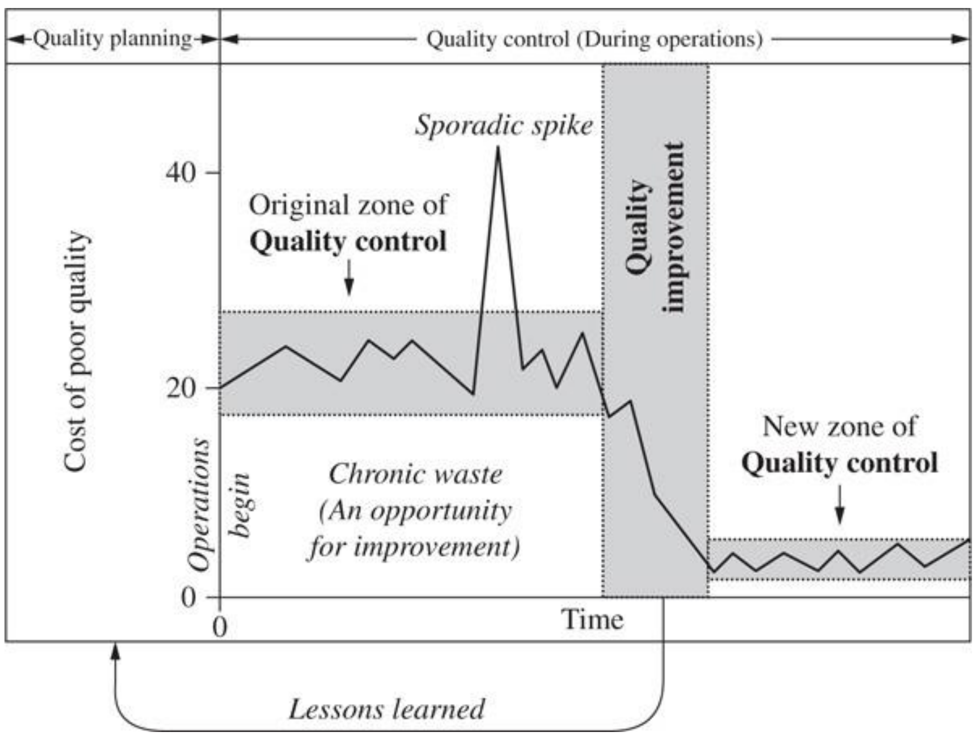
- **Recognise when a DoE might be useful**
- **Create, execute and analyse 2^k factorial DOE's (to an extent)**

Remember:

- We are only covering a small (but useful) portion of it. DoE is a much bigger subject
- "Do not try this alone" – consult with someone that has done it before
- DoE is not just for statisticians and chemical engineers. It's too powerful not to try.

DoE

- **Design of Experiments (DOE) is a structured, scientific approach to planning, conducting, analyzing, and interpreting controlled tests to determine the factors that influence a process or product's performance.**



DoE

- **Design of Experiments (DOE) is a structured, scientific approach to planning, conducting, analyzing, and interpreting controlled tests to determine the factors that influence a process or product's performance.**

- **It helps you learn efficiently — finding out *which factors matter most*, *how they interact*, and *what combination leads to the best outcome* — using the fewest possible experiments.**

DoE

Concept	Meaning
Factors	The inputs or variables you can control (e.g., temperature, pressure, grinding speed)
Levels	The different settings or values of each factor (e.g., 100° C, 120° C)
Response	The output you measure (e.g., defect rate, surface finish, yield)
Interactions	How two or more factors jointly influence the response (e.g., temperature and pressure combined affect surface finish)

Why?

From an industrial engineering perspective we're trying to use experimentation for the following purposes:

- reduce **time** to design/develop new products & processes
- improve **performance** of existing processes
- improve **reliability** and performance of products
- achieve product & process **robustness**
- perform **evaluation** of materials, design alternatives, setting component & system tolerances, etc.

