

Enhanced Array Manifold Matrices for L-Shaped Microphone Array-based 2-D DOA Estimation

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Abstract—This paper presents an alternative and efficient two-dimensional direction-of-arrival method to estimate multi-narrowband source directions from L-shaped microphone arrays. The problem of estimating direction-of-arrival method of multi-narrowband is considered and resolved by declaring new terms of source and central frequency in array manifold matrices. This new matrices are compatible with most classical subspace-based methods, such as, multiple signal classification and the other cross-correlation based two-dimensional direction-of-arrival estimation method. The performance is evaluated in terms of the root-mean-squared error over a range of the signal-to-noise ratio. Furthermore, simulation results demonstrate that proposed array manifold matrices with the classical subspace-based methods can efficiently handle with acceptable noise levels. In conclusion, the modified method provides a potential alternative to intelligent local positioning systems in voice control applications.

I. INTRODUCTION

The fundamental competence of source localization has been extensively implemented in navigation systems for the exploration of sources, which is known as direction-of-arrival (DOA) estimation. The application of DOA estimation has been widely implemented in many areas [1]–[6]. A number of methods have been introduced to improve the source localization efficiency, including, generalized cross correlation method [7]–[10], maximum cross-correlation methods [11] and subspace method [12]–[39].

Subspace methods are increasingly utilized for the DOA estimation of multiple sources. For one-dimensional (1-D) DOA estimation, the uniform linear arrays (ULAs) structure has been developed. The effective method of dealing with 1-D DOA estimation can be found in [12]–[15]. However, such the methods were unable to offer simultaneous angles of azimuth and elevation or the Euler angles.

Recently, the two-dimensional (2-D) DOA estimation using a 2-D geometrical structure of sensors has received considerable attention in the array signal processing and wide applications. Among various array geometrical structures such as the rectangular array, the parallel uniform linear array and circular array, the L-shaped array has attracted a lot of attention. Several L-shaped array-based 2-D DOA estimation methods have been proposed for narrowband sources [16]–[18]. Although these method calculate the directions of sources using the noise subspace via eigenvalue decomposition (EVD) which resulted in a superior multiple-source

localization. These method require spectrum peak search or 2-D intensive searching nonlinear optimization which resulted in huge computational cost and not suggestible for real-time applications. Due to its computational complexity, many computationally efficient algorithms for 2-D DOA estimation have been proposed.

Tayem et al. [20] and Wu et al. [21] initially proposed the computationally simple 2-D DOA estimation algorithm by utilizing the propagator method (PM) in order to avoid the EVD operation. However, the additional angle pairing is required [19]–[21]. There are two options to overcome this problem. One the one hand, the pair matching method of estimating 2-D angle was proposed to resolve this problem [22]. One the other hand, the automatic angle pairing algorithms were proposed to avoid this problem [23]–[35]. However, only narrowband sources can be localized. In order to estimate the DOA for wide-band sources, the coherent signal subspace (CSS) method was introduced [36], [37]. Nevertheless, It was previously found that the estimation performance is sensitive to the initial conditions. Poor initial conditions can lead to biased estimates.

This paper therefore aims to propose an alternative and efficient 2-D DOA method to estimate multi-narrowband source directions from L-shaped microphone arrays. The problem of estimating direction-of-arrival method of multi-narrowband is considered by introducing new terms of source and central frequency in the equation of phase difference, which is known as the array manifold matrix. In order to verified that proposed equation is compatible with most classical subspace-based methods, 2-D multiple signal classification (MUSIC) method [16], the Wang et al. [34] and Dong et al. [31] method are chosen. Furthermore, simulation results demonstrate that proposed array manifold matrices with the classical subspace-based methods can efficiently handle with acceptable noise levels.

Following are the notations used throughout this paper. The operations $|\cdot|$, $\ln(\cdot)$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^{-1}$, $(\cdot)^H$, \otimes , I_D , J_D , $E[\cdot]$, $A(:, i:j)$ and $\text{diag}(\cdot)$ and $\angle(\cdot)$ represent absolute value of a complex number, natural logarithm, conjugate, transpose, inverse, conjugate transpose, Kronecker product, a $D \times D$ identity matrix, a $D \times D$ exchange matrix, the statistical expectation, a submatrix consisting of the i^{th} to the j^{th} column of matrix A , diagonalization and the phase angle operator in radians, respectively.

