CS2001
Foundations of Computation
Lecture 11

Trees

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School of Computer Science
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Topics

- The tree data structure
- Implementation
- Complexity

Trees

- Generalisation of a linked list
  - each node can have multiple successors

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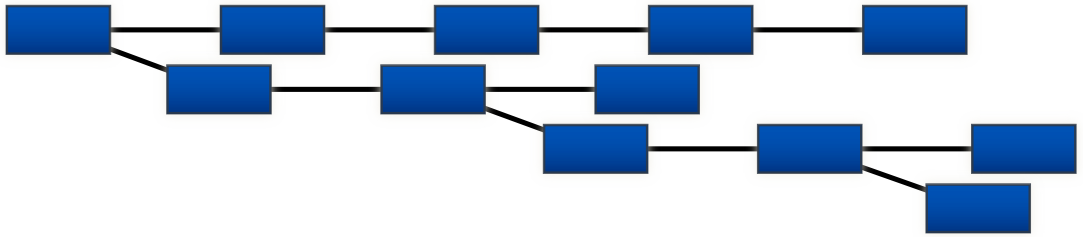

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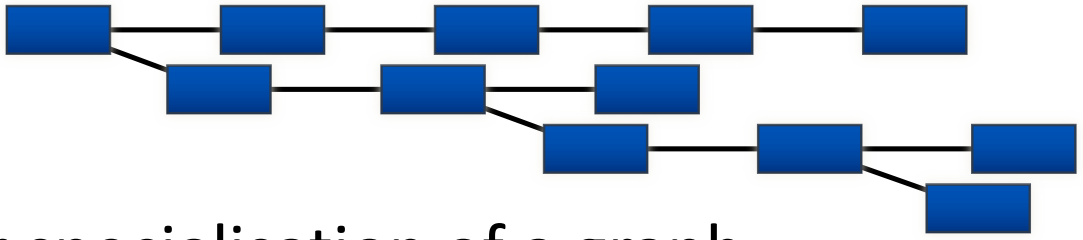

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  - directed, connected
  - no cycles
  - each node pointed at by at most one other

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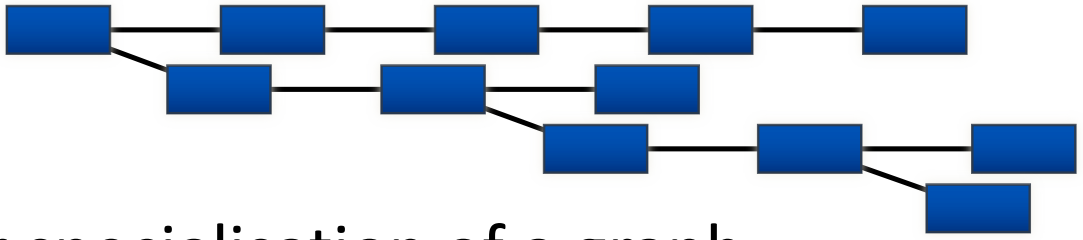

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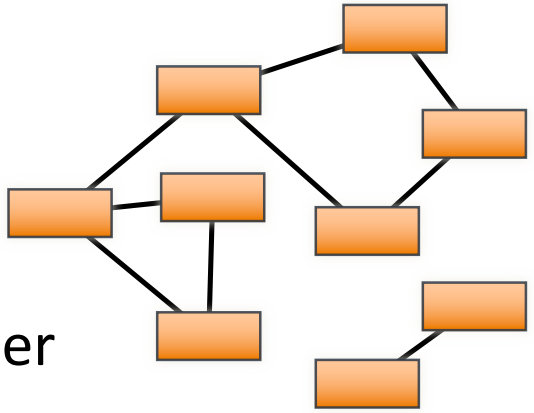

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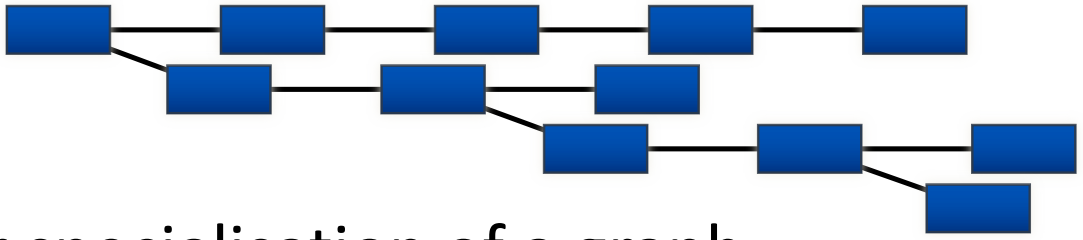

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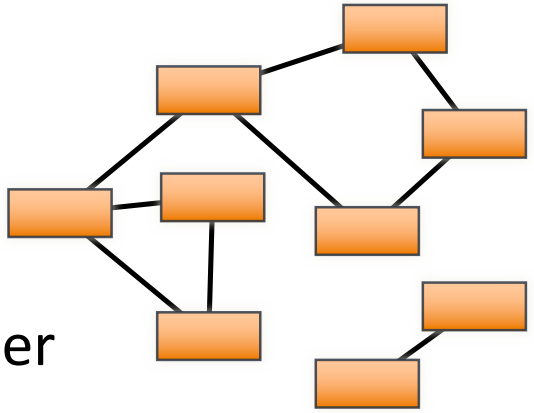

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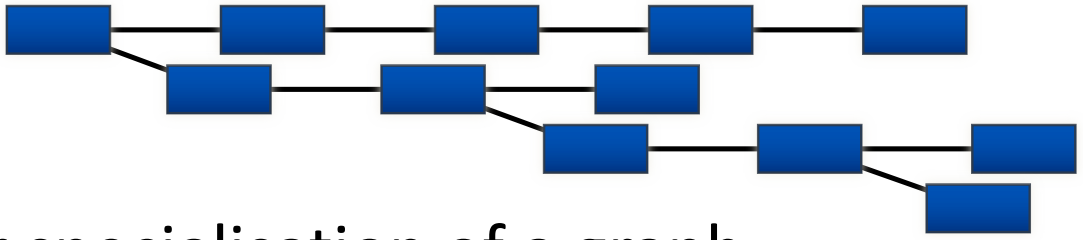

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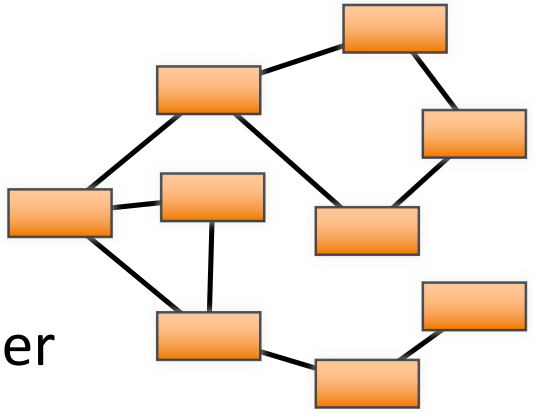

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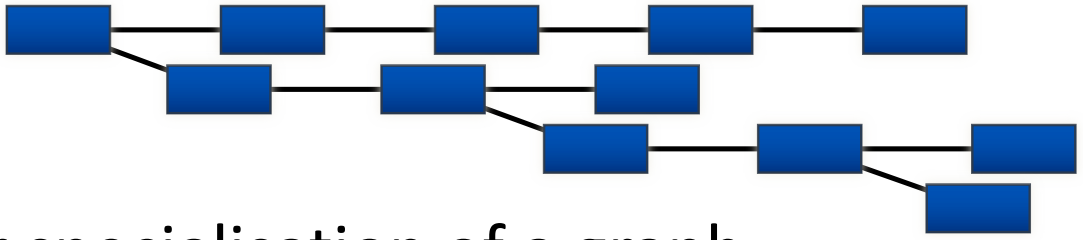