We always want to **fine tune or improve the process**. In today's global world this drive for competitiveness affects all of us both as consumers and producers

Example

- How does the following three factors impact fuel consumption?
  - Tire pressure
  - Speed driven
  - Fuel octane rating
- Set up test station in laboratory to control every variable
  - Sits on rollers
  - Fixed amount of fuel
  - Measure distance
  - Calculate fuel efficiency

Example

Trial	Factor Changed	Tire pressure	Speed	Octane	Miles per Gallon
1		35	55	85	24
2					
3					
4					

Example

Trial	Factor Changed	Tire pressure	Speed	Octane	Miles per Gallon
1		35	55	85	24
2	Tire Pressure	**40**	55	85	32
3					
4					

Example

Trial	Factor Changed	Tire pressure	Speed	Octane	Miles per Gallon
1		35	55	85	24
2	Tire Pressure	40	55	85	32
3	Speed	40	**65**	85	25
4					

Example

Trial	Factor Changed	Tire pressure	Speed	Octane	Miles per Gallon
1		35	55	85	24
2	Tire Pressure	40	55	85	32
3	Speed	40	65	85	25
4	Octane	40	55	**90**	25

Example

One Factor at A Time

Trial	Factor Changed	Tire pressure	Speed	Octane	Miles per Gallon
1		35	55	85	24
2	Tire Pressure	40	55	85	32
3	Speed	40	65	85	25
4	Octane	40	55	90	25

Example

Design of Experiment (DoE)

Trial	OFAT trial	Tire pressure	Speed	Octane	Miles per Gallon
1	X	35	55	85	24
2	X	40	55	85	32
3					
4	X	40	65	85	25
5					
6	X	40	55	90	25
7					
8					

Example

Design of Experiment (DoE)

Trial	OFAT trial	Tire pressure	Speed	Octane	Miles per Gallon
1	X	35	55	85	24
2	X	40	55	85	32
3		**35**	**65**	**85**	**38**
4	X	40	65	85	25
5		**35**	**55**	**90**	**36**
6	X	40	55	90	25
7		**35**	**65**	**90**	**29**
8		**40**	**65**	**90**	**37**

Example

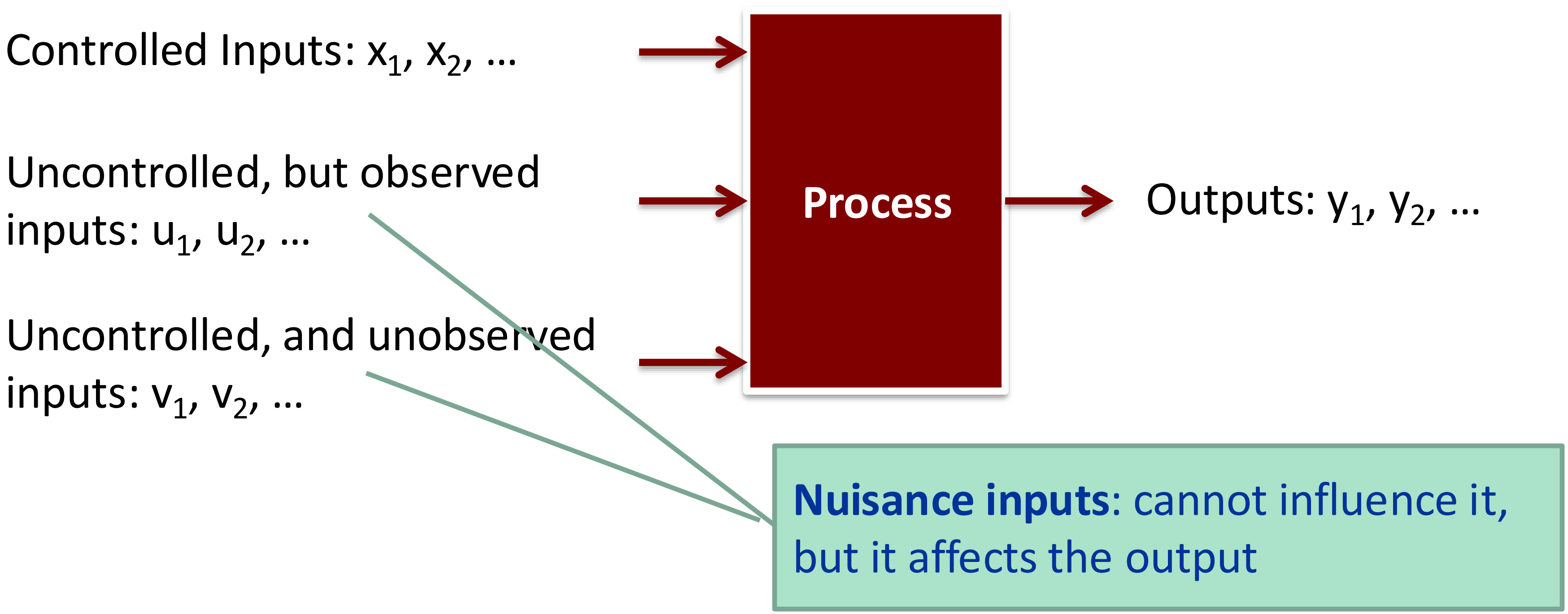
Design of Experiment (DoE)

