Department of Industrial Engineering
University of Stellenbosch

Simulasie 442 : Simulation 442
2025

MEMORANDUM

Tutoriaal 4 *Tutorial 4*	Punt: 85 *Mark:*	Ingeedatum: **15-08-2025** (10:00) B3003 *Due date:*
Instruksies:	Formatteer alle syfers sinvol. U mag in groepe van **twee** of minder werk om die vrae te beantwoord. Handig slegs een hardekopie van u antwoordstel in. Gebruik Excel, R of Matlab vir u berekenings. Die data vir hierdie tutoriaal is beskikbaar in die lêer Tut04_2025_RawData.xlsx. **Hierdie tutoriaal is verpligtend**. **Indien u nalaat om die vereistes betyds** **na te kom, sal u die module sak**.	
Instructions:	*Format all numbers sensibly.* *You may work in groups of **two** or less when* *answering the questions.* *Submit one hardcopy only.* *Use Excel, R or Matlab for your calculations.* The data for this tutorial is available in the file Tut04_2025_RawData.xlsx. ***This tutorial is compulsory.*** ***You will fail the module if you do not*** ***comply with the requirements, on time.***	

Question 1 [13]

Jamie frequently visits the Sanga coffee shop on campus during the busy morning rush to study for her upcoming exams. She becomes curious about customer behavior and decides to track how many coffee orders are placed every 15-minute interval during her study sessions. She meticulously records these observations and enters the data into the Excel file Tut04_2025_RawData.xlsx on the data sheet labelled 'Question 1'.

8	12	6	9	3	11	4	7
5	2	13	8	10	6	9	4
11	7	3	15	5	8	2	12
6	14	4	7	9	3	10	8
2	5	9	11	6	13	7	1

Table 1: Question 1

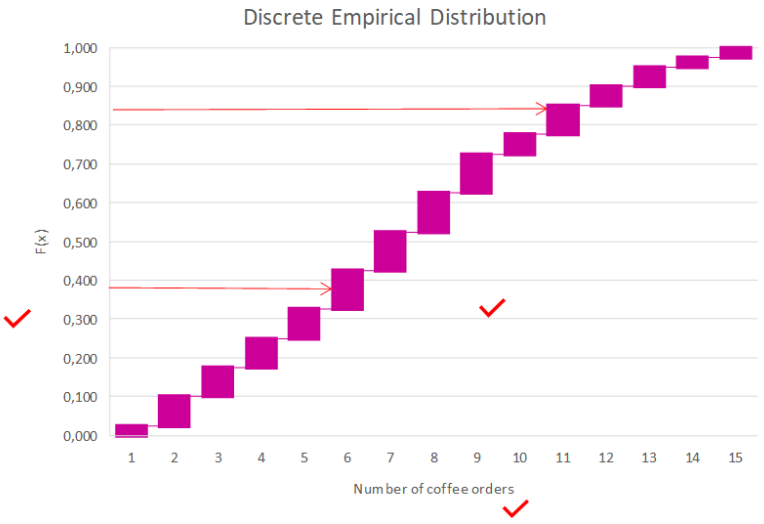
(a) Construct an empirical distribution for this dataset and create a graphical representation of the distribution. [9]

(b) Determine how many coffee orders correspond to the cumulative probabilities of 0.40 and 0.85. [4]

(a)

Min= 1
Max = 15

Classes	Frequency	Cumulative Frequency	Empirical Distribution	
1	1	1	0,025	0,025
2	3	4	0,100	0,075
3	3	7	0,175	0,075
4	3	10	0,250	0,075
5	3	13	0,325	0,075
6	4	17	0,425	0,100
7	4	21	0,525	0,100
8	4	25	0,625	0,100
9	4	29	0,725	0,100
10	2	31	0,775	0,050
11	3	34	0,850	0,075
12	2	36	0,900	0,050
13	2	38	0,950	0,050
14	1	39	0,975	0,025
15	1	40	1,000	0,025