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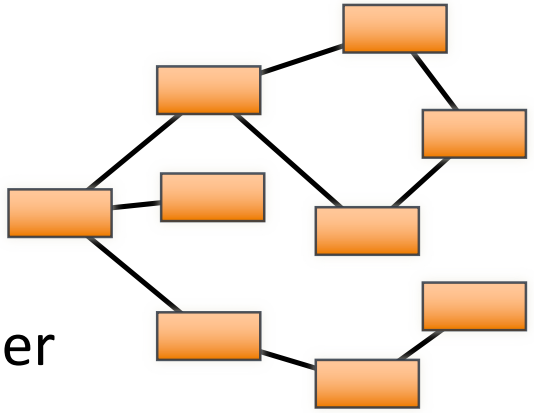

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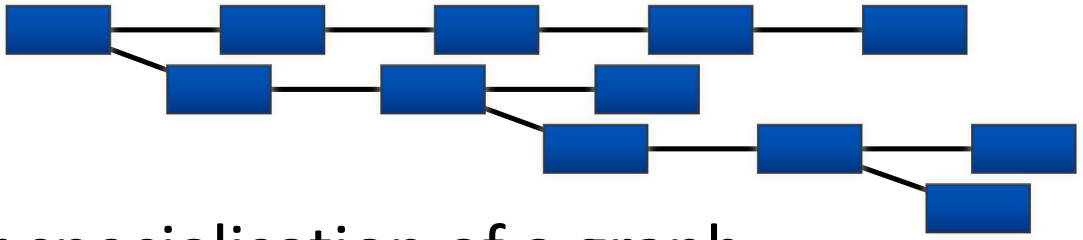

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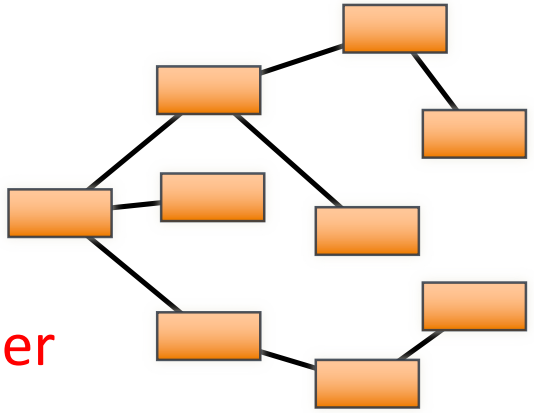