Trial	OFAT trial	Tire pressure	Speed	Octane	Miles per Gallon
1	X	35	55	85	24
2	X	40	55	85	32
3		**35**	**65**	**85**	**38**
4	X	40	65	85	25
5		**35**	**55**	**90**	**36**
6	X	40	55	90	25
7		**35**	**65**	**90**	**29**
8		**40**	**65**	**90**	**37**

Why do we experiment?

Understand relationship between inputs and outputs:

- **Modeling – understand the process, inputs vs outputs**
- **Optimisation – get the output that we want**
- **Control – what changes in input to move to new target value?**



Dealing with three types of inputs

Controlled Inputs (*x*)

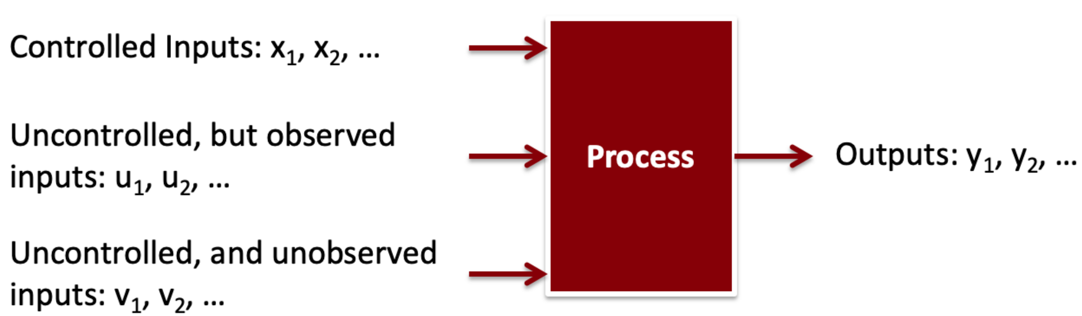
- Variation + **replication**: use our control over them to vary the inputs and repeat the experiment in a systematic way
- Called **factors,** which are changed to specific **'levels'**

Uncontrolled but Observed (*u*)

- **Blocking** – group experiments into blocks, each block having a fixed value of *u*.
- **Analysis of covariance** – model the impact of *u*.

Uncontrolled and unobserved (*v*)

- **Randomisation** – let impact of *v* average out to zero



Example: detergents

- Suppose you want to compare three detergents for their ability to clean clothes in a washing machine
- We will measure 'ability to clean' by level of 'whiteness'

Response Variable	Factor	Level	Uncontrolled but observed	Uncontrolled and unobserved
• Whiteness	• Detergent type	• Omo = A • Ariel = B • Sunlight = C	• Machine type • Water temperature • Washing time • Amount of detergent	• Wear & tear • Water hardness • Stain type & amount • Temperature

Dealing with three types of inputs

- **Controlled Inputs (*x*)**
  - Variation + **replication**: use our control over them to vary the inputs and repeat the experiment in a systematic way
  - Called **factors,** which are changed to specific **'levels'**

- **Uncontrolled but Observed (*u*)**
  - **Blocking** – group experiments into blocks, each block having a fixed value of *u.*
  - **Analysis of covariance** – model the impact of *u*.

