AMA1D04 Project Proposal 1st semester 2021/22

Name: ______ Student ID: __

Project Title: Game theory: Why do competing businesses often locate their stores next to one another, and how can we make use of it?

Topics related: (tick ☑ if appropriate, you can tick more than once)

☐ Social choice theory ☐ Voting system and political power ☑ Game theory

☑ Fairness and Division ☐ Other (Please specify: ____________________)

Abstract: (Please introduce the problem/situation raised in this research, how this can be simulated, the methodology used in your research, what knowledge you learn from this course can be applied, what kind of results do you expect to find, …)

Some time ago I was astonished to find that, near Homantin Hall where I'm living at, KFC and McDonald's are so adjacent! Less than 100 meters! Moreover, when I observe the surroundings more carefully, it is unexpectedly common with fast food stores, gas stations, mattress stores, coffee shops, large retailers, etc. More interestingly, this phenomenon could be well elaborated by Game Theory after I searched for more information online!

More specifically, it is Hotelling's Model of Spatial competition that enables logical elaboration. I'd like to analyze why it contributes to the result that competing businesses often locate their stores nearby, utilizing some good data in real lives and economics knowledge and analysis. And apart from that, I also like to point out some shortcomings of this model, since some factors like brand loyalty and, such that it's not always financially good for a firm locate its store near its competitor. Thus, a bigger picture of this model could be elaborated when we take its benefits and shortcomings into account.

Game theory: Why do competing businesses often locate their firms next to one another, and how can we make use of it?

Anthony

Introduction

Economies of agglomeration has been greatly contributing to some regions since the clustering of similar firms would lead to labor pooling, higher wages, and provide better social and civic opportunity (Glaeser, 2010). In daily lives, it is acknowledged that some firms are located nearby their competitors, and some instances can be seen in Figure 1 and 2. It is understandable that stores cluster together to increase their customer pulling power, however, it is quite confusing for the public to see the firms selling homogenous products tend to cluster together, instead of locating their firms far away from others to reduce competition. In this report, it would be illustrated by game theory, and its applicability would be further discussed.

Figure 1: The locations of KFC and McDonald's in Pu Dong Distinct, Shanghai City, P.R. China (Shuair, 2016). (Note: Red point: KFC, Yellow point: McDonald's)

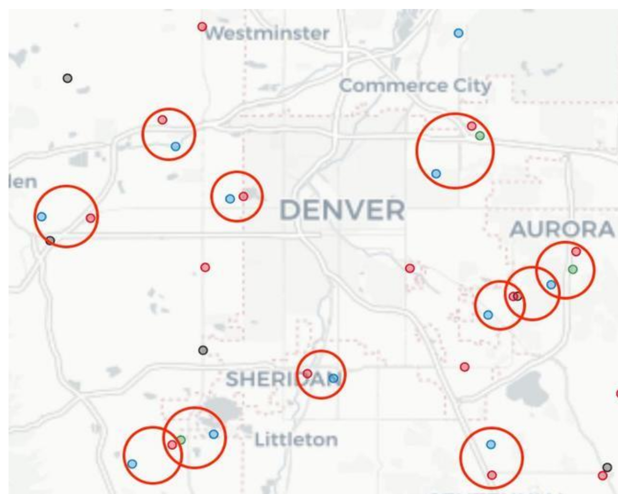


Figure 2: The locations of 4 major U.S. retail supermarket chains (Target, Walmart, JCPenney and Kohl) in Denver Metropolitan Area, Colorado (Yang & Xia, 2019).

Method

To examine the situation, the linear city model proposed by Hotelling (1929) would be firstly introduced, then integrated with the knowledge learnt in this subject. Moreover, analyzing the real-world data, Rossmann Store Sales, available in Kaggle could enable better understanding of Hotelling's law, and to prove and even optimize the model through real life's practice.

When merely considering the location —— Pure spatial competition

The original location model presented by Hotelling (1929) simplified the market or city into a straight, bounded line of a certain length, and the consumers are distributed uniformly in this straight. The other details would be provided in Table 1. In this report, we assume consumers live uniformly on the $[0, 1]$ interval.

