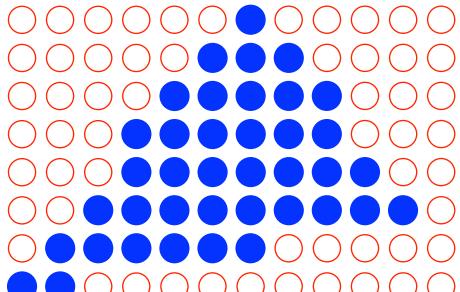


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COMP27112

Computer  
Graphics  
and  
Image Processing



## 4: Polygons and pixels

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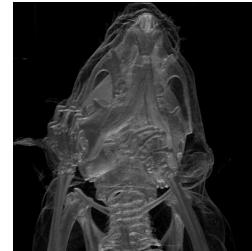
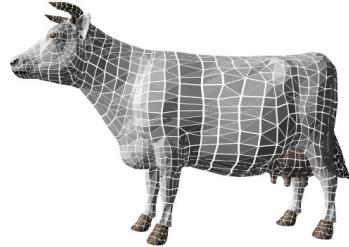
## Introduction

- We'll look at
  - Properties of polygons: convexity, winding, faces, normals
  - Scan conversion of polygons
  - Hidden surface removal with the Z-buffer
  - Data structures for polygonal models
  - Where do we get polygons from?

2

## Polygons as building blocks

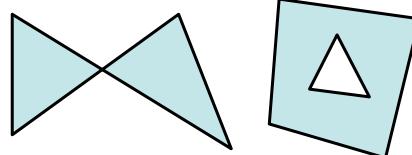
- The basic unit of 2D raster graphics is the pixel
- The most common basic unit of 3D graphics is the polygon
- The most common polygon is the triangle
- Many different types of 3D objects can be modelled well with meshes of polygons...
- ...but there are other techniques too which don't involve polygons (e.g., volume rendering, particle systems)



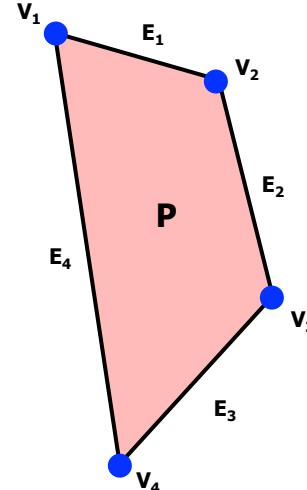
3

## What is a polygon?

- An ordered set of vertices ( $V_1 \dots V_4$ )
- A set of edges between each pair of vertices ( $E_1 \dots E_4$ )
- The polygon ( $P$ ) is then the space bounded by the vertices
- Some polygons are not so simple...



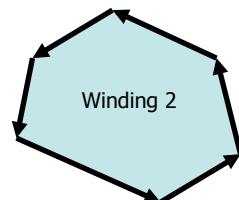
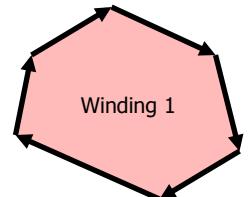
- So we avoid polygons like that



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## Polygon attributes: winding

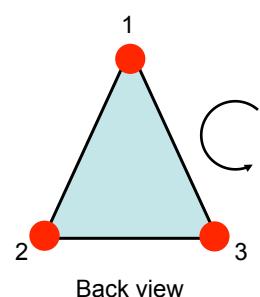
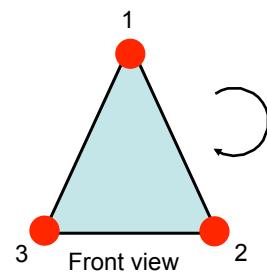
- The order in which the edges connect the vertices gives a polygon a “winding”
- There are two winding orders: **clockwise** and **counter clockwise** (but which is which depends on which side you view from)
- We need to use a consistent winding for **all** the polygons in a scene



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## Polygon attributes: faces

- If we have a consistent winding, then the polygon has two defined faces – a **front** and a **back**
- If all the vertices lie on plane, then we have a planar polygon (which is almost always what we want)
- Triangles are always planar

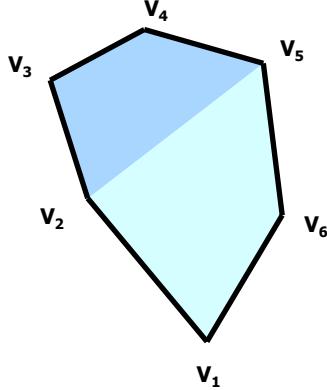


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## Polygon attributes: faces

- Non-planar polygons have vertices that do not all lie on a single plane
- Although they have two "sides" they don't have obvious faces in the same way
- This can cause problems with some graphical algorithms...
- ...so generally we assume our polygons are planar
- Most graphics systems will only guarantee to draw planar polygons



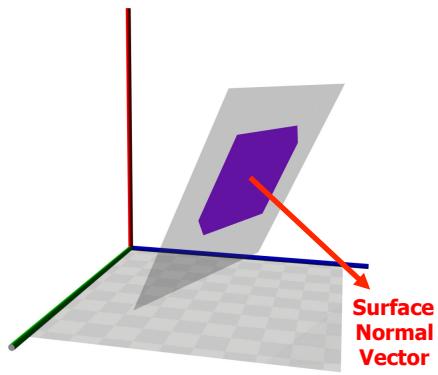
Here,  $[V_1 V_2 V_5 V_6]$  lie on one plane, but  $[V_2 V_3 V_4 V_5]$  lie on another. This is a non-planar polygon

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## Polygon attributes: surface normal

- With a consistent winding order, we can define a property called the **surface normal**
- This is a vector perpendicular to the plane of the polygon
- We can use this to give a polygon a distinguishable 'front' and 'back' ...
- ...and also to describe its **orientation** in 3D space
- Orientation is an essential property used extensively in lighting calculations, collisions, culling...