- **Uncontrolled and unobserved (*v*)**
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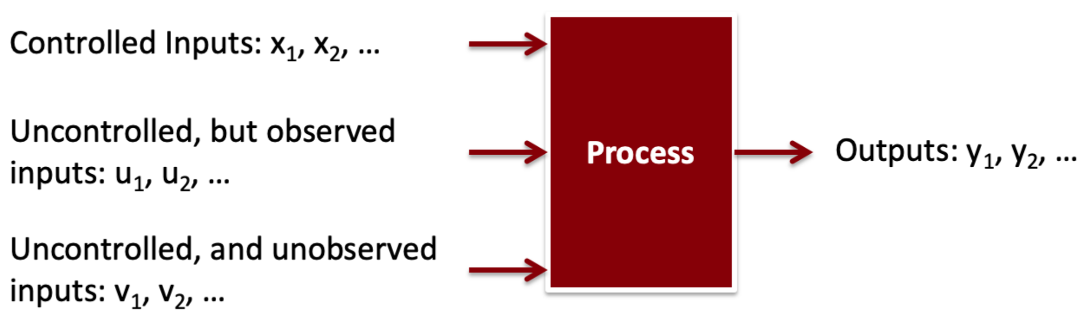


Example: detergents (classical design)

- The more **replicates**, the smaller the sampling error
- All uncontrolled but observed inputs held constant
  - **Blocking**
- **Randomise** the run order to deal with uncontrolled and unobserved inputs
- Conclusions about detergent only apply to the specific conditions of the experiment
  - Specific water temp
  - Specific type of machine
  - Specific washing time
  - Specific amount of detergent

Omo A	Ariel B	Sunlight C
X	X	X
X	X	X
X	X	X

Replicates

Example: detergents

- Introduce second factor at three levels, namely washing machine brand:
  - Type I = Samsung
  - Type II = Defy
  - Type III = LG
- Why is this a faulty design?
  - It would not be possible to determine whether an observed difference in the response variable was as a result of the detergent or the washing machine brand.

Type I	Type II	Type III
A	B	C
A	B	C
A	B	C

Replicates

Example: detergents

- **Completely randomised design**
  - Pure randomisation ✓
  - Also deals with uncontrolled and unobserved
- **Why is this a faulty design?**
  - Pure randomisation can eliminate otherwise useful information
    - Detergent A not used with machine brand III
    - ✗ Detergent B not used with machine brand I
    - Design is **not 'balanced'**

Type I	Type II	Type III
C	B	B
A	C	B
A	A	C

Block or control what you can and randomise what you cannot

Example: detergents

- **Randomised block design**
  - Each block is a machine brand
  - Detergents are run in random order within each block

Type I	Type II	Type III
B	A	C
C	C	A
A	B	B

- **Why is this a good design?**
  - Guards against possible bias from order in which detergents are used
    - I.e. uncontrolled and unobserved inputs
  - Hypothesis test can be run to compare detergent
  - Hypothesis test can be run to compare machine type
  - All nine observations used in each hypothesis test

Example: detergents

- **Fractional Factorial Experiments**
- Latin Square Design
  - Each detergent only once with each machine, and
  - Only once with each temperature
  - Remains balanced
- All three factors can be evaluated with only 9 tests
- Significant cost & time savings
- Weakness:
  - Assumes no interaction