Firstly, assuming only a store enters the market (represented by the straight, bounded line), and the store can locate anyway along this line, the consumer k's utility[1] from purchasing from file i:

$$U_{ki} = v - p_i - td_{ki}$$

where v is the value of consuming the product, p_i is the price of the only product, t is the unit transportation cost, and d_{ki} is the distance between consumer k and firm i's location, assuming all consumers could receive a nonnegative level of utility (Chen, 2014).

It also illustrates that the utility is lower for people who live farther from the firm. Using the knowledge of definite integral, i.e., $\int_0^1 U_{ki}\, dx$, it enables minimized consumers' average transportation cost (utility maximization) when the only one firm lies at the center of the market, as seen in Figure 3 and Figure 4. It is assumed that the firm should try to minimize the consumers' transportation costs.

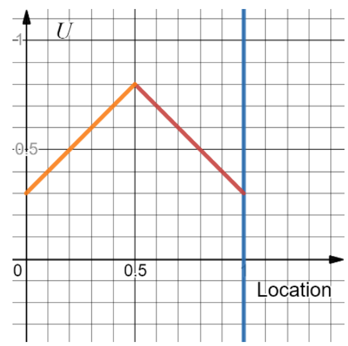


Figure 3

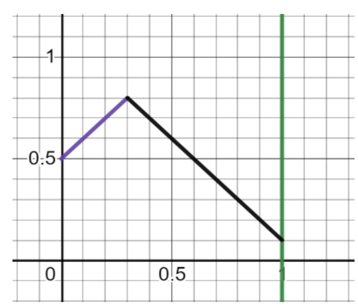


Figure 4

If there exist two competing firms, i.e., participant set is $N = \{i | i = 1,2\}$, as shown in Figure 5.

Figure 5: Linear city

[1] Utility is a term in economics referring to the total satisfaction received from consuming a good or service. It is usually assumed in some Economic theories that consumers will strive for utility maximization (Investopedia, 2021).

Table 1: the assumptions of the linear city.

Entity	Assumptions	Notes
The market	It is straight, bounded line of a certain length, i.e., **the** $\mathbf{[0,1]}$ **interval.**	The transportation costs for the consumers are a **constant rate** of the distance between the consumers and the sellers.
Consumers	Consumers are **distributed uniformly** in this straight line. Consumers would purchase the product which costs less regarding commodity price and transportation costs (Hotelling, 1929).	All the consumers are considered ideally the same without any difference in consumption custom and other factors. The price is not considered at this stage (Pure spatial competition).
Two sellers (A, B)	1. These two sellers sell identical products with the same, fixed or stable prices, and moreover they cannot cooperatively adjust the price. 2. Sellers are rational and aim at profit-maximization.	These two players are selfish, caring only about their own interest. There is also no coalition nor transfer of payoff between them.

The problem how to reach optimal Nash equilibrium. i.e., neither of them would try to change after considering the other one's choice, could be converted into division of contested sum, since consumers are uniformly distributed, and they would go to the nearest shop when the price is the same. Originally, they sell products at different locations as shown in Figure 6, assuming Firm A is on the left side of Firm B, the contested part is $(1-a-b)$, where a is the length of the A's location to $(0,0)$, b is the length of B's position to $(1,0)$.

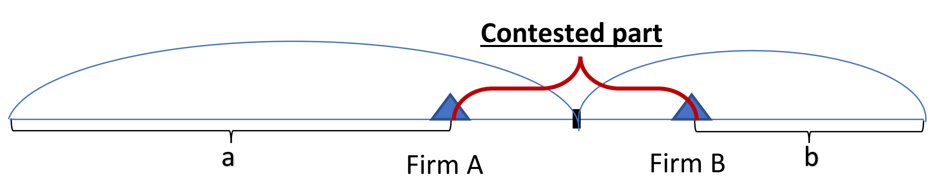


Figure 6

Thus, the consumers in this linear city would go to the nearest city to purchase products. Then Firm A can get $a+\frac{(1-a-b)}{2}=\frac{(1+a-b)}{2}$, Firm B can get $b+\frac{(1-a-b)}{2}=\frac{(1-a+b)}{2}$.