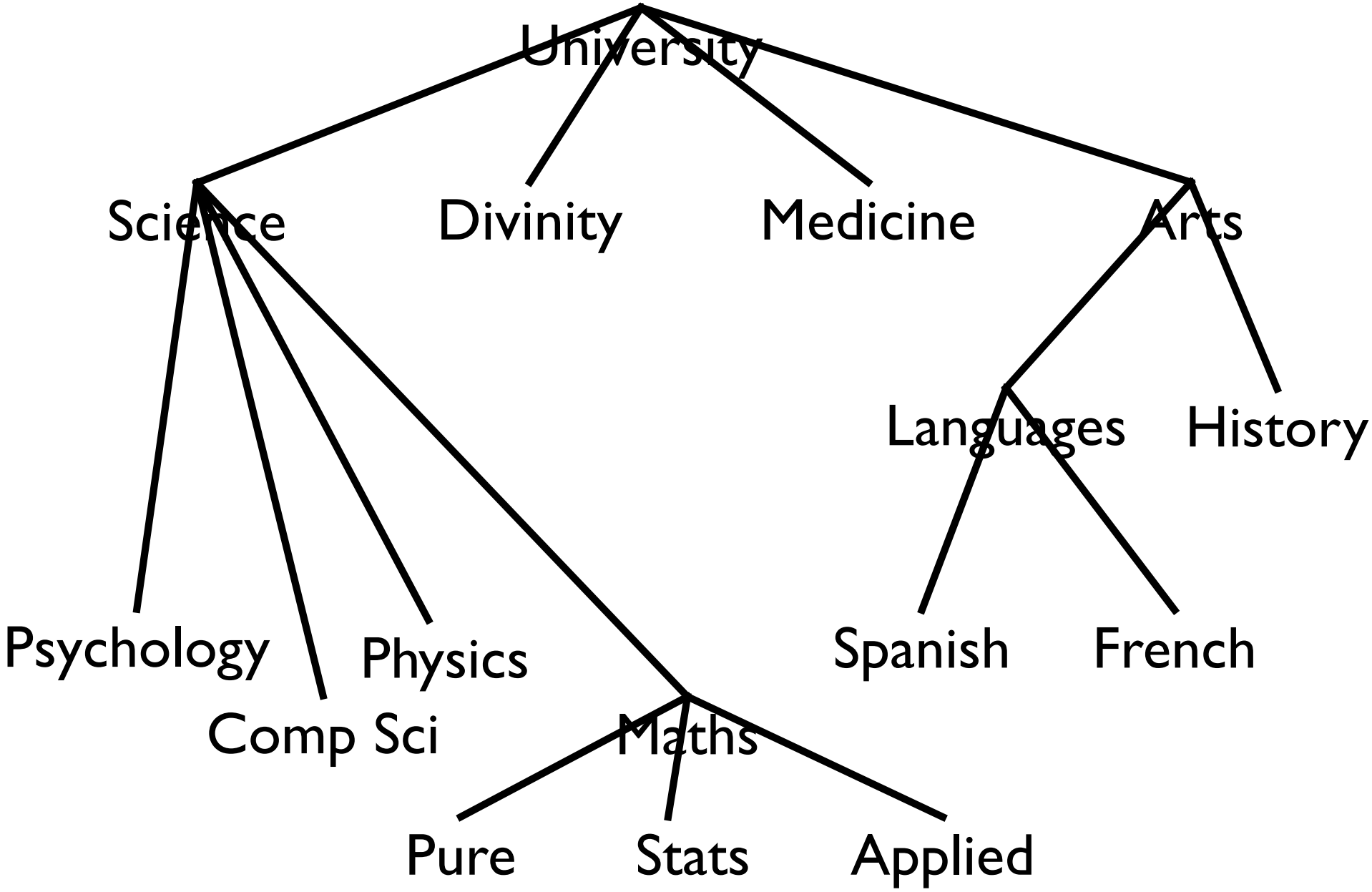
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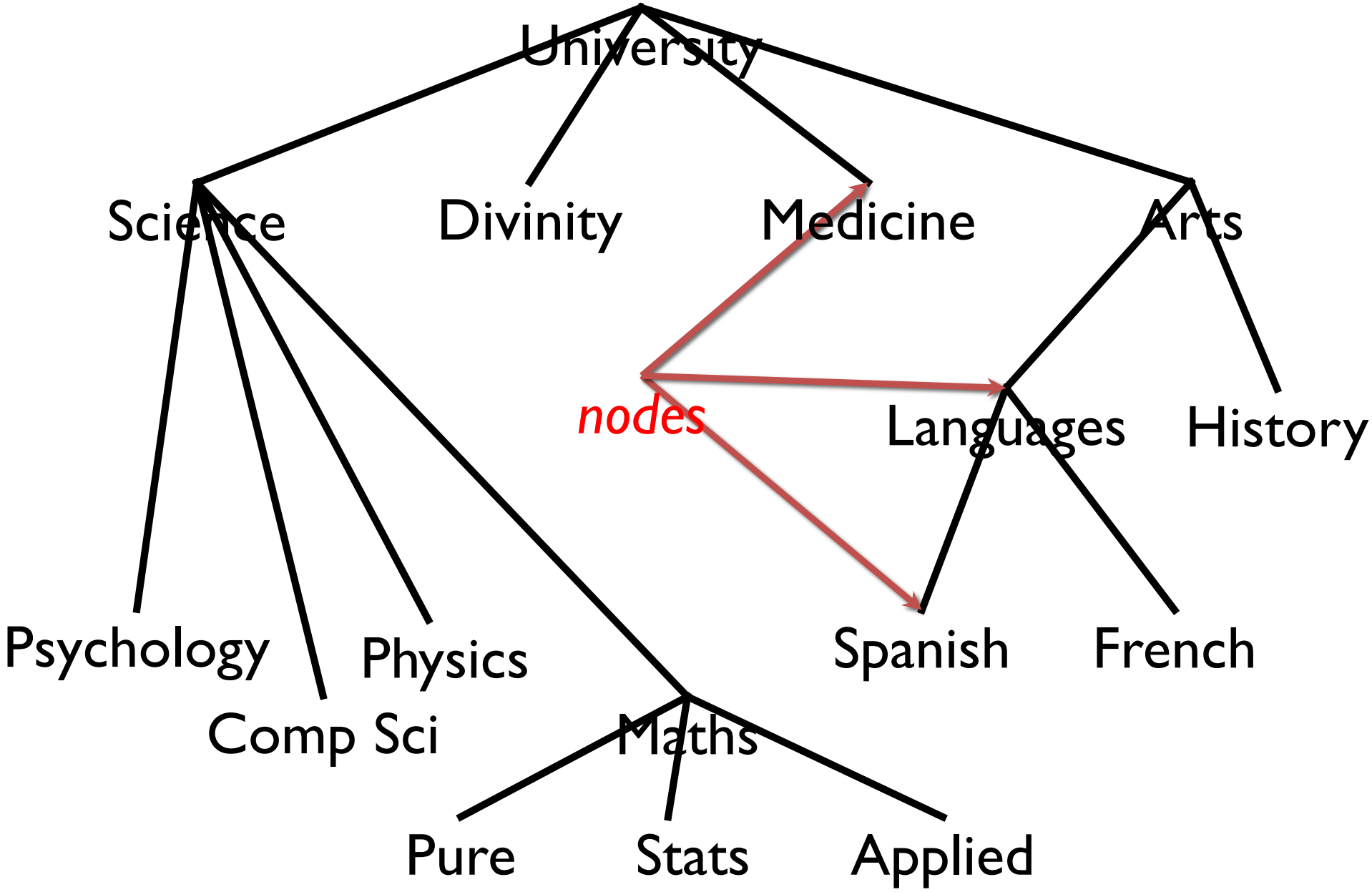
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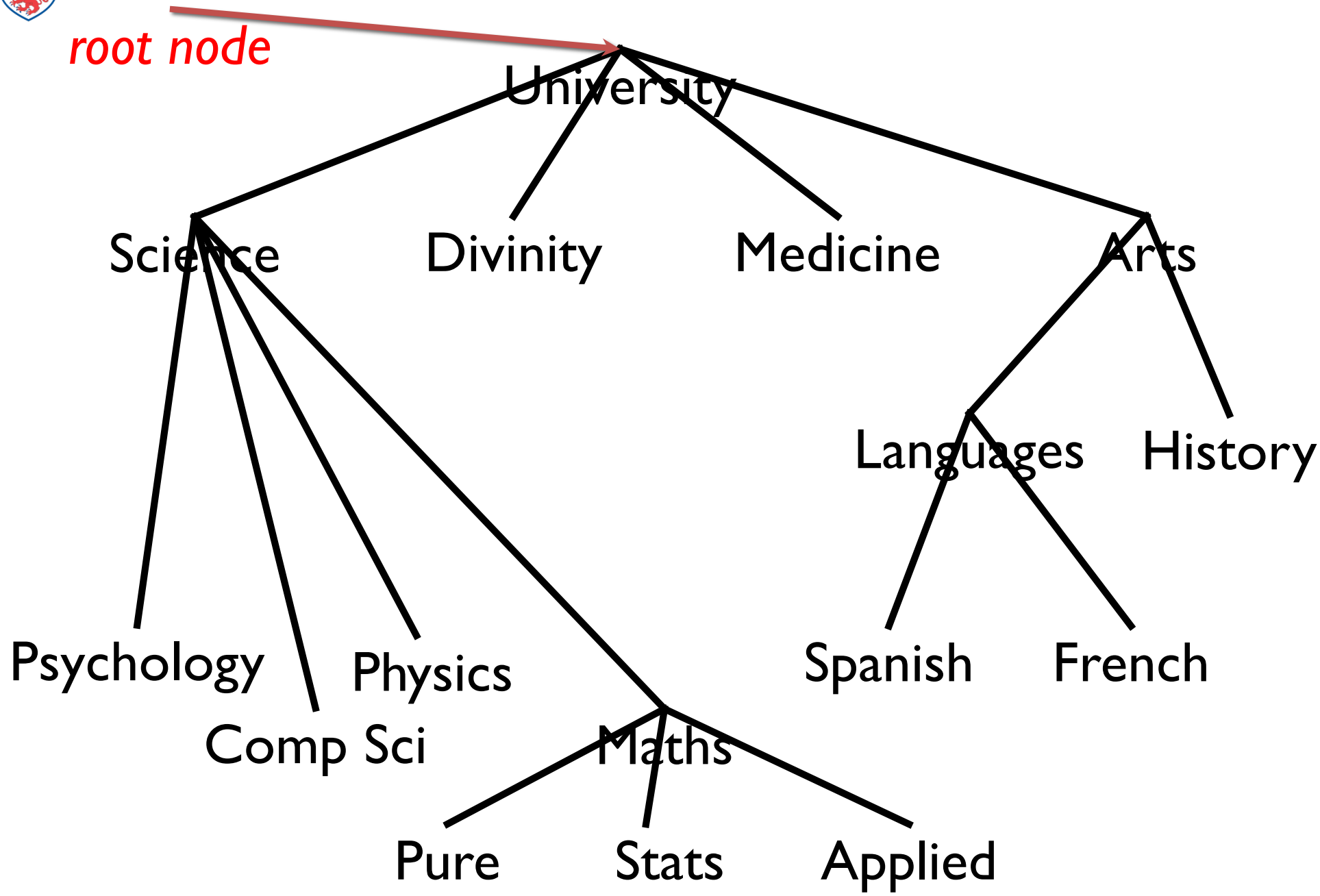
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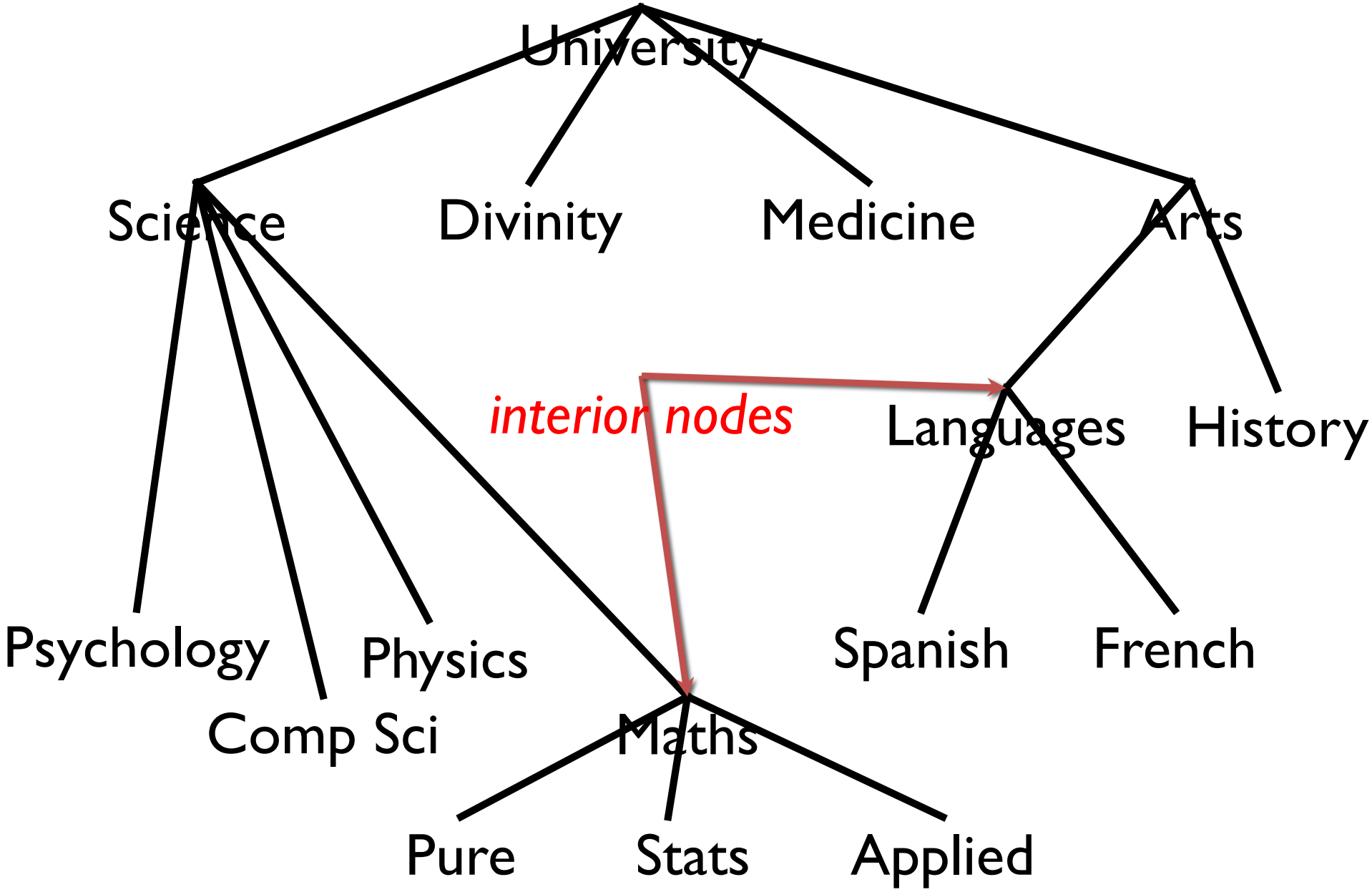
