Reliability Modelling / Betroubaarheidsmodelering

Series Systems

$R = \prod_{i=1}^{w} R_i$

Active Parallel Systems (k-out-of-w)

$R = \sum_{j=k}^{w} \frac{w!}{j!(w-j)!} R_i^j (1-R_i)^{w-j}$

Standby Parallel Systems

$R = 1 - \prod_{i=1}^{w} (1-R_i)$

Complex Systems

$R = \left[\sum_t \left(\prod_i R_i \prod_j F_i\right)\right] + \prod_{y=1}^{w} R_y$

Poisson Models

$R = (1+\lambda t)e^{-\lambda t}$

$R = 1 - \sum_{i=0}^{n-1} \frac{(-\lambda t)^i e^{-\lambda t}}{i!}$

$R = e^{-k\lambda t} \sum_{i=0}^{n-k} \frac{(k\lambda t)^i}{i!}$

Importance Measure

$l_i = \frac{\partial R_S}{\partial R_i}$

Laplace Trend Test / Tendenstoets

$U = \frac{\sum_{i=1}^{r-1} T_i - \frac{(r-1)T_r}{2}}{T_r \sqrt{\frac{1}{12(r-1)}}}$

Non-repairable Components / Nie-herstelbare Komponente

Weibull Distribution

Probability Density Function:

$f_X(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \cdot \exp \quad -\left(\frac{x}{\eta}\right)^{\beta}$

Cumulative Distribution Function:

$F_X(x) = 1 - \exp \quad -\left(\frac{x}{\eta}\right)^{\beta}$

Reliability Function:

$R_X(x) = \exp \quad -\left(\frac{x}{\eta}\right)^{\beta}$

Hazard Rate Function:

Expected Time to Next Failure

Conditional Expected Value (truncated):

$E\left[X_{r+1} \mid X_{r+1} \le X_P\right] = \frac{\int_x^{X_P} x \cdot f_X(x)\, dx}{\int_x^{X_P} f_X(x)\, dx}$

$\mu_{r+1} = E\left[X_{r+1} \mid X_{r+1} \le X_P\right] - x$

Unconditional Expected Value:

$E\left[X_{r+1}\right] = \frac{\int_0^{\infty} x \cdot f_X(x)\, dx}{\int_0^{\infty} f_X(x)\, dx}$

$\mu_{r+1} = E\left[X_{r+1}\right] - x$

$h_X(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$

Repairable Systems / Herstelbare Systems

Model ρ₁ (Exponential Trend Model)

Parameter Estimation:

$\min(\bar{\alpha}_0, \bar{\alpha}_1) = \sum_{i=1}^{r}[E[N(0 \to T_i)] - N(0 \to T_i)]^2$

Log-Likelihood Function:

$I_{\rho_1}(\alpha_0, \alpha_1) = r\alpha_0 + \alpha_1 \sum_{i=1}^{r} T_i - \frac{e^{\alpha_0}(e^{\alpha_1 T_r} - 1)}{\alpha_1}$

Intensity Function:

$\rho_1(t) = e^{\alpha_0 + \alpha_1 t}$

Expected Number of Failures:

$E[N(t_1 \to t_2)] = \frac{e^{\alpha_0+\alpha_1 t_2} - e^{\alpha_0+\alpha_1 t_1}}{\alpha_1}$

Reliability Function:

$R(t_1 \to t_2) = \exp\left(-\frac{e^{\alpha_0+\alpha_1 t_2} - e^{\alpha_0+\alpha_1 t_1}}{\alpha_1}\right)$

Expected Time to Next Failure:

$E[T_{r+1} \mid t = T_r] = \frac{\ln[(r+1)\alpha_1 + e^{\alpha_0}] - \alpha_0}{\alpha_1}$

Mean Time Between Failures:

$\mathrm{MTBF}_{\rho_1}(t_1 \to t_2) = \frac{\alpha_1(t_2 - t_1)}{e^{\alpha_0+\alpha_1 t_2} - e^{\alpha_0+\alpha_1 t_1}}$

Time Between Failures:

$\mu_{r+1} = T_{r+1} - T_r$

Model ρ₂ (Weibull Trend Model)

Parameter Estimation:

$\min(\bar{\lambda}, \bar{\delta}) = \sum_{i=1}^{r}[E[N(0 \to T_i)] - N(0 \to T_i)]^2$

Log-Likelihood Function:

$I_{\rho_2}(\lambda, \delta) = r(\ln\lambda + \ln\delta) - \lambda T_r^{\delta} + (\delta - 1)\sum_{i=1}^{r} \ln T_i$

Intensity Function:

$\rho_2(t) = \lambda\delta t^{\delta-1}$

Expected Number of Failures:

$E[N(t_1 \to t_2)] = \lambda(t_2^{\delta} - t_1^{\delta})$

Reliability Function:

$R(t_1 \to t_2) = e^{-\lambda(t_2^{\delta} - t_1^{\delta})}$

Mean Time Between Failures:

$\mathrm{MTBF}_{\rho_2}(t_1 \to t_2) = \frac{t_2 - t_1}{\lambda(t_2^{\delta} - t_1^{\delta})}$

Expected Time to Next Failure:

$E[T_{r+1} \mid t = T_r] = \left(\frac{1 + \lambda T_r^{\delta}}{\lambda}\right)^{1/\delta}$

Time Between Failures:

$\mu_{r+1} = T_{r+1} - T_r$

Availability / Beskikbaarheid

(MTBF = Mean Time Between Failures, MTTR = Mean Time To Repair)

Basic Availability Formulas

Failure Rate:

$$\lambda = \frac{1}{\text{MTBF}}$$

Repair Rate:

$$\mu = \frac{1}{\text{MTTR}}$$

Steady-State Availability:

$$A = \frac{\text{MTBF}}{\text{MTBF}+\text{MTTR}} = \frac{\mu}{\lambda+\mu}$$

Time-Dependent Availability:

$$A(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

Series System Availability:

$$A_{\text{Series}} = \prod_{i=1}^{w} \frac{\mu_i}{\lambda_i+\mu_i}$$

Parallel Configuration

Active Parallel System Availability:

$$A_{\text{Active}} = 1 - \prod_{i=1}^{w} \frac{\lambda_i}{\lambda_i+\mu_i}$$

Active k-out-of-w System Availability:

$$A_{\text{Active,k/w}} = 1 - \frac{1}{(\lambda+\mu)^w} \sum_{i=0}^{k-1} \binom{w}{i} \mu^i \lambda^{w-i}$$

Standby Parallel (2 components) Availability:

$$A_{\text{Standby,2}} = \frac{\mu^2+\mu\lambda}{\mu^2+\mu\lambda+\lambda^2}$$