Let's then consider their movement to secure more market share. x_1 and x_2 stand for the change of distance they would make correspondingly.

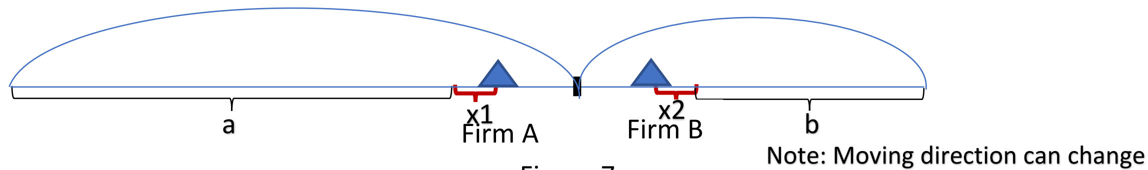


Figure 7

Table 2: the matrix describing the game

		Firm B		
		Move left x_2	Stay	Move right x_2
Firm A	Move left x_1	$\frac{(1+(a-x_1)-(b+x_2))}{2}, \frac{(1-(a-x_1)+(b+x_2))}{2}$	$\frac{(1+(a-x_1)-b)}{2}, \frac{(1-(a-x_1)+b)}{2}$	$\frac{(1+(a-x_1)-(b-x_2))}{2}, \frac{(1-(a-x_1)+(b-x_2))}{2}$
	Stay	$\frac{(1+a-(b+x_2))}{2}, \frac{(1-a+(b+x_2))}{2}$	$\frac{(1+a-b)}{2}, \frac{(1-a+b)}{2}$	$\frac{(1+a-(b-x_2))}{2}, \frac{(1-a+(b-x_2))}{2}$
	Move right x_1	$\frac{(1+(a+x_1)-(b+x_2))}{2}, \frac{(1-(a+x_1)+(b+x_2))}{2}$	$\frac{(1+(a+x_1)-b)}{2}, \frac{(1-(a+x_1)+b)}{2}$	$\frac{(1+(a+x_1)-(b-x_2))}{2}, \frac{(1-(a+x_1)+(b-x_2))}{2}$

Where $x_1 \geq 0, x_2 \geq 0$.

For Firm A, R_3 strongly dominates R_1 and R_2 since every entry in R_3 is greater than the corresponding entry in R_1 and R_2. Also, for Firm B, C_1 strongly dominates C_2 and C_3 since every entry in C_1 is greater than the corresponding entry in C_2 and C_3.

Therefore, a dominate strategy equilibrium exists and the outcome is

$$\frac{(1+(a+x_1)-(b+x_2))}{2}, \frac{(1-(a+x_1)+(b+x_2))}{2}$$

which means they would move towards one another, and this point should be a "dynamic" Nash equilibrium point based on my understanding, for x_1 and x_2 is unsure. Also, it is Pareto optimal, since there does not exist another outcome that offers a non-worse payoff for all players and better payoff for some players, while noting that under this circumstance, $a + b + x_1 + x_2 \leq 1$

Now considering the boundary situation, which is $a + b + x_1 + x_2 = 1$, we can convert the Nash equilibrium point into the point

$$(a + x, 1 - a - x)$$

where we replace x_2 with is $1 - a - b - x_1$, and we replace x_1 with x.

A great problem faced is that the relationship of two points $a + x$ and $1 - a - x$ is yet to know. After consulting many papers, I failed to find a suitable or understandable explanation, so I can only explain it based on my knowledge. In the very first beginning, we just consider position is that Firm A must be on the right of B. Note that the locations of two firms are interchangeable. In the beginner If $a + x > 1 - a - x$, which means that firm A could earn more than firm B. To maximize the profit, the owner of firm B will choose to move the firm to the right of firm A. Then the relative position of A and B is transposed, since apart from position, all factors of these two firms are considered the same. It changes the point into

$$(1 - a - x, \quad a + x)$$

And then, as the owner of Firm A is rational, it would like to transpose the position with Firm B. Equally, the owner of firm B will change again after that, leading to a repeated game. Again and again, this kind of exchange will reach a "static" Nash equilibrium point, that is, the company AB is at the center of this linear market.

