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Department of Mechanics

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Computational modelling of thermoplasts and usin in anchor systems

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Počítačové modelování reaktoplastů a jejich využití v kotevních systémech

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1 Drucker-Prager model of plasticity

1.1 Introduction

Drucker-Prager model of plasticity modified Mohr-Coulomb model. Unlike Mohr-Coulomb model is Drucker-Prager model yield criterion smooth and in space of the principal stresses have form of cylindrical cone. If laboratory results are in effective rather than total stress, criterion of damage become dependent on the hydrostatic and mean stress.

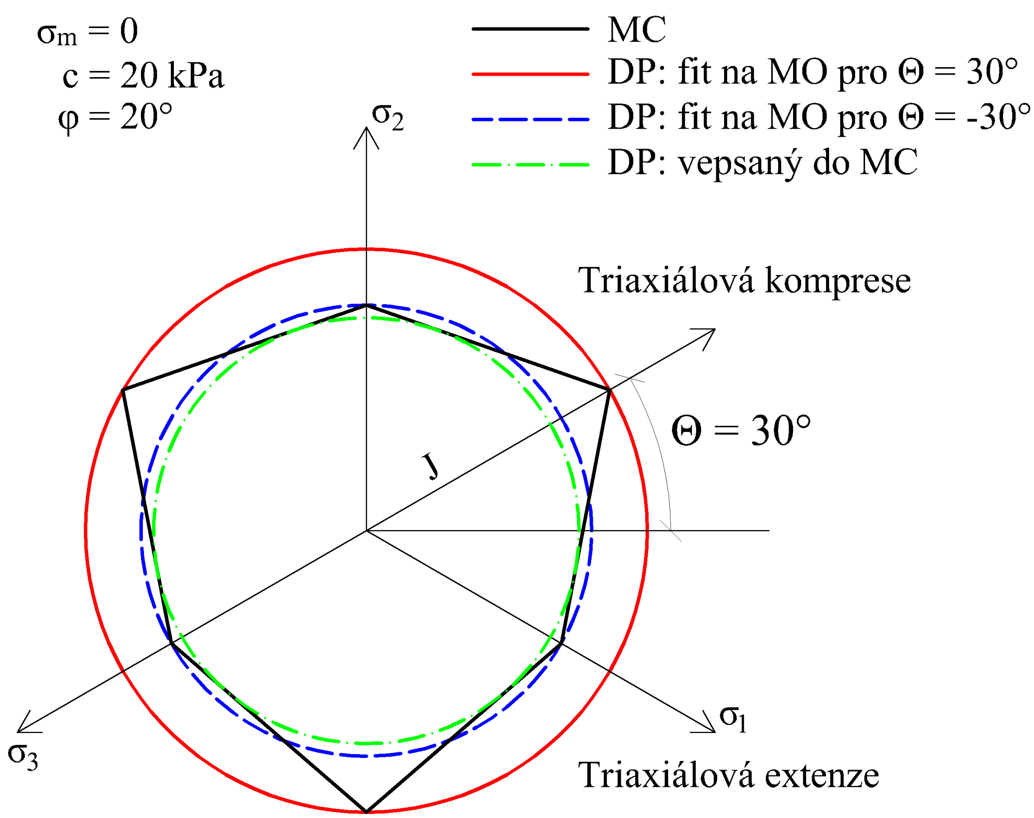

Figure 1.1: Drucker-Prager and Mohr-Coulomb yield criterion in space of principal stresses.

1.2 Drucker-Prager yield criterion

Drucker-Prager model is based on the von Mises model in that the mean stress (first invariant of stress tensor) is obtained in the yield criterion equation, which has the form

$$F(\sigma) = J + (\sigma_m - c)M_{JP}(\varphi) = 0, \tag{1.1}$$

where J is second invariant of stress tensor and M_{JP} is used for approximation to the Mohr-Coulomb model. Is defined as

$$M_{JP} = \frac{\sin(\varphi)}{cos(\theta) - \frac{\sin(\theta)\sin(\varphi)}{\sqrt{3}}}, \tag{1.2}$$

$$\theta = \arctan \frac{\sin\varphi}{\sqrt{3}}, \tag{1.3}$$

where φ is tangle of internal friction. Equations 1.2 and 1.3 are used to inscribe Drucker-Prager model into Mohr-Coulomb model. Equations (1.2) a (1.3) were taken from [1].

1.3 Calculation procedure and implementation

Total elastic stress can be calculated as

$$\sigma = D\varepsilon_e, \tag{1.4}$$

where D is stiffness matrix and ε_e is elastic deformation. Calculation is selected as implicit, so model is implemented with increments. Then can be equation modified as

$$\sigma^{n+1} = \sigma^n + D\mathrm{d}\varepsilon_e. \tag{1.5}$$

Next step in calculation is find out, if is (1.1) satisfied, shere σ_m a J is first respective second invariant of stress tensor and are defined as

$$J = sqrt(\frac{1}{2}\sigma^T P\sigma), \tag{1.6}$$

$$\sigma_m = m^T\sigma. \tag{1.7}$$

If is 1.1 satisfied, calculation continues with next deformation increment. If no, material become to plastic flow. Because of that is defined dλ, which is coefficient of plastic flow. Dependence of J and σ_m on the dλ is taken from [1] and have form

$$F(\sigma) = \overbrace{J - \mu \mathrm{d}\lambda}^{J^{n+1}} + (\overbrace{\sigma_m - KM_{JP}(\varphi)\mathrm{d}\lambda}^{\sigma_m^{n+1}} - c)M_{JP} = 0. \tag{1.8}$$

Coefficient of plastic flow dλ is iterating with Newton-Raphson method until is (1.8) satisfied and until are increments infinitesimal. Popsána je vztahem

$$\mathrm{d}\lambda^{n+1} = \mathrm{d}\lambda^n + \frac{F^n}{F'^n}. \tag{1.9}$$

Return to the yield of plasticity is going after normal to equation (1.1). Normal vector is counted as [1]

$$n = \frac{\delta F}{\delta\sigma} = \frac{1}{2J}P\sigma + M_{JP}m, \tag{1.10}$$

and its derivation

$$\frac{\delta n}{\delta\sigma} = \left(\frac{3}{2}\right)^{1/2}\frac{\sigma^T P\sigma P - P\sigma\sigma^T}{(\sigma^T P\sigma)^{3/2}}. \tag{1.11}$$

1.4 Return to yield of plasticity

Basic equations are

$$F(\sigma) = \overbrace{J^{res} - \mu \mathrm{d}\lambda}^{J^{n+1}} + [\overbrace{\sigma_m^{res} - KM_{PP}(\varphi^{n+1})\mathrm{d}\lambda}^{\sigma_m^{n+1}} - c^{n+1} cot(\varphi^{n+1})] M_{JP}(\varphi^{n+1}) = 0. \tag{1.12}$$

$$C_{E_d^{pl}} = c^{n+1} - c^{i-1} - h_c^n (E_d^{pl} - (E_d^{pl})^{i-1}) = 0 \tag{1.13}$$

$$\Theta_{E_d^{pl}} = \varphi^{n+1} - \varphi^{i-1} - h_\varphi^n (E_d^{pl} - (E_d^{pl})^{i-1}) = 0 \tag{1.14}$$

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