Retinal Ganglion Cells Modeling

Daniela Pamplona

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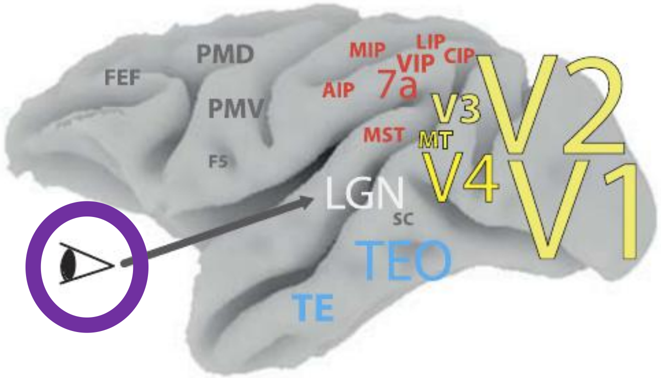
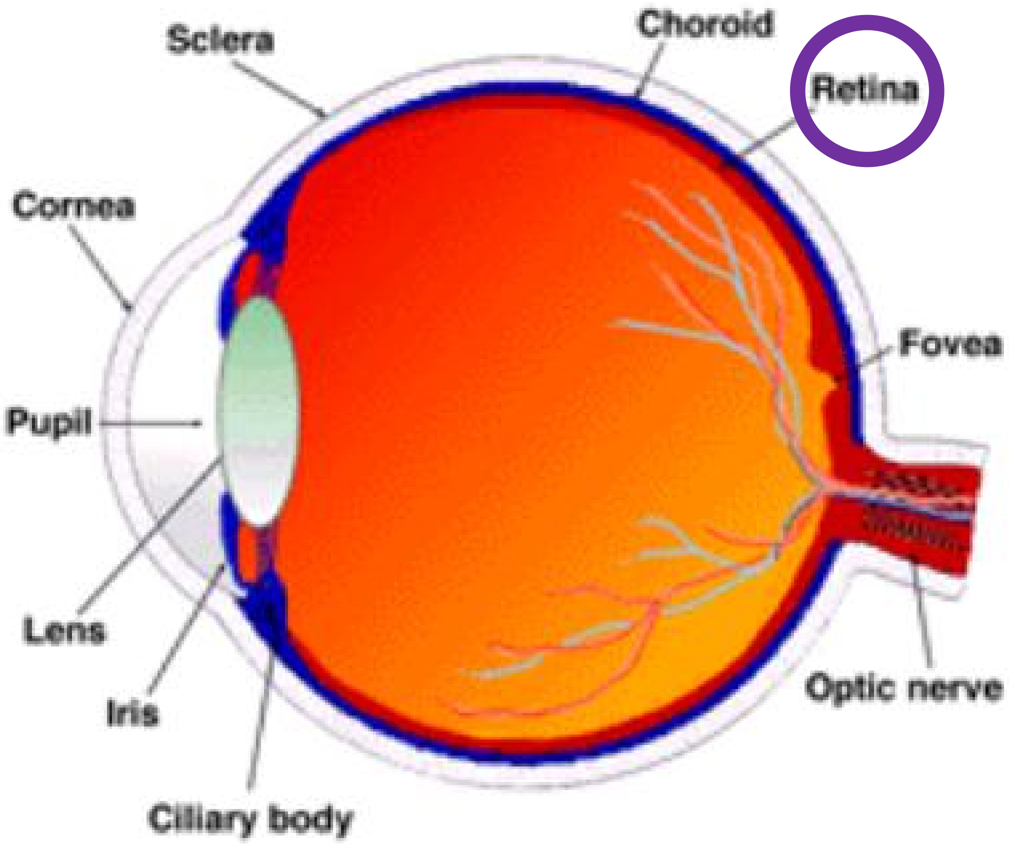
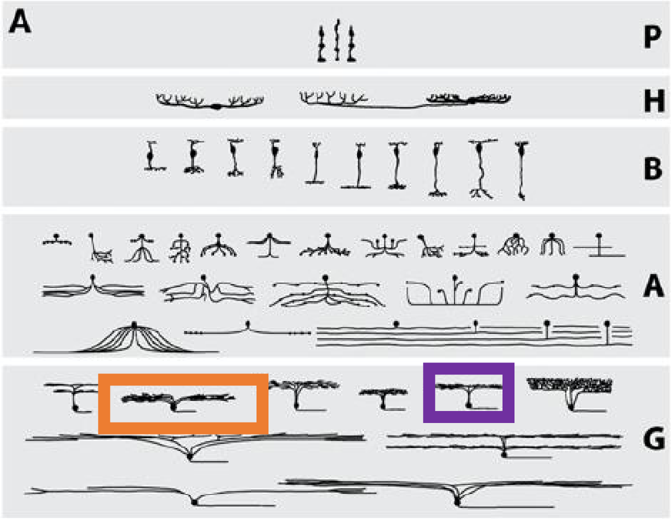
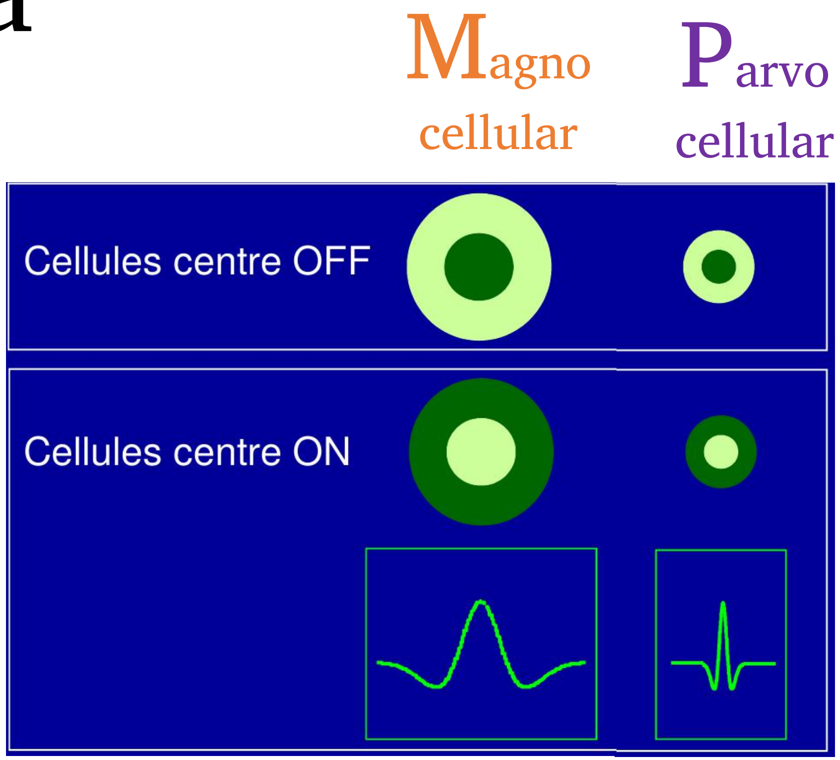
Where is the retina?





