Natural images statistics

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U2IS - ENSTA - IPParis

ecampus moodle: MI210 - Modèles neuro-computationnels de la vision (P4 - 2020-21)

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What is vision?

"Vision is the process of discovering from images what is present in the world, and where it is"

David Marr, 1982

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"Vision is the process of discovering from images what is present in the world, and where it is"

David Marr, 1982

... in order to solve tasks efficiently.

Daniela Pamplona, 2017

What are the requirements of a working visual system?

Constrains:

- To run real time
- To be robust to noise
- To adapt to lightness, contrast, etc
- To be energetically cheap
- To use limited memory
- To cope with imperfect imaging process

Functions:

- To select and extract relevant information from the environment to solve tasks
- To represent the environment for navigation, reasoning, memory,
- To learn new objects
- To predict location, motion, shape
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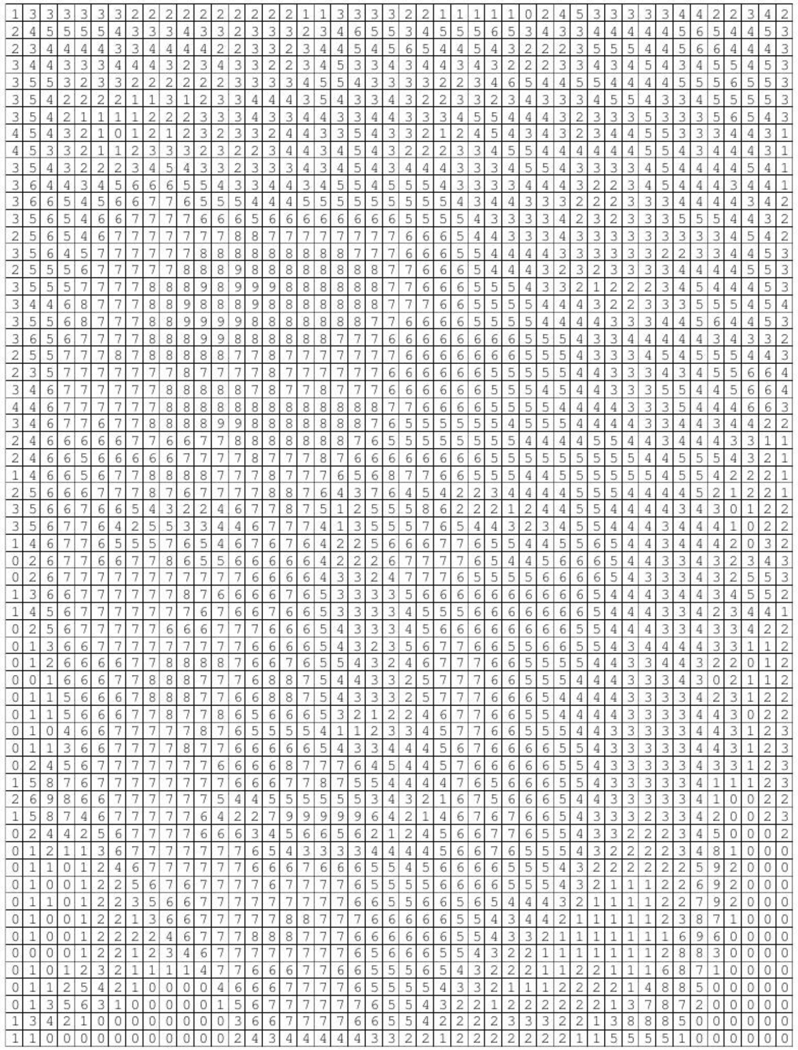
IMPOSSIBLE!

What is in here?