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## The Cross Product

- is a vector, defined as follows:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} y_1 \times z_2 - z_1 \times y_2 \\ x_1 \times z_2 - z_1 \times x_2 \\ x_1 \times y_2 - y_1 \times x_2 \\ 1 \end{bmatrix}$$

- For two vectors, their cross product is a third vector **perpendicular** to them both (forming a right handed system)

$V3 = V1 \times V2$

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## Finding the surface normal

- Choose a pair of sequential edges  $E_1$  and  $E_2$  and compute their vectors
- Invert the direction of the first so they now emanate from their shared vertex
- Their cross product gives the surface normal  $N$

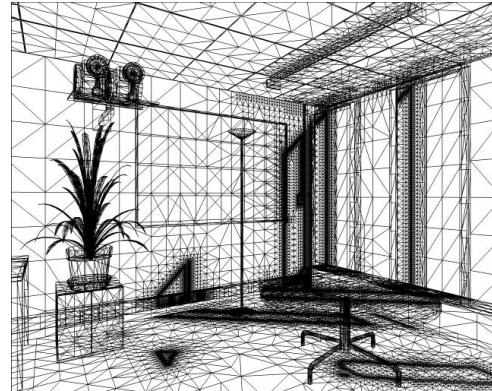
$$N = E_2 \times -E_1$$

- We then almost always normalize  $N$  (make its length 1)

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## Representing things with polygons

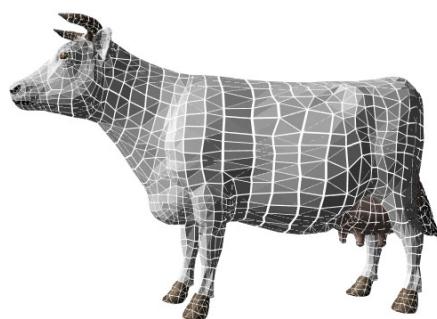
- How can we represent a scene using polygons?
- One option is to have a huge list of individual polygons, to colour them individually and to draw them all in order
- This is called **polygon soup** and has a number of undesirable properties...



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## Problems with polygon soup

- Huge waste of storage space... most models contain "surfaces" not individual polygons, so we could share vertices rather than replicate them per polygon
- Complete loss of semantics... does a polygon belong to a cow... or a table...?
- Leads to 'brute force' rendering, and makes interaction with the model complex

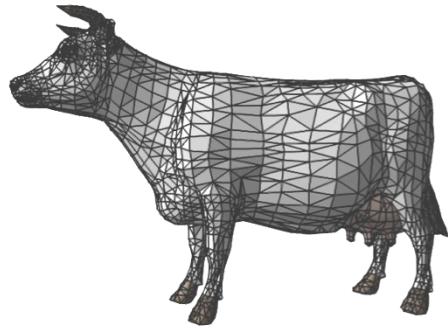


18,000 separate  
vertices, many repeated

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## Polygon meshes

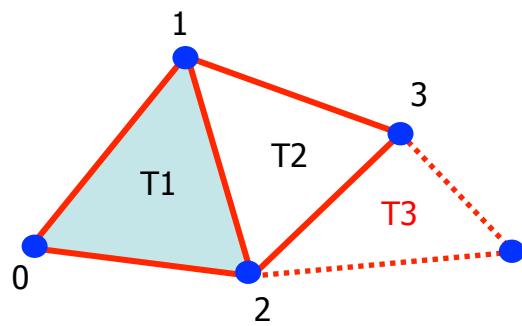
- Use linked groups of polygons, or **meshes**, to represent surfaces
- Retains semantics of “surface”...
- ...but reduces storage by sharing vertices and edges
- Helps with interaction
- Helps with structuring the model so we can manipulate it more easily



6,000 separate vertices,  
mostly unique/shared

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## Meshes: triangle strips

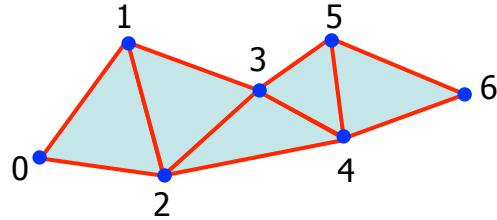


- Add one new **vertex**...
- ... get one new **triangle**

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## Meshes: triangle strips

- Collection of linked triangles
- Very widely used – efficient

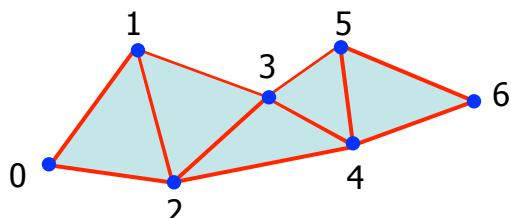


- **N** linked triangles can be defined using **N+2** vertices – compared with **3N** vertices if each triangle were defined separately

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## Triangle strip in OpenGL

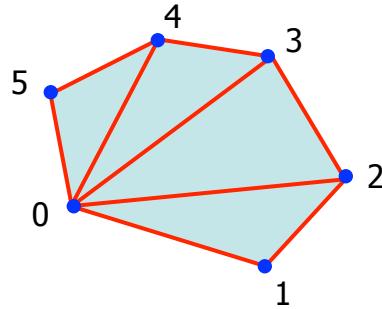
```
glBegin(GL_TRIANGLE_STRIP);
    glVertex3f(x0, y0, z0);
    glVertex3f(x1, y1, z1);
    glVertex3f(x2, y2, z2);
    glVertex3f(x3, y3, z3);
    /* and other vertices */
glEnd();
```



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## Meshes: triangle fan

- Collection of linked triangles

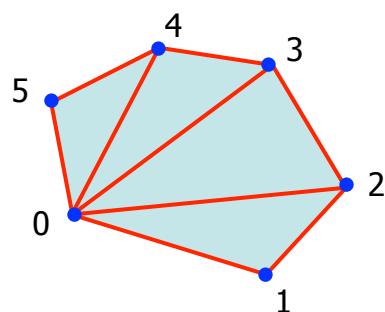


- **N** linked triangles can be defined using **N+2** vertices – compared with **3N** vertices if each triangle were defined separately

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## Triangle fan in OpenGL

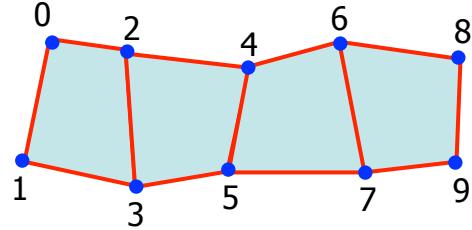
```
glBegin(GL_TRIANGLE_FAN);  
    glVertex3f(x0, y0, z0);  
    glVertex3f(x1, y1, z1);  
    glVertex3f(x2, y2, z2);  
    /* and other vertices */  
glEnd();
```



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## Meshes: quadrilateral strips

- Collection of linked quadrilaterals (a.k.a. quads)

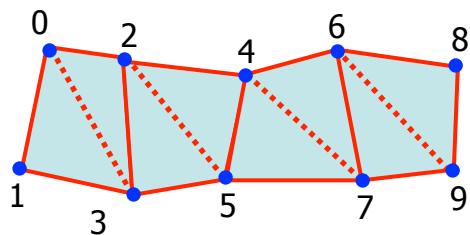


- N** quads can be defined using **2N+2** vertices, compared with **4N** separate vertices

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## Quad strip in OpenGL

```
glBegin(GL_QUAD_STRIP);
    glVertex3f(x0, y0, z0);
    glVertex3f(x1, y1, z1);
    glVertex3f(x2, y2, z2);
    glVertex3f(x3, y3, z3);
    /* and other vertices */
glEnd();
```



