Programming Assignment: Barnes-Hut Simulation

You have not submitted. You must earn 8/10 points to pass.

Deadline Pass this assignment by April 9, 11:59 PM PDT

Instructions

My submission

Discussions

Barnes-Hut Simulation

First, start by downloading the assignment zip file:
http://alaska.epfl.ch/~dockermoocs/parprog1/barneshut.zip

In this assignment, you will implement the parallel Barnes-Hut algorithm for *N-body simulation*. N-body simulation is a simulation of a system of *N* particles that interact with physical forces, such as gravity or electrostatic force. Given initial positions and velocities of all the *particles* (or *bodies*), the N-body simulation computes the new positions and velocities of the particles as the time progresses. It does so by dividing time into discrete short intervals, and computing the positions of the particles after each interval.

Before we study the Barnes-Hut algorithm for the N-body simulation problem, we will focus on a simpler algorithm -- the *direct sum N-body algorithm*. The direct sum algorithm consists of multiple iterations, each of which performs the following steps for each particle:

1. The particle position is updated according to its current velocity (delta is a short time period).x' = x + v_x * delta y' = y + v_y * delta
2. The net force on the particle is computed by adding the individual forces from all the other particles.F_x = F_1x + F_2x + F_3x + ... + F_Nx F_y = F_1y + F_2y + F_3y

+ ... + F_Ny

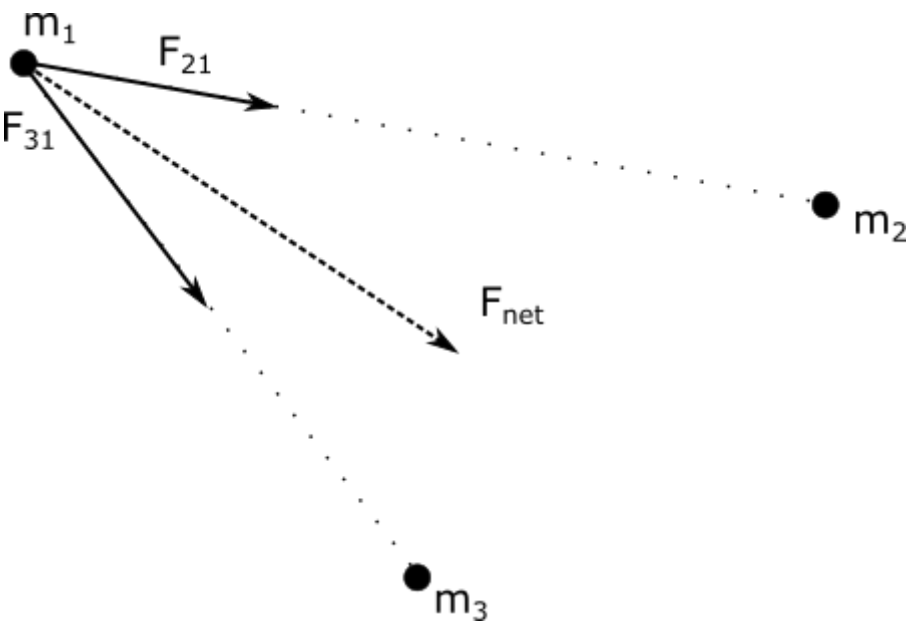
3. The particle velocity is updated according to the net force on that particle.v_x' = v_x + F_x / mass * delta v_y' = v_y + F_y / mass * delta

In this exercise, we will assume that the force between particles is the *gravitational force* from classical mechanics. Let's recall the formula for the gravitational force between two stellar bodies:

```
1   F = G * (m1 * m2) / distance^2
```

Above, F is the absolute value of the gravitational force, m1 and m2 are the masses of the two bodies, and r is the distance between them. G is the gravitational constant.

