Robust Adaptive Beamforming via Covariance Matrix Reconstruction Under Colored Noise

Huichao Yang, Pengyu Wang, and Zhongfu Ye

***Abstract*—Aimed at the performance degradation of the standard Capon beamformer (SCB) when the signal of interest (SOI) appearing in the training data under the colored noise, a novel interference-plus-noise covariance matrix (INCM) reconstruction method is proposed in this letter. The colored noise covariance matrix (CNCM) is estimated by the cross Capon power in the noise sector and the interference power is obtained via the quasi-orthogonality based on the conventional beamforming (CBF) between different sparse steering vectors (SVs) to project the sample covariance matrix for the INCM reconstruction. Then, the SV of the SOI is updated by the principal eigenvector of the reconstructed SOI covariance matrix. Simulation results show that the proposed method is robust against some mismatch errors under the colored noise.**

***Index Terms*—Robust adaptive beamforming, covariance matrix reconstruction, cross Capon power, quasi-orthogonality, colored noise.**

I. Introduction

As A spatial filter, adaptive beamformer can receive a signal from a certain direction. Meanwhile, it can suppress interference and noise by adjusting its weight vector according to the changes of the external environment. Therefore, it has been widely used in radar, sonar, wireless communication, and so on [1]–[3]. It is known that the standard Capon beamformer (SCB) is one of the optimal adaptive beamformers [4] but it is sensitive to model mismatch errors and causes performance degradation, especially when the signal of interest (SOI) is present in the training data [5]. To improve its robustness, many researchers have made great contributions to addressing this problem during the past decades, such as diagonal loading (DL) technique [6], eigenspace-based method [7], uncertainty set-based method [8] and so on.

Recently, some methods based on the interference-plus-noise covariance matrix (INCM) reconstruction to realize robust adaptive beamformers have been proposed [9]–[17]. The INCM was reconstructed through the Capon spectrum estimator integrated over a region separated from the desired signal direction in [9] and the steering vector (SV) of SOI was obtained through solving a quadratically constrained quadratic programming (QCQP) problem. However, the method was limited to the angle error and not suitable for other error types. To improve its applicability, the INCM based on the integration of the Capon spectrum over the surface of an annulus was brought up in [11], which could be regarded as a more generalized type of method in [9]. However, the methods based on the Capon spectrum integral were sensitive to the SV mismatch errors [12] and highly depended on the estimated spectra [13]. Considering the precision of the reconstructed INCM, a method using the powers based on the principle of maximum entropy power spectrum to replace Capon spectrum for the INCM reconstruction was proposed in [14]. The INCM was reconstructed based on searching interference SVs lying within the intersection of interference subspace and signal-interference subspace in [15]. A method based on gradient searching for more accurate SVs to reconstruct the INCM was proposed in [17] and realized good performance.

Most of the INCM methods are developed under white noise because it is easy to realize the noise covariance matrix reconstruction through the estimation of the noise power. However, colored noise is more common in practice but it is difficult to reconstruct the colored noise covariance matrix (CNCM) because it cannot be reconstructed only through the noise power estimation, which causes the performance degradation of the beamformer under colored noise. To improve the performance of the beamformer under this situation, a new beamformer is proposed in this letter. The initial direction-of-arrivals (DOAs) of the signal and interferences are estimated by the Capon spectrum. Through the property of the weight vector of the beamformer, the cross Capon power is firstly proposed to realize the CNCM reconstruction. Besides, eigenvalue decomposition is applied to further remove the residual SOI and interferences for a more accurate CNCM. Then, the interference power is estimated via the quasi-orthogonality between different SVs for the INCM reconstruction. Meanwhile, the SV of the SOI is updated by the principal eigenvector of the reconstructed covariance matrix of the SOI to improve the robustness.

The rest of this letter is arranged as follows. The signal model is built in Section II and the proposed method is demonstrated in Section III. In Section IV, the simulation results are provided

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to demonstrate the effectiveness and robustness of the proposed method and then Section V concludes this letter.