Tessellated into triangles during rendering

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## Meshes: quadrilateral meshes

- Collection of linked quadrilaterals
  - Used in terrain modelling, and for approximating curved surfaces
- $N \times M$  quads can be defined using  $(N+1) * (M+1)$  vertices, compared with  $4*M*N$  separate vertices

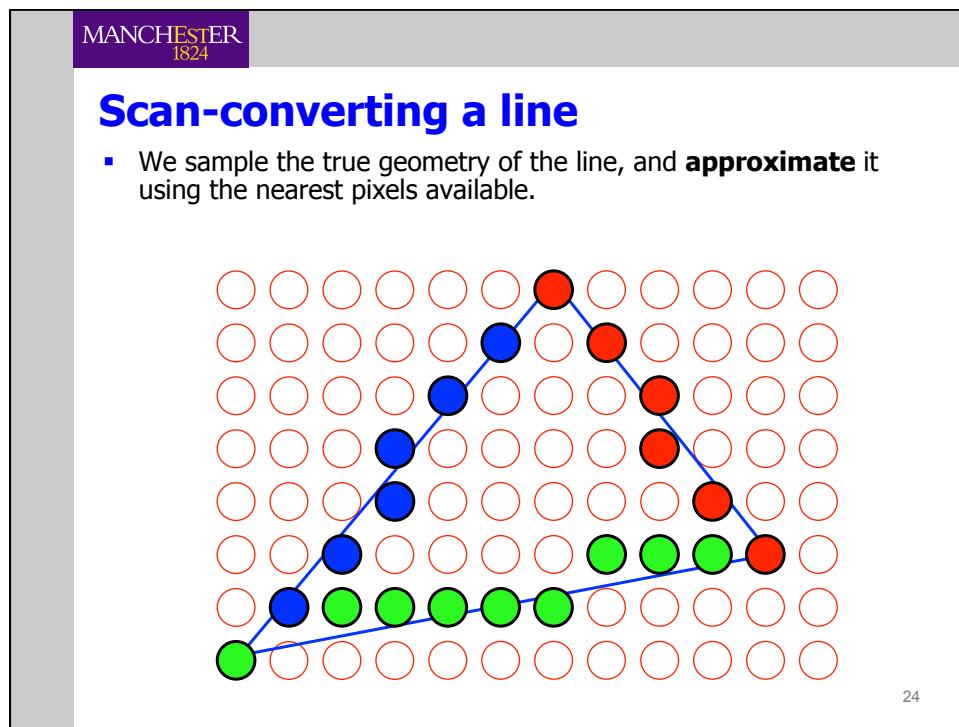
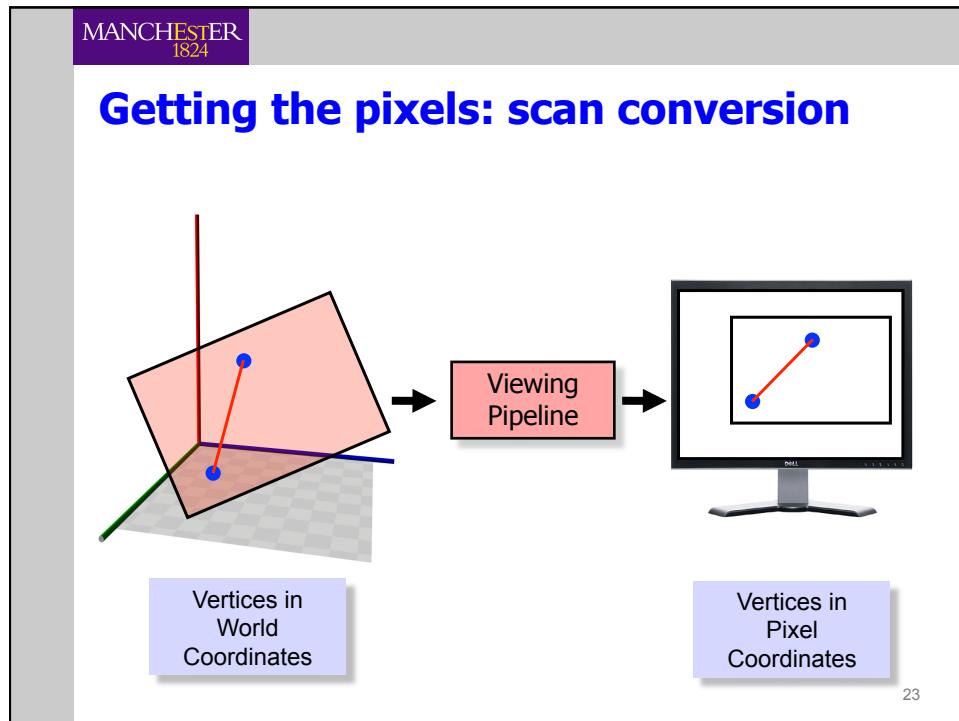
21

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## From the model to the display

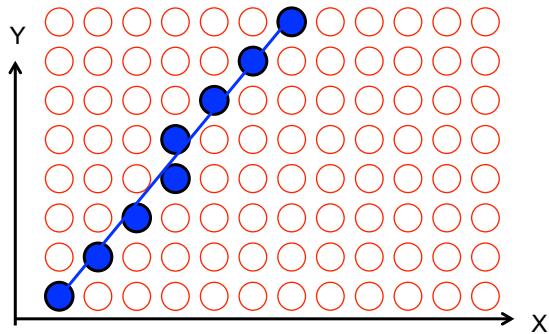
- We model in a 3D space, then take a view using a “camera” to create a 2D screen image

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## Bresenham's algorithm

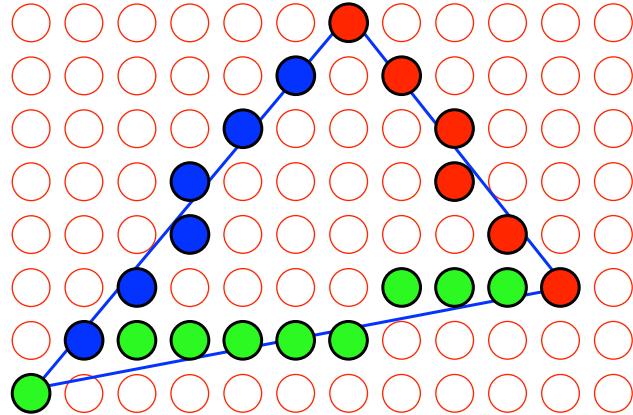
- $y = mx + c$
- As we move horizontally  $x$  changes by 1 pixel
- So  $y_{n+1} = y_n + m$
- Round  $y_{n+1}$  to the nearest pixel
- Need to swap  $x$  and  $y$  according to gradient of the line
- Bresenham (1965) developed a fast algorithm using only integer arithmetic
- Still in use today