For each particle, the net force is computed by summing the components of individual forces from all other particles, as shown in the following figure:



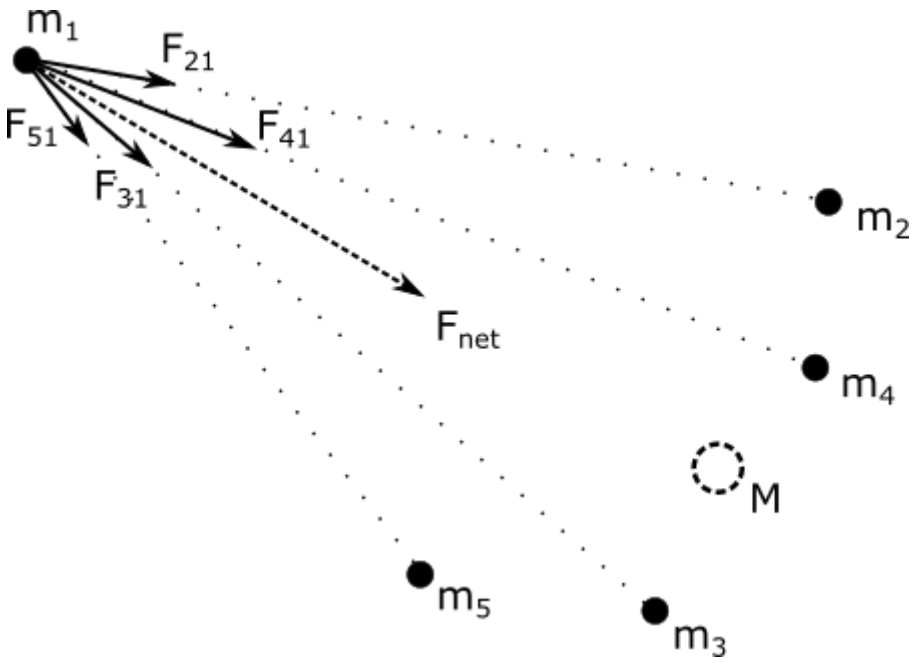
net-force

The direct sum N-body algorithm is very simple, but also inefficient. Since we need to update N particles, and compute N - 1 force contributions for each of those particles, the overall complexity of an iteration step of this algorithm is O(N^2). As the number of particles grows larger, the direct sum N-body algorithm becomes prohibitively expensive.

The Barnes-Hut algorithm is an optimization of the direct sum N-body algorithm, and is based on the following observation:

