Department of Industrial Engineering
University of Stellenbosch

Simulasie 442 : Simulation 442
2025

Tut 3: Memorandum

Tutoriaal 3 *Tutorial 3*	Punt: 73 *Mark:*	Ingeedatum: **08-08-2025** (10:00) B3003 *Due date:*

Instruksies:	Formatteer alle syfers sinvol. Ontwikkel die modelle individueel. U mag in groepe van **twee** of minder werk om die vrae te beantwoord. Handig slegs een hardekopie van u antwoordstel in . Gebruik Tecnomatix en Excel vir u berekenings. **Hierdie tutoriaal en prakties is verpligtend.** **Indien u nalaat om die vereistes betyds na te kom, sal u die module sak.**
Instructions:	*Format all numbers sensibly.* *Develop the models individually.* *You may work in groups of **two** or less when answering the questions.* *Submit one document only.* *Use Tecnomatix and Excel for your calculations.* ***This tutorial and practical are compulsory.*** ***You will fail the module if you do not comply with the requirements, on time.***

Question 1 [12]

Within the context of random number generation,

(a) List four properties of a random number generator. [4]

(b) What are the properties that define the behaviour and characteristics of discrete and continuous random variables? You can use the given table to answer this question. [8]

Property	**Discrete Random Variables**	**Continuous Random Variables**
Definition		
Distribution Function (CDF)		
Inverse CDF		
Examples		

(a)
- They can take only discrete values due to integer division. ✓
- A random number stream is periodic. ✓
- It can degenerate. ✓
- Requires a seed number. ✓

(b) One mark for each correct box = 8 marks ✓✓✓✓✓✓✓✓

Property	**Discrete Random Variables**	**Continuous Random Variables**
Definition	Takes on a countable number of distinct values	Takes on an uncountable number of values within a given range
Distribution Function (CDF)	Step Function	Continuous function
Inverse Transform Method	Maps uniform random numbers to discrete values using the CDF	Maps uniform random numbers to continuous values using the CDF
Examples	Number of arrivals, defect counts	Time between events, measurement errors

Question 2 [6]

Use the linear congruential method (LCG) to generate a set of 41 random numbers where $a = 3$, $c = 2$ and $Z_0 = 5$, with Z_0 the seed number of the series. Choose an appropriate m such that the string repeats itself after six numbers. *Hint: Use the required properties of m to narrow down the options during your search.* The formula for LCG is

$$Z_i = (aZ_{i-1} + c) \bmod m.$$

Generate the set of random numbers indicating the values of parameters, as well as the values of Z_i and U_i at each index.

One mark for correct Z at first index value ✓
one mark for correct Z_i values ✓
two marks for correct U_i values ✓✓
two marks for m = 7 ✓✓

Parameters	
a =	3
c =	2
Z_0 =	5
m =	7

Formula: $Z_i = (aZ_{i-1} + c) \bmod m$

String repeats itself after 6 numbers

i	Z_i	U_i
0	5	0,714286
1	3	0,428571
2	4	0,571429
3	0	0
4	2	0,285714
5	1	0,142857
6	5	0,714286
7	3	0,428571
8	4	0,571429
9	0	0
10	2	0,285714
11	1	0,142857
12	5	0,714286
13	3	0,428571
14	4	0,571429
15	0	0
16	2	0,285714
17	1	0,142857
18	5	0,714286
19	3	0,428571
20	4	0,571429
21	0	0
22	2	0,285714
23	1	0,142857
24	5	0,714286
25	3	0,428571
26	4	0,571429
27	0	0
28	2	0,285714
29	1	0,142857
30	5	0,714286
31	3	0,428571
32	4	0,571429
33	0	0
34	2	0,285714
35	1	0,142857
36	5	0,714286
37	3	0,428571
38	4	0,571429
39	0	0
40	2	0,285714
41	1	0,142857

Question 3 [9]

Determine the inverse transform of the following function and develop and expression for x.