In conclusion, not under this point, both rational owners will try to change the position, leading to the repeated game, since it is assumed that the firm should try to minimize the consumers' transportation cost and they cannot cooperate. What if under this balance point, if someone wants to break the balance, an extensive form game will happen. Let us assume that the last decision is made by Firm B, so we look at the subtree to find the most rational response of Firm B. Given the choice made by Firm B, Firm A makes the final choice after consideration. This process is repeated, and the order of choice making can be transposed.

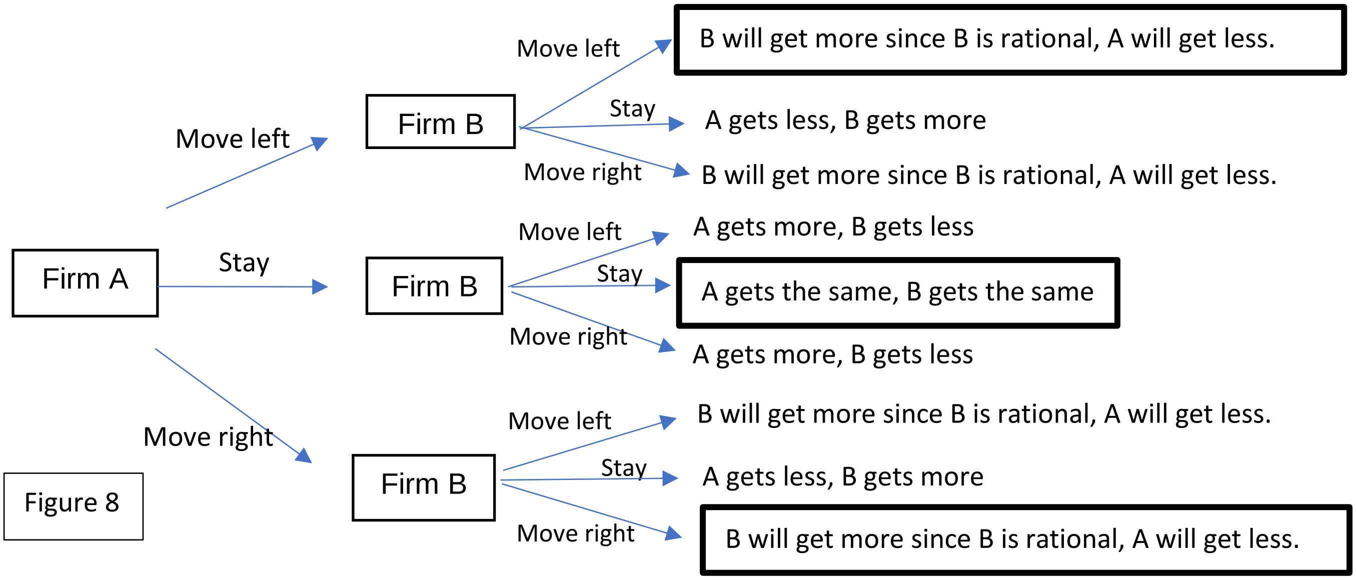


Figure 8

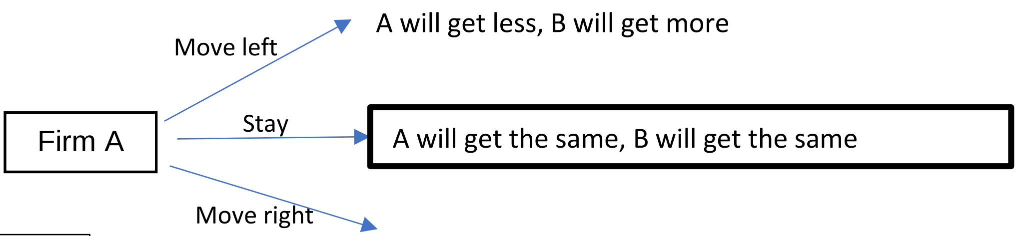


Figure 9

Finally, both will choose to stay at the middle point. Another better explanation is to make use of the utility function, as elaborated by Qin (2020). Participation set now is $N = \{i|i = 1, 2\}$, and the strategy set is $A_i = \{x_i|0 \le x_i \le 1\}$, and the utility function is $u_i(x_i, x_j) =$