II. Signal Model and Background

Assume that a uniform linear array (ULA) with M omnidirectional sensors receives $L+1$ uncorrelated far-field narrowband sources (one signal and L interferences), which are uncorrelated with noise. Then the observation at time k can be described as

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k)\\ &= s_0(k)\mathbf{a}_0 + \sum_{l=1}^{L} s_l(k)\mathbf{a}_l + \mathbf{x}_n(k)\end{aligned} \tag{1}$$

where $\mathbf{x}_s(k)$, $\mathbf{x}_i(k)$ and $\mathbf{x}_n(k)$ are the components of the SOI, interferences and noise, respectively. $s_0(k)$ and $\mathbf{a}_0$ are the SOI and its SV. $s_l(k)$ and $\mathbf{a}_l, l = 1, 2, \ldots, L$ are the l-th interference and its SV. Assume that the SOI, interferences and colored noise are statistically independent with each other.

The output of the beamformer can be expressed as $y(k) = \mathbf{w}^H\mathbf{x}(k)$, where $\mathbf{w} = [w_1, w_2, \ldots, w_M]^T$ is the weight vector. $(\cdot)^H$ and $(\cdot)^T$ denote Hermitian transpose and transpose operation respectively. The output signal-to-interference-plus-noise ratio (SINR) is usually used to measure the performance of the beamformer, which is defined as

$$\mathrm{SINR} = \frac{\sigma_s^2\left|\mathbf{w}^H\mathbf{a}_0\right|^2}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} \tag{2}$$

where $\sigma_s^2 = E\{|s_0(k)|^2\}$ is the power of the SOI and $\mathbf{R}_{i+n}$ is the theoretical INCM, which is expressed as

$$\begin{aligned}\mathbf{R}_{i+n} &= E\left\{\mathbf{x}_{i+n}(k)\mathbf{x}_{i+n}^H(k)\right\}\\ &= \sum_{l=1}^{L}\sigma_l^2\mathbf{a}_l\mathbf{a}_l^H + \mathbf{R}_n = \mathbf{R}_i + \mathbf{R}_n\end{aligned} \tag{3}$$

where $\mathbf{x}_{i+n}(k) = \mathbf{x}_i(k) + \mathbf{x}_n(k)$ and $\sigma_l^2, l = 1, 2, \ldots, L$ is the power of the l-th interference. $\mathbf{R}_i$ and $\mathbf{R}_n$ represent the interference covariance matrix and the colored noise covariance matrix, respectively. Through maximizing the beamformer's output SINR, the optimal weight vector can be obtained by solving the optimization problem

$$\min_{\mathbf{w}} \ \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w} \ \ s.t. \ \mathbf{w}^H\mathbf{a}_0 = 1 \tag{4}$$

The optimal solution of (4) is

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}_0}{\mathbf{a}_0^H\mathbf{R}_{i+n}^{-1}\mathbf{a}_0} \tag{5}$$

which is widely used and known as the minimum variance distortionless response (MVDR) beamformer. The weight vector of the Capon beamformer can be obtained by replacing $\mathbf{R}_{i+n}$ in (5) with $\mathbf{R}_x = E\{\mathbf{x}(k)\mathbf{x}^H(k)\}$ [1], [4], which has the same performance as MVDR. The output power based on this weight vector is known as Capon power or Capon spectrum, which is

$$P = \frac{1}{\mathbf{a}_0^H\mathbf{R}_x^{-1}\mathbf{a}_0} \tag{6}$$

Usually, $\mathbf{R}_x$ and $\mathbf{a}_0$ are replaced with the sample covariance matrix $\hat{\mathbf{R}}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k)$ with K snapshots and the nominal SV $\bar{\mathbf{a}}_0$ according to the known array structure. The Capon power becomes $\hat{P} = 1/\bar{\mathbf{a}}_0^H\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_0$. Correspondingly, the weight vector becomes

$$\mathbf{w}_{\text{SMI}} = \frac{\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_0}{\bar{\mathbf{a}}_0^H\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_0} \tag{7}$$

which is called the sample matrix inversion (SMI) beamformer.

III. Proposed Method

The weight vector of the SMI beamformer can enhance the SOI and suppress the interferences and the noise. If it is calculated in the noise sector, which means the noise is the enhanced component, the SOI and the interferences will be suppressed. Besides, the nominal SV of the interference can also strengthen itself and restrain others, which is based on the conventional beamforming (CBF) that has a weight vector $\mathbf{w} = \mathbf{a}_0/M$ [18]. The proposed method is based on these and the details are demonstrated in the following.