Network in the retina





Name	Prop	Dendritic Size	Conduction	Receptive Field Size	Receptive Field Shape	Color Sensitivity	Contrast Sensitivity
Midget	80%	Small	Slow	Small	Center-Surround	Strong	Weak
Parasol	10%	Large	Fast	Large	Center-Surround	Weak	Strong
Bistratified	5%	Very Small	Moderate	Very Large	Center	?	Medium

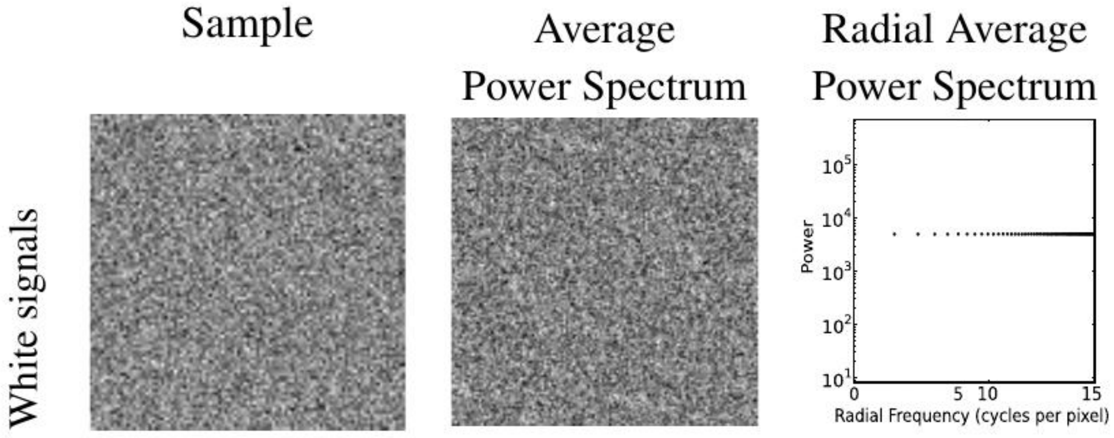
Gollish, eye's smarter than scientists believe

Flat retina: many functions on the base level

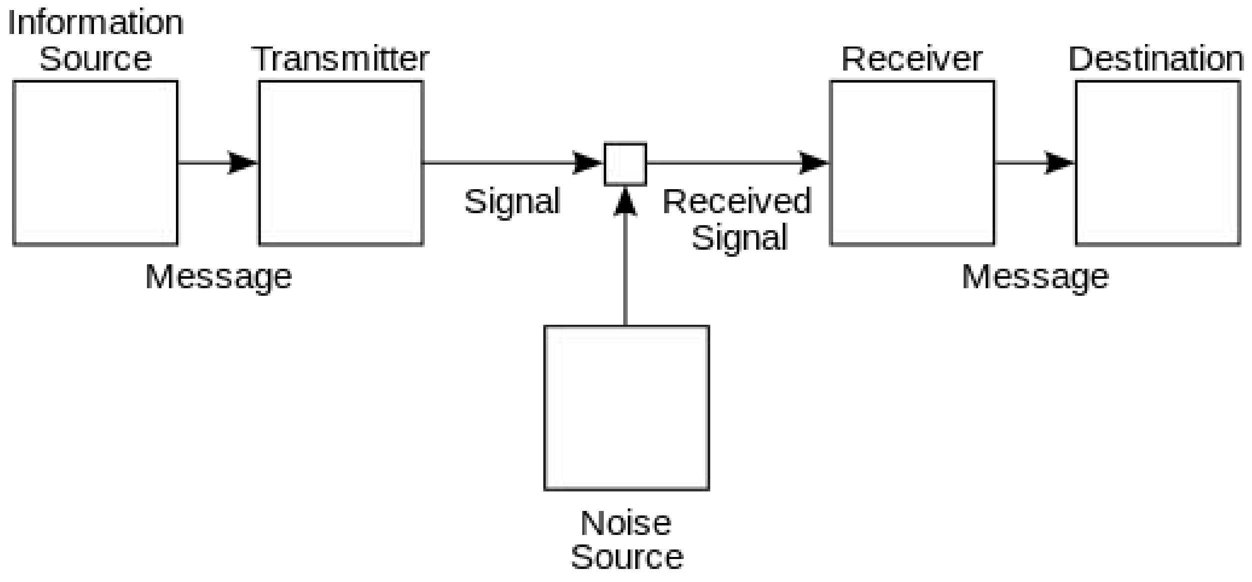
- Light detection
- Whitening
- Contrast and pattern adaptation
- Texture motion detection
- Object motion detection
- Approaching motion detection
- Motion extrapolation
- Omitted stimulus response
- …

White Signals

- White noise: random signal with flat average power spectrum
- White noise: autocorrelation is a diagonal matrix
- White noise: no second order redundancies



Why Whitening?



Example: Visual System

Information Source: Environment

Transmitter: Eye

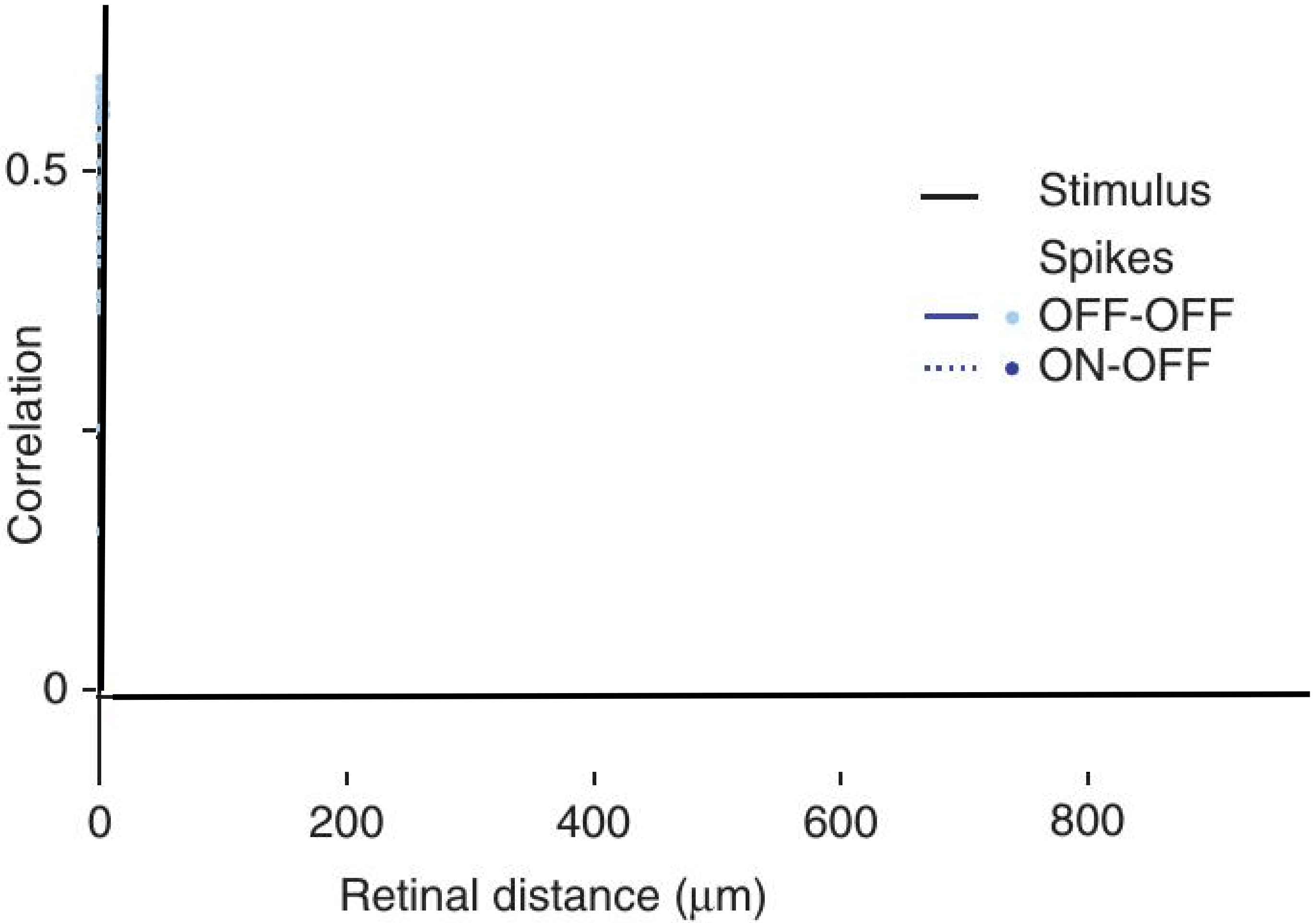
Channel: Early visual system

Noise: Unknown

Receiver: Higher areas (MT,TE,MIP,..)

