Lineaire Algebra Huiswerk

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Huiswerkset 2

14 september 2025

Opdracht 1.4.2(1)

Bewijs. Schrijf probleem in logische notatie zodat het makkelijker is.

$$||v|| = ||w|| \Leftrightarrow < v - w, v + w >= 0$$
$$\Leftrightarrow$$
$$(||v|| = ||w|| \implies < v - w, v + w >= 0) \quad \wedge \quad (< v - w, v + w >= 0 \implies ||v|| = ||w||)$$

Bewijs $||v|| = ||w|| \implies < v, w >= 0$:

$$\begin{aligned}
\boxed{||v|| = ||w||} &\implies ||v|| - ||w|| = 0 \\
&\implies ||v||^2 - ||w||^2 = 0 \\
&\implies < v, v > - < w, w >= 0 \\
&\implies < v, v > + < v, w > - < v, w > - < w, w >= 0 \\
&\implies v_1^2 + vw_1 - vw_1 - w_1^2 + v_2^2 + vw_2 - vw_2 - w_2^2 + \cdots + v_n^2 + vw_n - vw_n - w_n^2 = 0 \\
&\implies (v_1 + w_1)(v_1 - w_1) + \cdots + (v_n + w_n)(v_n - w_n) = 0 \\
&\implies \boxed{< v + w, v - w >= 0}
\end{aligned}$$

Bewijs nu voor $< v, w >= 0 \implies ||v|| = ||w||$:

$$\begin{aligned}
\boxed{< v + w, v - w >= 0} &\implies (v_1 + w_1)(v_1 - w_1) + \cdots + (v_n + w_n)(v_n - w_n) = 0 \\
&\implies v_1^2 + vw_1 - vw_1 - w_1^2 + v_2^2 + vw_2 - vw_2 - w_2^2 + \cdots + v_n^2 + vw_n - vw_n - w_n^2 = 0 \\
&\implies < v, v > + < v, w > - < v, w > - < w, w >= 0 \\
&\implies < v, v > - < w, w >= 0 \\
&\implies ||v||^2 - ||w||^2 = 0 \\
&\implies \boxed{||v|| - ||w|| = 0}
\end{aligned}$$

Dit voldoet aan $||v|| = ||w|| \Leftrightarrow < v - w, v + w >= 0$ en dus zijn we klaar. $\square$

Opdracht 1.6.5