A. INCM Reconstruction

The initial directions of the sources are obtained by the Capon spectrum search and then the whole space is divided into the SOI sector Θ_s, the interference sector Θ_i and the noise sector Θ_n. For α and β in the noise sector, the weight vector of the SMI can be obtained as

$$\mathbf{w}_\alpha = \frac{\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\alpha}{\bar{\mathbf{a}}_\alpha^H\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\alpha}, \ \ \mathbf{w}_\beta = \frac{\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\beta}{\bar{\mathbf{a}}_\beta^H\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\beta} \tag{8}$$

where $\bar{\mathbf{a}}_\alpha$ and $\bar{\mathbf{a}}_\beta$ are the nominal SVs associated with the angles α and β. The cross Capon power between α and β can be described as

$$\begin{aligned}\hat{P}_{\alpha,\beta} &= \frac{1}{K}\sum_{k=1}^{K}\left[\mathbf{w}_\alpha^H\mathbf{x}(k)\right]\left[\mathbf{w}_\beta^H\mathbf{x}(k)\right]^H\\ &= \hat{P}_\alpha\hat{P}_\beta\bar{\mathbf{a}}_\alpha^H\hat{\mathbf{R}}_x^{-1}\hat{\mathbf{R}}_x\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\beta\end{aligned} \tag{9}$$

where $\hat{P}_\alpha$ and $\hat{P}_\beta$ are the Capon power associated with α and β. Choose M sparse angles $\alpha_m, m = 1, 2, \ldots, M$ in the noise sector and the matrix form of (9) can be expressed as

$$\bar{\mathbf{A}}_N^H\hat{\mathbf{R}}_x^{-1}\hat{\mathbf{R}}_x\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{A}}_N = \mathbf{P} \tag{10}$$

where $\bar{\mathbf{A}}_N = [\bar{\mathbf{a}}_{\alpha_1}, \bar{\mathbf{a}}_{\alpha_2}, \ldots \bar{\mathbf{a}}_{\alpha_M}]$ is the SV matrix with full rank in the noise sector and $\mathbf{P}$ is the normalized cross Capon power matrix, which is as

$$\mathbf{P} = \begin{bmatrix}\frac{\hat{P}_{\alpha_1,\alpha_1}}{\hat{P}_{\alpha_1}\hat{P}_{\alpha_1}} & \cdots & \frac{\hat{P}_{\alpha_1,\alpha_M}}{\hat{P}_{\alpha_1}\hat{P}_{\alpha_M}}\\ \vdots & \cdots & \vdots\\ \frac{\hat{P}_{\alpha_M,\alpha_1}}{\hat{P}_{\alpha_M}\hat{P}_{\alpha_1}} & \cdots & \frac{\hat{P}_{\alpha_M,\alpha_M}}{\hat{P}_{\alpha_M}\hat{P}_{\alpha_M}}\end{bmatrix} \tag{11}$$

Since the weight vector is calculated in the noise sector, the SOI and interferences will be suppressed and (9) can be rewritten as

$$\hat{P}_{\alpha,\beta} \approx \frac{1}{K}\sum_{k=1}^{K}\left[\mathbf{w}_\alpha^H\mathbf{x}_n(k)\right]\left[\mathbf{w}_\beta^H\mathbf{x}_n(k)\right]^H = \hat{P}_\alpha\hat{P}_\beta\bar{\mathbf{a}}_\alpha^H\hat{\mathbf{R}}_x^{-1}\hat{\mathbf{R}}_n\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}_\beta \quad (12)$$

In the same way, it can be modeled as

$$\bar{\mathbf{A}}_N^H\hat{\mathbf{R}}_x^{-1}\hat{\mathbf{R}}_n\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{A}}_N \approx \mathbf{P} \quad (13)$$

Apply eigenvalue decomposition on $\mathbf{P}$ in (13) and discard $L+1$ small eigenvalues to further remove the residual SOI and interferences, where the reconstructed $\mathbf{P}_N$ can be obtained as

$$\mathbf{P}_N = \sum_{m=1}^{M-L}\eta_m\mathbf{u}_m\mathbf{u}_m^H \quad (14)$$

where η_m arranged in descending order is the eigenvalue of the $\mathbf{P}$ in (11) and $\mathbf{u}_m$ is the eigenvector corresponding to η_m. Then, replace $\mathbf{P}$ in (13) with $\mathbf{P}_N$ and the CNCM can be obtained as

$$\hat{\mathbf{R}}_n \approx \hat{\mathbf{R}}_x\bar{\mathbf{A}}^{-H}\mathbf{P}_N\bar{\mathbf{A}}^{-1}\hat{\mathbf{R}}_x \quad (15)$$