(b) F(0.4) = 6

F(0.85) = 11

Question 2 [16]

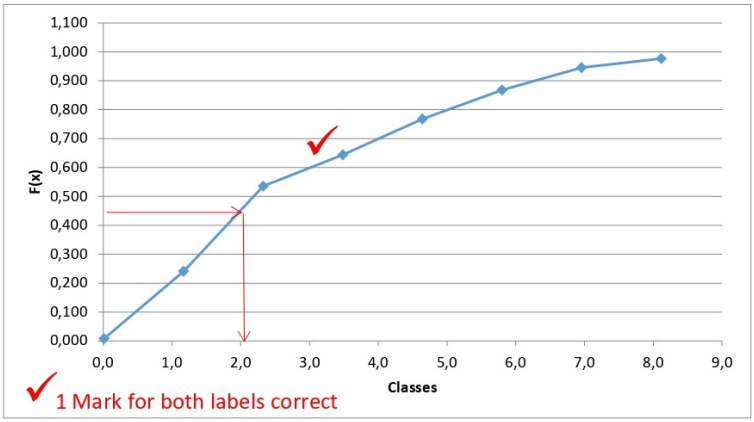
Luan collected data for his final-year project. He stores this data in the Excel file *Sim442_Tut04_2025_RawData.xlsx* on the data sheet labelled 'Question 2'. Luan claims that, according to the empirical distribution of the dataset, the data value associated with a probability of 0.45 is 2.2.

Investigate this claim, stating whether you agree or disagree with it. Provide evidence to substantiate your conclusion.

Min =	0,01216
Max =	9,27577
k =	8
Class w =	1,15795

Classes	Frequency	Cumulative frequency	Empirical distr.
0,01216	1	1	0,008
1,17012	30	31	0,240
2,32807	38	69	0,535
3,48602	14	83	0,643
4,64397	16	99	0,767
5,80192	13	112	0,868
6,95987	10	122	0,946
8,11782	4	126	0,977
9,27577	3	129	1,000
Sum =	129		

Data value at F(x), where x=0.45: x= 1,994
F(0.45) = 1,994 y= 0,45
(through interpolation)



Interpolation: (same 3 marks as on Excel image)

$$F^{-1}(0.45) = \left(\frac{0.45 - 0.24}{0.535 - 0.24} \times (2.32 - 1.17)\right) + 1.17 \quad \checkmark$$
$$= 1.994 \quad \checkmark\checkmark$$

As $F^{-1}(0.45) = 1.994$ and not 2.2, I disagree with the claim. $\checkmark\checkmark$

Question 3 [19]

Trucks arrive at a weigh bridge according to the data in the Excel file *Sim442_Tut04_2025_RawData.xlsx* provided, on sheet 'Question 3'. Determine if the number of arrivals per day follows a Poisson distribution with $\lambda = 4$, at the 5% level of significance. Note: λ was not estimated from the data.

H0: The observations for the average number of trucks arriving per day, is Poisson distributed with lambda = 4.

lambda = 4 n= 98

Classes	Number of observations	Expected frequency	Adjusted expected frequency	Adjusted observed frequency	χ^2-sum terms
0	0	1,795	8,975	10	0,117
1	10	7,180	14,359	11	0,786
2	11	14,359	19,146	15	0,898
3	15	19,146	19,146	10	4,369
4	10	19,146	15,317	13	0,350
5	13	15,317	10,211	8	0,479
6	8	10,211	5,835	13	8,798
7	13	5,835	5,011	18	33,668
8	12	2,917			
9	6	1,297			
>9	0	0,797			

χ^2-statistic =	49,465
α =	0,05
Number of classes =	8
Parameters estimated =	0
Degrees of freedom =	7
Critical value =	**14,067**

χ^2-statistic > Critical value, therefore Reject H0

Question 4 [21]

The organisers of the Stellenbosch Street Soiree want to understand the demand for tickets. As soon as tickets go on sale, they record the inter-arrival times (in seconds) of users logging into their website. The data can be found in the Excel file *Sim442_Tut04_2025_RawData.xlsx* provided, on sheet 'Question 4'.