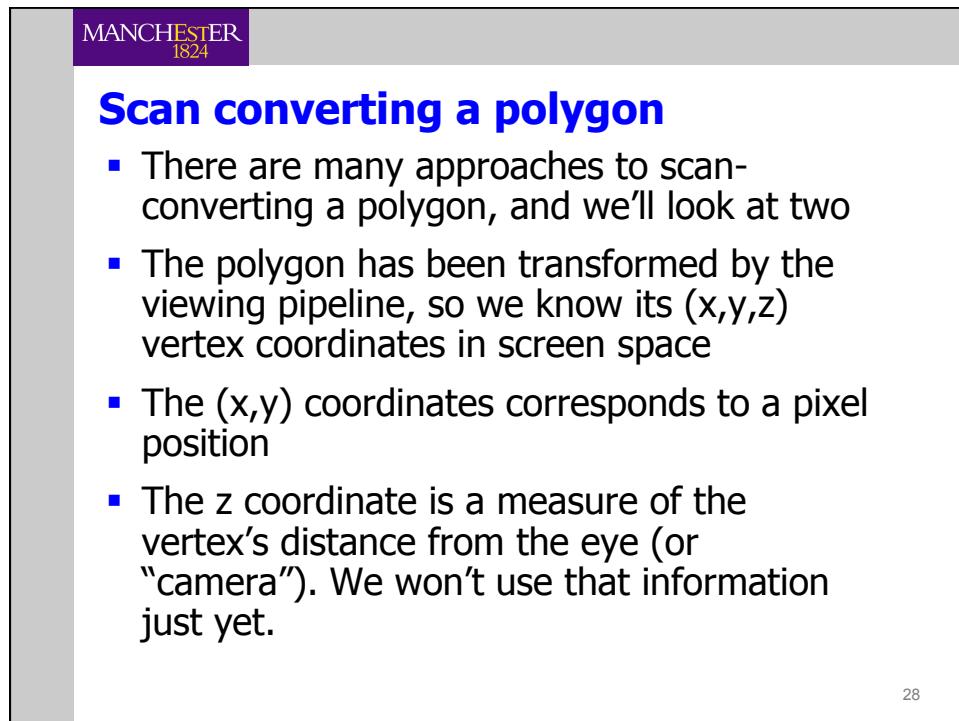
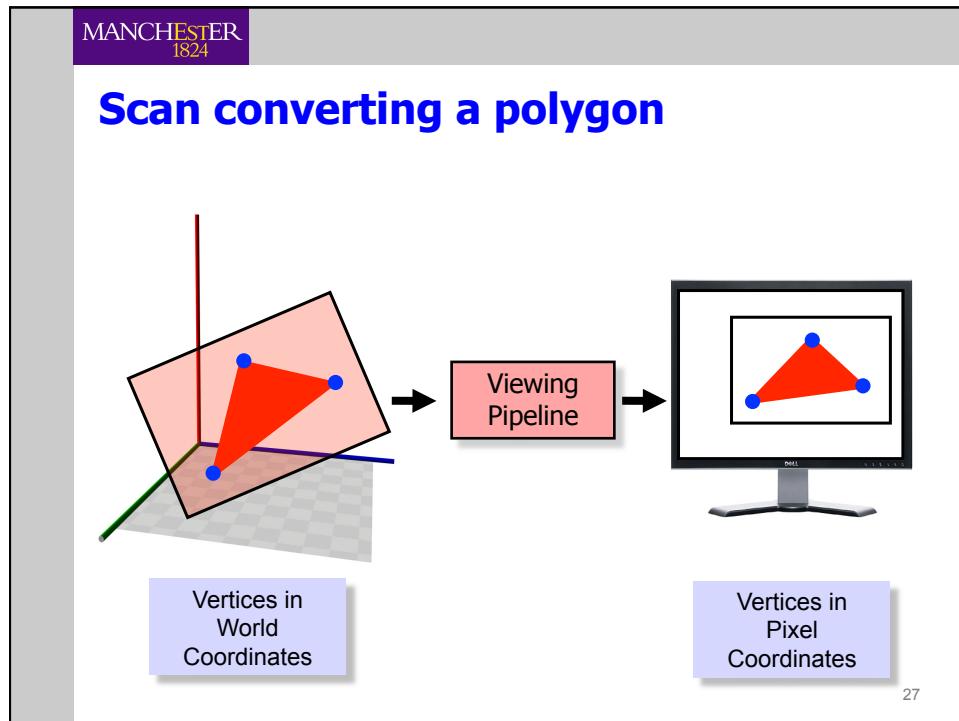
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## Scan-converting a line

- We sample the true geometry of the line, and **approximate it** using the nearest pixels available.

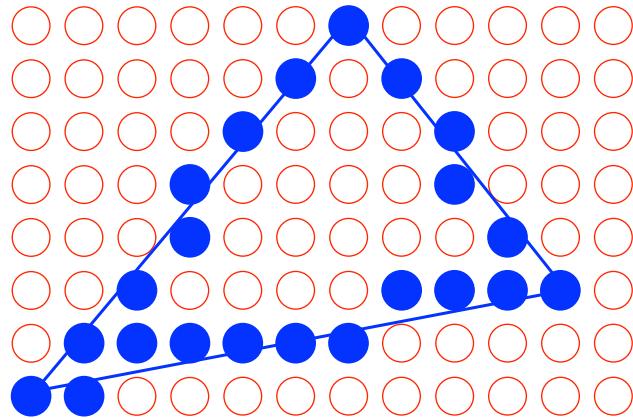


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## Scan converting a triangle

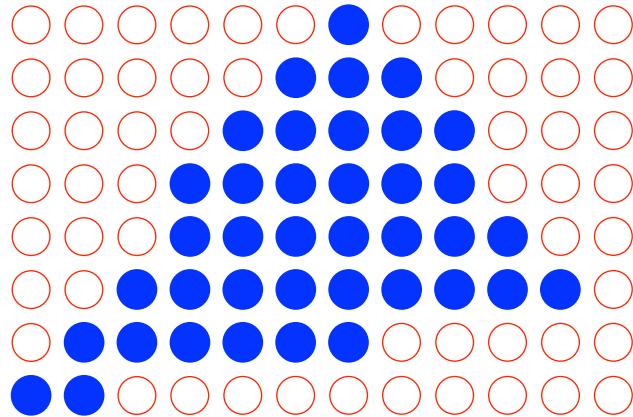
- We can scan-convert each of the edges



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## Scan converting a triangle