The SV with error can be modeled as [9], [11]

$$\mathbf{a}_l = \bar{\mathbf{a}}_l + \mathbf{e}_l,\ l = 0,1,\ldots,L \quad (16)$$

where $\mathbf{e}_l$ is the component of the mismatch error. Bring (16) to (1) and use the nominal interference SV $\bar{\mathbf{a}}_l = [1, e^{-j2\pi d\sin\theta_l/\lambda}, \ldots, e^{-j2\pi(M-1)d\sin\theta_l/\lambda}]^T$, $l = 1,2,\ldots,L$ based on the estimated directions to project $\mathbf{x}(k)$ and it can be obtained as

$$\mathbf{x}_l(k) = \bar{\mathbf{a}}_l^H\mathbf{x}(k) \approx \bar{\mathbf{a}}_l^H\bar{\mathbf{a}}_l s_l(k) + \bar{\mathbf{a}}_l^H\mathbf{x}_n(k) \quad (17)$$

The approximate orthogonality still works though the SV has errors because the nominal SV plays a role like the weight vector of CBF. The sample covariance matrix of (17) is

$$\hat{\mathbf{R}}_{x_l} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_l(k)\mathbf{x}_l^H(k) = \hat{\sigma}_l^2\bar{\mathbf{a}}_l^H\bar{\mathbf{a}}_l\bar{\mathbf{a}}_l^H\bar{\mathbf{a}}_l + \bar{\mathbf{a}}_l^H\hat{\mathbf{R}}_n\bar{\mathbf{a}}_l \quad (18)$$

Note that $\hat{\mathbf{R}}_{x_l}$ is a 1×1 matrix because $\mathbf{x}_l(k)$ has a dimension of 1×1. The power of the l-th interference $\hat{\sigma}_l^2$ can be obtained as

$$\hat{\sigma}_l^2 = \frac{\hat{\mathbf{R}}_{x_l} - \bar{\mathbf{a}}_l^H\hat{\mathbf{R}}_n\bar{\mathbf{a}}_l}{\bar{\mathbf{a}}_l^H\bar{\mathbf{a}}_l\bar{\mathbf{a}}_l^H\bar{\mathbf{a}}_l} \quad (19)$$

Via the estimated powers of the interferences, the INCM can be reconstructed as

$$\hat{\mathbf{R}}_{i+n} = \sum_{l=1}^{L}\hat{\sigma}_l^2\bar{\mathbf{a}}_l\bar{\mathbf{a}}_l^H + \hat{\mathbf{R}}_n \quad (20)$$

Though the proposed method is based on the quasi-orthogonality by a ULA, it is also suitable for other complex arrays as long as the CBF and SMI methods are valid.

B. SV Estimation

Similar to the SOI SV estimation in [15], the reconstructed covariance matrix of the SOI can be obtained as

$$\hat{\mathbf{R}}_s = \sum_{r=1}^{R}\frac{1}{\bar{\mathbf{a}}^H(\theta_r)\hat{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(\theta_r)}\bar{\mathbf{a}}(\theta_r)\bar{\mathbf{a}}^H(\theta_r) \quad (21)$$

where $\bar{\mathbf{a}}(\theta_r)$ is the nominal SV associated with the direction $\theta_r \in \Theta_s$, $r = 1,2,\ldots,R$. Apply eigenvalue decomposition on $\hat{\mathbf{R}}_s$ and it can be seen that the eigenvalues work like the powers of the SOI and the eigenvectors work like the SVs. Since the largest eigenvalue λ_1 of $\hat{\mathbf{R}}_s$ contains the most information of the reconstructed SOI covariance matrix, the estimated SV of the SOI can be updated by its principal eigenvector $\mathbf{v}_1$, which is described as

$$\hat{\mathbf{a}}_0 = \sqrt{M}\mathbf{v}_1 \quad (22)$$

Based on the INCM and the updated SV, the weight vector of the proposed method can be described as