$$f(x) = 3x + \frac{5}{8}, \quad 0 \leq x \leq \frac{-5 + \sqrt{409}}{24}$$

$$
\begin{aligned}
F(x) &= \int_0^x (3y + \frac{5}{8})\mathrm{d}y \quad \checkmark\checkmark \\
&= \frac{3y^2}{2} + \frac{5y}{8}\bigg|_0^x \\
&= \frac{12x^2 + 5x}{8} \quad \checkmark \\
&= U. \quad \checkmark
\end{aligned}
$$

Now we rearrange to get

$$
\begin{aligned}
12x^2 + 5x - 8U &= 0 \quad \checkmark \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \checkmark \\
&= \frac{-5 \pm \sqrt{5^2 + (4 \times 12 \times 8U)}}{24} \quad \checkmark \\
&= \frac{-5 \pm \sqrt{25 + 384U}}{24}. \quad \checkmark
\end{aligned}
$$

Last mark is student went further and saw that the only valid expression for x is

$$
x = \frac{-5 + \sqrt{25 + 384U}}{24}. \quad \checkmark
$$

Question 4 [8]

The Sanga is a food truck serving the Stellenbosch University Engineering faculty. Since their main customers are university students and staff, they experience varying preparation times for different orders throughout the day. Let x represent the number of minutes required to prepare a customer's order. The preparation time follows the probability density function

$$
f(z) = 12z^2 e^{3z^3 - 2} \quad \text{where} \quad 0 \le z \le \sqrt[3]{\frac{1}{3}\ln(\frac{2}{3} + e^{-2}) + \frac{2}{3}}.
$$

Generate *five* observations from the distribution by means of the inverse transform method and the first five random numbers generated in Question 2. Show all the steps to determine the inverse transform.

$$\begin{aligned}
F(x) &= \int_0^x 12z^2 e^{3z^3-2}\mathrm{d}z \quad \checkmark \\
&= \frac{12}{9}e^m \bigg|_{-2}^{3x^3-2} \quad \checkmark\checkmark \\
&= \frac{4}{3}e^{-2}(e^{3x^3}-1) \checkmark \\
&= U
\end{aligned}$$

When m is chosen as $3z^3-2$, $\mathrm{d}m = 9z^2\ \mathrm{d}z$, the lower bound $m = -2$ and the upper bound is $m = 3z^3 - 2$.

$$\begin{aligned}
X &= F^{-1}(U) \\
&= \sqrt[3]{\frac{\ln\left(\frac{3}{4}e^2U + 1)\right)}{3}} \quad \checkmark\checkmark
\end{aligned}$$

The generated observations (by replacing U with the random numbers) are therefore,
0,811144;
0,740150;
0,780635;
0;
0,6813903 (for every wrong answer minus half a mark with minimum mark being 0.) ✓✓

Question 5 [12]

A delivery drone operates in a city, making package deliveries. The drone's battery level drops by Unif(8, 25) percentage points (no decimals) after each delivery, and it takes Unif(3, 18) minutes to travel to each delivery. The drone starts with a full battery (100%). Use

the random numbers in the order given below to simulate the drone's operation.
Questions:

1. How much time did the drone take to make all five deliveries?
2. What is the battery percentage after the deliveries?

Random numbers to use in order:

0.45; 0.12; 0.89; 0.33; 0.76; 0.04; 0.61; 0.95; 0.28; 0.82

We use the continuous uniform (for the time to recharge) as well as the discrete uniform (for the level drop) distributions; discrete uniform inverse transform $x = a + \text{INT}(U(b-a+1))$ and continuous uniform inverse transform $t = a + U(b-a)$.

Simulation of convoluted distributions.