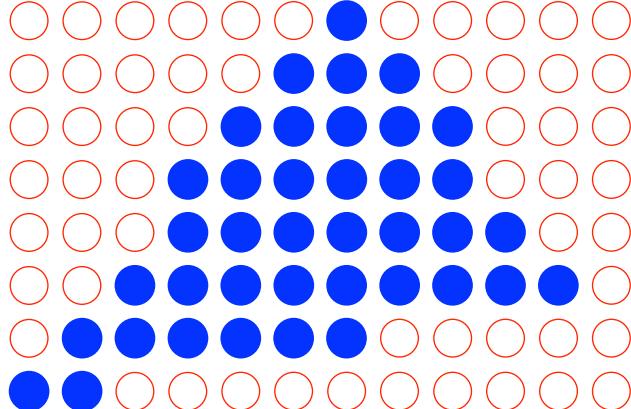
- Now we can process each row of pixels and fill in the remaining interior pixels



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## Scan converting a triangle

- In practice, this naïve approach is never used. There are far more efficient methods, which can be implemented in hardware

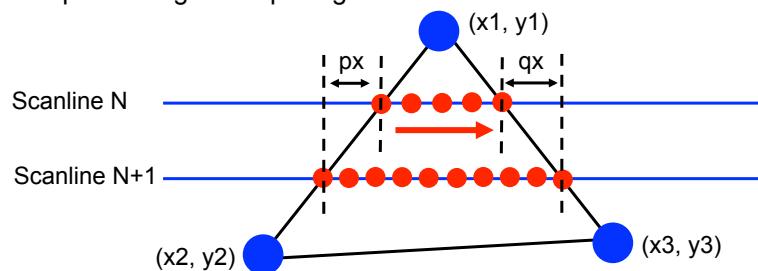


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## Scan converting a triangle efficiently

- One example is the “sweep-line” algorithm

The algorithm steps down a pair of edges, starting in this example at  $(x_1, y_1)$ , then goes down scanline by scanline, finding the start and end of the part of the scanline inside the triangle. Then it fills the pixels inside the triangle for that scanline. Efficient because we only need to compute the slopes of the edges ( $px$  and  $qx$ ) once, at the beginning. It is, however, a floating point algorithm, and we have to keep rounding to the pixel grid.

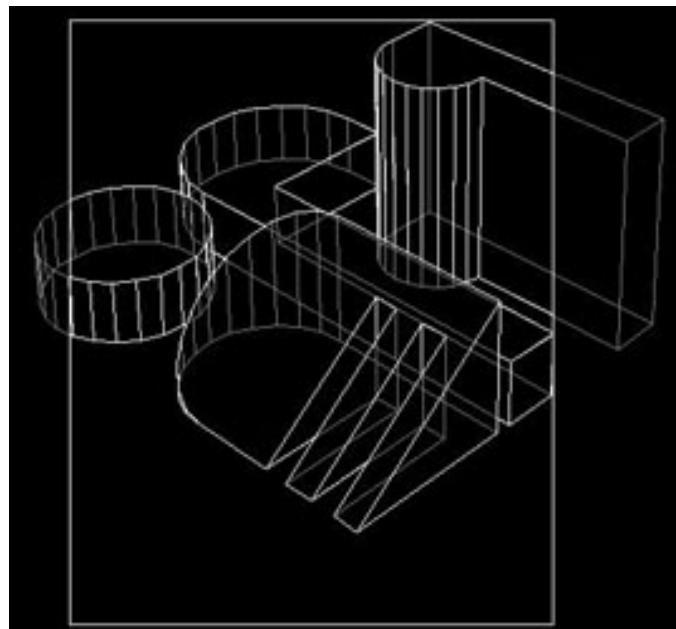


32

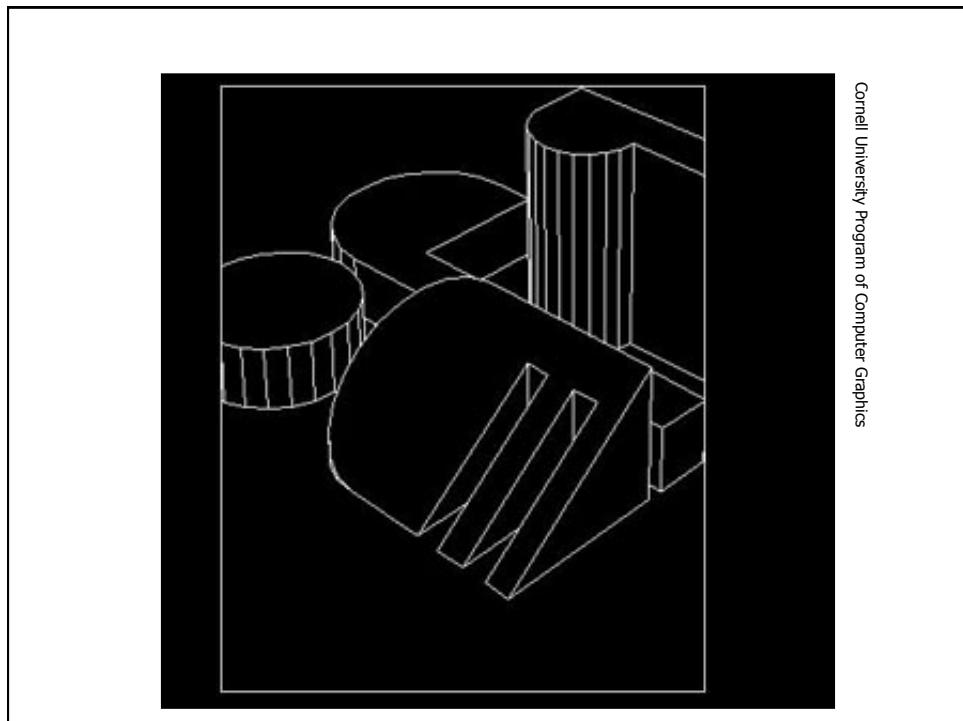
## Hidden surface removal

- Question: viewing the world from a particular viewpoint, which parts of the world can we see, and which parts can we not see, because other objects (partially) block them?
- There are two fundamentally different approaches that we can take to solving this problem:
  - We can solve it in **world space**. We can try to work it out geometrically in 3D, and then draw the result. This was the first approach used, 1960-1980ish, and it is extremely hard
  - We can do it in **display space**. During scan-conversion, whenever we generate a pixel **P**, we determine whether some **other** 3D object, nearer to the eye, **also** maps to **P**. This is now the standard approach.