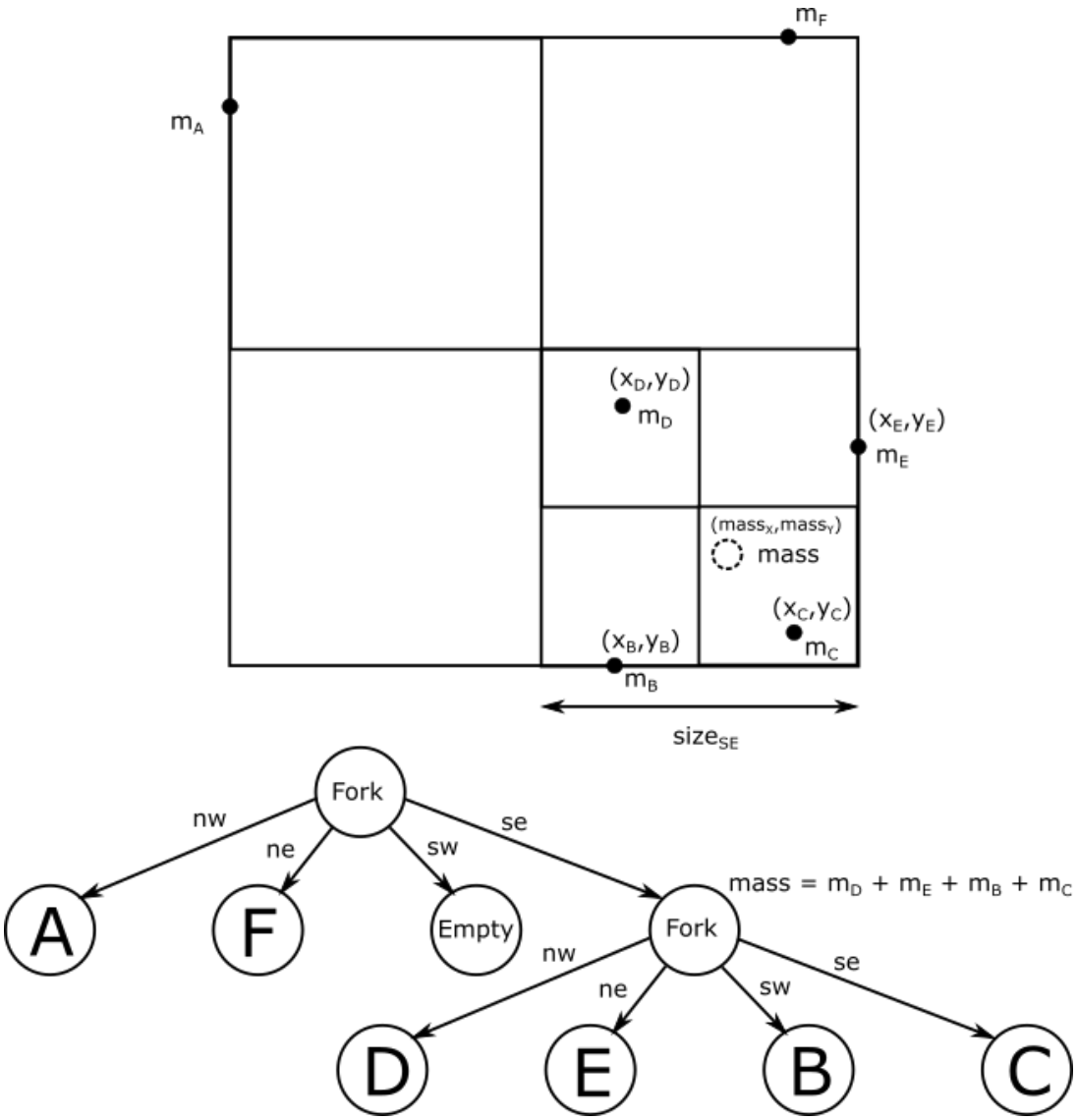
If a cluster of bodies is sufficiently distant from a body *A*, the net force on *A* from that cluster can be approximated with one big body with the mass of all the bodies in the cluster, positioned at the center of mass of the cluster.

This is illustrated in the following figure:



observation

To take advantage of this observation, the Barnes-Hut algorithm relies on a *quadtree* -- a data structure that divides the space into cells, and answers queries such as 'What is the total mass and the center of mass of all the particles in this cell?'. The following figure shows an example of a quadtree for 6 bodies:



quadtree

Above, the total force from the bodies *B*, *C*, *D* and *E* on the body *A* can be approximated by a single body with *mass* equal to the sum of masses *B*, *C*, *D* and *E*, positioned at the center of mass of bodies *B*, *C*, *D* and *E*. The center of mass (massX, massY) is computed as follows:

```
1   mass = m_B + m_C + m_D + m_E
2   massX = (m_B * x_B + m_C * x_C + m_D * x_D + m_E * x_E) / mass
3   massY = (m_B * y_B + m_C * y_C + m_D * y_D + m_E * y_E) / mass
```

An iteration of the Barnes-Hut algorithm is composed of the following steps:

1. Construct the quadtree for the current arrangement of the bodies.
2. Determine the *boundaries*, i.e. the square into which all bodies fit.
3. Construct a quadtree that covers the boundaries and contains all the bodies.
4. Update the bodies -- for each body:
5. Update the body position according to its current velocity.
6. Using the quadtree, compute the net force on the body by adding the individual forces from all the other bodies.