Temp	Type I	Type II	Type III
1	C	A	B
2	B	C	A
3	A	B	C

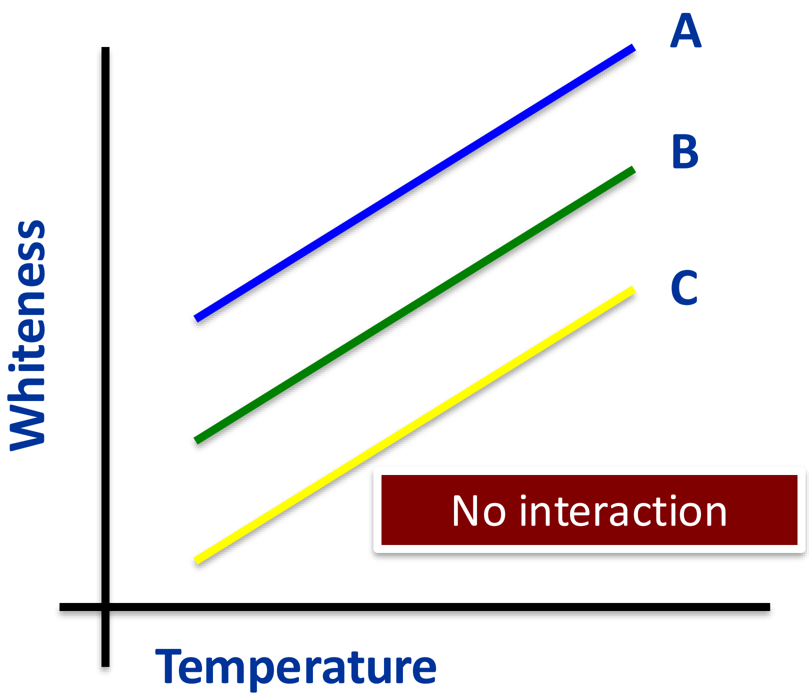
Example: detergents

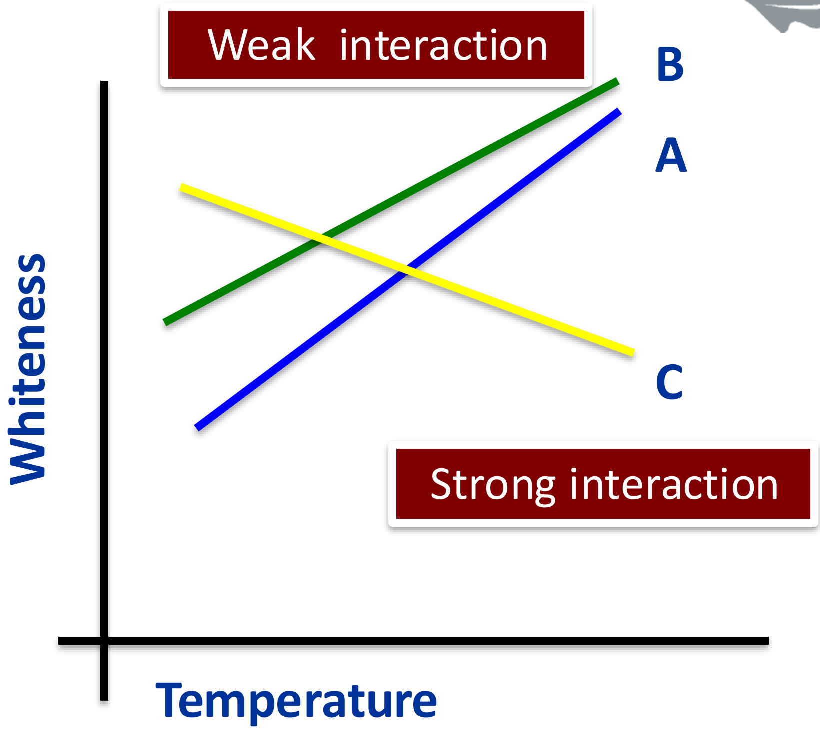
- Suppose we want to study 3rd factor, such as water temp?
  - Each block is a machine brand
  - Detergents are run in random order within each block
  - Water temp levels = 1, 2 & 3
- Why is this a good design?
  - Factorial ✓
    - One test for every condition
  - Separate hypothesis test for each main factor using all 27 runs. ✓
  - Calculate **interaction** effects ✓
    - Detergent x machine
    - Detergent x temp
    - Machine x temp
    - Detergent x machine x temp

3 x 3 x 3 = 27 runs!