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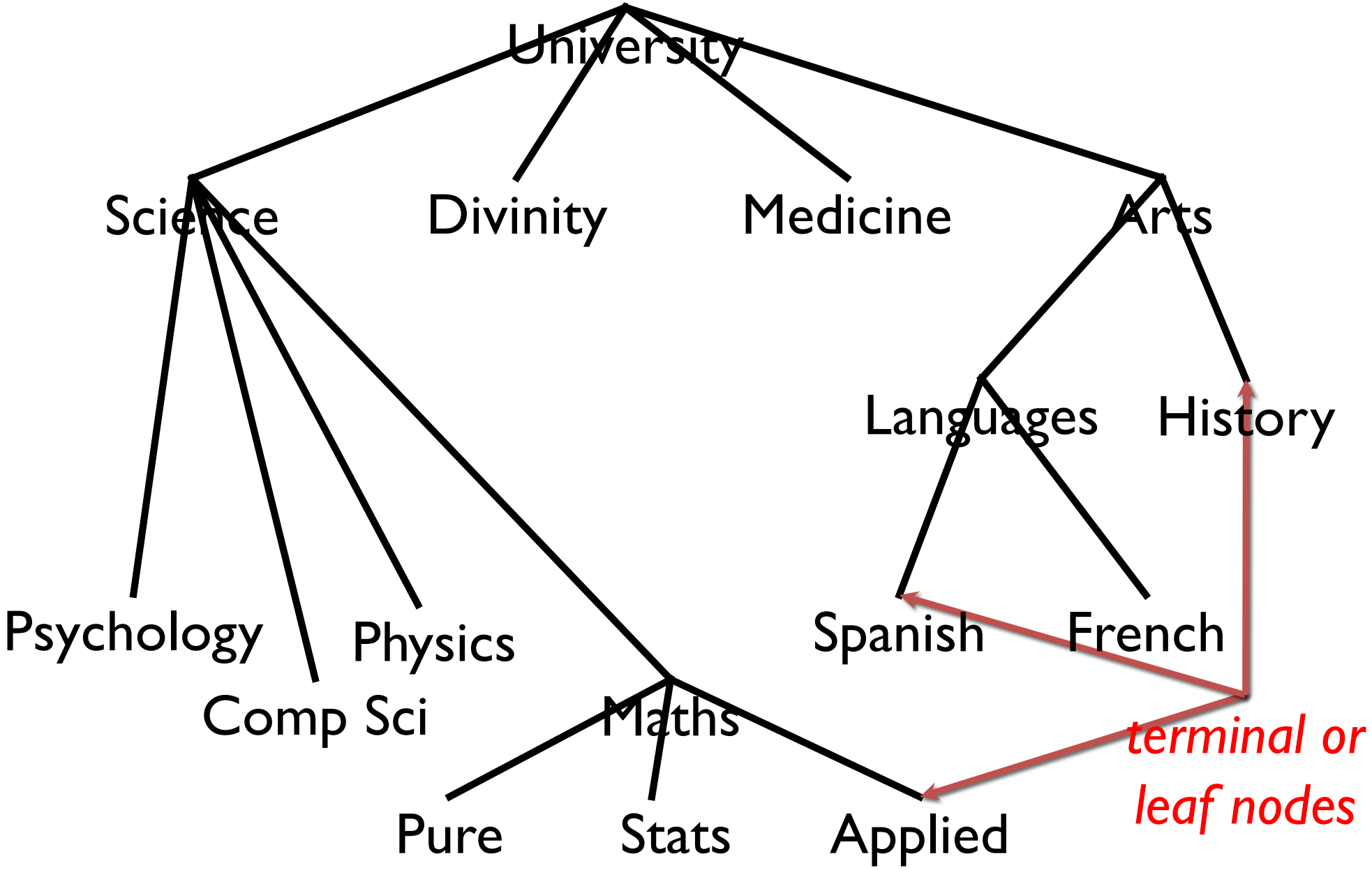
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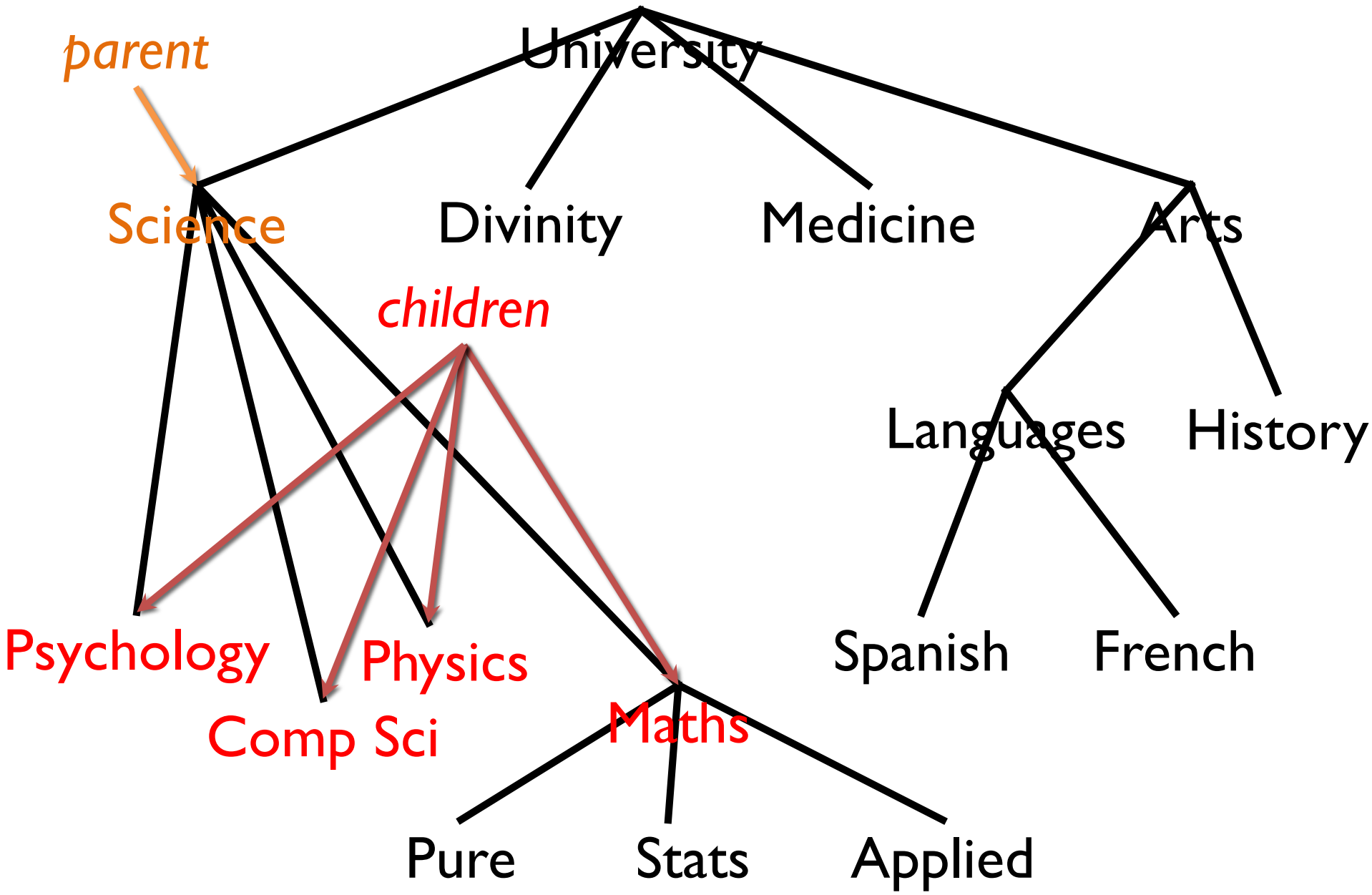
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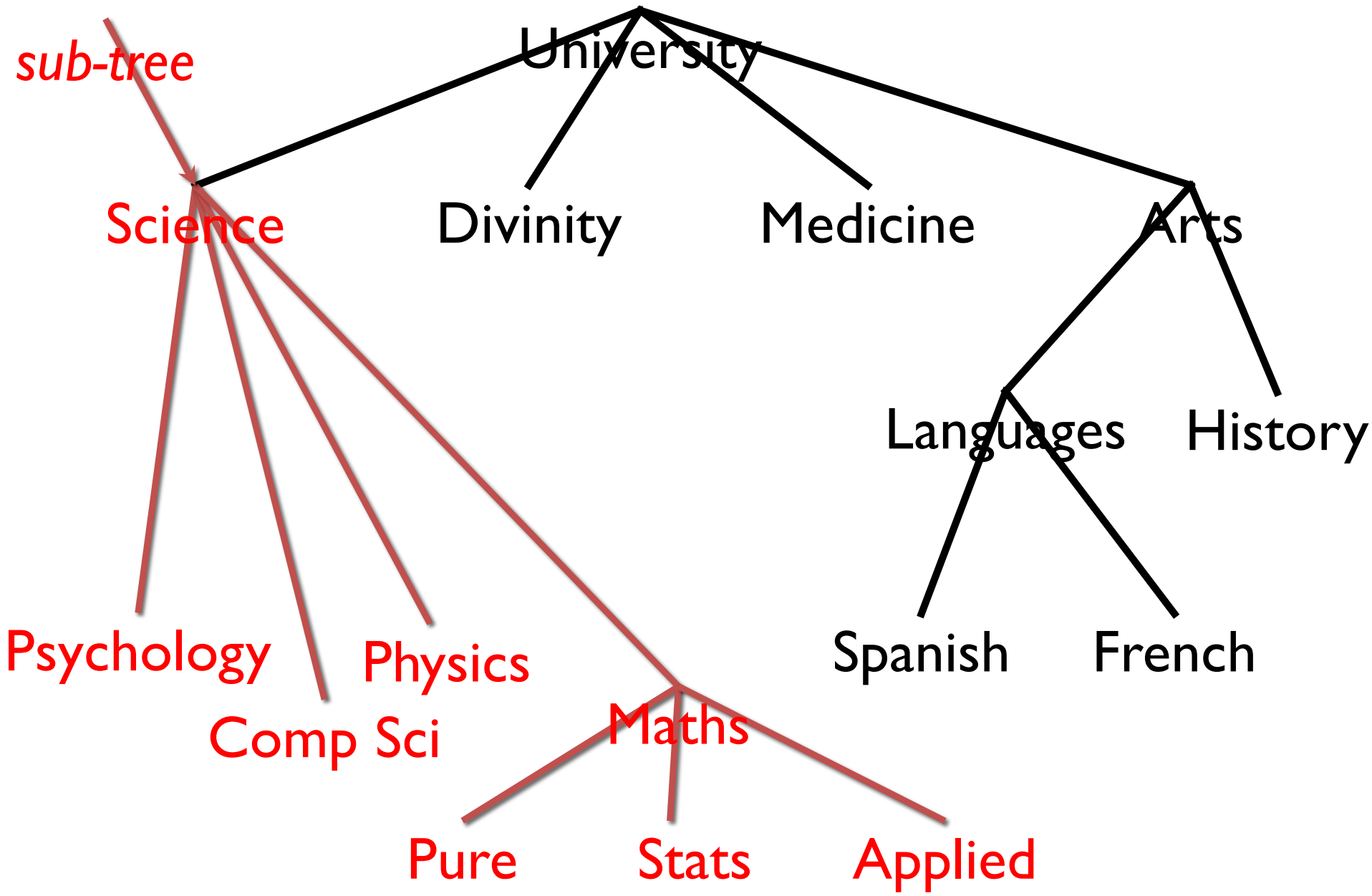
