DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION

Course Title: **Mathematical Analysis for Computer Science**

Course Code: CSE-3201

3rd Year 2nd Semester

Admission Session: 2018-19

(Answer Any Five Questions)

Time: 3 Hours Marks: 60

1. a) Show that it's possible to construct a Venn diagram for all 2^n possible subsets of n given sets, using n convex polygons that are congruent to each other and rotated about a common center. [3]

b) A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is .95; by device B, .90; and by both devices, 0.88. Now, [3]

i. If smoke is present, find the probability that the smoke will be detected by either device A or B or both devices.

ii. Find the probability that the smoke will be undetected.

c) If W_n is the minimum number of moves needed to transfer a tower of n disks from one peg to another when there are four pegs instead of three, show that [3]

$$W_{n(n+1)/2} \le 2W_{n(n-1)/2} + T_n, \quad \text{for } n > 0.$$

(Here $T_n = 2^n - 1$ is the ordinary three-peg number. Use this to find a closed form $f(n)$ such that $W_{n(n+1)/2} \le f(n)$ for all $n \ge 0$.

d) Josephus had a friend who was saved by getting into the next-to-last position. What is I(n), the number of the penultimate survivor when every second person is executed? [3]

2. a) Write down the following questions with necessary explanation. [3]

A. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

i. 43 ii. 47 iii. 51 iv. 55 v. 59

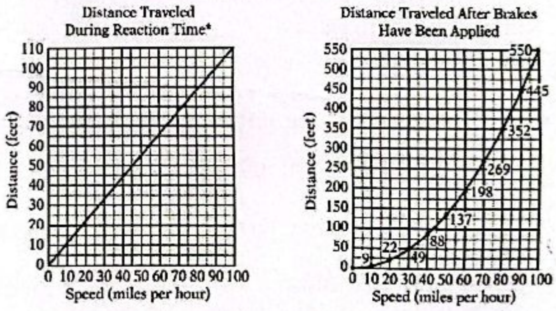
(B) Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

i. 130 ii. 110 iii. 90 iv. 70 v. 40

(C) Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

i. 50 ii. 55 iii. 60 iv. 65 v. 70

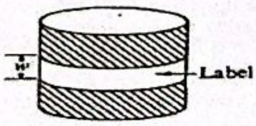
DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR'S SPEED WHEN THE DRIVER IS SIGNALED TO STOP



*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

b) Prove that "Circumference (c) of a circle is directly proportional to its radius (r)". Establish the relationship between 'w' with 'r' [2]



A rectangular label is attached to a right circular cylinder with radius r. The label, which encircles the cylinder without overlap, has width w and an area equal to the area of the base of the cylinder.

c) A certain pet store sells only dogs and cats. In March, the store sold twice as many dogs as cats. In April, the store sold twice the number of dogs that it sold in March, and three times the number of cats that it sold in March. If the total number of pets the store sold in March and April combined was 500, how many dogs did the store sell in March? [2]

d) At a recent dog show, there were 5 finalists. One of the finalists was awarded "Best in Show" and another finalist was awarded "Honorable Mention." In how many different ways could the two awards be given out? [3]

e) What do you mean by expectation? [2]

3. a) Consider the following process. We have two coins, one of which is fair, and the other of which has heads on both sides. We give these two coins to our friend, who chooses one of them at random (each with probability 1/2). During the rest of the process, she uses only the coin that she chose. She now proceeds to toss the coin many times, reporting the results. We consider this process to consist solely of what she reports to us. [6]

   i. Given that she reports a head on the nth toss, what is the probability that a head is thrown on the $(n+1)^{st}$ toss?

   ii. Consider this process as having two states, heads and tails. By computing the other three transition probabilities analogous to the one in part (a), write down a "transition matrix" for this process.

   iii. Now assume that the process is in state "heads" on both the (n−1)st and the nth toss. Find the probability that a head comes up on the (n+ 1)st toss.

   iv. Is this process a Markov chain?

b) Three tanks fight a three-way duel. Tank A has probability 1/2 of destroying the tank at which it fires, tank B has probability 1/3 of destroying the tank at which it fires, and tank C has probability 1/6 of destroying the tank at which it fires. The tanks fire together and each tank fires at the strongest opponent not yet destroyed. Form a Markov chain by taking as states the subsets of the set of tanks. Find *N, Nc, and NR*, and interpret your results. [4]

c) Suppose that two candidates, Daisy and Oscar, are running for office, and $n \in N$ voters cast their ballots. Votes are counted by the same official, one by one, until all n of them have been processed (like in the old days). After each ballot is opened, the official records the number of votes each candidate has received so far. At the end, the official announces that Daisy has won by a margin of $m > 0$ votes, i.e., that Daisy got $(n + m)/2$ votes and Oscar the remaining $(n - m)/2$ votes. What is the probability that at no time during the counting has Oscar been in the lead? [4]

4. a) Define Tower of Hanoi. If W_n is the minimum number of moves needed to transfer a tower of n disks from one peg to another when there are four pegs instead of three, show that [2]

$$W_{n(n+1)/2} \le 2W_{n(n-1)/2} + T_n, \quad \text{for } n > 0.$$

(Here $T_n = 2^n - 1$ is the ordinary three-peg number. Use this to find a closed form $f(n)$ such that $W_{n(n+1)/2} \le f(n)$ for all $n \ge 0$.