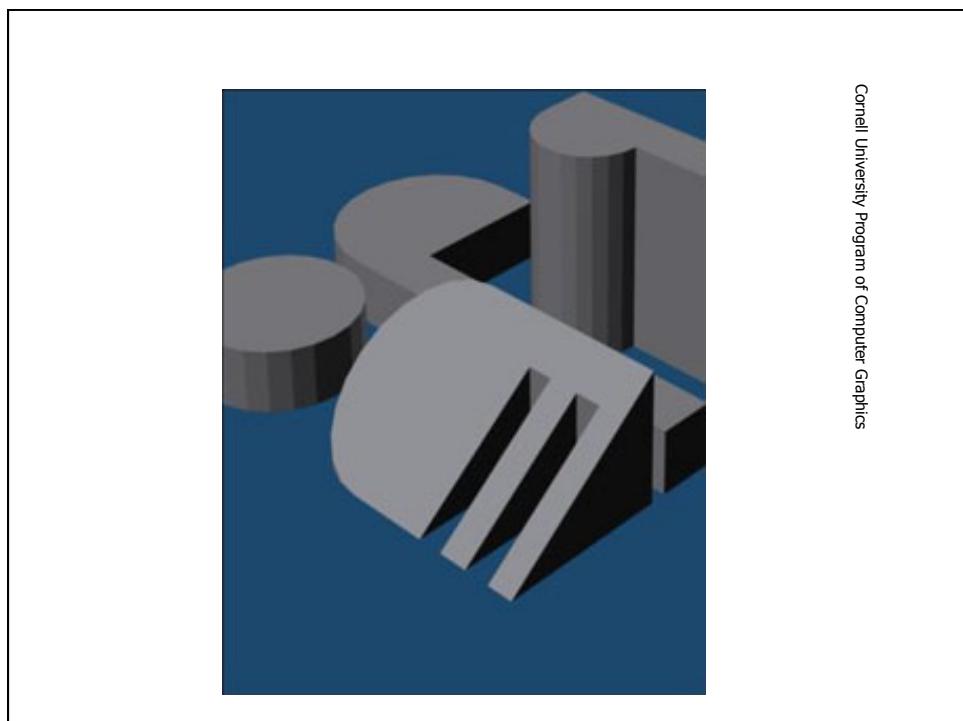
33



Cornell University Program of Computer Graphics



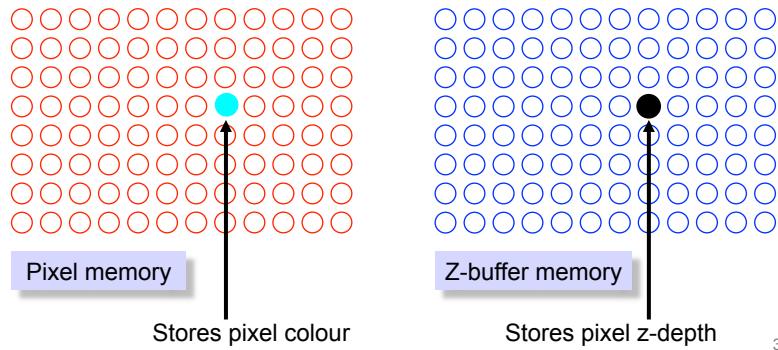
Cornell University Program of Computer Graphics



Cornell University Program of Computer Graphics

## The Z-buffer (or depth-buffer)

- We introduce a new data structure called the Z-buffer
- For every pixel in the display memory, there is a corresponding entry in the Z-buffer
- The Z-buffer is used to keep a record of the z-value of each pixel



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## The Z-buffer algorithm

1. Initialise each pixel to desired background colour
2. Initialise each Z-buffer entry to MAXDEPTH (biggest possible number)
3. For each pixel  $P$  generated during scan-conversion of an object:
  - IF z-coordinate of  $P < \text{Z-BUFFER}[P]$  THEN
    - // i.e., the part of the 3D object that mapped to  $P$
    - // is nearer to the eye than any other part of the world
    - // that has so far mapped to  $P$
    - Compute colour of  $P$  // lighting, texture... later in the course
    - Store colour in  $P$
    - Store (i.e., update) z-coordinate of  $P$  in  $\text{Z-BUFFER}[P]$
  - ELSE
    - // something else has already mapped to  $P$  and is nearer to us
    - // so don't change  $P$

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## The Z-buffer: Z-fighting

- Lack of precision in the Z-buffer leads to incorrect rendering of pixels with similar z-values
- Especially horrible when animated
- “Solution” : `glPolygonOffset()`

Grant James, zeusand.com

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## Modelling with multiple objects

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## Structured models

- We can represent complex models using a hierarchical structure
- ~~Later in the course we'll look at how to give structure to complex models, using scene graphs (no, I've decided not to cover this.)~~
- For now we'll consider data structures for keeping track of polygons

```
graph TD; M[Molecule] --> A[Atoms]; M --> B[Bonds]; A --> S[Spheres]; B --> C[Cylinders]; S --> T[Triangles]; C --> Q[Quads]
```

Ocw.ox.ac.com

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## Structured polygons

- Maintain a hierarchy of structure

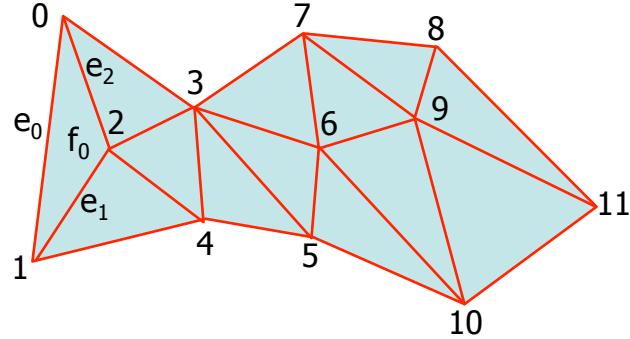
Model → Object → Surfaces → Polygons → Edges/Vertices

- Object
- Surface
  - Polygons
  - Edges
  - Vertices

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## General polygon mesh

- Flexible way to define linked polygons



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## Mesh data structure

- vertex list**
- edge list**, indexing into the **vertex** list
- face list**, indexing into the **edge** list

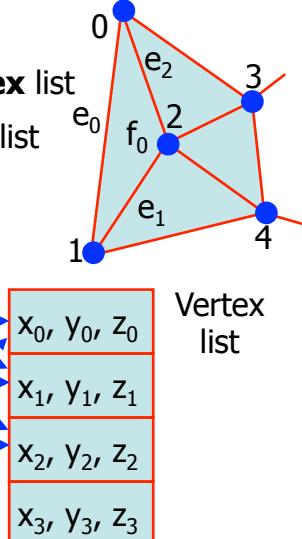
Face list

$f_0$

Edge list

$e_0$   
 $e_1$   
 $e_2$

Vertex list



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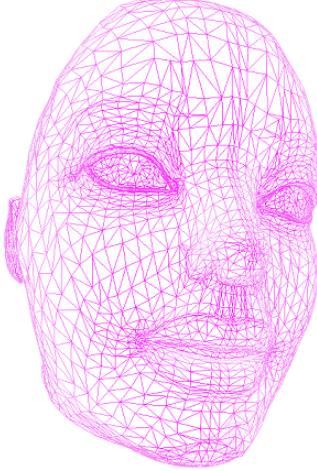
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## Practicalities: file formats