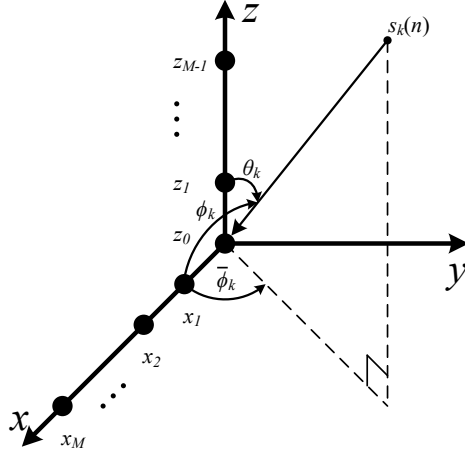


Fig. 1. Geometrical configuration of the L-shaped array for 2-D DOA estimation.

II. PHASE DIFFERENCE EQUATION

The fundamental phase difference equation can be simply described in Euler's formula as $e^{j2\pi ft}$, where f is the source frequency and t is time difference of arrival due to the distance between the source and microphones. Assume that two microphones in an x-z plane, The phase difference equation of the x-z plane arrays can be defined as

$$q(\phi_k, \theta_k) = e^{\frac{j2\pi f(\epsilon_x \cos \phi_k + \epsilon_z \cos \theta_k)}{c}} \quad (1)$$

where ϕ_k is the angle between the x axis and the sources, θ_k is the angle between the z axis and the sources, ϵ_x and ϵ_z are spacing of the microphone elements in the x and z axis, c is the speed of sound. According to [38], the spacing of the microphone elements in any axis should be set to $\frac{\lambda}{2}$ to avoid confusion caused by many grating lobes and the lower power beam-width of the lobe in its structure radiation where $\lambda = c/f_c$ is the wavelength and f_c is the frequency corresponding to the wavelength.

Assuming that the power beam-width of the grating lobes in the radiation patterns depends on f and f_c , the proposed phase difference function is redefined as

$$q(\phi_k, \theta_k) = e^{\left(\frac{f}{f_c}\right) \cdot \left(\frac{j2\pi(\epsilon_x \cos \phi_k + \epsilon_z \cos \theta_k)}{\lambda}\right)}. \quad (2)$$

Consider an L-shaped array as shown in Fig. 1 consisting of two orthogonal M -element ULAs in the x-z plane where spacing of the microphone elements is d and the spacing between z_0 and x_1 is also d . The reference point is pinned at the origin of coordinates z_0 . Form Eq. (2), the magnitude of the L-shaped array factor represents the radiation patterns and correspond to

$$AF = \frac{1}{2M} \left| \sum_{m=1}^M \alpha_m(\phi_k) + \sum_{m=1}^M \beta_m(\theta_k) \right| \quad (3)$$

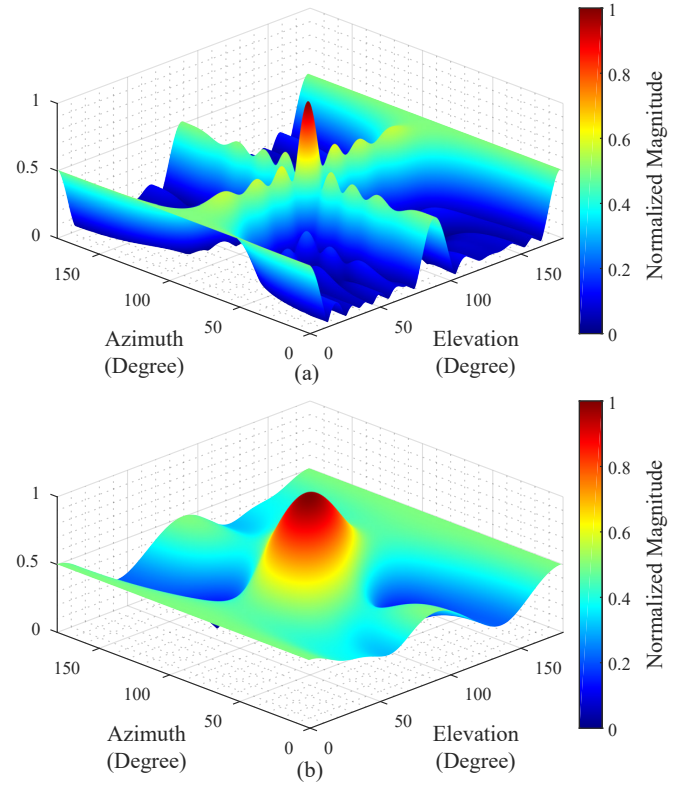


Fig. 2. Normalized radiation patterns of the L-shaped arrays where $M = 8$, $d = \lambda/2$, (a) $f = f_c$ and (b) $f = f_c/4$.