Hyvarinen et al, Natural Image Statistics, 2009

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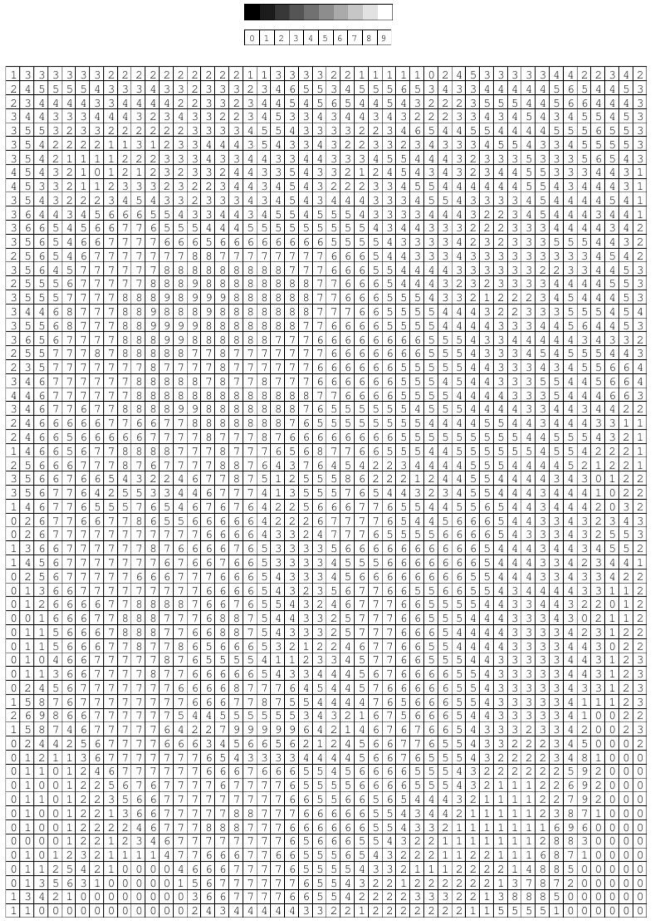

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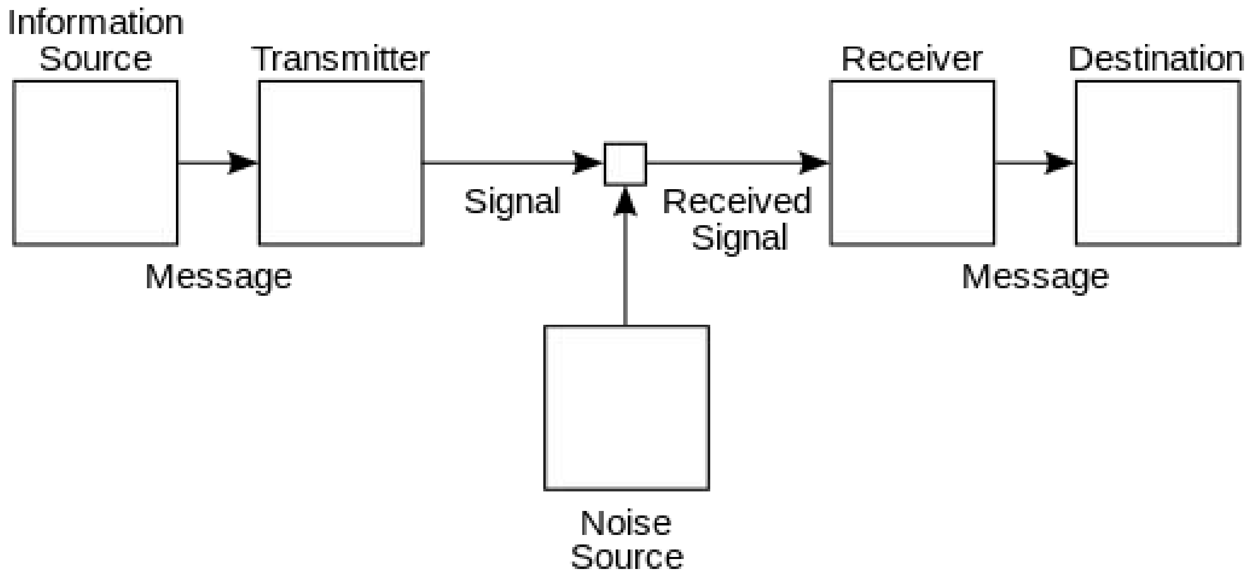
General Communication System