- Meshes are often big
- Many different file formats
- The example here is a Wavefront “obj” file:
  - 2953 vertices

```
## Three-D Library generated .obj file
## data/womanheadH
##
v 0.698214 -0.204674 2.690314
v 0.685697 -0.198298 2.709758
v 0.715357 -0.148323 2.717670
v 0.716248 -0.176445 2.695745
v 0.666189 -0.129036 2.739556
v 0.704262 -0.140100 2.727099
v 0.674602 -0.190075 2.719187
v 0.655446 -0.197328 2.735766
v 0.791905 -0.143497 2.688993
```

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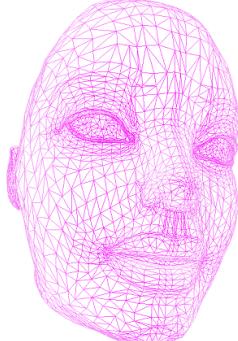


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## Example: the obj file format

- List of vertices
  - **v** x y z
- List of vertex normals
  - **vn** x y z
- List of vertex texture coordinates
  - **vt** s t
- List of faces
  - **f** v1/vt1/vn1 v2/vt2/vn2 v3/vt3/vn3...
- List of groups
  - **g** f1 f2 f3
- Browse the full details, and get a feel for the problem, at <http://www.dcs.ed.ac.uk/home/mxr/gfx/3d/OBJ.spec>

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### Creating Geometry: (1) by hand

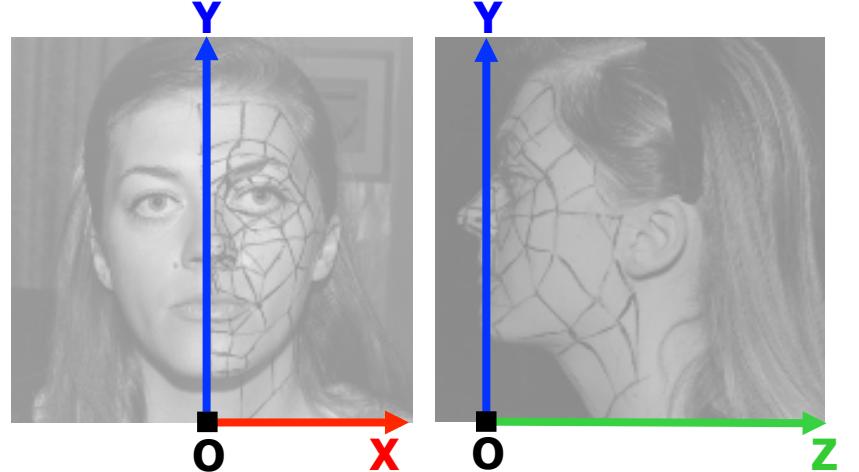


- Making a polygon mesh by hand – Mme Sylvie Gouraud (1971)

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### Creating Geometry: by hand



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## Mme Gouraud (1971)



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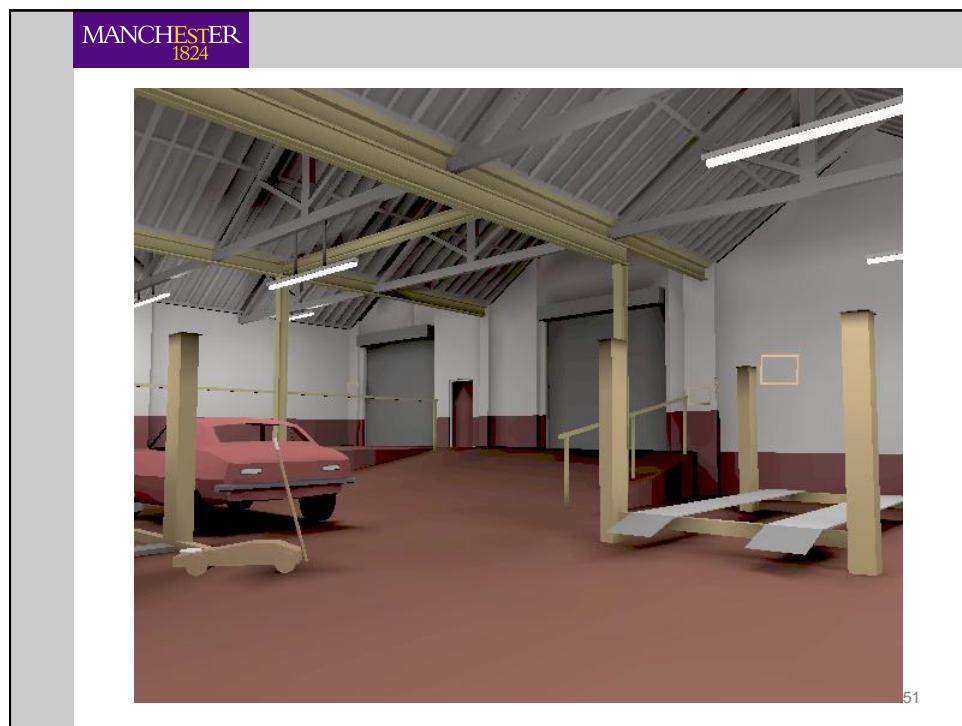
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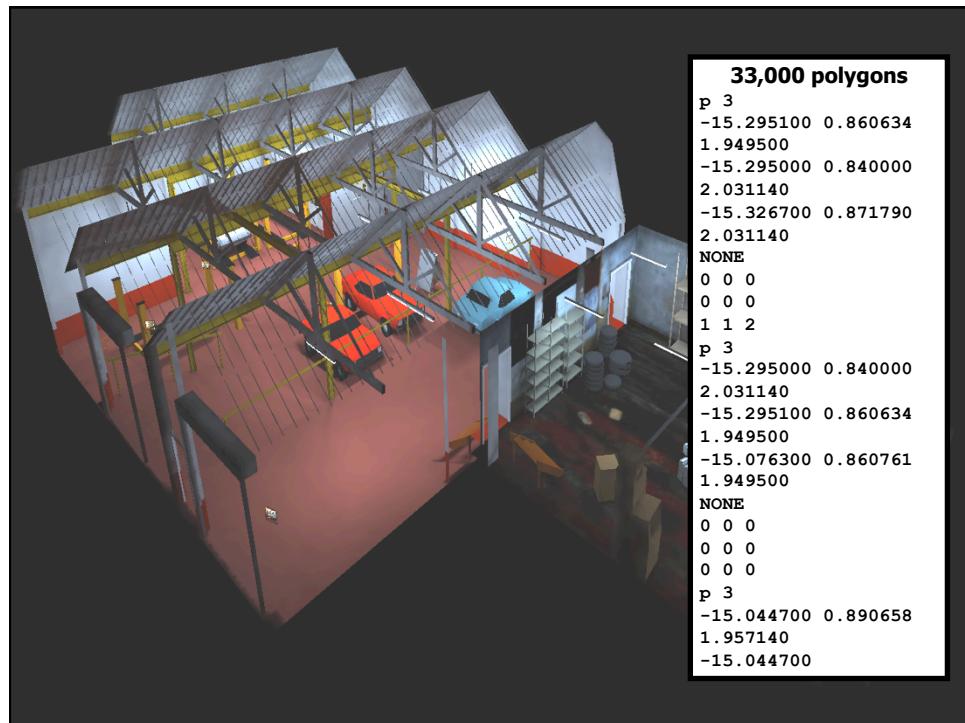
### Case Study: manual reconstruction