b) Write down the recursive formula for the following problems, [3]

   i. Fibonacci Number

   ii. Lines in the Plane

   iii. GCD

c) A number in decimal notation is divisible by 3 if and only if the sum of its digits is divisible by 3. Prove this well-known rule, and generalize it. [3]

d) The number 1111111111111111111 is prime. Prove that, in any radix b, $(11\ldots1)_b$ can be prime only if the number of 1's is prime. [2]

e) What do you mean by complex numbers? Prove that √2 is irrational number. [2]

5. a) What do you mean by divergence and convergence along with its usage? [2]

b) What do you mean by p values and harmonic series? Is harmonic series divergent or convergent? Justify your answer. [3]

c) Write down the formula for the Collatz conjecture number. It is conjectured (but not yet proven) that this algorithm will terminate at n = 1 for every integer n. Still, the conjecture holds for all integers up to at least 1,000,000. For an input n, the cycle-length of n is the number of numbers generated up to and including the 1. Now count the cycle length for 22, 1000, 1 and 7. [2]

d) Determine whether the following series converges or diverges: [3]

i.	ii.	iii.	iv.	v.
$\sum_{n=0}^{\infty}(-1)^n(\frac{5}{n})$	$\sum_{n=1}^{\infty}\frac{1}{n^2+n}$	$\sum_{n=1}^{\infty}\frac{1}{2^n-1}$	$\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+n}$	$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{n^3+1}$

e) Is there any differences between stochastic process and random process? Justify your answer. [2]

a) Write down the importance of Game Theory in computer science and engineering. [2]

b) Write down the meaning of competitive situation, strategy, two person zero sum game, maximin-minimax principle. [3]

c) Define Saddle point. Solve the following pay-off matrix to find the following [3]
i. Saddle point
ii. Best strategy for player A and B

Player A	Player B			
	Strategies	I	II	III
	I	6	8	6
	II	4	12	2

d) Find a range of values of a and b for which the following pay-off matrix will a saddle point at (2, 2) position. [4]

Player A	Player B			
	Strategies	I	II	III
	I	2	4	5
	II	10	7	b
	III	4	a	6

7. a) Write short notes on reducibility, irreducible, periodic, classes, and stationary distribution in marcov chain. [2]

b) Find out the transition matrix for the following scenario [3]

a) The President of the United States tells person A his or her intention to run or not to run in the next election. Then A relays the news to B, who in turn relays the message to C, and so forth, always to some new person. We assume that there is a probability that a person will change the answer from yes to no when transmitting it to the next person and a probability b that he or she will change it from no to yes. We choose as states the message, either yes or no.

b) In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.

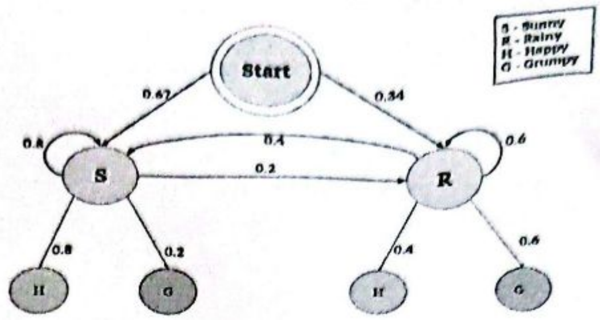
c) Define Bayes theorem. What do you mean by priori and posterior probability. A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag I. [2]

d) Write down the following

i. Define hidden states and observed variable

ii. Find the stationary distribution for the figure along with the transition matrix.

iii. Find out the probability where observed mood sequences are HHG with SSR sequence. Write down the program to find the most likely weather sequence for the observed mood sequence.



8. a) Define Queuing network. Write short notes on different customer behavior in queuing model. [3]

b) The arrival rate of customers is 120 per hour and the service rate of each server is 180 per hour. Find the following. **All times need to be converted in minutes.** [4]

a) The average number of customers in the system. $L_s = \lambda W_s$

b) The average waiting time in the queue. $W_q = \frac{\lambda}{\mu} W_s = \frac{\lambda}{\mu}\left(\frac{1}{\mu-\lambda}\right)$

c) The system utilization. $\rho_b = \frac{\lambda}{\mu}$

d) The probability of a customer finding the server busy. a

e) The probability of customers waiting in the queue.

f) The probability of customers being blocked.

c) Define the types of queue in Kendall's form from the scenario, [3]

i. Write down the Kendall's notation for the queuing model.

ii. A queue with an exponential distribution for the inter-arrival times of customers, an exponential distribution for service times of customers with *m* number of server server.

iii. A queue with an exponential distribution for the inter-arrival times of customers, a general distribution for service times of customers with infinite server.

iv. A queueing system with an exponential distribution for the inter-arrival times of customers and the service times of customers, m servers, a maximum of K customers in the queueing system at once, and N potential customers in the calling population.A company has positions for three employees. Applicants for these jobs appear at a Poisson rate of two per year. If all jobs are filled, applicants look elsewhere. Those who hold a job do so for an exponentially distributed time with mean 1 / 2 year. Describe the situation, using Kendall's notation, as a queueing system.

d) On an average, 6 customers reach a telephonic booth every hour to make calls. Determine the probability that exactly 4 customers will reach in 30 min period, assuming that arrivals follow poisson distribution. Correct answer is between ' 0.160, 0.170'. Briefly explain the answer? [2]

****Good Luck** Best Wishes for You**Be Safe****