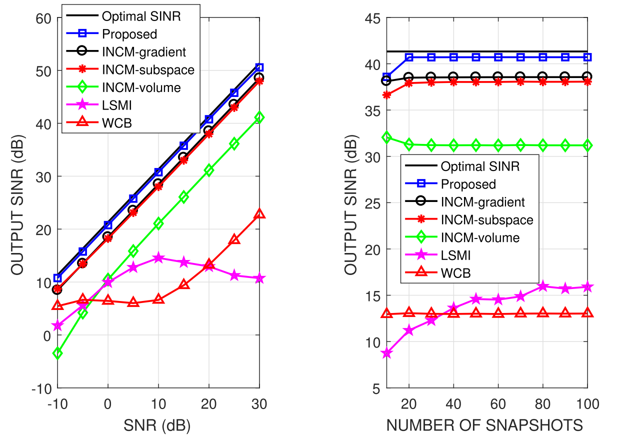
$$\mathbf{w}_{\text{proposed}} = \frac{\hat{\mathbf{R}}_{i+n}^{-1}\hat{\mathbf{a}}_0}{\hat{\mathbf{a}}_0^H\hat{\mathbf{R}}_{i+n}^{-1}\hat{\mathbf{a}}_0} \quad (23)$$

It can be seen that the main computation cost of the proposed method lies in the eigenvalue decomposition with $O(M^3)$, the inverse operation with $O(M^3)$ and confirming the initial DOAs of the sources, which is not involved in any convex optimization problem. Let J stand for the searching times and the computational complexity of confirming DOAs is $O(JM^2)$, where the computational complexity of the SV estimation is contained. Generally, to have a precise estimation of the sources, J is bigger than M. Thus, the computational complexity of the proposed method is $O(JM^2)$.

IV. Simulation Results

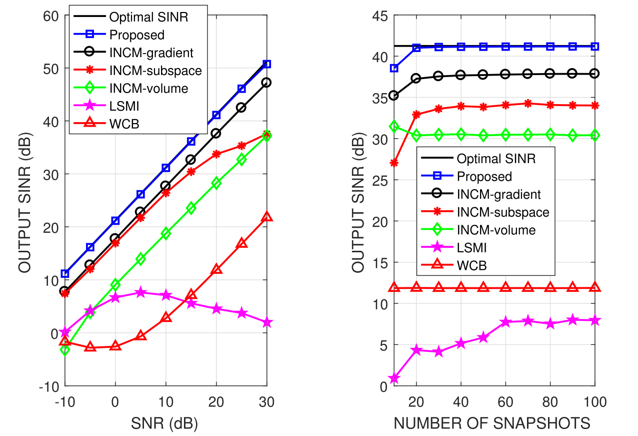
Some simulations are performed in this section. Consider a ULA composed of $M = 10$ sensors spaced half a wavelength. Three sources are impinging onto the array from directions $\theta_0 = -5°, \theta_1 = -25°$, and $\theta_2 = 25°$, which stand for one signal and two interferences. Besides, the directions are estimated as $\bar{\theta}_0 = -8°$, $\bar{\theta}_1 = -28°$, and $\bar{\theta}_2 = 23°$ with $3°$ errors. The signal sector and interference sector is $\Theta_s = [\bar{\theta}_0 - 8°, \bar{\theta}_0 + 8°]$ and $\Theta_i = \Theta_{i1}\bigcup\Theta_{i2} = [\bar{\theta}_1 - 8°, \bar{\theta}_1 + 8°]\bigcup[\bar{\theta}_2 - 8°, \bar{\theta}_2 + 8°]$. The number of snapshots is set as 30 when considering the performance versus SNR and the interference-to-noise ratio (INR) is 20 dB in all the simulations. At the same time, the sectors are uniformly sampled with an interval of $0.1°$. The colored noise $\mathbf{x}_n(k)$ is generated by a second-order AutoRegressive (AR) model $\mathbf{x}_n(k) = \mathbf{w}(k) - 0.6\mathbf{x}_n(k-1) - 0.8\mathbf{x}_n(k-2)$, where $\mathbf{w}(k)$ is the white noise with zero mean and unit variance. 200 Monte-Carlo simulations are performed for each case.