- Chestergate: Project with Greater Manchester Police



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The image contains two photographs of a workshop interior. The left photo shows a room with wooden shelving, a compressor, and stacks of boxes. The right photo shows a similar scene with white 3D models of the shelving and boxes placed in their approximate positions. A purple header bar reads "MANCHESTER 1824". Below the images is a bulleted list discussing manual reconstruction.

- Manual reconstruction may omit much detail
  - This is very difficult to avoid (but how could we improve this?)

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## Chestergate: Manual reconstruction

- Reconstruction performed entirely manually
  - Measurements from architectural plans
    - CAD model created in textual format
  - Measurements from photographs
  - Images and textures scanned from photographs
  - 20 person-months of effort (approx)
  - How to do better (Research Topic)

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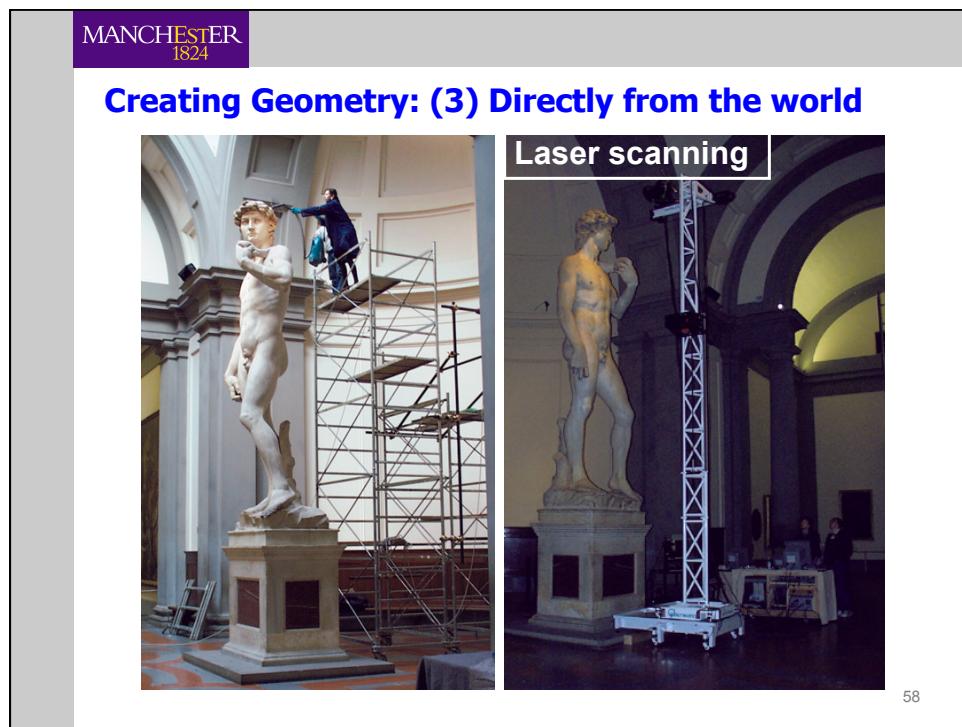
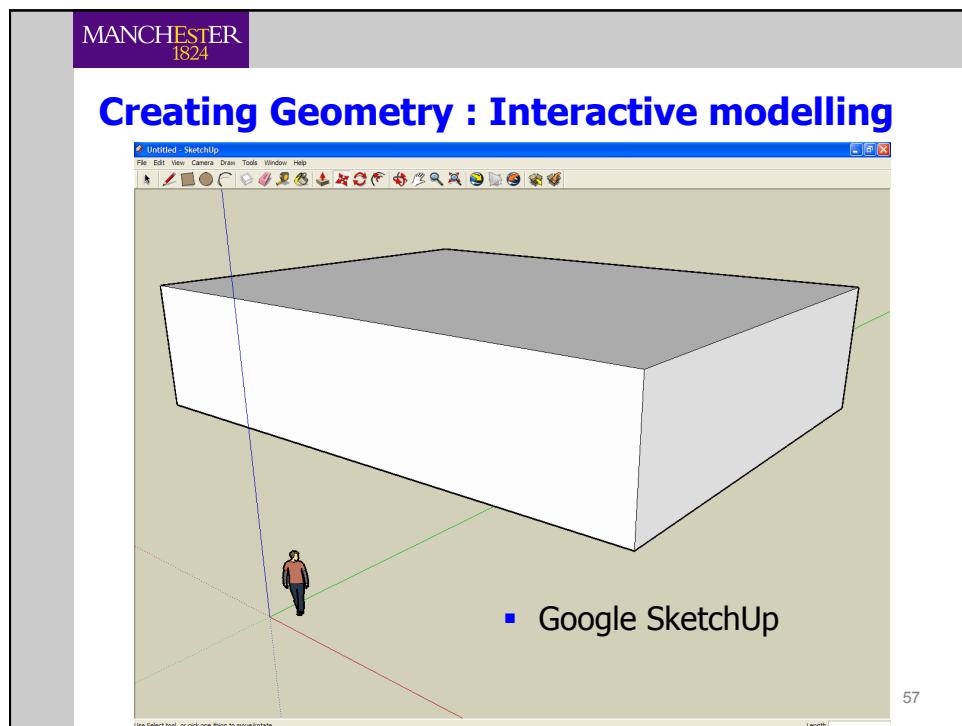
## Creating Geometry : (2) Interactive modelling

The screenshot shows a 3D modeling application with four viewports:

- Front (XY):** Displays a wireframe model of a ring and a sphere.
- Side (ZY):** Displays a wireframe model of the same objects.
- Plan (XZ):** Displays a wireframe model of the objects.
- 3D View:** Displays the final rendered model, which is a white ring with a red sphere inside it, centered in a coordinate system.

The interface includes a toolbar at the top with various tools like File, Edit, View, Object, Surface, Vertex, 3D, Tools, Help, and a status bar at the bottom.

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## Mme Gouraud (1999)



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- Capture geometry directly from images/video
- Important area of ongoing research (see COMP37111)



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