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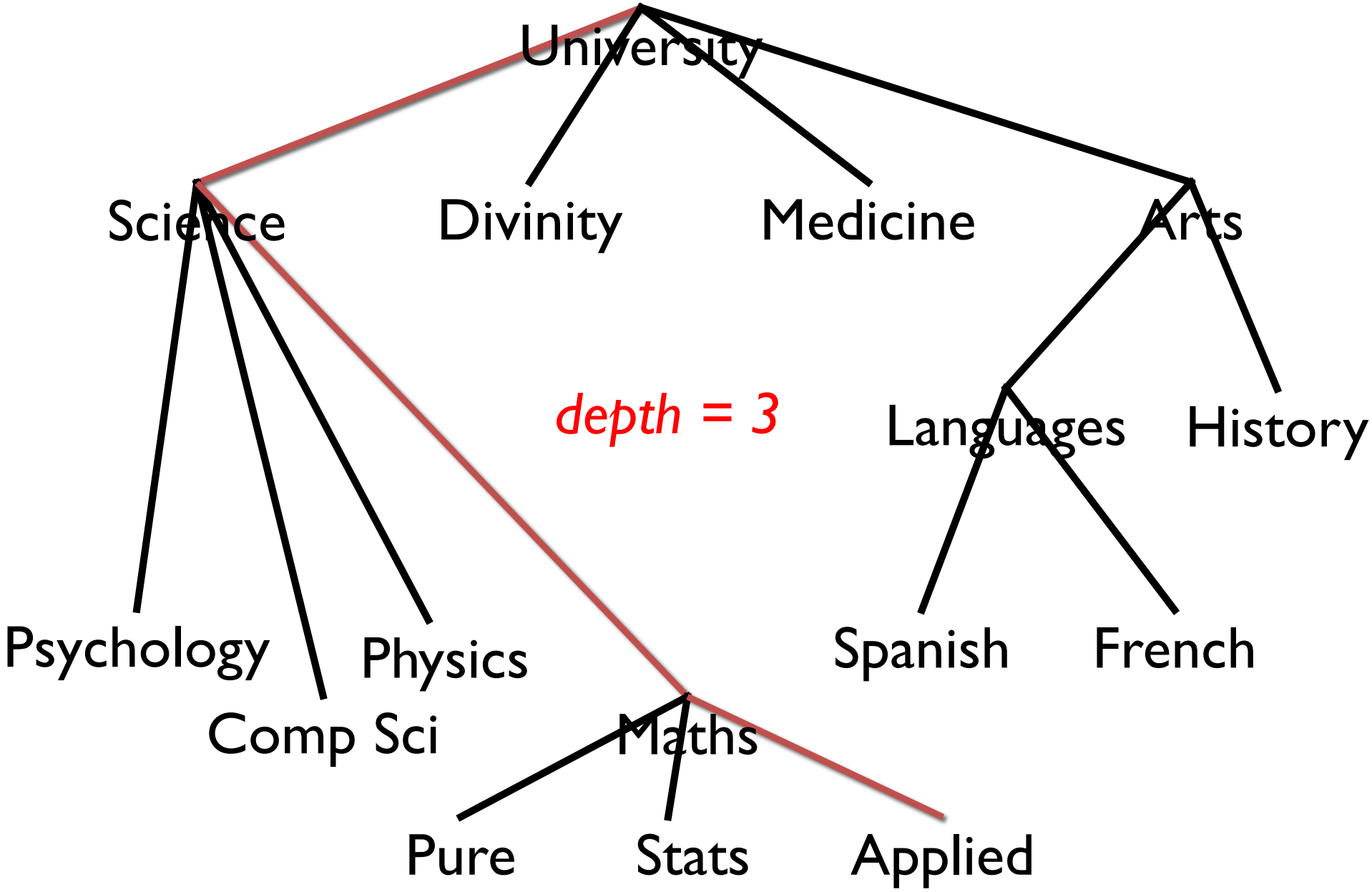
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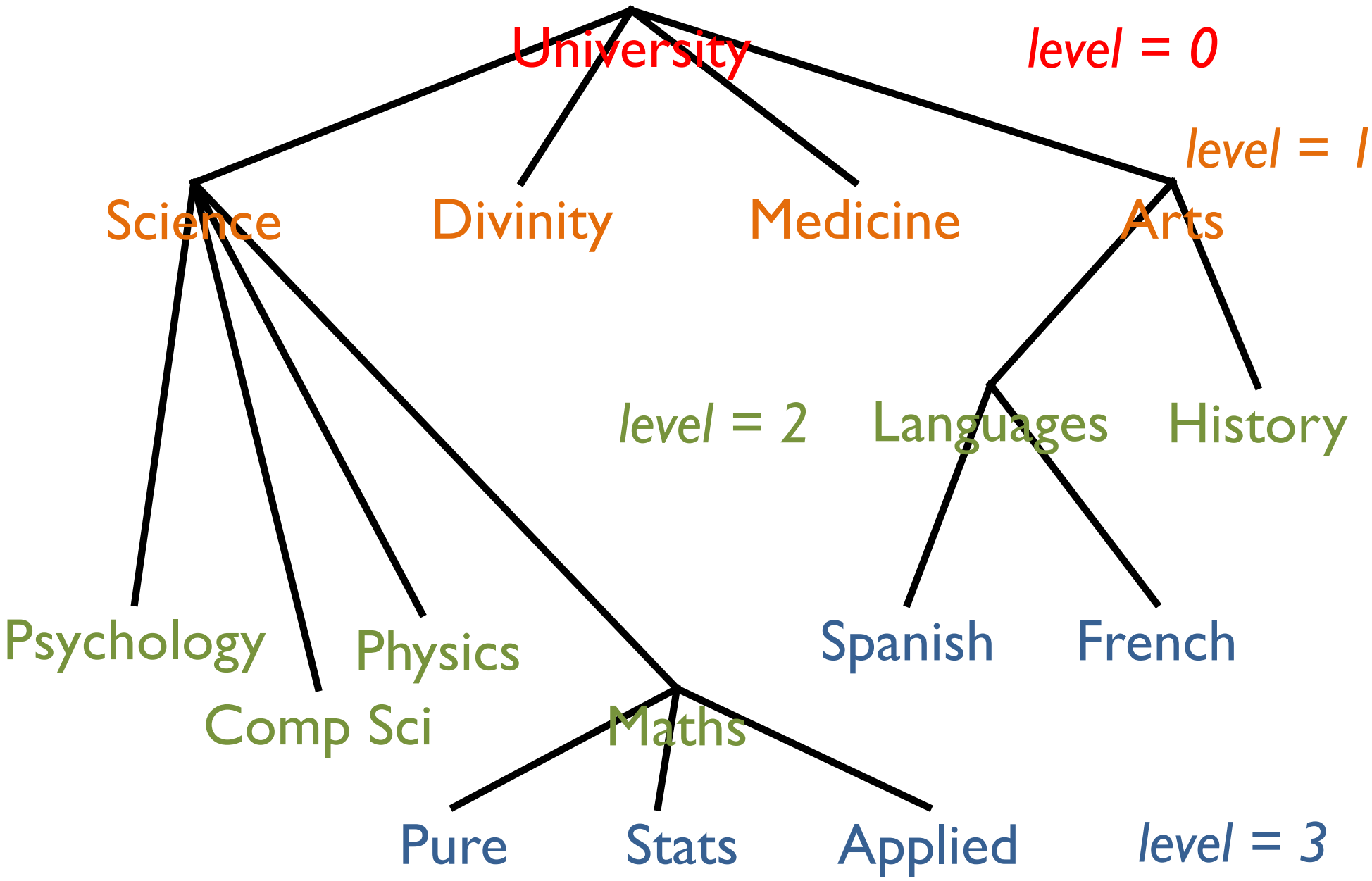
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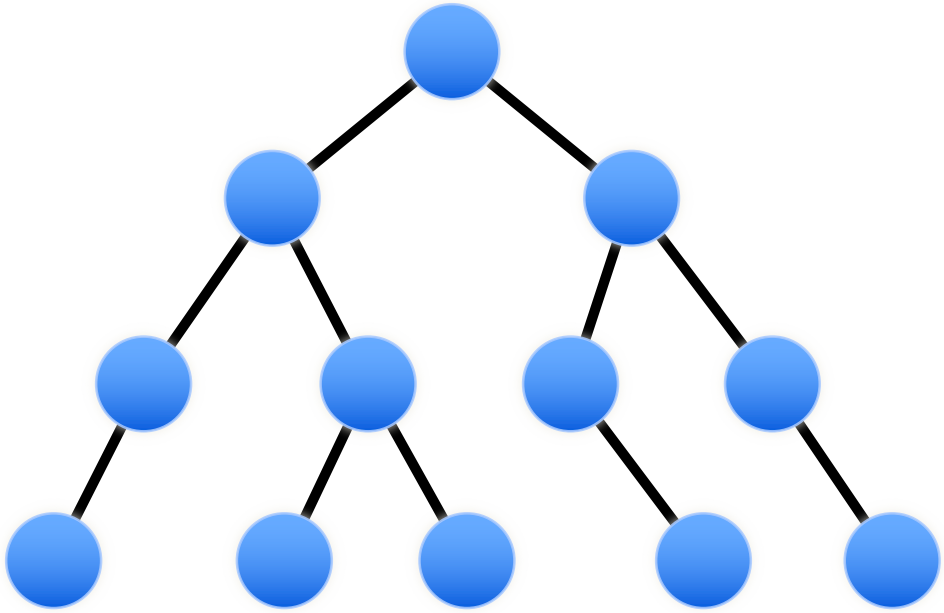
Binary Trees