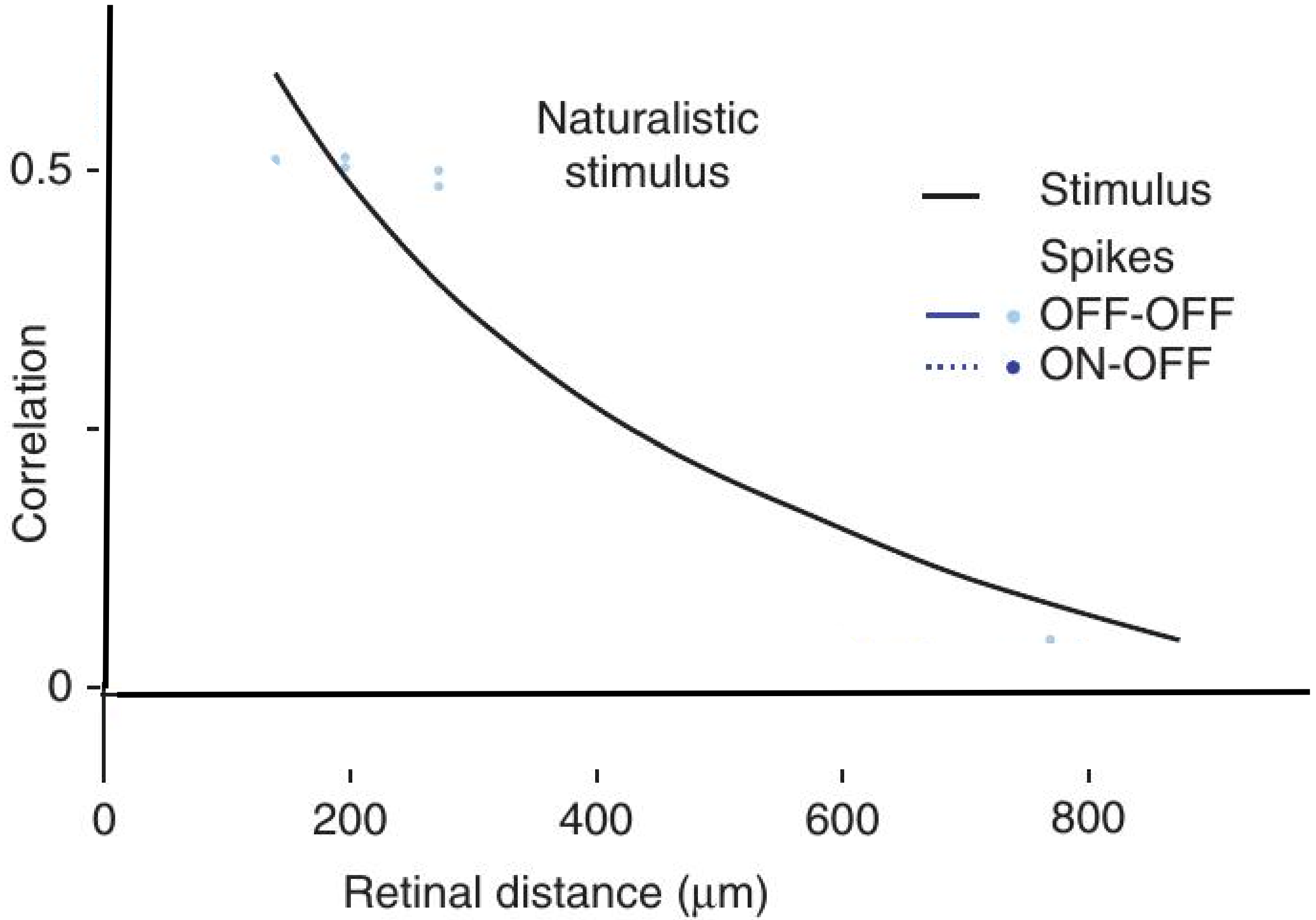
Destination: Other brain areas (PMC,..)

Stimulus Decorrelation by the Retina



Pitkow and Meister (2012)

Stimulus Decorrelation by the Retina



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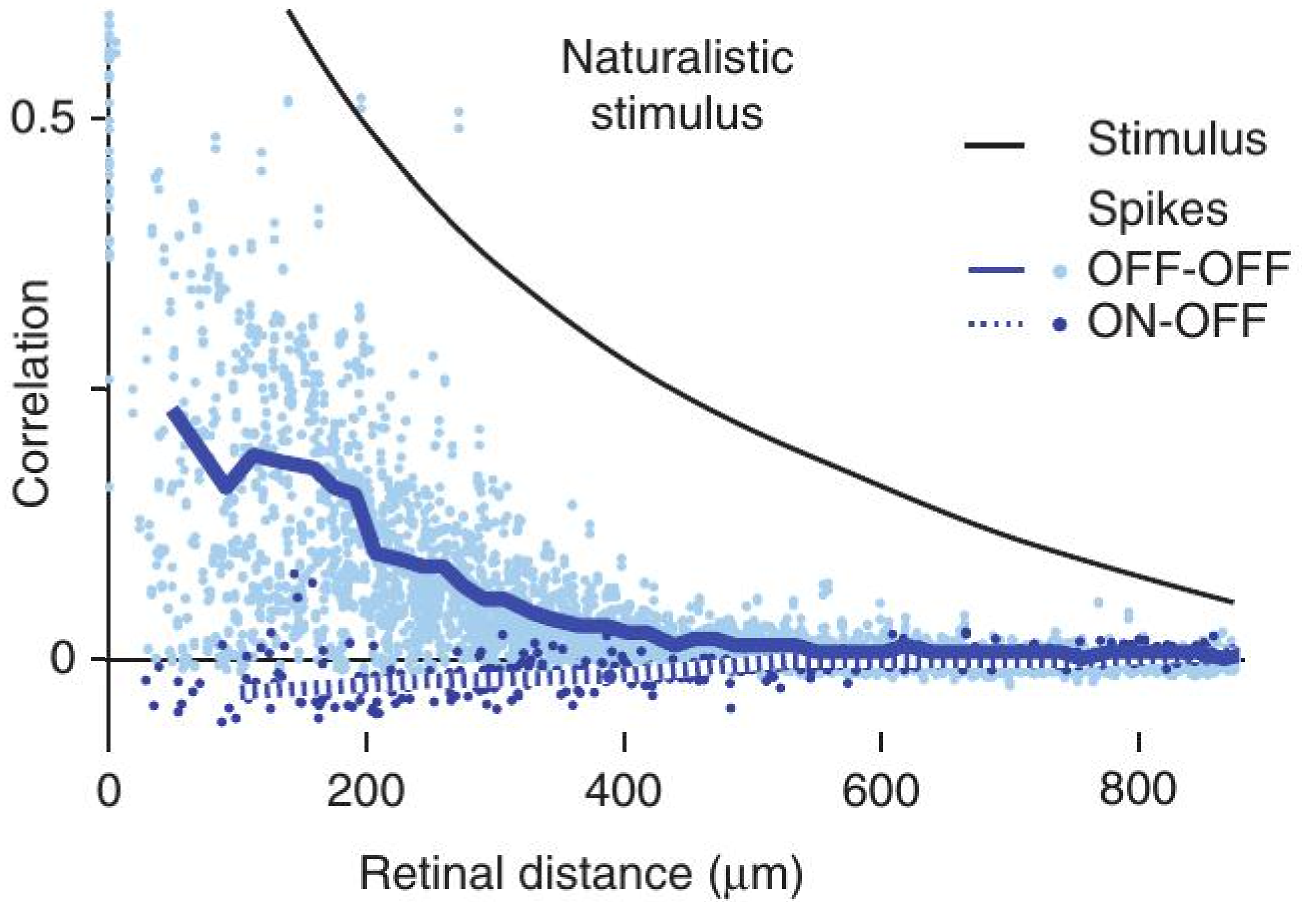