	Type I			Type II			Type III		
	A	B	C	A	B	C	A	B	C
1	X	X	X	X	X	X	X	X	X
2	X	X	X	X	X	X	X	X	X
3	X	X	X	X	X	X	X	X	X

Interaction





Features of Factorial Designs: Balance

- **Balance**: Each of the variables in the experiment have the same number of runs at the high (+1) and low (-1) levels.
- This property helps to simplify the mathematical analysis of the design by giving each level of the variable an equal impact on the final result.

Run	A	B
1	-1	-1
2	+1	-1
3	-1	+1
4	+1	+1
Σ	**0**	**0**

Features of Factorial Designs: Orthogonality

- **Orthogonality**: If we multiply the signs of each of the variable columns, we get the “AB” column at the right side of the design below.
  - This column represents the interaction between A and B.
- The property of “Orthogonality” ensures that the plus and minus signs are arranged in the main effects columns (A and B) such that their product column will be balanced which, in turn, ensures that each of the factors will be independent of each other.

Run	A	B	AB
1	-1	-1	+1
2	+1	-1	-1
3	-1	+1	-1
4	+1	+1	+1
Σ	**0**	**0**	**0**

Example: detergents

- Full Factorial designs are very powerful, but number of runs grows very quickly as more factors are added
- Two strategies:
  - Confine initial experiments to only two levels (**2^k designs**)
    - Assumes linearity over ‘small region’ of analysis
    - Include centre points if not
    - Response surface modelling
  - **Fractional factorial** designs used for screening

3 x 3 x 3 = 27 runs!

	Type I			Type II			Type III		
	A	B	C	A	B	C	A	B	C
1	X	X	X	X	X	X	X	X	X
2	X	X	X	X	X	X	X	X	X
3	X	X	X	X	X	X	X	X	X

When to use Fractional Factorial Design

- Used when **time and resources are limited**.
- Advantages are simplicity and **economy**
- Eliminates Higher Order Interactions for less runs.
- Narrowing the Xs down to direct focus on the vital few.
- Design consists of a **fraction of the runs** of a full factorial design.
- Typically used for **screening** experiments in order to identify a few important factors and the lower order interactions.
- Runs are chosen so that if some factors are confounded with others, the fractional factorial becomes a higher resolution design for remaining factors.

What is DoE?

- Design of Experiments (DoE) is a **scientific process** of planning an experiment that will yield statistically useful results
  - An **experiment** is the deliberate variation of one or more process variables while observing the effect on one or more response variables
  - In an **observation**, we observe both the process variables and the response variables

- DoE allows the experimenter to study the effect of **many factors** that may influence the product or process **simultaneously**

- The goal of DoE is to get the most information from the least amount of data, i.e. **minimise time and cost**

When to use DoE / types of DoE?

Fractional Factorial DoE's

- Screening DoE's
- To find the key factors that affect a response
- "Screen" many factors at one time in order to determine which are worthy of deeper investigation