$$\begin{cases} \frac{x_i + x_j}{2} & if\ x_i < x_j \\ 1 - \frac{x_i + x_j}{2} & if\ x_i > x_j \\ \frac{1}{2} & if\ x_i = x_j \end{cases}$$

as Qin demonstrates, will lead to Nash equilibrium, which is exactly two stores will lie in the middle point. And he also points out, there did not exist Nash equilibrium when there are three firms in this linear city. Apart from that, it should be noted that only considering the location would fail to describe the reality, and the price factor shall also be considered.

When considering both the location and the price

When merely considering the price, as Hotelling and Aspremont et al. (1979) contend, it would trigger a process of price adjustment that would lead to a stable equilibrium. Yet as Hu and Tang (2014) argues, not reducing the price is the Pareto efficiency to balance the game. It would be a Pareto efficiency to balance the game. Neither of them would tend to take the first or the subsequent price reduction strategy. What if both factors are considered?

The first stage is each firm chooses a location, and the nest stage considered here is that firms compete in price given the locations of both sellers. Still, based on Figure 3, there is maximal differentiation when $a = b = 0$, and $a + b = 1$ corresponds no (minimal) differentiation (Iida & Matsubayashi, 2011). And we still consider the transportation cost in the linear city is the linear rate of the unit length. We have

$$\begin{cases} p_1 + t(a - x) = p_2 + t(1 - b - x) \quad x \leq a, i.e., if\ the\ consumer\ is\ on\ the\ left\ of\ firm\ A \\ p_1 + t(x - a) = p_2 + t(1 - b - x)\ a < x < 1 - b, i.e., if\ the\ consumer\ is\ in\ between\ firm\ A\ and\ B \\ p_1 + t(x - a) = p_2 + t[x - (1 - b)]\ x \geq 1 - b, i.e., if\ the\ consumer\ is\ on\ the\ right\ of\ firm\ B \end{cases}$$

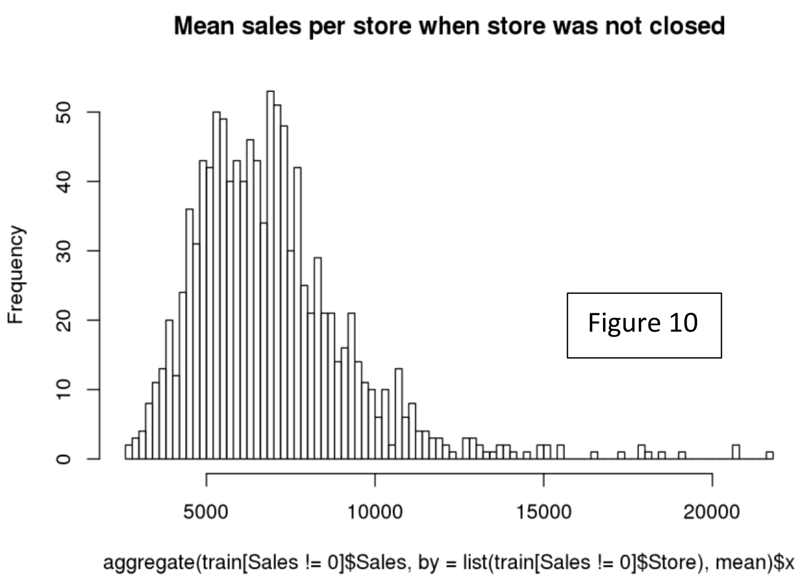
Aspremont et al. (1979) imply that there exists no pure-strategy Nash equilibrium when both locations and prices are choice variables in linear market. The reason is that stores benefit from distancing themselves from one another which dampens price competition. Yet at the same time, each store also tends to inch closer to its competitor to steal the customers. Therefore, there exists no equilibrium when transportation costs are linear, considering the location and the price.