where

$$\begin{aligned} \alpha(\phi_k) &= [1 \ e^{j\alpha_k} \ \dots \ e^{j(M-1)\alpha_k}]^T, \\ \beta(\theta_k) &= [e^{j\beta_k} \ e^{j2\beta_k} \ \dots \ e^{jM\beta_k}]^T, \\ \alpha_k &= \left(\frac{f}{f_c}\right) \cdot \left(\frac{2\pi d \cos \phi_k}{\lambda}\right), \\ \beta_k &= \left(\frac{f}{f_c}\right) \cdot \left(\frac{2\pi d \cos \theta_k}{\lambda}\right). \end{aligned} \quad (4)$$

It can be seen that power beam-width of the grating lobes in Eq. (3) is apparently affected by f/f_c as illustrated in Fig. 2. As the ratio of f and f_c decreases from 1 to 1/4, the overall power beam-width decreases. These results suggest that range of source frequency f should be selected to be less than or equal center frequency f_c . In order to avoid the confusion when a lower source frequency is chosen, the number of microphone elements M should be increased.

III. DATA MODEL

Consider P far-field multi-narrowband sources impinging on the L-shaped arrays as shown in the previous section, the output of each microphone is sampled and decomposed into n snapshots by a short-time Fourier transform (STFT), with each frame containing a set of frequencies f . Therefore, the received signal vector vectors in x and z subarrays can be represented as follows:

$$\mathbf{X}(n, f) = \mathbf{A}_x(\phi) \mathbf{S}(n, f) + \mathbf{W}_x(n, f) \quad (5)$$

$$\mathbf{Z}(n, f) = \mathbf{A}_z(\theta) \mathbf{S}(n, f) + \mathbf{W}_z(n, f) \quad (6)$$

$$\mathbf{A}_x(\phi) = [\alpha(\phi_1) \quad \alpha(\phi_2) \quad \dots \quad \alpha(\phi_P)] \quad (7)$$

$$\mathbf{A}_z(\theta) = [\beta(\theta_1) \quad \beta(\theta_2) \quad \dots \quad \beta(\theta_P)] \quad (8)$$

where matrices in Eqs. (7) and (8) can be expressed as

$$\begin{aligned} \mathbf{X}(n, f) &= [x_1(n, f) \quad x_2(n, f) \quad \dots \quad x_M(n, f)]^T, \\ \mathbf{Z}(n, f) &= [z_0(n, f) \quad z_1(n, f) \quad \dots \quad z_{M-1}(n, f)]^T, \\ \mathbf{S}(n, f) &= [s_1(n, f) \quad s_2(n, f) \quad \dots \quad s_P(n, f)]^T, \\ \mathbf{W}_x(n, f) &= [w_{x1}(n, f) \quad w_{x2}(n, f) \quad \dots \quad w_{xM}(n, f)]^T, \\ \mathbf{W}_z(n, f) &= [w_{z0}(n, f) \quad w_{z1}(n, f) \quad \dots \quad w_{zM-1}(n, f)]^T. \end{aligned}$$

Note that $\mathbf{X}(n, f)$ and $\mathbf{Z}(n, f)$ represent sum of received signal vectors in x and z subarrays for all P sources with different angles. $\mathbf{S}(n, f)$ is a signal source vector. $\mathbf{W}_x(n, f)$ and $\mathbf{W}_z(n, f)$ are additive noise vectors in x and z subarrays for all microphone elements. $\mathbf{A}_x(\phi)$ and $\mathbf{A}_z(\theta)$ are proposed array manifold matrices of x and z subarrays. n is a snapshot and f is the source frequency. The relationship between the azimuth angle $\bar{\phi}$, elevation angle θ and x subarray angle ϕ is expressed as

$$\cos \phi_k = \sin \theta_k \cos \bar{\phi}_k. \quad (9)$$

IV. DOA ESTIMATION SCHEME

In this section, in order to verified those Eqs. (7) and (8) are compatible with most classical subspace-based methods, 2-D MUSIC method [16], the Wang et al. [34] and Dong et al. [31] method are chosen and reconstructed from the conventional array manifold matrix to the proposed array manifold matrix which explains in the next three subsections.