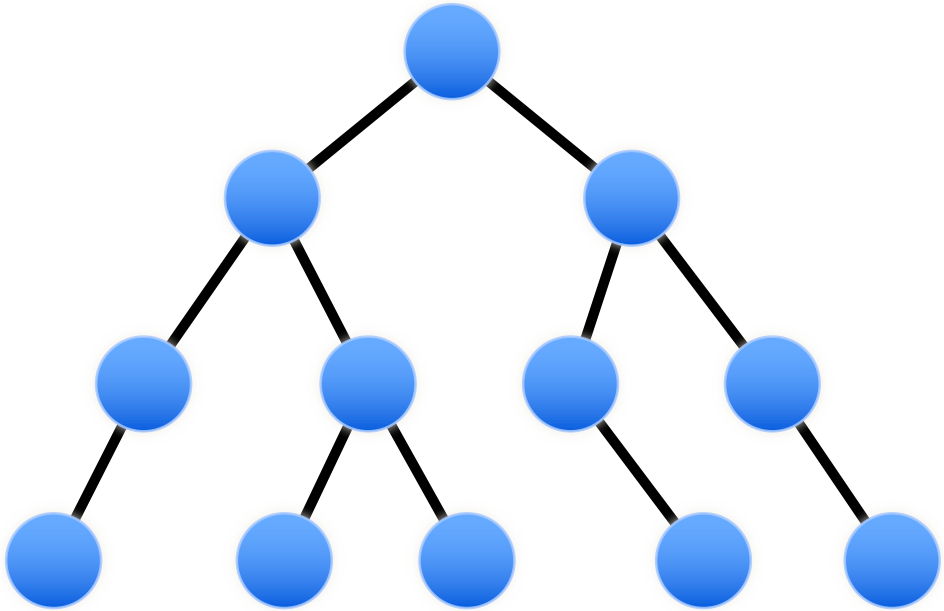
- Every node has at most two child nodes



```
public class BinaryTreeNode {

  public Object element;

  public BinaryTreeNode left;

  public BinaryTreeNode right;
}

public class BinaryTreeCollection implements ICollection {
  private BinaryTreeNode root = null;
  ...
}
```

