



DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION

Course Title: Mathematical Analysis for Computer Science

Course Code: CSE-3201

3rd Year 2nd Semester

Session: 2017-18

Time: 3 Hours

[Answer Any FIVE Questions]

Marks: 60

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Mathematical analysis is the key to opportunity. No longer just the language of science, now it contributes in direct and fundamental ways to business, finance, health, and defense. For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological community. Now your task is to,

- a) How mathematical analysis is fundamental to computer science and engineering? Justify your answer. [3]
- b) List the first three terms for the following series from a_1 to a_4 . [3]

$$a_n = \sum_{i=1}^n (2i - 1)$$

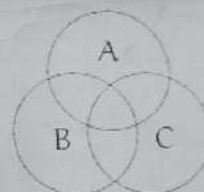
$$a_n = \sum_{i=1}^n (3i^2 - 3i + 1)$$

$$a_n = \sum_{i=1}^n \sum_{j=1}^n 1$$

$$a_n = \sum_{i=1}^n \sum_{j=1}^n i$$

$$a_n = \sum_{i=1}^n \sum_{j=1}^n j$$

- c) What do you mean by conditional probability and expectation? A box contains 4 black chips and 2 blue chips. If two chips are selected at random without replacement, what is the probability that the chips are different colors? [3]
- d) A "Venn diagram" with three overlapping circles is often used to illustrate the eight possible subsets associated with three given sets. Can the sixteen possibilities that arise with four given sets be illustrated by four overlapping circles? Justify your answer. [3]



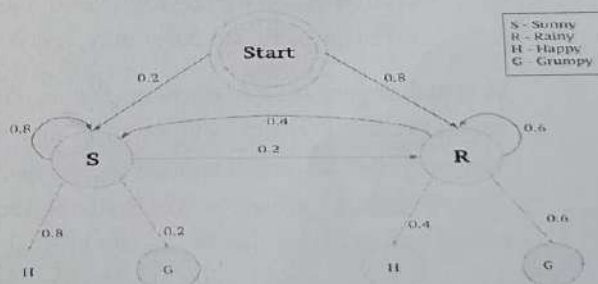
- 2/ a) Write down the differences among dynamic programming, markov chain and monte carlo method with examples. [3]

- b) What do you mean by hidden markov model? Write down the following questions. [6]

i) Draw the transition matrix with the probability if emission of 3 days are G, H, G given that S, R and S respectively.

ii) What will be the probability if emission of 3 days are H, G, H considering first day S and second day R are fixed but third day is random. What is the most possible outcome for the event.

- c) Write short notes on transient, recurrent and classes in Markov chain with examples. "A Markov chain is called irreducible if and only if all states belong to one communication class" justify your answer. [3]



- 3/ a) Probability is the measure of the likelihood that an event will occur. Probability quantifies as a number between 0 and 1, where, loosely speaking, 0 indicates impossibility and 1 indicates certainty. [4]
- A die is rolled, find the probability that the number obtained is greater than 4.
 - Two coins are tossed, find the probability that one head only is obtained.
 - Two dice are rolled, find the probability that the sum is equal to 5.
 - A card is drawn at random from a deck of cards. Find the probability of getting the

- b) Is there any differences between stochastic process and random process? Justify your answer.
- c) Write down the program for B2 sequence and look and say sequence.
- d) How does Euclid's algorithm work to find the GCD? Explain with examples.

4. Number theory is a branch of pure mathematics devoted primarily to the study of the integers and integer-valued functions. German mathematician Carl Friedrich Gauss said, "Mathematics is the queen of the sciences and number theory is the queen of mathematics. Number theorists study prime numbers as well as the properties of objects made out of integers or defined as generalizations of the integers.

- a) What do you mean by number theory? [2]
- b) A number in decimal notation is divisible by 3 if and only if the sum of its digits is divisible by 3. Prove this well-known rule, and generalize it. [2]
- c) Compare w with r using the following figure. [2]



A rectangular label is attached to a right circular cylinder with radius r . The label, which encircles the cylinder without overlap, has width w and an area equal to the area of the base of the cylinder.

- d) Prove that there exists at least one prime number between n and $n!$ [3]
- e) Suppose $a_0 = 0$, $a_1 = 2$ and $a_{n+2} = 4a_{n+1} - 4a_n + n^2 - 5n + 2$ where n divides and for all $n \geq 0$ 1. Find value for n where value is 5 and 6. [3]

5. a) Verify the following relations for a single server queue, [4]
- $L = L_q + 1 - p_0$
 - $L = L_q + \rho$
 - $p_0 = 1 - \rho$

- b) Define the types of queue in Kendall's form from the scenario, [4]
- Define Kendall's notation for the queuing model.
 - A queue with an exponential distribution for the inter-arrival times of customers, an exponential distribution for service times of customers with m number of server.
 - A queue with an exponential distribution for the inter-arrival times of customers, a general distribution for service times of customers with infinite server.
 - A queueing system with an exponential distribution for the inter-arrival times of customers and the service times of customers, m servers, a maximum of K customers in the queueing system at once, and N potential customers in the calling population.