A. Extension of 2-D MUSIC Method

Under the assumptions of data model from [34], [38], [39] and assuming a noise free environment, the cross-correlation matrix from all received signal vector in Eqs. (5) and (6) can be described as follows:

$$\begin{aligned} \mathbf{R}_{yy} &= E \left[\begin{bmatrix} \mathbf{X}(n, f) \\ \mathbf{Z}(n, f) \end{bmatrix} \begin{bmatrix} \mathbf{X}(n, f) \\ \mathbf{Z}(n, f) \end{bmatrix}^H \right] \\ &= \begin{bmatrix} \mathbf{A}_x(\phi) \\ \mathbf{A}_z(\theta) \end{bmatrix} E \left[\mathbf{S}(n, f) \mathbf{S}(n, f)^H \right] \begin{bmatrix} \mathbf{A}_x(\phi) \\ \mathbf{A}_z(\theta) \end{bmatrix}^H \quad (10) \\ &= \begin{bmatrix} \mathbf{A}_x(\phi) \\ \mathbf{A}_z(\theta) \end{bmatrix} \mathbf{R}_{ss} \begin{bmatrix} \mathbf{A}_x(\phi) \\ \mathbf{A}_z(\theta) \end{bmatrix}^H \end{aligned}$$

where \mathbf{R}_{ss} is the auto-correlation matrix of sources $\mathbf{S}(n, f)$. Applying EVD, \mathbf{R}_{yy} can be represented as the factorization of a matrix into a canonical form as follows:

$$\mathbf{R}_{yy} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_w \mathbf{\Lambda}_w \mathbf{U}_w^H \quad (11)$$

where \mathbf{U}_s and $\mathbf{\Lambda}_s$ are a matrix of eigenvector and eigenvalue in the signal subspace. \mathbf{U}_w and $\mathbf{\Lambda}_w$ are a matrix of eigenvector and eigenvalue in the noise subspace. Note that noise subspace \mathbf{U}_w estimated by the smallest eigenvalues of \mathbf{R}_{yy} [16].

After \mathbf{U}_w is estimated, we can obtain ϕ_k and θ_k by maximizing the following cost function with Eq. (4) as follows:

$$\left\{ \hat{\phi}_k, \hat{\theta}_k \right\}_{k=1}^P = \arg \max_{\phi, \theta} (\ln |f_{\text{MUSIC}}(\phi, \theta)|) \quad (12)$$

where

$$f_{\text{MUSIC}}(\phi, \theta) = \frac{1}{\left[\begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix}^H \mathbf{U}_w \mathbf{U}_w^H \begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix} \right]}.$$

B. Extension of Wang et al. Method

The computationally efficient subspace-based 2-D DOA estimation method without EVD was proposed by Wang et al [34]. In the similar way as [16], this method began by constructing the cross-correlation matrix \mathbf{R}_{yy} which already described in Eq. (10). In summary [34], the modified cost function from Eq. (4) is given by

$$\left\{ \hat{\phi}_k, \hat{\theta}_k \right\}_{k=1}^P = \arg \min_{\phi, \theta} \left(\begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix}^H \hat{\Pi} \begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix} \right) \quad (13)$$

or

$$\left\{ \hat{\phi}_k, \hat{\theta}_k \right\}_{k=1}^P = \arg \max_{\phi, \theta} \left(- \begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix}^H \hat{\Pi} \begin{bmatrix} \alpha(\phi) \\ \beta(\theta) \end{bmatrix} \right) \quad (14)$$

where

$$\begin{aligned} \hat{\Pi} &= \hat{\mathbf{Q}} \left(\mathbf{I}_{2M-P} - \hat{\mathbf{P}}^H (\hat{\mathbf{P}} \hat{\mathbf{P}}^H + \mathbf{I}_P)^{-1} \hat{\mathbf{P}} \right) \hat{\mathbf{Q}}^H, \\ \hat{\mathbf{P}} &= \left(\hat{\mathbf{G}}_1 \hat{\mathbf{G}}_1^H \right)^{-1} \hat{\mathbf{G}}_1 \hat{\mathbf{G}}_2^H, \\ \hat{\mathbf{Q}} &= \left[\hat{\mathbf{P}}^T \quad -\mathbf{I}_{2M-P} \right]^T, \\ \mathbf{R}_{yy} &= \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix}. \end{aligned}$$

Note that size of matrices \mathbf{G}_1 and \mathbf{G}_2 are $2M \times P$ and $2M \times 2M - P$, respectively.