add

- Where can a new element be added?



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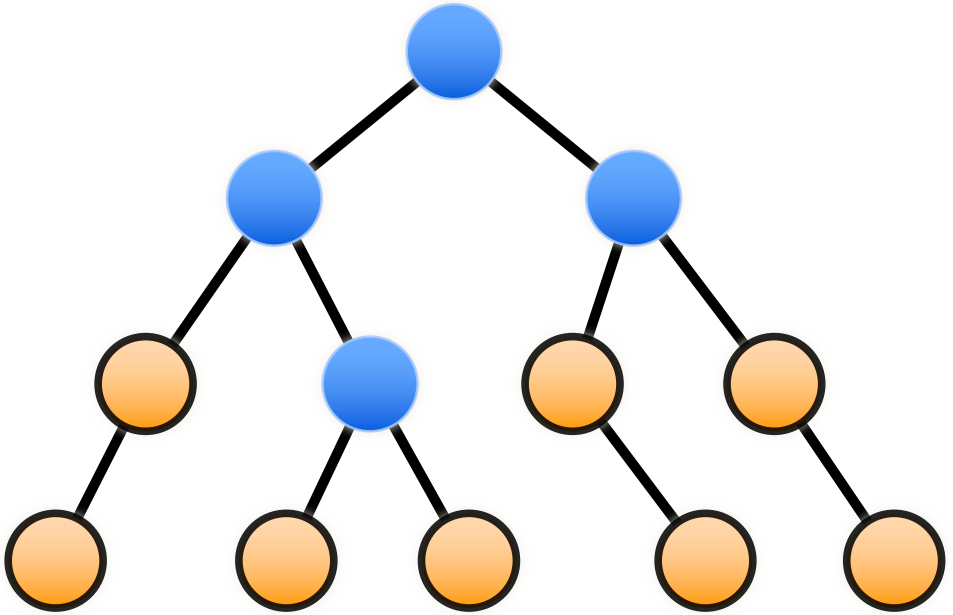
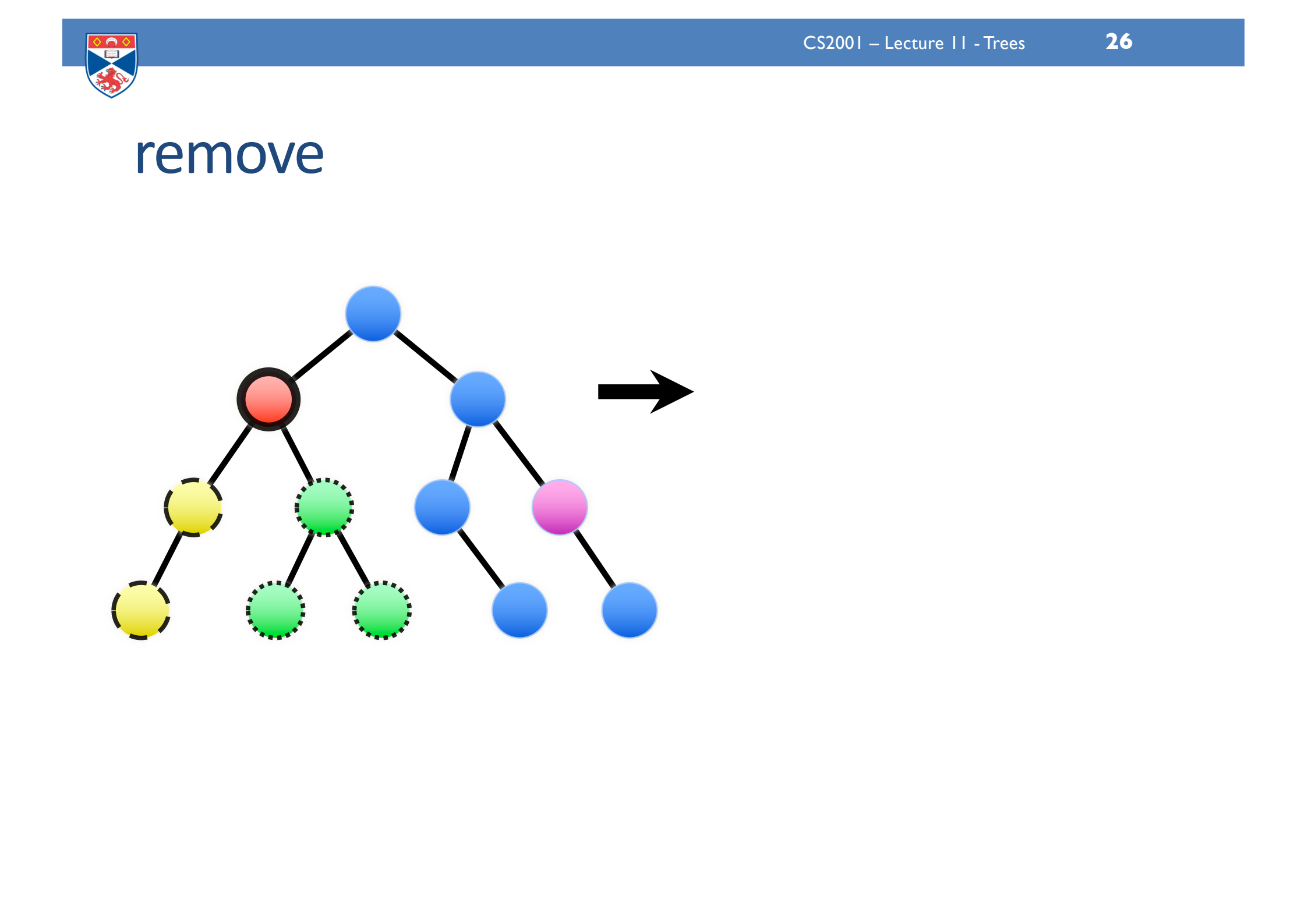
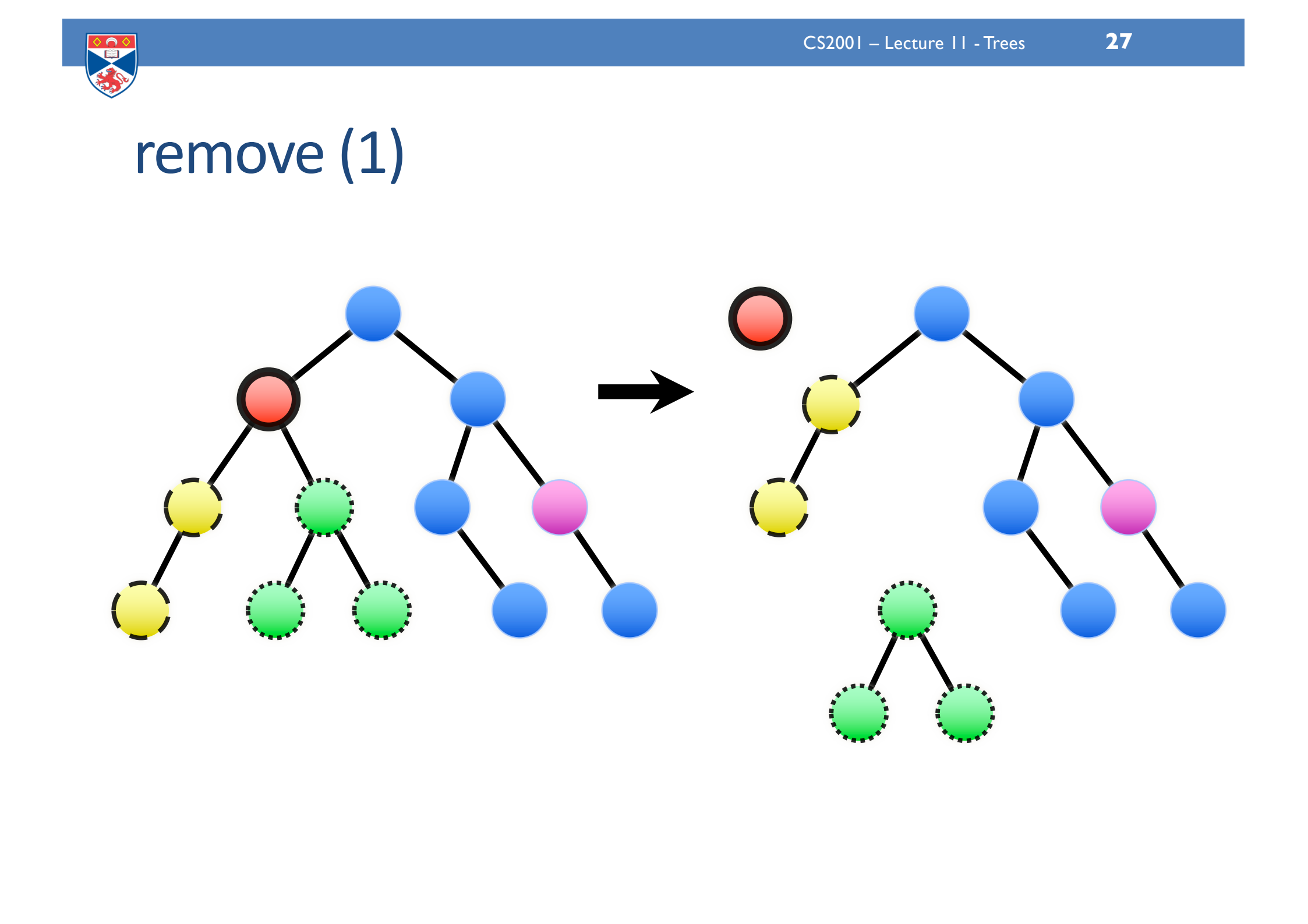