- c) A dental surgery has two operation rooms. The service times are assumed to be independent, exponentially distributed with mean 15 minutes. Andrew arrives when both operation rooms are empty. Bob arrives 10 minutes later while Andrew is still under medical treatment. Another 20 minutes later Caroline arrives and both Andrew and Bob are still under treatment. No other patient arrives during this 30-minute interval. [4]

- What is the probability that Andrew will be ready before Bob?
- What is the probability that Caroline will be ready before Andrew?
- What is the probability that Caroline will be ready before Bob?

6. a) Write down the customer behavior of waiting in the queuing system. [3]
- b) Every even number greater than or equal to 4 can be expressed as the sum of two prime numbers. [3]

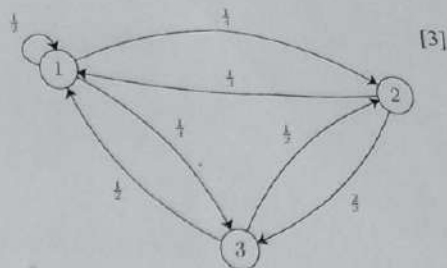
For example:

- $8 = 3 + 5$. Both 3 and 5 are odd prime numbers.
- $20 = 3 + 17 = 7 + 13$.
- $42 = 5 + 37 = 11 + 31 = 13 + 29 = 19 + 23$.

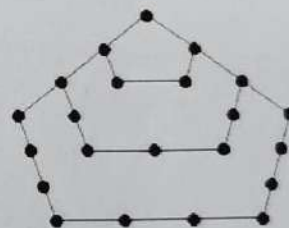
Now your task is to write the program to find out the output for a given number.

- c) What do you mean by queuing network? Briefly describe the classification of queuing [3]

- d) Consider the Markov chain shown in Figure
- Is this chain irreducible
 - Is this chain aperiodic?
 - Find the stationary distribution for this chain.



7. a) Cribbage players have long been aware that $15 = 7 + 8 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5$. Find the number of ways to represent 1050 as a sum of consecutive positive integers. [4]
- b) Assume that a man's profession can be classified as professional, skilled laborer, or unskilled laborer. Assume that, of the sons of professional men, 80 percent are professional, 10 percent are skilled laborers, and 10 percent are unskilled laborers. In the case of sons of skilled laborers, 60 percent are skilled laborers, 20 percent are professional, and 20 percent are unskilled. Finally, in the case of unskilled laborers, 50 percent of the sons are unskilled laborers, and 25 percent each are in the other two categories. Assume that every man has at least one son, and form a Markov chain by following the profession of a randomly chosen son of a given family through several generations. Now your task is to create a Markov chain with transition probability and transition matrix. Also Find the probability that a randomly chosen grandson of an unskilled laborer is a professional man. [3]
- c) Find the grey code value from decimal number 45 to 50. [5]
8. a) Wrote down the labels of all the edges: 0111101100101000. Now you can use the Eulerian path: (000, 000, 001, 011, 111, 111, 110, 101, 011, 110, 100, 001, 010, 101, 010, 100, 000) and draw the Bruijn graph for the state. [6]
- d) Define p - values. Write down the differences between divergent and convergent. Is harmonic series divergent or convergent? Explain. [3]
- e) The sequence of pentagonal numbers $\langle 1, 5, 12, 22, \dots \rangle$ generalizes the triangular and square numbers in an obvious way. Now, Your task is to, [3]
- What is the general formula for the pentagonal number
 - Write down the triangular number along with formula.



Appendix:

- A Gray code is an encoding of numbers so that adjacent numbers have a single digit differing by 1.
- Queuing Model:



- A **rational number** is a number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q .
- $\rho = \lambda/(c\mu)$: utilization of the server; also the probability that the server is busy or the proportion of time the server is busy.
- L_q : mean number of customers in the queue.
- An infinite sequence of positive integers $1 \leq b_1 < b_2 < b_3 < \dots$, also called a Sidon sequence, such that all pairwise sums $b_i + b_j$ for $i \leq j$ are distinct called b2 sequence.
- In mathematics, the look-and-say sequence is the sequence of integers beginning as follows: 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, 31131211131221
- Look and say sequence building process 1 is read off as "one 1" or 11 and 11 is read off as "two 1s" or 21.

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