C. Extension of Dong et al. method

In a different way to estimates DOA sources without the 2-D search, Dong et al. proposed the efficient method with only 1-D search [31]. This method began by constructing the cross-correlation matrix \mathbf{R}_{xz} between x and z subarrays as follows:

$$\begin{aligned} \mathbf{R}_{xz} &= E \left[\mathbf{X}(n, f) \mathbf{Z}(n, f)^H \right] \\ &= \mathbf{A}_x(\phi) E \left[\mathbf{S}(n, f) \mathbf{S}(n, f)^H \right] \mathbf{A}_z^H(\theta) \quad (15) \\ &= \mathbf{A}_x(\phi) \mathbf{R}_{ss} \mathbf{A}_z^H(\theta) \end{aligned}$$

According to [32], the conventional array manifold matrix, which is known as a Vandermonde matrix. As the conjugate system property of the conventional array manifold matrix, array manifold matrices in Eqs. (7) and (8) can also be mentioned as follows:

$$\mathbf{J}_M(\mathbf{A}_x(\phi))^* = \mathbf{A}_x(\phi) \tilde{\mathbf{\Phi}}_{xr}(\phi) \quad (16)$$

$$\mathbf{J}_M(\mathbf{A}_z(\theta))^* = \mathbf{A}_z(\theta) \tilde{\mathbf{\Phi}}_{zr}(\theta) \quad (17)$$

$$\mathbf{A}_{z2}(\theta) = \mathbf{A}_{z1}(\theta) \Phi_z(\theta) \quad (18)$$

where

$$\begin{aligned} \tilde{\Phi}_{xr}(\phi) &= \text{diag}([e^{-j(M-1)\alpha_1} \quad \dots \quad e^{-j(M-1)\alpha_P}]), \\ \tilde{\Phi}_{zr}(\theta) &= \text{diag}([e^{-j(M-1)\beta_1} \quad \dots \quad e^{-j(M-1)\beta_P}]), \\ \Phi_z(\theta) &= \text{diag}([e^{j\beta_1} \quad e^{j\beta_2} \quad \dots \quad e^{j\beta_P}]), \end{aligned}$$

$\mathbf{A}_{z1}(\theta)$ and $\mathbf{A}_{z2}(\theta)$ are the first and the last $M-1$ rows of $\mathbf{A}_z(\theta)$. Hence, the new cross-correlation matrix can be reconstructed with similar to [25], [31] as follows:

$$\begin{aligned} \mathbf{Y} &= \begin{bmatrix} \mathbf{Y}_1, \mathbf{J}_M \mathbf{Y}_2^* \\ \mathbf{Y}_2, \mathbf{J}_M \mathbf{Y}_1^* \end{bmatrix} \\ &= \mathbf{A}_g(\phi, \theta) \mathbf{S}_g(\phi, \theta) \end{aligned} \quad (19)$$

where

$$\begin{aligned} \mathbf{A}_g(\phi, \theta) &= \begin{bmatrix} \mathbf{A}_x^T(\phi) & (\mathbf{A}_x(\phi) \Phi_z^*(\theta))^T \end{bmatrix}^T, \\ \mathbf{S}_g(\phi, \theta) &= [\mathbf{R}_{ss} \mathbf{A}_{z1}^H(\theta) \quad \Phi_z(\theta) \tilde{\Phi}_{xr}(\phi) \mathbf{R}_{ss} \mathbf{A}_{z1}^T(\theta)], \\ \mathbf{Y}_1 &= \mathbf{A}_x(\phi) \mathbf{R}_{ss} \mathbf{A}_{z1}^H(\theta) = \mathbf{R}_{xz}(:, 1:M-1), \\ \mathbf{Y}_2 &= \mathbf{A}_x(\phi) \mathbf{R}_{ss} \mathbf{A}_{z2}^H(\theta) = \mathbf{R}_{xz}(:, 2:M). \end{aligned}$$