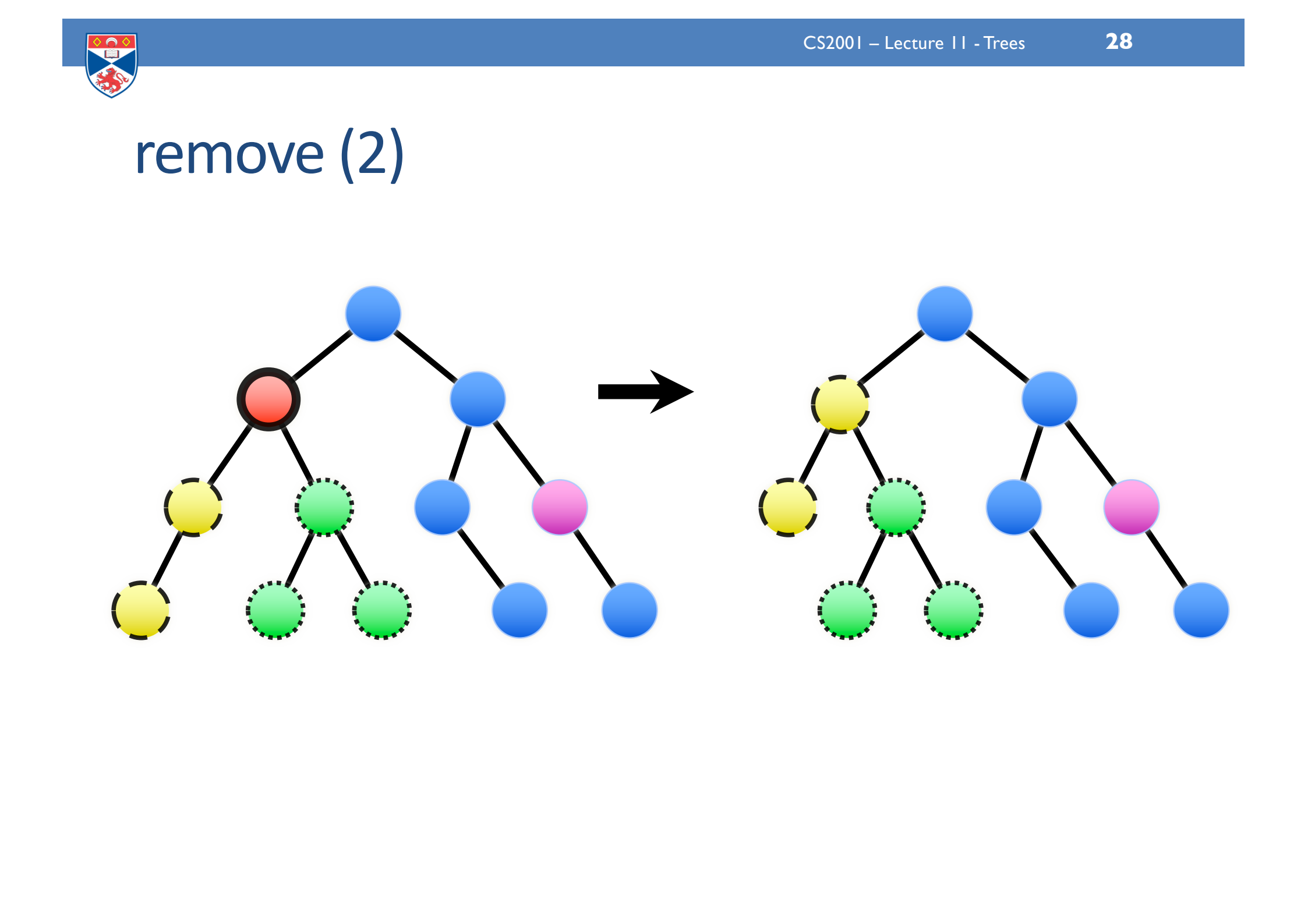
```
public void add(Object element) {
  BinaryTreeNode node = new BinaryTreeNode(element);
  if (root == null) root = node;
  else add(node, root);
}

private void add(BinaryTreeNode subtree, BinaryTreeNode parent) {
  if (subtree != null)
    if (parent.left == null) parent.left = subtree;
    else if (parent.right == null) parent.right = subtree;
         else if (size(parent.left) < size(parent.right))
                   add(subtree, parent.left);
              else add(subtree, parent.right);
}
```



remove



remove (1)



remove (2)

```
public void remove(Object element) {
  BinaryTreeNode parent = findParent(element);
  BinaryTreeNode node = findNode(element);

  if (node != null) {
    if (parent == null) {
      root = node.left;
      add(node.right, root);
    }
    else {
      if (parent.left == node) parent.left = node.left;
      else parent.right = node.left;
      add(node.right, parent);
    }
  }
}
```

```
public int size() {
  return size(root);
}

private int size(BinaryTreeNode subtree) {
  if (subtree == null) return 0;
  else return 1 + size(subtree.left) + size(subtree.right);
}
```

Trees: Complexity

- insert linear
- contains linear
- remove linear
- size linear

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Key Points

- Trees suitable for storing hierarchical data
- Many applications for in-memory and on-disk data structures
- Binary trees restrict nodes to two children
- Many applications use sorted trees: still to come

Read More

- Sedgewick sections 5.4-5.7
- Goodrich & Tamassia section 7.1-7.3