As Eaton and Lipsey (2005) suggest, when the transportation cost turns to be a quadratic rate of distance, there would exist an equilibrium. Moreover, Karas (2020) proposes that when three firms are in a circle city, there would be countless Nash equilibrium. As the relative formula is difficult for me who has no Economics background student, so I decide not to dive into them, but to discuss the applicability of Hotelling’s law utilizing some meaningful real-world data.

Case study[2] —— Rossmann Firm Sales

To verify if the hotelling's theory is fact-supported, some empirical data or observation in real life could be helpful for testing it. In the era of big data, it makes it easier to collect the relevant data online. Rossmann, as one of the largest drug firm chains in Europe, now operates over 4000 drug firms in the world, and the historical data for 1115 Rossmann firms in some periods are provided in Kaggle for data analysis competition. To the best of my knowledge, this individual report is the first literature that correlates the hotelling's theory with the Rossmann Firm Sales data.

For the Data preprocessing, the data is stored in three .csv files, so the data should be firstly imported into the program. There are some missing values. Some skills learned in other courses, for example missing data handling (no value stands for missing instead of no), would be applied to polish this work as well. Also, some libraries have been utilized, say NumPy for scientific calculation, Matplotlib, ggplot2 for data visualization. Data visualization makes it easier to analyze the significance of multiple factors to the sales of different stores.



As shown in Figure 10, **the overall trend is that average sales per store increase first and then reduce as the competition distance increases. The critical point is around 5000 – 7000. It indicates that for drug stores, keeping a certain distance to the competitors may be a better choice to maximize the profits, which to some extent contrasts with Hotelling's idea.**

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2 The program for data visualization is available in my GitHub repository: https://github.com/RepublicHo/Rossmann-Store-Sales. Since this part is for verifying the Hotelling's model, the program references a lot to the program and guidance of GGEP (2015), ELENA (2017) and Huy (2021).

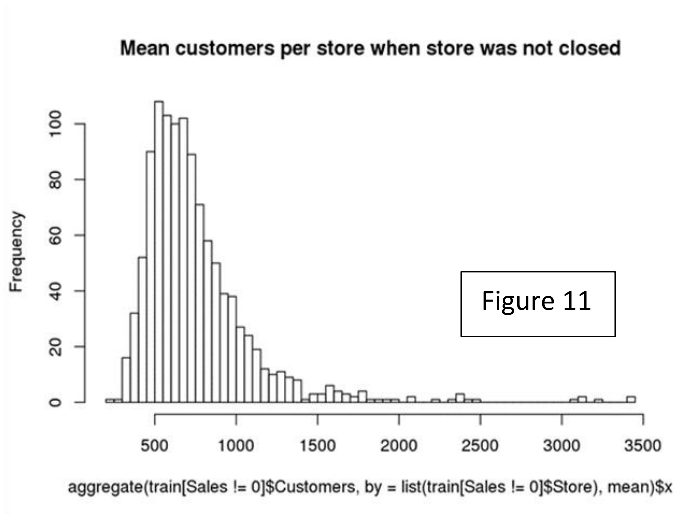

Figure 11

As shown in Figure 11, **the average customers per store have similar trends, increasing fast first and then reducing as the competition distance increases. Yet the critical point is around 500. It also shows that for drug stores, keeping relatively close distance to their competitors may be a better approach to secure more customers.**