To highlight the performance of the proposed method, some compared methods are chosen. They are including the covariance matrix reconstruction based on gradient searching (INCM-gradient) [17], the INCM reconstruction based on subspace (INCM-subspace) [15], the INCM based on volume integration (INCM-volume) [11], the loading sample matrix inverse



(a) (b)

Fig. 1. Output SINR versus (a) input SNR and (b) number of snapshots in the case of look direction error.



(a) (b)

Fig. 2. Output SINR versus (a) input SNR and (b) number of snapshots in the case of incoherent local scattering error.

(LSMI) [6] and the worst-case-based beamformer (WCB) [8]. Though the methods are based on the assumption of white noise, they still work under colored noise. The searching range is set as 10° in [17] and the threshold ρ is 0.9 in [15]. Besides, $\epsilon = \sqrt{0.1}$ is employed in [11] and the DL factor is set as ten times the noise power in [6]. The degree interval is 0.1° in the simulations. The CVX toolbox is used to solve the optimization problem in other methods [19].

Example 1: *Mismatch due to look direction error:* In this example, assume that the random look direction error of the SOI and interference is uniformly distributed in $[-4^\circ, 4^\circ]$ for each simulation, which means that the direction of the SOI is subject to a uniform distribution in $[-9^\circ, -1^\circ]$ and the directions of two interferences are subject to uniform distributions in $[-29^\circ, -21^\circ]$ and $[21^\circ, 29^\circ]$, respectively.

As it depicts in Fig. 1(a), the performance of the proposed method and the compared INCM methods improves with the increase of input SNR because the INCM can be estimated more accurately at high SNRs. The proposed method shows the best performance among the tested methods and almost reaches the optimal SINR, which benefits from the estimation of CNCM and more accurate interference powers. The performance of the LSMI and WCB method degrades at high SNRs because the SOI appears in the training data but it plays a weak impact at low SNRs. Fig. 1(b) illustrates the output SINR versus the number of snapshots. All the tested beamformers show a stable performance because more information can be obtained when the number of snapshots increases. Besides, it is obvious that the proposed method still outperforms other methods.

Example 2: *Mismatch due to incoherent local scattering error:* The incoherent scattered source is mainly derived from multi-path scattering effects caused by the local scatterers. Assume that the desired signal has a time-varying SV and it can be modeled as $\tilde{\mathbf{x}}_s(k) = s_0(k)\mathbf{a}_0(\theta_0) + \sum_{p=1}^{4} s_p(k)\bar{\mathbf{a}}(\theta'_p)$ [9], [11], where the real SV of the SOI is time-varying because it can be regarded as a combination of $\mathbf{a}_0$ and $s_p(k)\bar{\mathbf{a}}(\theta'_p)$. $\mathbf{a}_0$ represents the direct path corresponding to the direction θ_0 while $\bar{\mathbf{a}}(\theta'_p)$ is the incoherent local scattering SV, where θ'_p, $p = 1, 2, 3, 4$ are subject to Gaussian distribution with $\theta_0 = -5^\circ$ mean and 2° standard deviation. $s_0(k)$ and $s_p(k)$, $p = 1, 2, 3, 4$ are independent and identically subject to the complex Gaussian distribution with zero mean and unit variance.

As input SNR increases in Fig. 2(a), the proposed method shows an outstanding performance improvement versus the compared methods at low and high SNRs for its more INCM reconstruction. The performance of the INCM-subspace method and WCB method slows down at high SNRs, which is because the actual SV is magnified owing to the incoherent local scattering error and leads to the slow growth of the output SINR in Fig. 2(a) but the INCM-volume has a steady improvement because the integral region is an annulus that contains more error information. Similarly, the performance of the LSMI method still degrades at high SNRs. Meanwhile, it is distinct in Fig. 2(b) that all the tested beamformers have a stable performance when the snapshots change and the proposed method has a close output SINR with the optimal SINR when the number of snapshots increases.

V. Conclusion

In this letter, a robust adaptive beamforming method based on covariance matrix reconstruction under colored noise has been proposed to reduce the impact on the performance of the Capon beamformer when the SOI is present in the training data. Through the computation of the cross Capon power in the noise sector, the CNCM has been reconstructed and the interference power has been estimated based on the projection of the received data. The SV of the SOI has been replaced by the principal eigenvector of the reconstructed SOI covariance matrix. Simulation results have shown the effectiveness and robustness of the proposed method.

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