DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION
Course Title: Mathematical Analysis for Computer Science
Course Code: CSE-3201
3rd Year 2nd Semester
Session: 2016-17(Admission)
(Answer Any Five Questions)

Time: 3 Hours Marks: 60

1. a) Write down the reason of studying mathematical analysis in Computer Science and Engineering. [2]

   b) Define standard deviation and variance. In a certain set of numbers, 12.5 is 1.5 units of standard deviation above the mean, and 8.9 is 0.5 units of standard deviation below the mean. What is the mean of the set? [2.5]

   c) What do you mean by conditional probability? A box contains 4 red chips and 2 blue chips. If two chips are selected at random without replacement, what is the probability that the chips are different colors? [3.5]

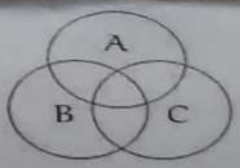
   d) Probability is the measure of the likelihood that an event will occur.Probability quantifies as a number between 0 and 1, where, loosely speaking, 0 indicates impossibility and 1 indicates certainty. [4]
      i. A die is rolled, find the probability that the number obtained is greater than 4.
      ii. Two coins are tossed, find the probability that one head only is obtained.
      iii. Two dice are rolled, find the probability that the sum is equal to 5.
      iv. A card is drawn at random from a deck of cards. Find the probability of getting the King of heart.

2. a) For all positive integers x, the function f is defined as follows: [2]

$$f(x) = \begin{cases} x & if\ x\ is\ a\ prime \\ x-1 & if\ x\ is\ not\ prime \end{cases}$$

   If, $f(a)\ x\ f(a+1) = a^2$; write down the conditions for which the overall conditions hold.

   b) A "Venn diagram" with three overlapping circles is often used to illustrate the eight possible subsets associated with three given sets, Can the sixteen possibilities that arise with four given sets be illustrated by four overlapping circles? Justify your answer. [3]



   c) Write down the recursive formula and complexity analysis of the Tower of Hanoi Problem. [3]

   d) Using taylor's series method, find y at x=0.1,0.2 correct up to three significant digits by solving $\frac{dy}{dx} = x - y\ with\ y(0) = 1$. [4]

3. a) Set Q contains 20 positive integer values. The smallest value in Set Q is a single digit value and the largest value in Set Q is a three digit value. What are possible values for the range of Set Q? Find out the minimum and maximum range for set Q.[Range in a set means, $Range_Q = Q_{max} - Q_{min}$.] [2]

   b) Write down the recursive formula for the following problems, [4]
      i. Fibonacci Number
      ii. Factorial Number
      iii. GCD

   c) A number in decimal notation is divisible by 3 if and only if the sum of its digits is divisible by 3. Prove this well-known rule, and generalize it. [3]

   d) The length of human pregnancies from conception to birth approximates a normal distribution with a mean of 266 days and a standard deviation of 16 days. What proportion of all pregnancies will last between 240 and 270 days (roughly between 8 and 9 months)? [3]

4. a) Draw the transition diagram for each case. Is it irreducible? For both yes or no, each cases justify your answer with proper explanation. [4]

(a) $P = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$ (b) $P = \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}$ (c) $P = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{pmatrix}$

(d) $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (e) $P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

b) Using modified Euler's method find the value of y when $y(0) = 1$ *and* $\frac{dy}{dx} = x^2 + y$. *with* $x = 0.1$ [4]

c) Solve the system of linear equations [3]

$3X+Y+2Z=3, \quad 2X-3Y-Z=-3, \quad X+2Y+Z=4$

d) What do you mean by ordinary and partial differential equation? [1]

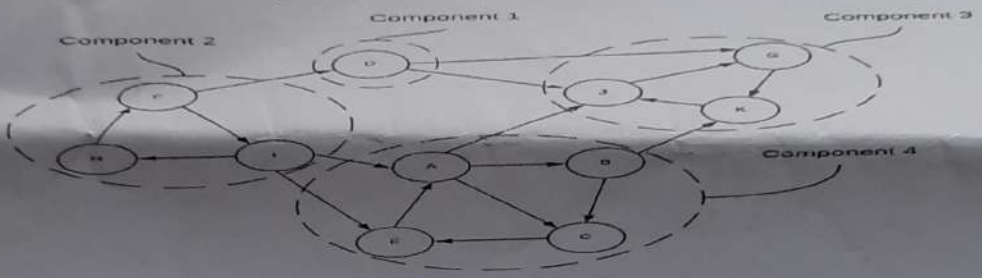
5. a) Let, $T_0 = 2, T_1 = 3, T_2 = 6,$ [3]
and for $n \geq 3, \quad T_n = (n+4)\,T_{n-1} - 4nT_{n-2} + (4n-8)\,T_{n-3}$. Is it possible to find out the series from the above information?