7. Update the velocity according to the net force on that body.

It turns out that, for most spatial distribution of bodies, the expected number of cells that contribute to the net force on a body is log n, so the overall complexity of the Barnes-Hut algorithm is O(n log n).

Now that we covered all the necessary theory, let's finally dig into the implementation! You will implement:

- a quadtree and its combiner data structure
- an operation that computes the total force on a body using the quadtree
- a simulation step of the Barnes-Hut algorithm

Since this assignment consists of multiple components, we will follow the principles of *test-driven development* and test each component separately, before moving on to the next component. That way, if anything goes wrong, we will more precisely know where the error is. It is always better to detect errors sooner, rather than later.

Data Structures

We will start by implementing the necessary data structures: the quadtree, the body data-type and the sector matrix. You will find the stubs in the package.scala file of the barneshut package.

Quadtree Data Structure

In this part of the assignment, we implement the quadtree data structure, denoted with the abstract data-type Quad. Every Quad represents a square cell of space, and can be one of the following node types:

- an Empty node, which represents an empty quadtree
- a Leaf node, which represents one or more bodies
- a Fork node, which divides a spatial cell into four quadrants

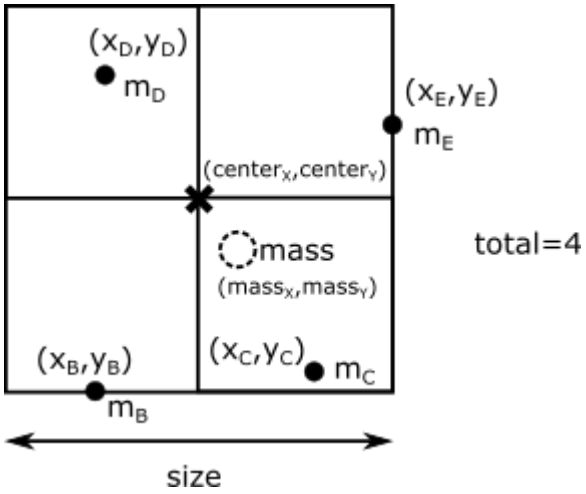
The definition of Quad is as follows:

```
1  sealed abstract class Quad {
2    def massX: Float
3    def massY: Float
4    def mass: Float
5    def centerX: Float
6    def centerY: Float
7    def size: Float
8    def total: Int
9    def insert(b: Body): Quad
10 }
```

Here, massX and massY represent the center of mass of the bodies in the respective cell, mass is the total mass of bodies in that cell, centerX and centerY are the coordinates of the center of the cell, size is the length of the side of the cell,

and total is the total number of bodies in the cell.

Note that we consider the top left corner to be at coordinate (0, 0). We also consider the x axis to grow to the right and the y axis to the bottom.



cell

The method insert creates a new quadtree which additionally contains the body b, and covers the same area in space as the original quadtree. Quadtree is an *immutable* data structure -- insert does not modify the existing Quad object. Note that Body has the following signature:

```
1  class Body(val mass: Float, val x: Float, val y: Float, val xspeed: Float, val
     yspeed: Float)
```

In this part of the exercise, you only need to know about body's position x and y.

Let's start by implementing the simplest Quad type -- the empty quadtree:

```
1  case class Empty(centerX: Float, centerY: Float, size: Float) extends Quad
```

The center and the size of the Empty quadtree are specified in its constructor. The Empty tree does not contain any bodies, so we specify that its center of mass is equal to its center.

Next, let's implement the Fork quadtree:

```
1  case class Fork(nw: Quad, ne: Quad, sw: Quad, se: Quad) extends Quad
```

This node is specified by four child quadtrees nw, ne, sw and se, in the northwest, northeast, southwest and southeast quadrant, respectively.

The northwest is located on the top left, northeast on the top right, southwest on the bottom left and southeast on the bottom right.

The constructor assumes that the children nodes that represent four adjacent cells of the same size and adjacent to each other, as in the earlier figure. The center of the Fork quadtree is then specified by, say, the lower right corner of the quadtree nw. If the Fork quadtree is empty, the center of mass coincides with the center.