They estimate that the data follows an exponential distribution. Determine whether the data follows the proposed distribution or not using a 5% level of significance.

Ho: The data is distributed according to an Exponential distribution with
$\beta = 0.54$.

0,035311	1,122055	0,512421	0,40028658	0,308746	1,328039	0,419075	0,32598	0,278171	0,302189
0,443926	0,137337	0,168599	0,14200982	0,654902	0,034282	0,204962	1,655573	1,94815	0,364483
0,091841	0,229159	0,592838	0,11211325	0,298831	0,517626	0,06552	0,802368	0,09903	0,707123
0,174265	0,701397	0,011368	0,13285299	0,670718	0,77431	0,183575	1,073778	1,17336	1,117294
0,029842	0,133858	1,042332	2,20079044	0,732881	1,260047	0,321943	1,50264	0,138197	1,08939
0,227484	0,781652	0,439699	1,90836061	0,067566	0,547591	0,194667	0,058103	0,313896	0,786934
0,450141	0,152513	0,945851	0,01766453	0,164741	0,031462	0,041483	0,278137	0,380991	2,031843
0,308794	0,008323	0,03724	0,54490166	0,171972	0,01153	0,064606	0,787322	0,20898	1,004243
0,551801	0,02842	0,658731	0,22749197	0,080427	0,283001	0,88922	0,071977	0,287297	0,497473
0,650005	1,88194	0,082399	0,95306414	0,434162	0,536402	1,661094	0,298114	1,22498	0,929933

n =	100
min =	0,01
max =	2,2
average=	0,54
# classes =	7
class width =	0,31

Interval end point (h)	Oi	Theor prop	Ei	Ei'	Oi'	χ^2 terms
0,008323262	1	0,015	1,531			
0,32	47	0,449	43,359	44,889	48	0,216
0,63	18	0,692	24,267	24,267	18	1,618
0,95	15	0,827	13,581	13,581	15	0,148
1,26	10	0,903	7,601	7,601	10	0,757
1,57	2	0,946	4,254	9,661	9	0,045
1,89	3	0,970	2,381			
2,20	3	0,983	1,333			
10000,00	1	1,000	1,694			
Total	100	6,785	678,524		χ^2 calc =	2,784

Exponential Distribution Parameters:

β =	0,54

classes =	5
α =	5%
m =	1
v (d.o.f) =	3
p-value =	0,4261
χ^2 crit =	7,815

Thus no reason to reject Ho since chi-square calc < chi-square crit

Question 5 [16]

The product delivery interval rate of a machine is described by

$$F(x) = 1 - \exp^{-\frac{x}{116}}.$$

Determine if the following data points were taken from the product delivery time sheet.

122 133 106 128 135 126

We use the K-S test because we have few data points and the case "All parameters known" applies because the distribution is specified.

Ho: The data is from F(x)=1-e-x/116

Beta = 116

n = 6

Xi sorted	$F(x)=1-e^{-x/116}$	$F_n(x) = \#X_i \leq x/n$	F(x)	D-	D+	D
106	0,5990	0,1667	0,599	0,599	0,432	0,599
122	0,6507	0,3333	0,651	0,484	0,317	
126	0,6625	0,5000	0,663	0,329	0,163	
128	0,6683	0,6667	0,668	0,168	0,002	
133	0,6823	0,8333	0,682	0,016	0,151	
135	0,6877	1,0000	0,688	0,146	0,312	

$$\left(D_n - \frac{0.2}{n}\right)\left(\sqrt{n} + 0.26 + \frac{0.5}{\sqrt{n}}\right) > c''_{1-\alpha}$$

α = 0,05

**c = 1,094

Dn = 0,599

$$\left(D_n - \frac{0.2}{n}\right)\left(\sqrt{n} + 0.26 + \frac{0.5}{\sqrt{n}}\right) = 1{,}648 > 1.094$$ **Reject Ho**