Pitkow and Meister (2012)

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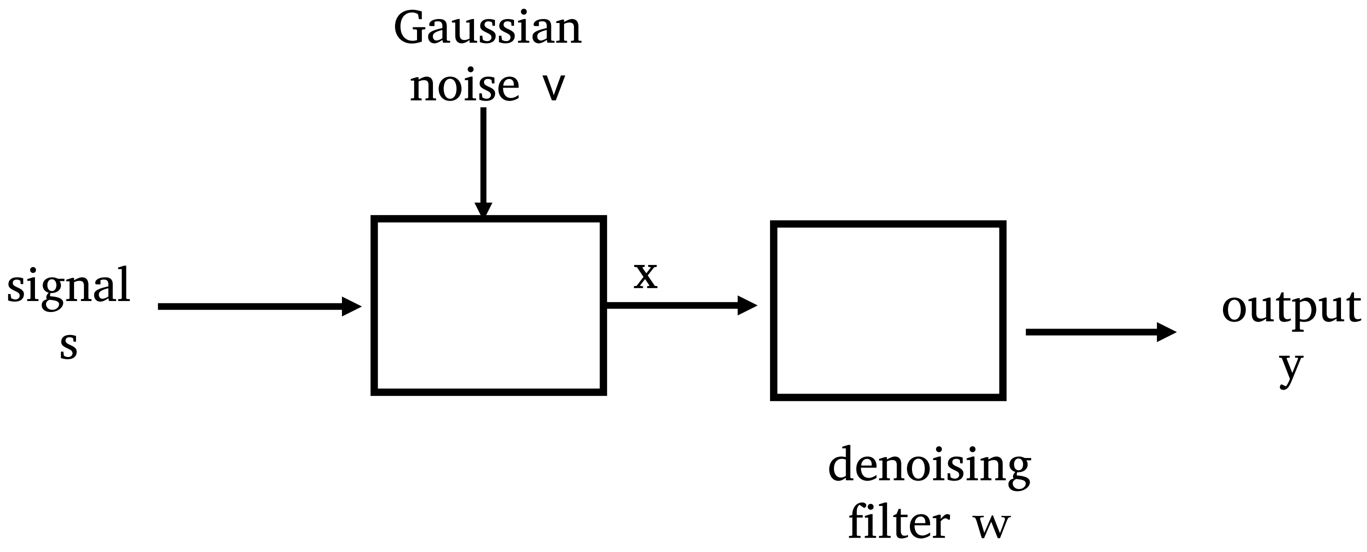
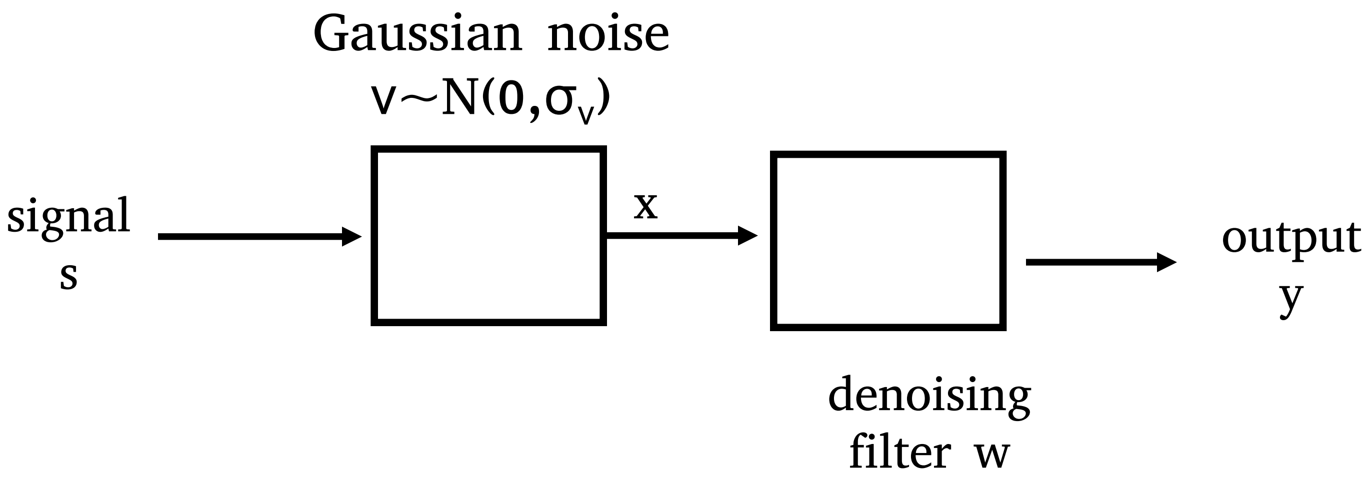
Can we build a denoising filter?



Goal:

$$\omega = \underset{\omega}{\operatorname{argmin}} : ||\, \mathbb{E}[\mathrm{PS}(y)] - \mathbb{E}[\mathrm{PS}(s)]||^2$$

Can we build a denoising filter?