Definitions

Information Source: Produces messages

Transmitter: Transform/encode the message into a signal

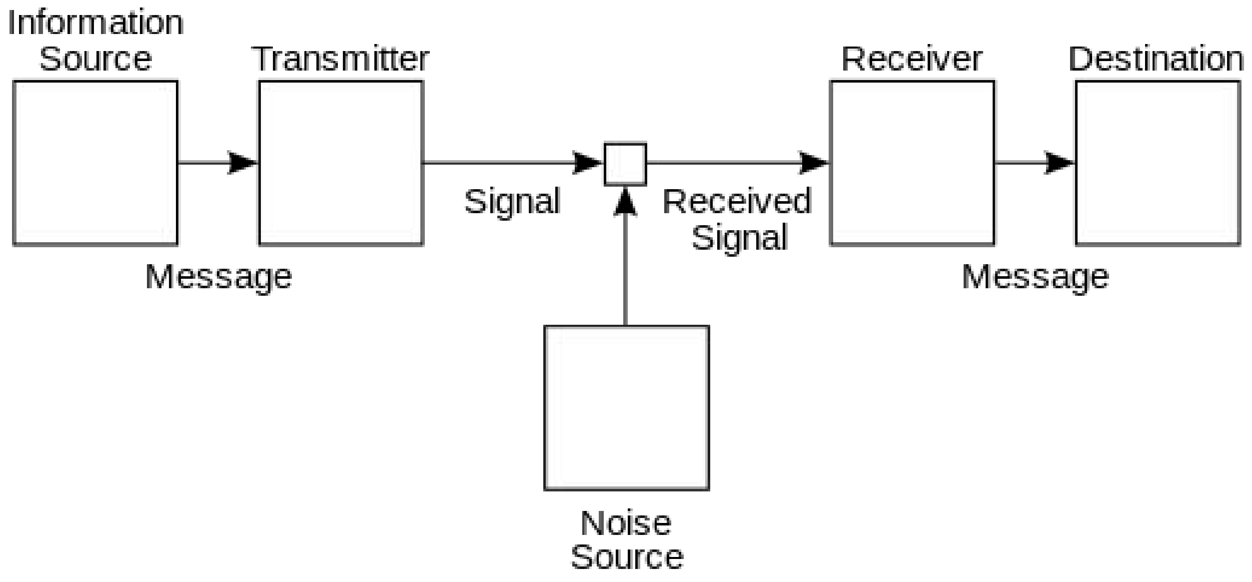
Channel:Medium to carry the message

Receiver: Transforms/decode the signal into a message

Destination: Entity that the message is intended

Shannon A mathematical theory of communication

General Communication System



Example: WhatsApp

Information Source: Alice

Transmitter: Mobile phone

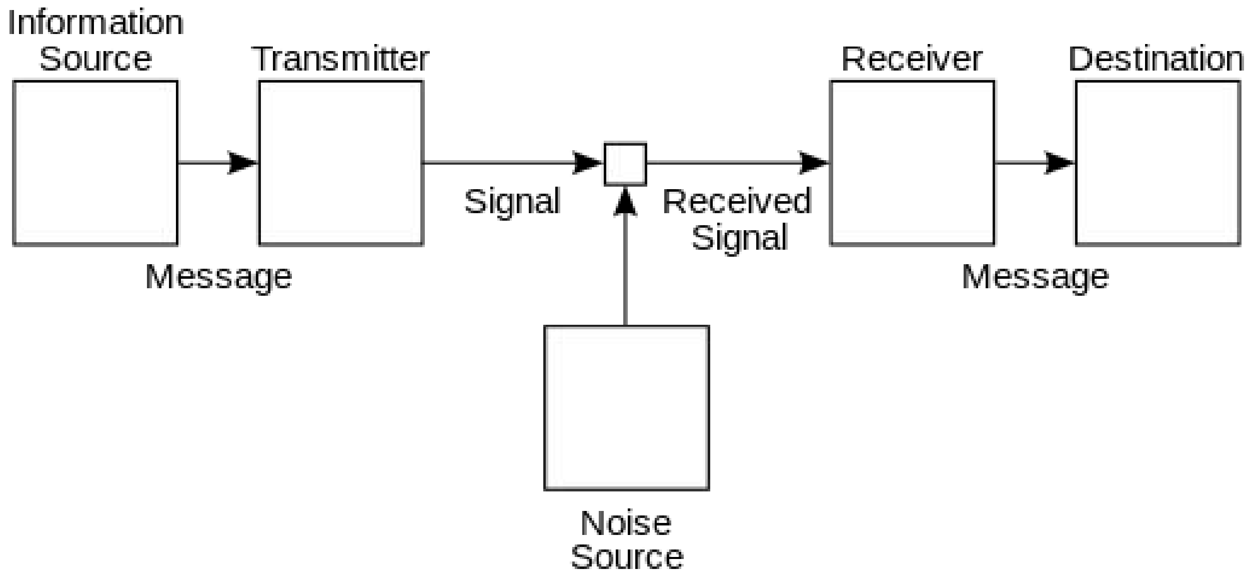
Channel:WiFi

Noise: SNR:30dB

Receiver: Mobile phone

Destination: Bob

General Communication System



Example: Visual System

Information Source: Environment

Transmitter: Eye

Channel: Early visual system

Noise: Unknown

Receiver: Higher areas (MT,TE,MIP,..)