b) Cribbage players have long been aware that $15 = 7 + 8 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5$. Find the number of ways to represent 1050 as a sum of consecutive positive integers. [3]

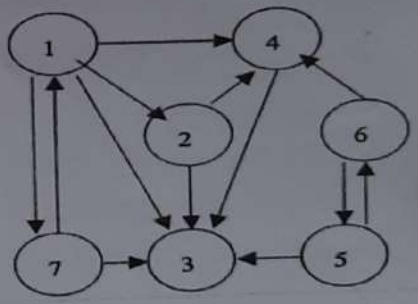
c) Define stochastic process. Write short notes on pdf and cdf. [4]

d) If 724 is the greatest common divisor of positive integers A and B, and 726 is the least common multiple of A and B, then AB=? [2]

6. a) Define Strongly Connected Components. Prove that the following split of the graph to components are properly splitted. Briefly describe your answer. [5]



b) Apply depth first search to the following graph. Show the graph after a new edge has been identified as a tree, forward, or cross edge, back edge (indicate one of the four types). Show also the current color of all the nodes. (NOTE: assume starting from node 1, and go to node 2 first). [4]



c) Solve the following problems, [3]

i. If $f(x) = 5 - 2x$ and $f(3k) = f(k+1)$, then $f(k) = ?$

ii. What is the sum of each distinct prime factor of 9999?

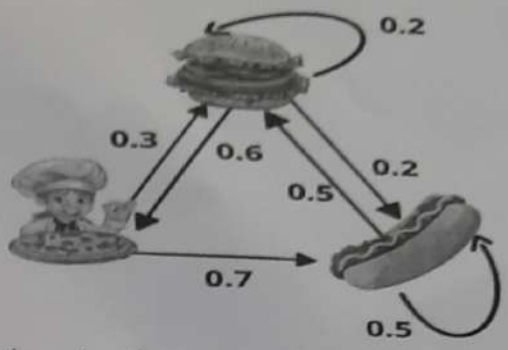
iii. If x is a positive integer and x+2 is divisible by 10, what is the remainder when x^2+4x+9 is divided by 10?

7. a) Dr. Strong is a dentist who schedules all her patients for 30 minutes appointments. Some of them take more or less than 30 minutes depending on the type of dental work to be done. Following summary shows the various categories of work, their probabilities and the time actually needed to complete the work: [5]

Category	Time required (Minutes)	No. of patients	Probability of category
Filing	45	40	
Crown	60	15	
Cleaning	15	15	
Extraction	45	10	
Checkup		20	

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time, starting at 8.00 am. Use the following random numbers in handling the above problem: 40, 82, 11, 34, 35, 25, 66, 17, and 79.

b) From the following diagram find out the transition matrix. Suppose we are in the pizza day, find the 5th day probability for the overall scenario. [4]



c) Define harmonic and p – series with examples. Is harmonic series divergent or convergent? Justify your answer. [3]

8. a) A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage, there is a probability p that the digit that enters this stage will be changed when it leaves and a probability q=1−p that it won't. Form a Markov chain to represent the process of transmission by taking as states the digits 0 and 1. What is the matrix of transition probabilities? [4]

b) Assume that a man's profession can be classified as professional, skilled laborer, or unskilled laborer. Assume that, of the sons of professional men, 80 percent are professional, 10 percent are skilled laborers, and 10 percent are unskilled laborers. In the case of sons of skilled laborers, 60 percent are skilled laborers, 20 percent are professional, and 20 percent are unskilled. Finally, in the case of unskilled laborers, 50 percent of the sons are unskilled laborers, and 25 percent each are in the other two categories. Assume that every man has at least one son, and form a Markov chain by following the profession of a randomly chosen son of a given family through several generations. Now your task is to create a marcov chain with transition probability and transition matrix. Also Find the probability that a randomly chosen grandson of an unskilled laborer is a professional man. [5]

c) Write short notes on transient, recurrent and classes in Marcov chain with examples. [3]

****Good Luck** Best Wishes for You**Be Safe****