Exercise 2: Bayesian modeling of visual perception

Objective: The goal of this TD is to learn to make Bayesian models of visual perception, from mathematics to computational predictions.

Components: There is a probability theory component, where you have to derive MAP estimates and their distribution; a modeling component, where mostly you have to write and describe the variables.

Submission: You should submit this in the moodle until April 06. The file should be called MI210_E2_GroupX.pdf, where X is your group number. This exercise weights 2.5 values in your final grade.

Part I: Bayes Theorem

1. If A is the event "a person is old," and B is the event "a person suffers from Alzheimer's disease," is p(A|B) less than, equal to, or greater than p(B|A)? Why?

Part II: Bayesian Brain

2. Consider the visual localization task, defined in the class. Subjects are facing a projection screen that displays a horizontal line stretching across the width of the screen. Located behind the screen, at the same elevation as the line, is a densely spaced array of very many tiny flashlights. A flash will originate from one of these flashlights. The setup is described in Figure 1. The subject task is to report with a cursor the location from which you perceived the flash to emanate. This task is repeated many times; each repetition is called a trial. In the class, we have built the Bayesian model of this task assuming Gaussian priors and likelihoods. If the prior is not Gaussian, the posterior might not be Gaussian either. Can you construct a prior that would give rise to a posterior with two local maxima? Show mathematically that your claim is true.

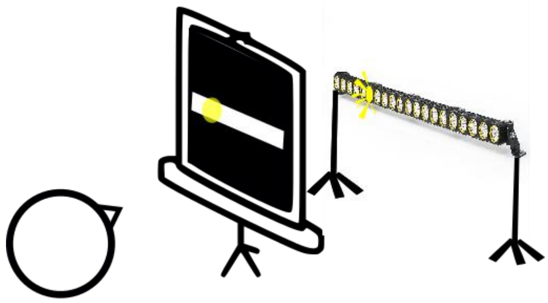

Figure 1: Experimental set up of question 2.

3. A research article entitled “The Easter bunny in October: Is it disguised as a duck?” explained that “Very little is known about the looks of the Easter bunny on his non-working days.” To investigate, the authors showed an “ambiguous drawing of a duck/rabbit to 265 subjects on Easter Sunday and to 276 different subjects on a Sunday in October of the same year.” The authors report that “Whereas on Easter the drawing was significantly more often recognized as a bunny, in October it was considered a bird by most subjects.” The drawing shown by the authors in their study was similar to the following in Figure 2. Provide a Bayesian perceptual explanation for the authors’ results by first defining all the relevant variables, then the generative process, and finally the graphical model of the problem.

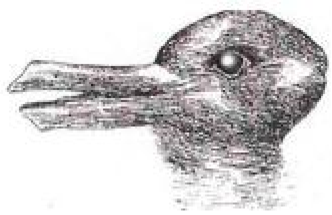

Figure 2: Duck/Rabbit illusion

4. Recall the problem of integration of two Gaussian cues (auditory and visual) described in the class (section 2, slide 42). Starting from the generative model described in the class, derive mathematically the µcombined and σcombined estimators for the posterior distribution of the integrated cues.

5. In this problem, we examine suboptimal estimation in the context of cue combination. Suppose an observer estimates a stimulus s from two conditionally independent, Gaussian-distributed measurements, xH and xV, centered in the stimulus s.
   a) Now suppose the observer uses an estimator of the form $\hat{s}$ = w xH+ (1 − w) xV. Show that this estimate is unbiased (just like the MAP estimate); this means that the mean (expected value) of the estimate is equal to s.
   b) What is the variance of this estimate as a function of w? Plot this function. At which value of w is it minimal, and does this value make sense? State your conclusion in words.
   c) Which of the conclusions in (a) and (b) break down when we consider estimates that are general linear combinations of measurements, $\hat{s}$ = wH xH + wV xV ? Explain.

6. In Fig 3, is shown a simplified version of the Ponzo illusion, described in section 2, slide 52. The goal of this question is to analyze this illusion in a Bayesian way. The key is to interpret the scene as truly three-dimensional as if you are looking down on a road with two horizontal lines drawn on it. The relevant variables are the true length of the upper horizontal line, its length on the retina, its vertical position, the distance it is away, the true viewing angle with respect to the ground plane, and the context (the angle that each of the two tilted lines makes with the vertical). Give these variables names and put them into a graphical model..

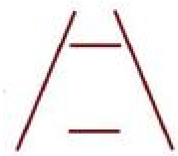

Fig. 2: Ponzo illusion

Part 3 Life long learning

7. Recall the Bayesian life long learning (recursive) spoken in the class for the random dot motion (section 3, slide 86). Recall that, in this model, the decision is made when the logarithm of the posterior ratio reaches one of the bounds. Show that the decision is also recursive.