In the same way of 2-D MUSIC method, applying EVD to $\mathbf{R}_{YY} = \mathbf{Y} \mathbf{Y}^H$, \mathbf{R}_{YY} can be factorized as summation of two matrices, including, the matrix of eigenvector and eigenvalue in the signal subspace and noise subspace. After the noise subspace $\mathbf{U}_{w, \mathbf{R}_{YY}}$ is estimated, modified cost function from Eq. (4) is described as follows:

$$\{\hat{\phi}_k, \hat{\theta}_k\}_{k=1}^P = \arg \min_{\phi, \theta} \left(\frac{\mathbf{q}^H(\theta) \mathbf{F}(\phi) \mathbf{q}(\theta)}{\mathbf{q}^H(\theta) \mathbf{q}(\theta)} \right) \quad (20)$$

or

$$\{\hat{\phi}_k, \hat{\theta}_k\}_{k=1}^P = \arg \max_{\phi, \theta} \left(-\frac{\mathbf{q}^H(\theta) \mathbf{F}(\phi) \mathbf{q}(\theta)}{\mathbf{q}^H(\theta) \mathbf{q}(\theta)} \right) \quad (21)$$

where

$$\begin{aligned} \mathbf{q}(\theta) &= [1 \quad e^{-j\beta}]^T, \\ \mathbf{F}(\phi) &= (\mathbf{I}_2 \otimes \alpha(\phi))^H \mathbf{U}_{w, \mathbf{R}_{YY}} \mathbf{U}_{w, \mathbf{R}_{YY}}^H (\mathbf{I}_2 \otimes \alpha(\phi)). \end{aligned}$$

In order to solve Eq. (21) without 2-D search, Dong et al. described the relationship between eigenvalue and determinant [32] which can be rewritten as 1-D search optimization problem as follows:

$$\hat{\phi}_k = \arg \max_{\phi} \left(\frac{1}{\mathbf{F}(\phi)} \right). \quad (22)$$

$\hat{\theta}_k$ can be estimated via the eigenvector corresponding to the minimum eigenvalue of $\mathbf{F}(\hat{\phi}_k)$ as

$$\hat{\phi}_k = \arccos \left(\text{angle} \left(\frac{\mathbf{e}_{\min}^1}{\mathbf{e}_{\min}^2} \right) \cdot \left(\frac{\lambda}{2\pi d} \right) \cdot \left(\frac{f_c}{f} \right) \right) \quad (23)$$

where \mathbf{e}_{\min}^1 and \mathbf{e}_{\min}^2 are the first and second elements of the eigenvector corresponding the minimal eigenvalue of $\mathbf{F}(\hat{\phi}_k)$. Those Eqs. (22) and (23) showed that method can pair the angles automatically without 2-D search.

