

Department of Industrial Engineering University of Stellenbosch

Simulasie 442 : Simulation 442 2025

Tutoriaal 3 <i>Tutorial 3</i>	Punt: 73 <i>Mark:</i>	Ingeedatum: 08-08-2025 (10:00) B3003 <i>Due date:</i>
Instruksies:	Formateer alle syfers sinvol. Ontwikkel die modelle individueel. U mag in groepe van twee of minder werk om die vrae te beantwoord. Handig slegs een hardekopie van u antwoordstel in. Gebruik Tecnomatix en Excel vir u berekenings. Hierdie tutoriaal en prakties is verpligtend. Indien u nalaat om die vereistes betyds na te kom, sal u die module sak.	
<i>Instructions:</i>	<i>Format all numbers sensibly.</i> <i>Develop the models individually.</i> <i>You may work in groups of two or less when answering the questions.</i> <i>Submit one document only.</i> <i>Use Tecnomatix and Excel for your calculations.</i> <i>This tutorial and practical are compulsory.</i> <i>You will fail the module if you do not comply with the requirements, on time.</i>	

Question 1 [12]

Within the context of random number generation,

- (a) List four properties of a random number generator. [4]
- (b) What are the properties that define the behaviour and characteristics of discrete and continuous random variables? You can use the given table to answer this question. [8]

Property	Discrete Random Variables	Continuous Random Variables
Definition		
Distribution Function (CDF)		
Inverse CDF		
Examples		

Question 2 [6]

Use the linear congruential method (LCG) to generate a set of 41 random numbers where $a = 3$, $c = 2$ and $Z_0 = 5$, with Z_0 the seed number of the series. Choose an appropriate m such that the string repeats itself after six numbers. *Hint: Use the required properties of m to narrow down the options during your search.* The formula for LCG is

$$Z_i = (aZ_{i1} + c) \bmod m.$$

Generate the set of random numbers indicating the values of parameters, as well as the values of Z_i and U_i at each index.

Question 3 [8]

Determine the inverse transform of the following function and develop an expression for x .

$$f(x) = 3x + \frac{5}{8}, \quad 0 \leq x \leq \frac{5 + \sqrt{193}}{12}$$

Question 4 [8]

The Sanga is a food truck serving the Stellenbosch University Engineering faculty. Since their main customers are university students and staff, they experience varying preparation times for different orders throughout the day. Let x represent the number of minutes required to prepare a customer's order. The preparation time follows the probability density function

$$f(z) = 12z^2 e^{3z^3-2} \quad \text{where } 0 \leq z \leq \sqrt[3]{\frac{1}{3} \ln\left(\frac{2}{3} + e^{-2}\right) + \frac{2}{3}}.$$

Generate *five* observations from the distribution by means of the inverse transform method and the first five random numbers generated in Question 1. Show all the steps to determine the inverse transform.

Question 5 [12]

A delivery drone operates in a city, making package deliveries. The drone's battery level drops by $\text{Unif}(8, 25)$ percentage points (no decimals) after each delivery, and it takes $\text{Unif}(3, 18)$ minutes to travel to each delivery. The drone starts with a full battery (100%). Use the random numbers in the order given below to simulate the drone's operation.

Questions:

1. How much time did the drone take to make all six deliveries?
2. What is the battery percentage after the deliveries?

Random numbers to use in order:

0.45; 0.12; 0.89; 0.33; 0.76; 0.04; 0.61; 0.95; 0.28; 0.82

Question 6 [10]

Using an **efficient** algorithm, explain stepwise how would you generate four observations from the distribution given below using the numbers from the given set of uniformly distributed, independent random numbers 0.28; 0.74; 0.15; 0.92. [10]

$$f(x) = \begin{cases} 0.18 & x = 1 \\ 0.07 & x = 2 \\ 0.46 & x = 3 \\ 0.21 & x = 4 \\ 0.08 & x = 5 \end{cases}$$

Question 7 [17]

Develop an expression to estimate each of the following integrals using Monte-Carlo simulation. You may leave your final answer in factorised form.

(a)

$$I = \int_1^4 (\sqrt{x} + 3x)dx$$

[5]

(b)

$$I = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} dx.$$

[12]

Total: Cross-check: 73