Inserting into a Fork is recursive -- it updates the respective child and creates a new Fork.

Finally, the Leaf quadtree represents one or more bodies:

```
1  case class Leaf(centerX: Float, centerY: Float, size: Float, bodies: Seq[Body])
2  extends Quad
```

If the size of a Leaf is greater than a predefined constant minimumSize, inserting an additonal body into that Leaf quadtree creates a Fork quadtree with empty children, and adds all the bodies into that Fork (including the new body). Otherwise, inserting creates another Leaf with all the existing bodies and the new one.

The Body Data-Type

Next, we can implement the Body data-type:

```
1  class Body(val mass: Float, val x: Float, val y: Float, val xspeed: Float, val
     yspeed: Float) {
2    def updated(quad: Quad): Body = ???
3  }
```

Here, xspeed and yspeed represent the velocity of the body, mass is its mass, and x and y are the coordinates of the body.

The most interesting method on the Body is updated -- it takes a quadtree and returns the updated version of the Body:

```
1  def updated(quad: Quad): Body
```

This method is already half-completed for you -- you only need to implement its nested method traverse, which goes through the quadtree and proceeds casewise:

- empty quadtree does not affect the net force
- each body in a leaf quadtree adds some net force
- a fork quadtree that is sufficiently far away acts as a single point of mass
- a fork quadtree that is not sufficiently far away must be recursively traversed

When are we allowed to approximate a cluster of bodies with a single point? The heuristic that is used is that the size of the cell divided by the distance dist between the center of mass and the particle is less than some constant theta:

```
quad.size / dist < theta
```

Hint: make sure you use the distance to compute distance between points, the theta value for the condition, and addForce to add force contributions!

Before proceeding, make sure to run tests against your Quad and Body implementations.

The Sector Matrix

The last data structure that we will implement is the *sector matrix*. In this data structure, we will use the auxiliary class Boundaries, which contains the minX, maxX, minY and maxY fields for the boundaries of the scene:

```
class Boundaries {
  var minX: Float
  var minY: Float
  var maxX: Float
  var maxY: Float
  def size = math.max(maxX - minX, maxY - minY)
}
```

We will also rely on the ConcBuffer data structure, mentioned in the lecture:

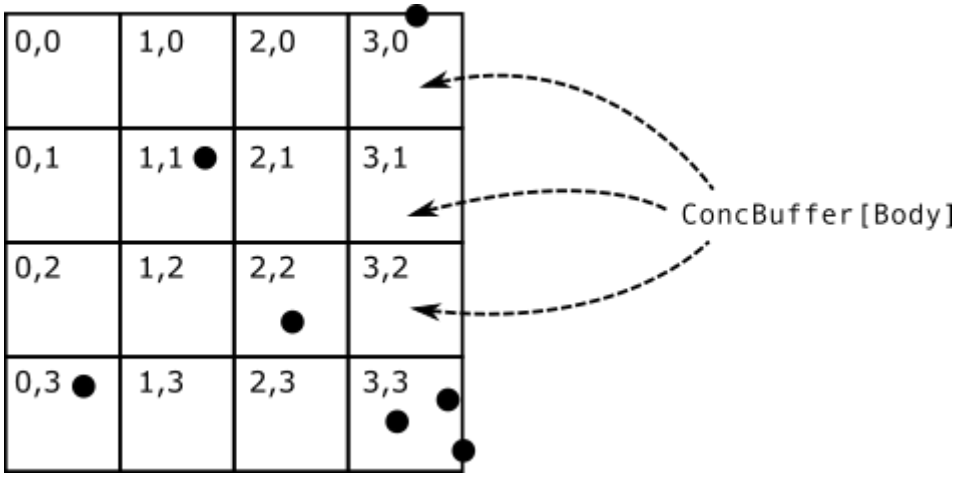
```
class ConcBuffer[T]
```

The ConcBuffer class comes with efficient +=, combine and foreach operations, which add elements into the buffer, combine two buffers and traverse the buffer, respectively. The sector matrix additionally has the toQuad method, which returns a quadtree that contains all the elements previously added with the += method. Recall from the lectures that this combination of methods make the ConcBuffer a *combiner*.