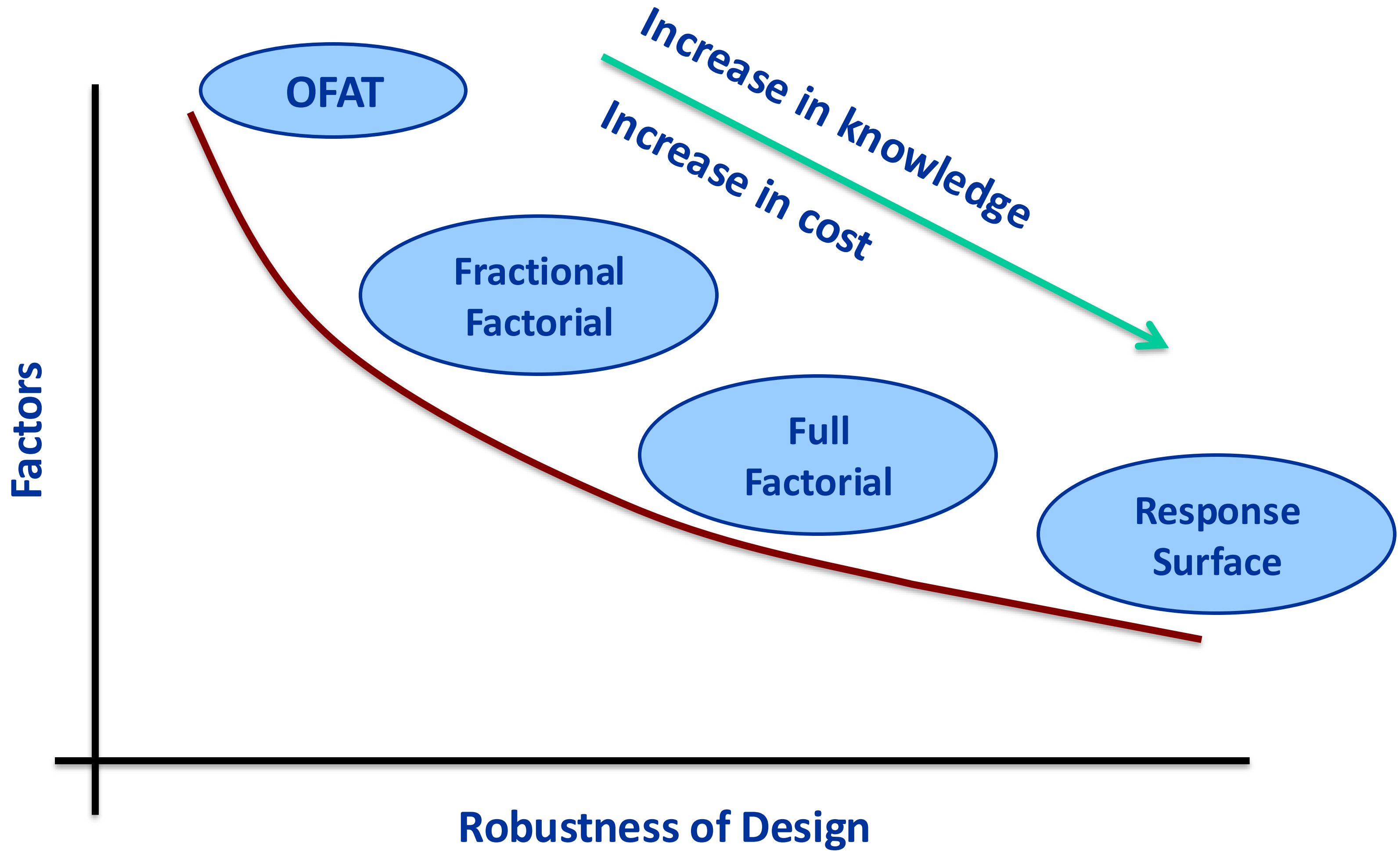
Full Factorial DoE's

- Test factors across all possible combinations in order to determine which factors are statistically significant
- Characterisation studies
- Take longer to complete and cost more money

Response Surface Modelling (RSM) DoE's

- Optimisation DoE's
- Help us identify optimal factor settings in order to hit specific targets
- Most advanced, but extremely powerful

Approach to DoE



Why use DoE?

- **To establish cause and effect**
  - DOE avoids false correlations because you literally change the inputs and see (or don't see) measurable change in output
- **To find interaction between variables**
  - OFAT experiments may miss key interactions
  - Joint effects can only be seen by changing multiple inputs simultaneously
- **DOE is economical**
  - Each run provides information on each input
  - Design the experiment to get the most information from the least amount of data