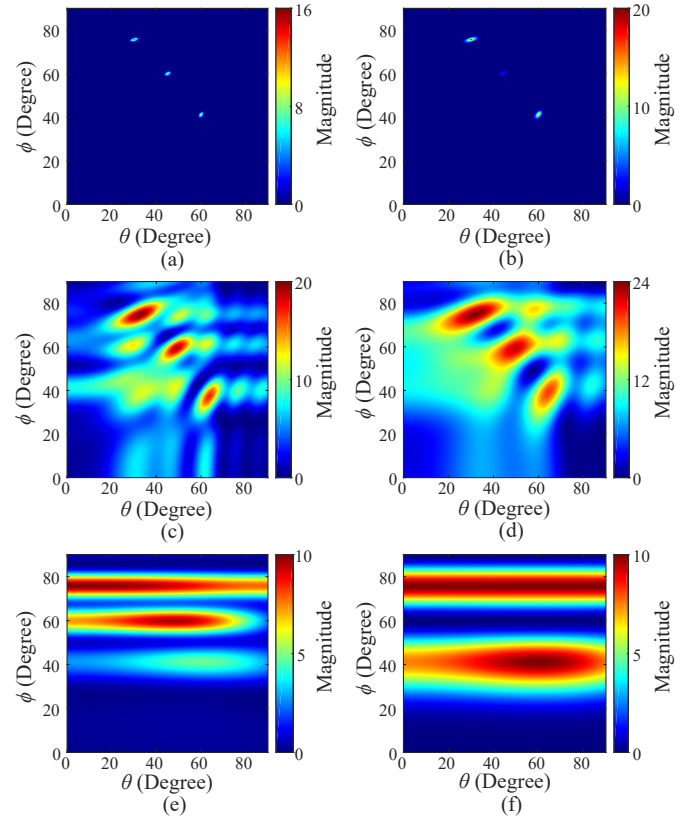


Fig. 3. Estimation results of the proposed array manifold matrices with three types of cost functions, including, (a), (b) 2-D MUSIC method in Eq. (12), (c), (d) the Wang et al. method in Eq. (14) and (e), (f) Dong et al. method in Eq. (21), where the source frequency in (a), (c) and (e) are 3.4 kHz and (b), (d) and (f) are 2.26 kHz.

V. NUMERICAL SIMULATION

In this section, the performance of the proposed array manifold matrix was demonstrated, including, 2-D MUSIC method [16], the Wang et al. [34] and Dong et al. [31] method. We merged the proposed array manifold matrix and all the 2-D DOA estimation method to support the multi-narrowband localization as shown in the previous section. All the method was tested by computer simulation. All the method was implemented in MATLAB software. The output of each microphone was decomposed into 192,000 snapshots. The sampling frequency was 192 kHz, the center frequency was 3.4 kHz, the number of subarray elements M was 12, the spacing of microphone elements was 5 cm and speed of sound was assumed to be 340 m/s. The 2-D search ranges for azimuth and elevation are $[0^\circ, 180^\circ]$ and $[0^\circ, 180^\circ]$ with interval 0.01. The definition of root-mean-square error (RMSE) of the 2-D DOA estimation was expressed as

$$\text{RMSE} = \sqrt{\frac{1}{LP} \sum_{l=1}^L \sum_{k=1}^P \left(\left(\bar{\phi}_k^{\text{DOA}} - \bar{\phi}_k^{(l)} \right)^2 + \left(\bar{\theta}_k^{\text{DOA}} - \bar{\theta}_k^{(l)} \right)^2 \right)} \quad (24)$$

where L is the number of times of the Monte Carlo experiment. $\bar{\phi}_k^{(l)}$ and $\bar{\theta}_k^{(l)}$ are the l^{th} estimated azimuth and elevation

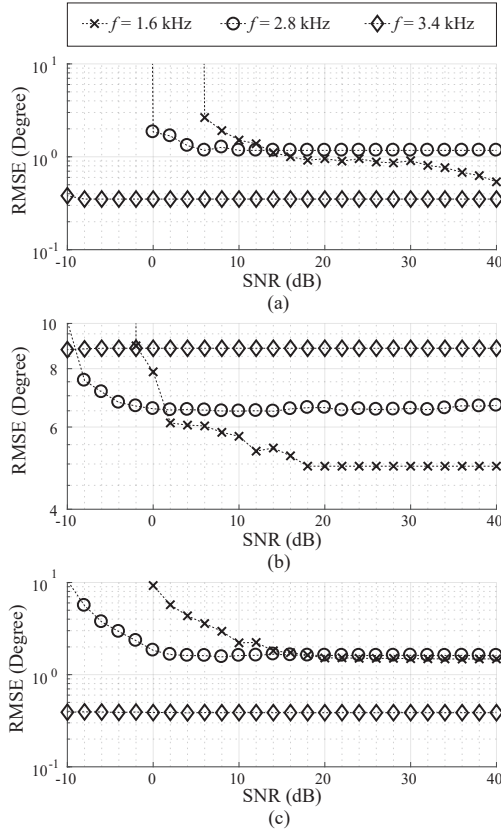


Fig. 4. RMSE performance evaluation with three cost functions, (a) 2-D MUSIC method in Eq. (12), (b) the Wang et al. method in Eq. (14) and (c) Dong et al. method in Eq. (21).

angles of the k^{th} source. $\bar{\phi}_k^{\text{DOA}}$ and θ_k^{DOA} are the true DOA azimuth and elevation angles of the k^{th} source.