The SectorMatrix is just a square matrix that covers a square region of space specified by the boundaries:

```
class SectorMatrix(val boundaries: Boundaries, val sectorPrecision: Int) {
  val sectorSize = boundaries.size / sectorPrecision
  val matrix = new Array[ConcBuffer[Body]](sectorPrecision * sectorPrecision)
  for (i <- 0 until matrix.length) matrix(i) = new ConcBuffer
  def apply(x: Int, y: Int) = matrix(y * sectorPrecision + x)
}
```

The sectorPrecision argument denotes the width and height of the matrix, and each entry contains a ConcBuffer[Body] object. Effectively, the SectorMatrix is a *combiner* -- it partitions the square region of space into sectorPrecision times sectorPrecision buckets, called *sectors*.



sectormatrix

Combiners such as the SectorMatrix are used in parallel programming to partition the results into some intermediate form that is more amenable to parallelization. Recall from the lecture that one of the ways to implement a *combiner* is by using a bucket data structure -- we will do exactly that in this part of the exercise! We will add three methods on the SectorMatrix that will make it a combiner. We start with the += method:

```
1  def +=(b: Body): SectorMatrix = {
2    ???
3    this
4  }
```

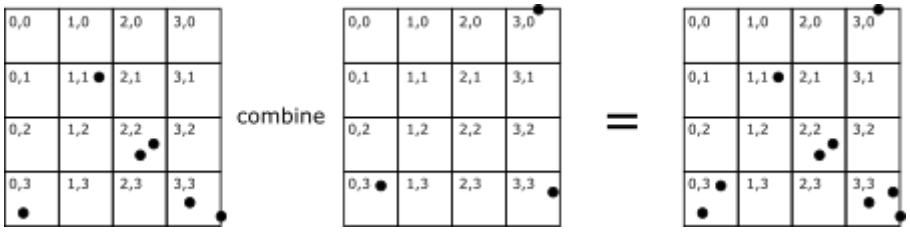
This method should use the body position, boundaries and sectorPrecision to determine the sector into which the body should go into, and add the body into the corresponding ConcBuffer object.

Importantly, if the Body lies outside of the Boundaries, it should be considered to be located at the closest point within the Boundaries for the purpose of finding which ConcBuffer should hold the body.

Next, we implement the combine method, which takes another SectorMatrix, and creates a SectorMatrix which contains the elements of both input SectorMatrix data structures:

```
1  def combine(that: SectorMatrix): SectorMatrix
```

This method calls combine on the pair of ConcBuffers in this and that matrices to produce the ConcBuffer for the resulting matrix. You can safely assume that combine will only be called on matrices of same dimensions, boundaries and sector precision.



combine

The nice thing about the sector matrix is that a quadtree can be constructed in parallel for each sector. Those little quadtrees can then be linked together. The toQuad method on the SectorMatrix does this:

```
1  def toQuad(parallelism: Int): Quad
```

This method is already implemented -- you can examine it if you would like to know how it works.

Congratulations, you just implemented your first combiner! Before proceeding, make sure to run those unit tests.

Implementing Barnes-Hut

Now that we have all the right data structures ready and polished, implementing Barnes-Hut becomes a piece of cake.

Take a look at the file Simulator.scala, which contains the implementation of the Barnes-Hut simulator, and in particular the step method. The step method represents one step in the simulation:

```
1  def step(bodies: Seq[Body]): (Seq[Body], Quad) = {
2    // 1. compute boundaries
3    val boundaries = computeBoundaries(bodies)
4
5    // 2. compute sector matrix
6    val sectorMatrix = computeSectorMatrix(bodies, boundaries)
7
8    // 3. compute quadtree
9    val quad = computeQuad(sectorMatrix)
10
11   // 4. eliminate outliers
12   val filteredBodies = eliminateOutliers(bodies, sectorMatrix, quad)
13
14   // 5. update body velocities and positions
15   val newBodies = updateBodies(filteredBodies, quad)
16
17   (newBodies, quad)
18 }
```

The pre-existing code in step nicely summarizes what this method does.