Goal:

$$\omega = \underset{\omega}{\operatorname{argmin}} : || \, \mathbb{E}[\mathrm{PS}(y)] - \mathbb{E}[\mathrm{PS}(s)] \, ||^2$$

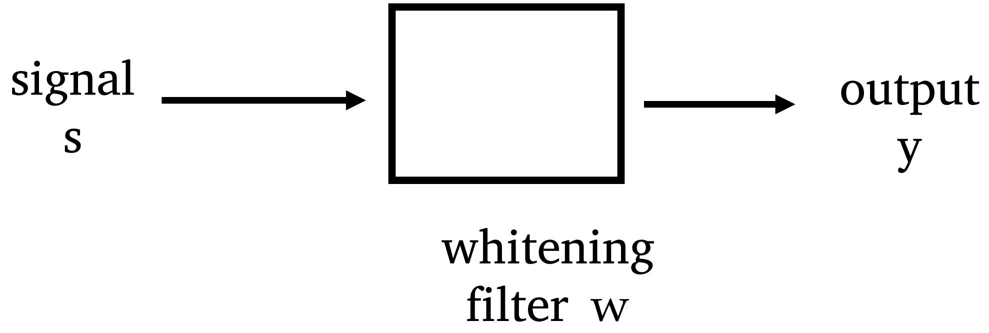
Solution: [Wiener filter]

$$w = \mathscr{F}^{-1} \left\{ \left\lfloor \frac{\mathbb{E}[\mathrm{PS}(x)] - \sigma_\nu^2 M}{\mathbb{E}[\mathrm{PS}(x)]} \right\rfloor \right\}$$

Total number of Pixels

$\lfloor x \rfloor = x$ if $x > 0$ and $\lfloor x \rfloor = 0$ otherwise.

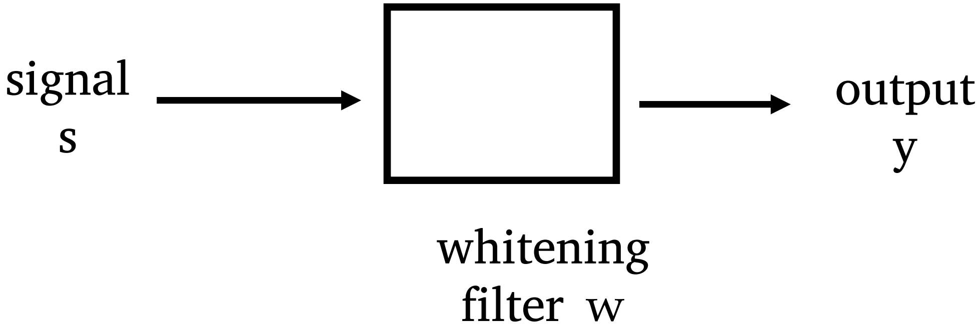
Can we build a whitening filter?



Goal:

$$\omega = \underset{\omega}{\operatorname{argmin}} : ||\,\mathbb{E}[\mathrm{PS}(y)] - \mathbf{K}||^2$$

Can we build a whitening filter



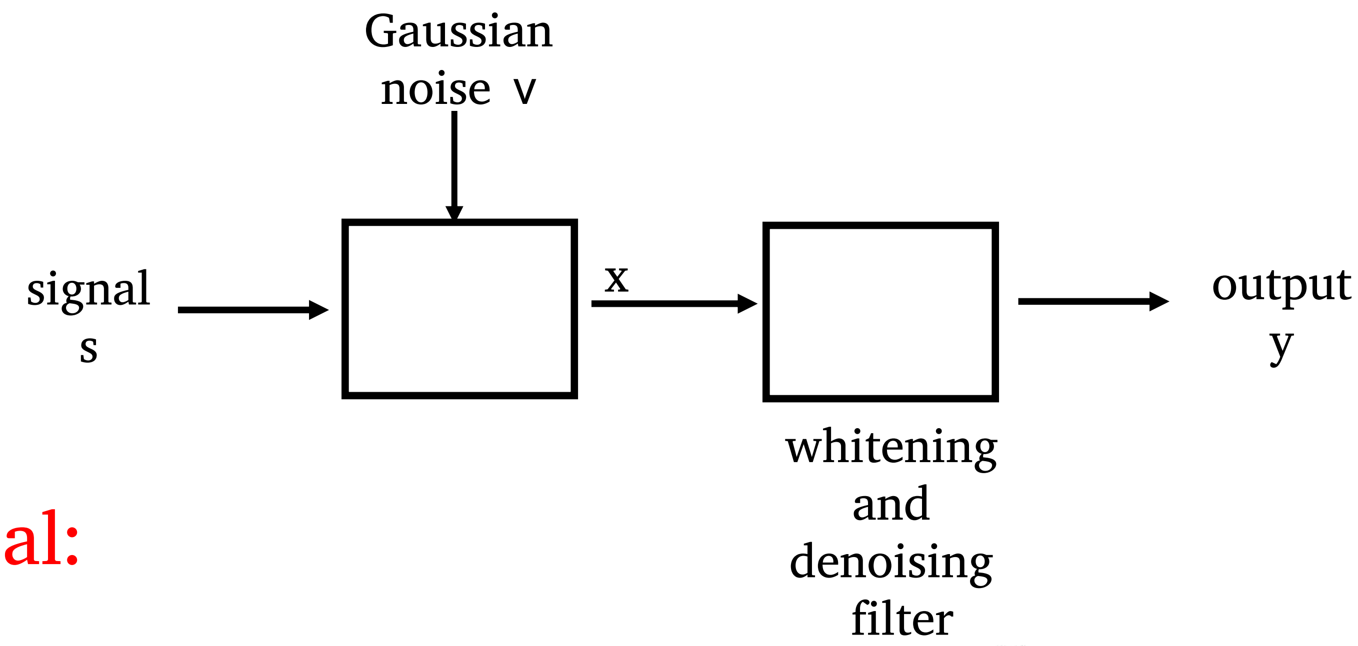
Goal:

$$\omega = \underset{\omega}{\mathrm{argmin}} : ||\,\mathbb{E}[\mathrm{PS}(y)] - \mathbf{K}||^2$$

Solution:

$$w = \mathscr{F}^{-1}\left\{ \frac{1}{\sqrt{\mathbb{E}[\mathrm{PS}(s)]}} \right\}$$

Can we build a denoising whitening filter?



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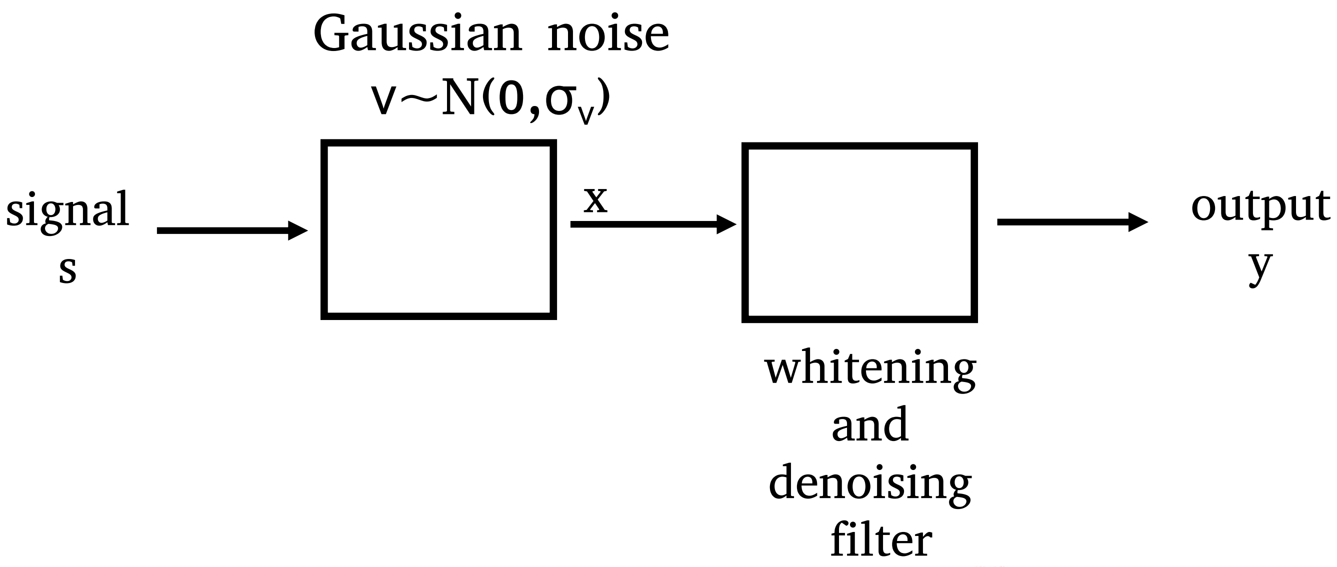


