Calculus Probability theory

Assignment 1

Brigel Pineti

April 2018

Exercise 6

a) x - x^3 = x(1-x^2) = x(1+x)(1-x) then we have x $\in$ {-1, 0, 1}

b) We have (1-x^2) > 0 => | x | < 1 meaning that x $\in$ (-1, 1).
Thus by resulting on 4 cases:
1) x < -1 => x < 0, (1-x^2) < 0, thus f(x) > 0
2) x $\in$ (-1, 0) => x < 0, (1-x^2) > 0, thus f(x) < 0
3) x $\in$ (0, 1) => x > 0, (1-x^2) > 0, thus f(x) > 0
4) x > 1 => x > 0, (1-x^2) < 0, thus f(x) < 0

Exercise 7

3 cases are to be distinguished considering the values of a and b:

1) a == b then y = {a}

2) a > b then (b-a) < 0. Keeping in mind that x $\in$ (0,1) then y = a + (b-a)x < a and y = a + (b-a)x > a + (b-a) = b.
Therefore, y will take values in (b,a). The question to be answered now is if y does reach every value in this interval

$$\mathrm{x}_u = \frac{u-a}{b-a}$$

Assume that u is part of the interval (a,b). This means that x_u falls in (0,1). To rephrase, $\exists$ x_u $\in$ (0,1) | v = a + (b-a)x_u. Thus, y will run throughout all the values in (b,a).

3) a < b then (b-a) > 0. Keeping in mind that x $\in$ (0,1) then y = a + (b-a)x > a and y = a + (b-a)x < a + (b-a) = b.
Therefore, y will take values in (a,b). The question to be answered now is if y

does reach every value in this interval

$$x_v = \frac{v - a}{b - a}$$

Assume that v is part of the interval (a,b). This means that x_v falls in (0,1). To rephrase, $\exists\ x_v \in$ (0,1) | v = a + (b-a)x_v. Thus, y will run throughout all the values in (a,b).

Exercise 8

a) f(-x) = 3(-x) - $(-x)^3$ = -3x + x^3 and -f(x) = -3x + x^3
Since f(-x) $\neq$ f(x) then the function is **not even**.
Since f(-x) -f(x) then the function is **odd**.

b) f(-x) = $\sqrt[3]{(1+x)^2}$ + $\sqrt[3]{(1-x)^2}$ and -f(x) = - $\sqrt[3]{(1-x)^2}$ - $\sqrt[3]{(1+x)^2}$
Since f(-x) f(x) then the function is **even**.
Since f(-x) $\neq$ -f(x) then the function is **not odd**.

Exercise 9

a) To find the domain, the expression inside the square root needs to be greater or equal to zero. Therefore, 7 - $x^2 \geq$ 0. In order for the former mentioned condition to be true, x has to fall inside this range: -$\sqrt{7} \leq$ x $\leq \sqrt{7}$.
So, **D(f) = [-$\sqrt{7}$, $\sqrt{7}$]**.

Knowing the values of x allowed for the expression inside the square root, it is possible to determine the values y may take => 0 $\leq \sqrt{7-x^2} \leq \sqrt{7}$. In f(x) the only difference is right part where *+1* is added leading to 0**+1** $\leq \sqrt{7-x^2}$ $\leq \sqrt{7}$**+1**. In order words, **R(f) = [1, $\sqrt{7}$ + 1]**.

b) Since in a division operation the denominator needs to be $\neq$ 0 then
D(f) = R \{0}. Additionally, the absolute value will guarantee that f(x) will always evaluate to a positive value without caring about the cases when x is positive or negative. Hence, **R(f) = (0, +∞)**.

Exercise 10

a) Transform expression to y(cx + d) = ax + b
> ycx + yd = ax + b {place x on one side to group}
> ycx -ax = b - yd {now group by x}
> x(yc - a) = b - yd {computer for x}
> x = $\frac{b - yd}{yc - a}$

Therefore, the inverse function is given by g(x) $= \dfrac{b - xd}{xc - a}$

b) When d = -a then g is equal to f. To be noted is that g is defined for all x besides x $= \frac{a}{c}$. f(x) being equal to $\frac{a}{c}$ means that ad - bd = 0, thus by saying that g is defined for the whole range of f.

Exercise 11

a) For all x $\neq$ 2 :
$> \dfrac{x-2}{(x-2)(x+3)}$
$> \dfrac{x-2}{(x-2)(x+3)}$
$> \dfrac{1}{x+3}$

So:

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x+3)} = \lim_{x\to 2} \frac{1}{x+3} = \frac{1}{5}$$

b) For all x $\neq$ 1 :
$> \dfrac{x^2-4x+3}{x^2+x-2}$
$> \dfrac{(x-1)(x-3)}{(x-1)(x+2)}$
$> \dfrac{x-3}{x+2}$

So:

$$\lim_{x\to 1} \frac{x^2-4x+3}{x^2+x-2} = \lim_{x\to 2} \frac{x-3}{x+2} = -\frac{2}{3}$$