Computing the Scene Boundaries

First, we must compute the boundaries of all the bodies in the scene. Since bodies move and the boundaries dynamically change, we must do this in every iteration of the algorithm. The computeBoundaries method is already implemented -- it uses

the aggregate combinator on the sequence of bodies to compute the boundaries:

```
def computeBoundaries(bodies: Seq[Body]): Boundaries = {
  val parBodies = bodies.par
  parBodies.tasksupport = taskSupport
  parBodies.aggregate(new Boundaries)(updateBoundaries, mergeBoundaries)
}
```

How does this work? The aggregate method divides the input sequence into a number of chunks. For each of the chunks, it uses the new Boundaries expression to create the accumulation value, and then folds the values in that chunk calling updateBoundaries on each body, in the same way a foldLeft operation would. Finally, aggregate combines the results of different chunks using a reduction tree and mergeBoundaries.

So, we need the updateBoundaries method:

```
def updateBoundaries(boundaries: Boundaries, body: Body): Boundaries
```

Given an existing boundaries object and a body, the updateBoundaries updates the minX, minY, maxX and maxY values so that the boundaries include the body.

Next, the mergeBoundaries method creates a new Boundaries object, which represents the smallest rectangle that contains both the input boundaries:

```
def mergeBoundaries(a: Boundaries, b: Boundaries): Boundaries
```

Question: Is mergeBoundaries associative? Is it commutative? Does it need to be commutative?

Implement these two methods, and test that they work correctly!

Building the Quadtree

Next, we need to build a Quad tree from the sequence of bodies. We will first implement the computeSectorMatrix method to get the SectorMatrix:

```
def computeSectorMatrix(bodies: Seq[Body], boundaries: Boundaries): SectorMatrix
```

Hint: aggregate the SectorMatrix from the sequence of bodies, the same way it was used for boundaries. Use the SECTOR_PRECISION constant when creating a new SectorMatrix.

Test that these methods work correctly before proceeding!

Eliminating Outliers

During the execution of the Barnes-Hut algorithm, some of the bodies tend to move far away from most of the other bodies. There are many ways to deal with such *outliers*, but to keep things simple, we will eliminate bodies that move too fast and too far away.

We will not go into details of how this works, but if you'd like to know more, you can try to understand how the eliminateOutliers method works.

Updating Bodies

The updateBodies method uses the quadtree to map each body from the previous iteration of the algorithm to a new iteration:

```
1   def updateBodies(bodies: Seq[Body], quad: Quad): Seq[Body]
```

Recall that we already implemented the updated method which updates a single body.

Running Barnes-Hut

At last, the parallel Barnes-Hut algorithm is implemented. Note that, despite all the work, we kept our Barnes-Hut algorithm implementation simple and avoided the details that a more realistic implementation must address. In particular:

- we represented each body as a single point in space
- we restricted the simulation to two-dimensional space
- we ignored close encounter effects, such as body collision or tearing
- we ignored any relativistic effects, and assume classical mechanics
- we ignored errors induced by floating point computations

You can now run it as follows:

```
1   > runMain barneshut.BarnesHut
```

To visualize the quadtree, press the *Show quad* button, and then hit the *Start/Pause* button.

Play with the parallelism level and the number of bodies, and observe the average speedups in the lower right corner. Then sit back, and enjoy the show!

How to submit

Copy the token below and run the submission script included in the assignment download. When prompted, use your email address **huajianmao@gmail.com**.

Generate new token

Your submission token is unique to you and should not be shared with anyone. You may submit as many times as you like.