U	Time	% Drop
0.45	9,75 ✓	
0.12		10 ✓
0.89	26.1 ✓	
0.33		13 ✓
0.76	40.5 ✓	
0.04		8 ✓
0.61	52.65 ✓	
0.95		25 ✓
0.28	59.85 ✓	
0.82		22 ✓

Total time: 59.85 minutes ✓
Battery % at end: (100% -78%) = 22% ✓

Question 6 [9]

Using an **efficient** algorithm, explain stepwise how would you generate four observations from the distribution given below using the numbers from the given set of uniformly distributed, independent random numbers 0.28; 0.74; 0.15; 0.92. [10]

$$f(x) = \begin{cases} 0.18 & \text{x} = 1 \\ 0.07 & \text{x} = 2 \\ 0.46 & \text{x} = 3 \\ 0.21 & \text{x} = 4 \\ 0.08 & \text{x} = 5 \end{cases}$$

1. If U ≤ 0.46, return $x = 3$. ✓
2. If U ≤ 0.67, return $x = 4$. ✓
3. If U ≤ 0.85, return $x = 1$. ✓
4. If U ≤ 0.93, return $x = 5$. ✓
5. Else return $x = 2$. ✓

1. Since $0 < U = 0.28 < 0.46$, return $x = 3$. ✓
2. Since $0.67 < U = 0.74 < 0.85$, return $x = 1$. ✓
3. Since $0 < U = 0.15 < 0.46$, return $x = 3$. ✓
4. Since $0.85 < U = 0.92 < 0.93$, return $x = 5$. ✓

Question 7 [17]

Develop an expression to estimate each of the following integrals using Monte-Carlo simulation. You may leave your final answer in factorised form.

(a)

$$I = \int_1^4 (\sqrt{x} + 3x)\mathrm{d}x$$

[5]

(b)

$$I = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x.$$

[12]

(a)

$$I = \int_1^4 (\sqrt{x} + 3x)\mathrm{d}x$$

Do the transformation with $a = 1$ and $b = 4$:

$$y = \frac{x-a}{b-a}$$
$$x = a + (b-a)y$$
$$= 3y + 1 \checkmark$$
$$\mathrm{d}x = 3\mathrm{d}y \quad \checkmark$$

$$I = \int_1^4 (\sqrt{x} + 3x)\mathrm{d}x$$
$$= 3\int_0^1 (\sqrt{3y+1} + 9y + 3)\mathrm{d}y \quad \checkmark\checkmark\checkmark$$

(b)

$$I = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x.$$

$$I = \int_{-\infty}^{0} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x + \int_{0}^{\infty} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x. \checkmark\checkmark$$

$$I = I_1 + I_2$$

Addressing I_1 with $x \to -\infty, y \to 0$ and if $x = 0, y = 1$:

$$\begin{aligned} y &= \frac{1}{1-x} \\ x &= 1 - \frac{1}{y} \\ \mathrm{d}x &= \frac{\mathrm{d}y}{y^2} \quad \checkmark\checkmark \end{aligned}$$

Then,

$$\begin{aligned} I &= \int_{-\infty}^{0} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x. \\ &= \int_0^1 (1 - \frac{1}{y}) \frac{1}{y^2\sqrt{2\pi}} \mathrm{d}y \quad \checkmark\checkmark\checkmark \end{aligned}$$

Addressing I_2 with $x = 0, y = 1$ and $x \to \infty, y \to 0$:

$$\begin{aligned} y &= \frac{1}{x+1} \\ x &= \frac{1}{y} - 1 \\ \mathrm{d}y &= -y^2 \mathrm{d}x \\ \mathrm{d}x &= -\frac{\mathrm{d}y}{y^2} \quad \checkmark\checkmark \end{aligned}$$

Then,

$$\begin{aligned} I &= \int_0^{\infty} x \frac{1}{\sqrt{2\pi}} \mathrm{d}x. \\ &= \int_0^1 (\frac{1}{y} - 1) \frac{1}{y^2\sqrt{2\pi}} \mathrm{d}y \quad \checkmark\checkmark\checkmark \end{aligned}$$

Total: Cross-check: 73