Goal:

$$\omega = \underset{\omega}{\operatorname{argmin}} : ||\mathbb{E}[PS(y)] - \mathbf{K}||^2$$

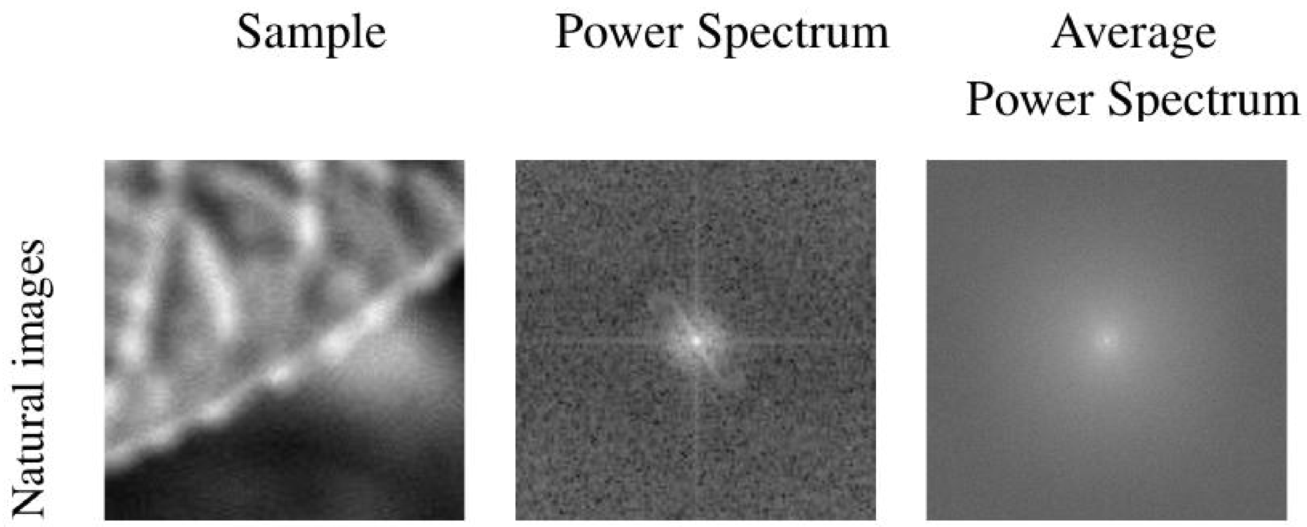
Total number of Pixels

Solution:

$$w = \mathscr{F}^{-1}\left\{ \frac{1}{\sqrt{\mathbb{E}[PS(x)]}} \left\lfloor \frac{\mathbb{E}[PS(x)] - \sigma_\nu^2 M}{\mathbb{E}[PS(x)]} \right\rfloor \right\}$$

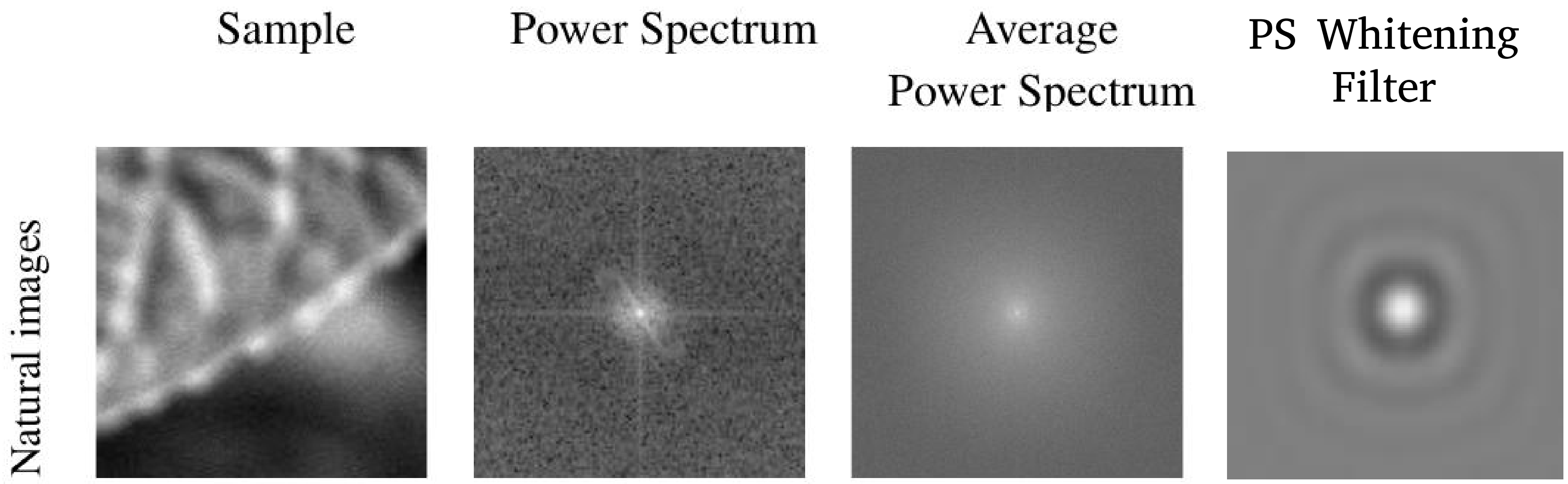
Dong and Atick, 1994

Power spectrum of natural images



Dong and Atick, 1994

Whitening filters of natural images



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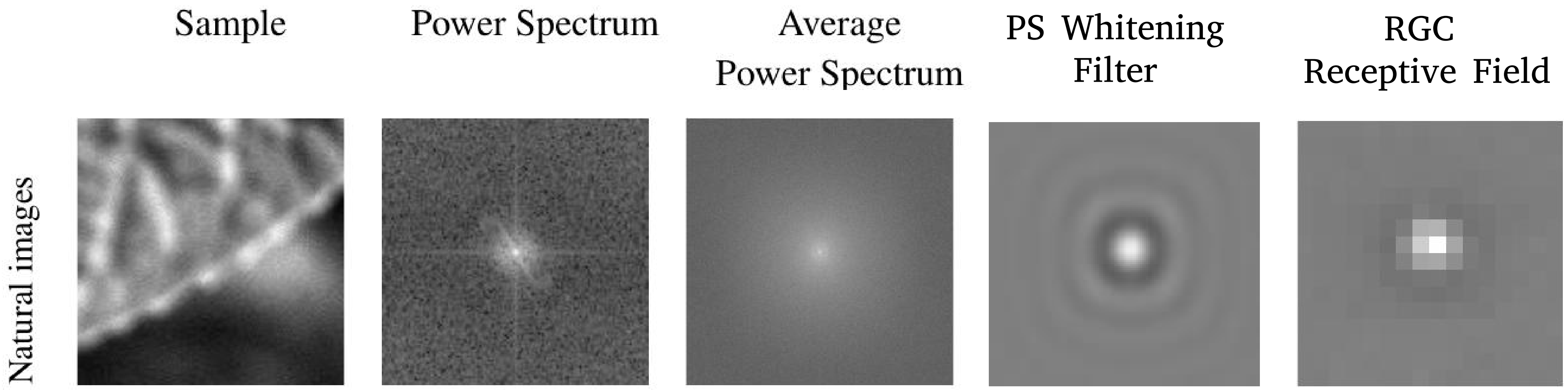

RGCs RFs have shape like whitening filters of natural images!