A. Robustness of the Cost Functions

Fig. 3 shows example of estimation results by the three types of cost functions, including, 2-D MUSIC method in Eq. (12) as showed in Fig. 3 (a) and (b), the Wang et al. method in Eq. (14) as showed in Fig. 3 (c) and (d) and Dong et al. method in Eq. (21) as showed in Fig. 3 (e) and (f). Where as the horizontal axis was represented as θ or elevation angle. The vertical axis was represented as ϕ . The relationship between the azimuth angle ϕ , elevation angle θ and ϕ was given in Eq. (9). Three sources were placed $(\theta_k^{\text{DOA}}, \phi_k^{\text{DOA}})$ at $(30^\circ, 75.52^\circ)$, $(45^\circ, 60^\circ)$ and $(60^\circ, 41.41^\circ)$, where three sources were human voices with containing the source frequency range from 100 to 16,000 Hz. Two simulation conditions were considered. First scenario, Fig. 3 (a), (c) and (e) were conducted with source frequency f was 3.4 kHz. Second scenario, Fig. 3 (b), (d) and (f) were conducted with source frequency f was 2.26 kHz. As can be seen from Fig. 3 (b) and (d) that as expected, the proposed array manifold matrix with Eqs. (12) and (14) can handle the source frequency f lower than the center frequency f_c successfully. Although, the surface of cost functions in Fig. 3 (d) and (e) spread out over a wide true DOA areas, which is led to uncorrected

estimation as shown in Fig. 3 (e). Since the computationally intensive and EVD process is avoided in Wang et al. method, the surface of cost function Eq. (14) can be influenced by the asymptotic mean-square-error expressions [34], which is shown in Fig. 3 (c) and (d). Therefore, the surface of cost functions of 2-D MUSIC method spread out less than Wang et al. method due to utilizing EVD in Eqs. (11) and (12). In case of Dong et al. method, it can be seen from Eq. (21) that surface of cost function may be dominated by a Rayleigh quotient problem [31], in other words, the surface should be spread out over a elevation angle θ . However, all confusion may be avoided by increases the number of microphone elements M and employs the constraint of $0 < f \leq f_c$ as shown in Eq. (3).

B. RMSE versus SNR

Three methods were tested for comparison with the proposed array manifold matrix: 2-D MUSIC method in Eq. (12) as illustrated in Fig. 4 (a), the Wang et al. method in Eq. (14) as illustrated in Fig. 4 (b) and Dong et al. method in Eqs. (22) and (23) as illustrated in Fig. 4 (c). Where as the horizontal axis was represented as signal-to-noise ratio (SNR). This SNR varies from -10 to 40 dB with interval 2 dB. The vertical axis was represented as RMSE, which was calculated by Eq. (24) where 100 Monte Carlo trials have been conducted for every fixed SNR. Three sources were placed $(\theta_k^{\text{DOA}}, \bar{\phi}_k^{\text{DOA}})$ at $(60^\circ, 30^\circ)$, $(45^\circ, 45^\circ)$ and $(30^\circ, 60^\circ)$, where the source frequencies were 1.6, 2.8 and 3.4 kHz, respectively. In this scenario, the number of sources P was assumed to be 1 in each narrowband. The simulation results in Fig. 4 indicated that the performance of all the method is significantly dominated by the source frequencies. As the source frequency is 2.8 kHz, all the methods exhibits good performance at SNR range from 0 to 40 dB. Likewise, in case of the source frequency is 1.6 kHz, all the methods exhibits good performance at only SNR range from 8 to 40 dB. This problem may be caused by the interfered power beam-width of the side lobe in L-array structure radiation pattern. In the end, DOA estimation for multi-narrowband sources can be done with acceptable SNRs and source frequencies.

VI. CONCLUSIONS

An alternative and efficient 2-D DOA method to estimate multi-narrowband source directions from L-shaped microphone arrays has been presented. The problem of estimating direction-of-arrival method of multi-narrowband was considered and resolved by the proposed array manifold matrix. Extension of most classical subspace-based methods for multi-narrowband source estimations were described and reconstructed. As demonstrated by simulation results, the proposed array manifold matrices with the classical subspace-based methods has achieved the multi-narrowband DOA estimation with acceptable noise levels. The modified method provides a potential alternative to intelligent local positioning systems in voice control applications. Future work, the efficient solution to deal with the lower source frequency and algorithm for determine a number of wideband sources are considered.

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