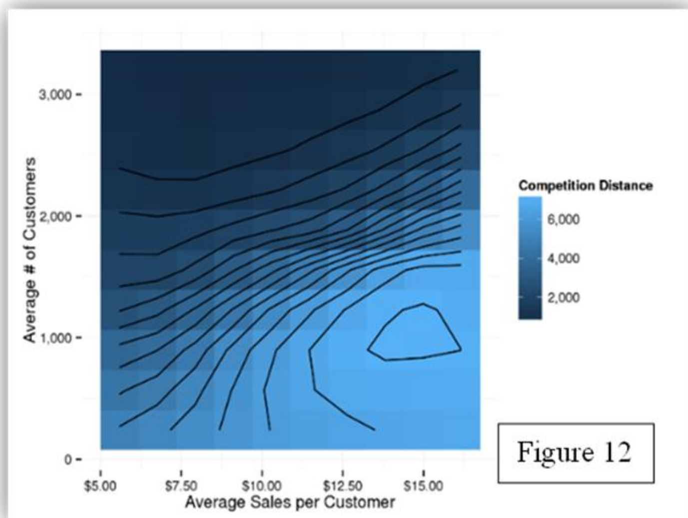

Figure 12

Another inspiring finding is, as the heatmap in Figure 12 implies (the stores with close competition are in the upper left), the average sales per customer increase when the competition distance increases. The possible reason to explain that is drug stores with longer competition distance are more likely to be in relatively remote areas, for example in the suburb or residential area, and thus less frequently visited by customers. The scale of these stores may be larger. Therefore, the fact is **once customers pay a visit to this kind of store, they might tend to purchase more in terms of the amount of money** one time owing to the relatively higher transportation cost and other complex factors. **Moreover, the overall sales of these relatively remote stores are relatively higher** than the stores with close competition and the stores with far longer competition.

The common trend may demonstrate that, the equilibrium in our daily lives is not the same as the model in the linear city proposed by Hotelling. Differently, to maximize the sales, opening their store relatively farther might be a better choice. However, there still may exist some special equilibrium when considering complex factors in daily lives. **It reveals that for better sales of drug stores, not the closer to their competitors the better, but be relatively close and keep a**

certain distance (around 5000-7000 meters), the better. As shown in the data, this opening store strategy may help attain better sales.

Yet this case study failed to deny the correctness of Hotelling's model and its result. However, it corroborates hotelling's model correctness in real life to a great extent since it reflects the Economies of agglomeration. **My personal understanding is that originally, to maximize the profits, similar stores would lie relatively closer in some areas, where the city gradually develops. And later in the developed urban areas, there are more consumers, and thus customers of the stores there are higher. Yet there is no strong positive correlation between the Sales and Customers for the drug store. It should be noted that, for drug stores, keeping a proper distance from the competitors may be a better strategy to gain more sales.**

Moreover, we find some distinction between the conclusion of Hotelling's model and the real lives' practice. It should be attributed to the complex factors in daily lives. Some rest datasets could be utilized to further discuss.