Contents

Matrix eigenvalues and eigenvectors

- A is a square matrix NxN
- λ is an eigenvalue of A and v the corresponding eigenvector if

$$Av = \lambda v$$

Petersen and Pedersen 2012

Matrix eigenvalues and eigenvectors

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- λ is an eigenvalue of A and v the corresponding eigenvector if

$$Av = \lambda v$$

How to find the eigenvalues/eigenvectors of A?

Algorithm Eigenvalues and eigenvectors

solve $\det(A - \lambda I) = 0$ to get the eigenvalues
for each λ_i **do**
  solve $(A - \lambda_i I)\nu_i = 0$ to get the corresponding eigenvector
end for
return λ, ν

Petersen and Pedersen 2012

Eigenvalues decomposition

- Q $\equiv$ Matrix of the eigenvectors of A
- Λ $\equiv$ Diagonal matrix of eigenvalues
- If Q is linearly independent, A can be decomposed:

$$A = Q\Lambda Q^{-1}$$

Eigenvalues decomposition

- $Q \equiv$ Matrix of the eigenvectors of A
- $\Lambda \equiv$ Diagonal matrix of eigenvalues
- If Q is linearly independent, A can be decomposed:

$$A = Q\Lambda Q^{-1}$$

- If A is symmetric and real, A can always be decomposed and Q is orthogonal and unitary:

$$A = Q\Lambda Q^{T}$$

Petersen and Pedersen 2012

Extracting components from data

Linearity is simplicity

Model

$$x = x(t)$$

Energy function

$$E = \sum_t \Big\{ \underset{\text{input}}{x} - \sum_i \underset{\text{code}}{\alpha_i(x)} \underset{\text{decoder}}{W} \Big\}^2$$

Goal

$$A, W = \operatorname{argmin} \mathbb{E}\left[E\right]$$

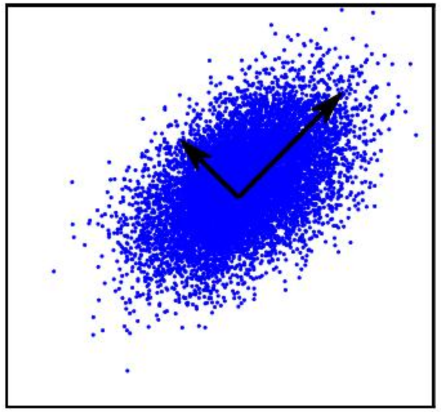
Principal Components Analysis (PCA)

$X_{k,n}$: k samples of a random variable of size n

Goal: Find basis $A = \{ f_1 , f_2, \ldots, f_n\}$ such:

- f_1 direction maximizes the data variance
- f_2 direction maximizes the data variance and it is orthogonal to f_1
- f_3 direction maximizes the data variance and and it is orthogonal to f_1 and f_2
- …

X



Jolliffe, 2002

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- f_1 direction maximizes the data variance
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- ...

Formally:

$$f_i = \operatorname{argmax} V[Xf_i]$$
$$\text{subject to } f_i^T f_i = 1 \text{ and } \forall_{j<i} f_i^T f_j = 0$$

Jolliffe, 2002

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- ...

Formally:

$$f_i = \operatorname{argmax} V[Xf_i]$$

subject to $f_i^T f_i = 1$ and $\forall_{j<i} f_i^T f_j = 0$

Solution:

Formalize in terms of a Lagrangian $\mathcal{L}(f_i, \lambda) = f_i^T C f_i + \lambda(1 - f_i^T f_i)$

The Lagrangian is maximized when $C f_i = \lambda f_i$

Oh! But this is the eigenvalues/eigenvectors problem! The solution is A = Q

Jolliffe, 2002

PCA algorithm

Algorithm Pre-process

if $\dim(X) > 2$ **then**
    for all $x_{i,:} \in X$ **do**
        vectorize $x_{i,:}$
    end for
end if
for all $x_{:,j} \in X$ **do**
    $x_{:,j} = x_{:,j} - \mathbb{E}[x_{:,j}]$
end for
return X

Algorithm PCA

get X
pre-process X
get eigenvalues
get eigenvectors
return eigenvectors

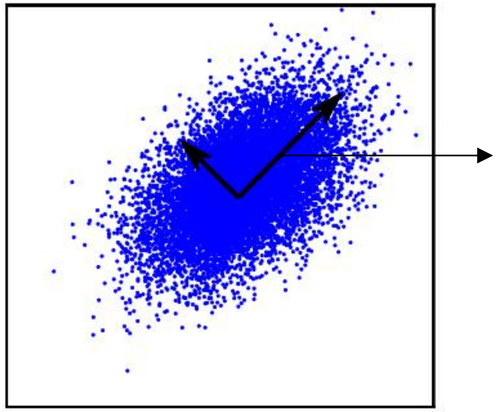
Principal Components Analysis (PCA)

$X_{k,n}$: k samples of a random variable of size n

Goal: Find basis $A = \{ f_1 , f_2 , ..., f_n\}$ such:

- f_1 direction maximizes the data variance
- f_2 direction maximizes the data variance and it is orthogonal to f_1
- ...

X



f_1:orientation given by v_1
length given by λ_1^2

Jolliffe, 2002

Principal Components Analysis (PCA)

$X_{k,n}$: k samples of a random variable of size n

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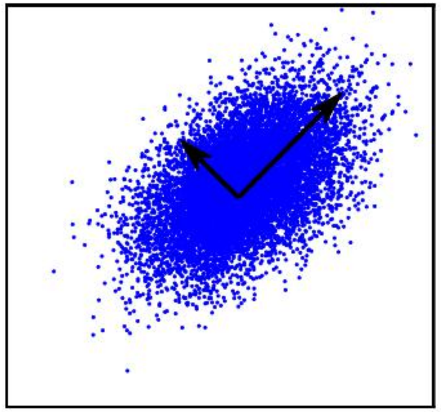
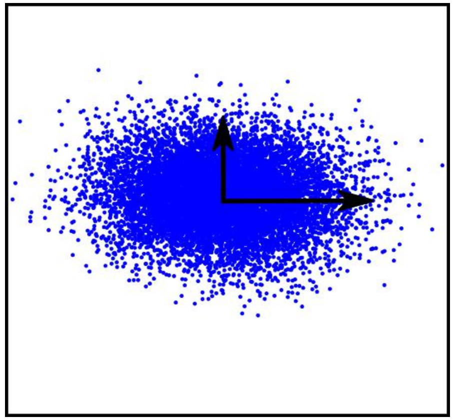
- f_2 direction maximizes the data variance and it is orthogonal to f_1
- …