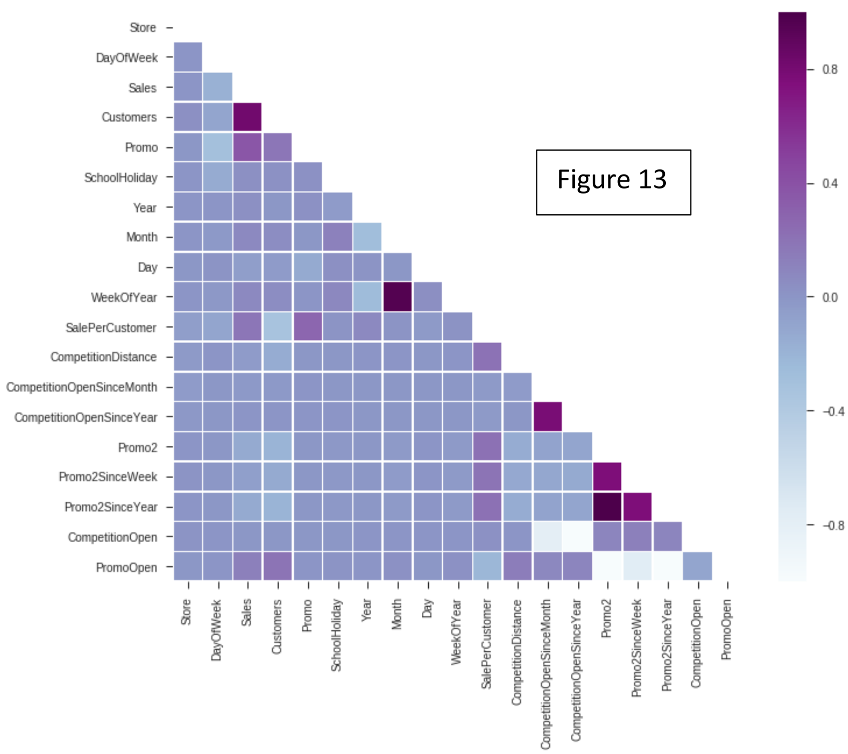

Figure 13

As shown in Figure 13, the sales fluctuate with the time in the day, month and year. And the opening data of the competitors have a great influence on the sales of the store owing to possible brand power and old customer. Also, it is noted that there is a positive correlation between a running promotion and the number of Customers. However, ELENA (2017) finds "as soon as the store continues a consecutive promotion (Promo2 equal to 1) the number of Customers and Sales seems to stay the same or even decrease, represented by the pale negative correlation on the heatmap. The same negative correlation is observed between the presence of the promotion in the store and the day of a week”. It may be explained by the price war. Furthermore, the hours of store operation, as well as the competitors’, count a great deal for the sales.

The conclusion of this data is still limited to some extent. For example, it may not be applied to the situations in Hong Kong, as the drug stores in Hong Kong sell far more other types of products other than medicine. Also, it may fail to apply to other types of stores.

Discussion

Hotelling’s law, as a scientific model, is considered to explain the phenomena in the real life by greatly simplifying reflections of reality. Although the scientific model may fail to represent the real world, since the details are ignored, the simulation should not produce some theoretical consequence that is contrary to reality.

It seems the consequence of Hotelling’s model relies overly upon its assumption, which makes it hard to generalize in real life. Even if it is logically valid, its robustness may fail to meet the requirements for it not considering complex social factors. Some of the factors are listed below.

1. There are **far more firms in society** than that in the model. Apart from the huge number, various types of stores and firms shall be considered.
2. There is not an optimal bounded linear market where consumers were evenly distributed. On the contrary, **the distribution is complex, the taste of consumers is different, and even the transportation cost is neither linear rate nor quadratic rate of the length.**
3. The demand has many types, which may lead to different results. If the demand is elastic, the result of the model would be different (Iida & Matsubayashi, 2011).
4. **No consideration of STP**, i.e., 1) Market Segmentation; 2) Market Targeting; 3) Market Position, as well as the relocation costs, brand influence, product differences, store opening time, old customers, the creativity of the product, etc.

Despite its inadequate robustness that hinders the applicability of the model to some extent, **the case study (Rossmann Firm Sales) still demonstrates the correctness of hotelling's model in terms of the relationship of the consumers and the competition distance.** Moreover, it could help elaborate the Economies of agglomeration. For drug store, keeping relatively close to the competitors (not the closer the better) may be attracting more consumers, yet it is not the best way of maximizing the profits. Whilst there may exist some situation for other types of stores where more consumers are directly proportional to the sales given the number of consumers is the is inversely proportional to competition distance. Then it would corroborate Hotelling's law.

Conclusion and Recommendations

Although this report fails to deal with sophisticated economic formulas, the idea within the model still could be elaborated by the knowledge learned in this subject. It is taboo to learn theories and their application with the one-sided conclusions of the models which overly simplifies the reality. The complexity of the world should be seriously considered, and some interdisciplinary knowledge could be utilized for a better decision.

And I have some suggestions for the person who wants to set up or currently runs a store, based on the points I benefit from this learning process.

1. For the newcomers to a new area, exploit it at every opportunity. For the latecomers, do not be frustrated, make use of the existed store there, seriously considering setting a store relatively close to the competitors to steal the consumers.
2. Keep an appropriate distance to your competitors still, based on every complex factor that you master in real lives.
3. To gain better sales, self and the environment are both of significance.
4. There are some competitors nearby, but it is not serious as expected, not to decide to move implusively, which will make you lose old consumers. More consumers there may be a major reason. Make use of it and the sales could be better if good managing and operating strategies are taken.

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