X



XA



Jolliffe, 2002

PCA Whitening

$X_{k,n}$: k samples of a random variable of size n

Goal: Find basis W = { g_1 , g_2 , …, g_n} such the covariance of XW is the identity matrix

Formally:

$$W = \arg_W C[XW] = I$$

Jolliffe, 2002

PCA Whitening

$X_{k,n}$: k samples of a random variable of size n

Goal: Find basis $W = \{ g_1 , g_2 , \ldots, g_n\}$ such the covariance of XW is the identity matrix

Formally:

$$W = \arg_W C[XW] = I$$

Solution:

We know that $W = Q$, then the data is rotated.
if $W = Q\Lambda^{-1/2}$ then the data is rotated and scaled

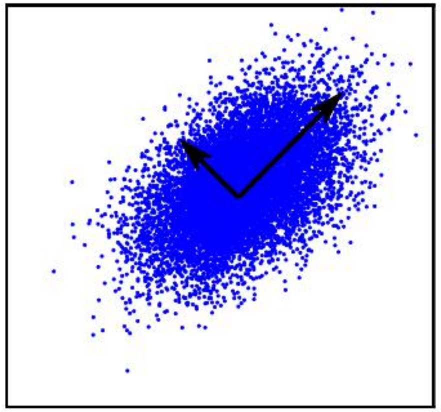
Jolliffe, 2002

PCA Whitening

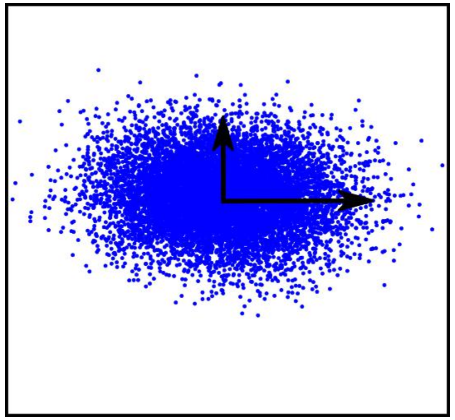
$X_{k,n}$: k samples of a random variable of size n

Goal: Find basis B = { g_1 , g_2 , …, g_n} such the covariance of XW is the identity matrix

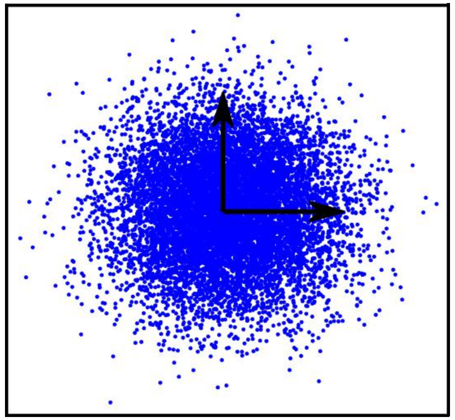
X



XA



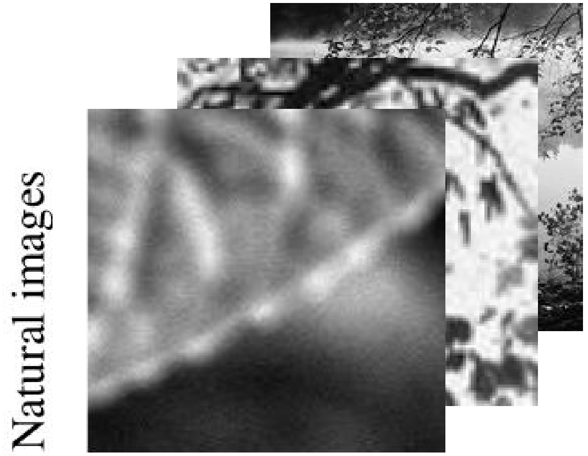
XB



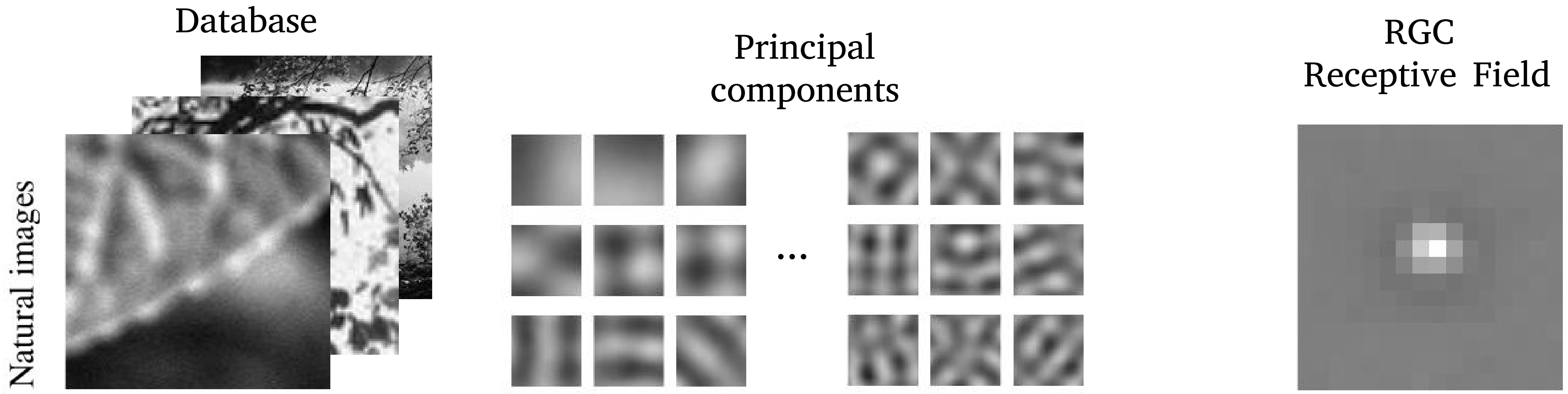
Jolliffe, 2002

Principal components whitening of natural images

Database



Principal components of natural images



Whitening methods

Power Spectrum

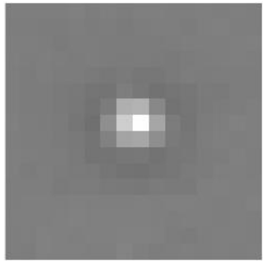
- Simple to model
- Under the shift invariance assumption
- Simple to implement
- Whiten on real time
- Filter is similar to RFs of RGC

Principal Components Analysis

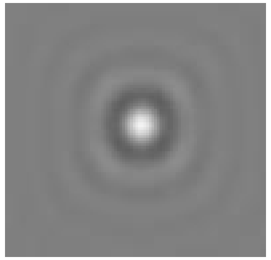
- Less simple to model
- No shift invariance assumption
- Simple to implement
- Whiten on real time (but slower)
- Components are not similar to RFs of RGC

Summary

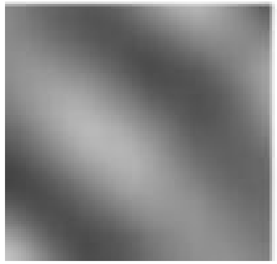
1. Ganglion cells receptive fields



2. Natural images power spectrum whitening



3. Natural images PCA whitening



Bibliography

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- Dong and Atick, 1995, Temporal decorrelation: a theory of lagged and non-lagged responses in the lateral geniculate nucleus
- Pitkow, Meister, 2012, Decorrelation and efficient coding by retinal ganglion cells
- Petersen and Pedersen, 2012, The Matrix Cookbook
- Jolliffe, 2002, Principal Component Analysis