Destination: Other brain areas (PMC,..)
(ultimatly the environment)

Self-Information of a message

Information content of an event, $h(E)$, with probability $P\{E\}$ must:

1. be a decreasing function of $P\{E\}$: more an event is likely, the less information its occurrence brings to us.

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$$h(E) = \log \frac{1}{P\{E\}} = -\log P\{E\}$$

Entropy

- Entropy: measure of randomness of a random variable (r. v.)
- Entropy is the expected value of self information

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- Entropy is the expected value of self information

X : discrete r.v. taking values in $\{x_1, x_2, \ldots, x_n\}$ with $p_i = P\{X = x_i\}$

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i$$

Maximum entropy of a discrete r.v.

- X: Bernoulli r. v.,

  $H_{max}(X) = 1$ with $p = 0.5$

- X : discrete r.v. taking values in $\{ x_1 , x_2 ,..., x_n \}$ with $p_i = 1/n$

  $H_{max}(X) = Log(N)$

Redundancy of a r.v.

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  $r = 1 - H(X)/H_{max}(X)$

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REDUCE THE REDUNDANCY <=> INCREASE THE ENTROPY

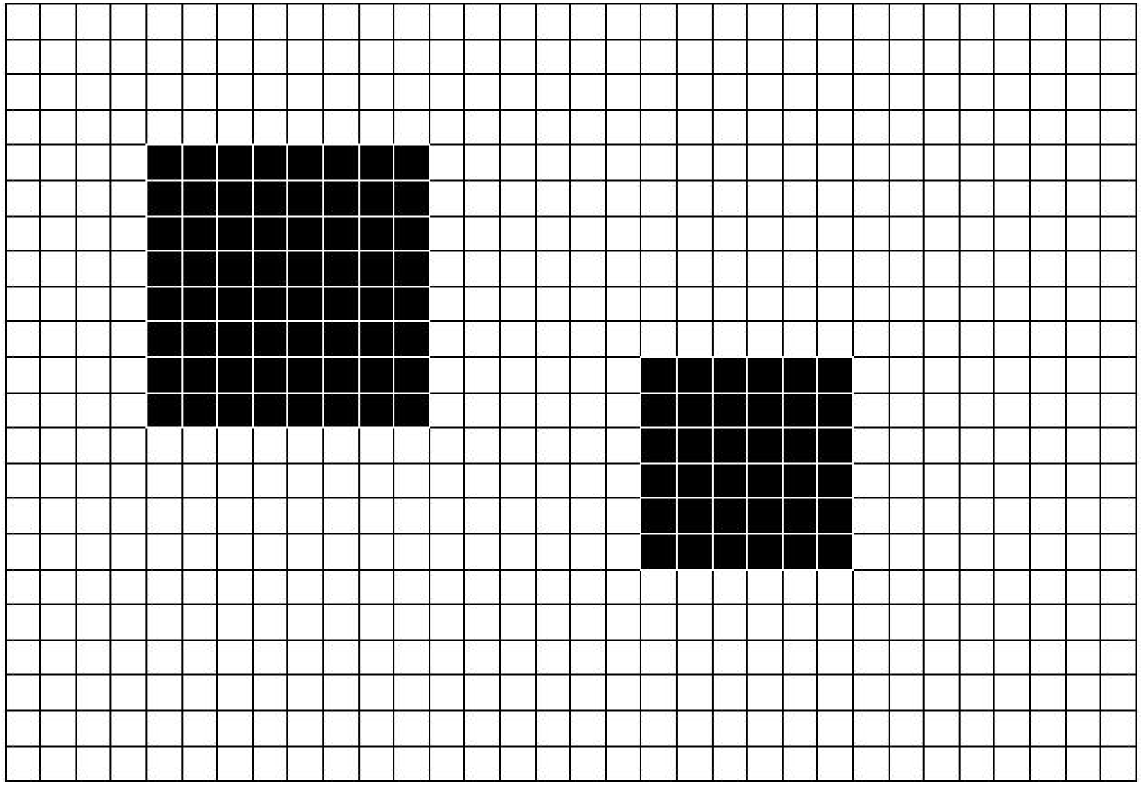
Contents

What is the function of the early visual system?

They [The sensory relays] recode sensory messages, extracting signals of high relative entropy from the highly redundant sensory input.

Barlow 1961

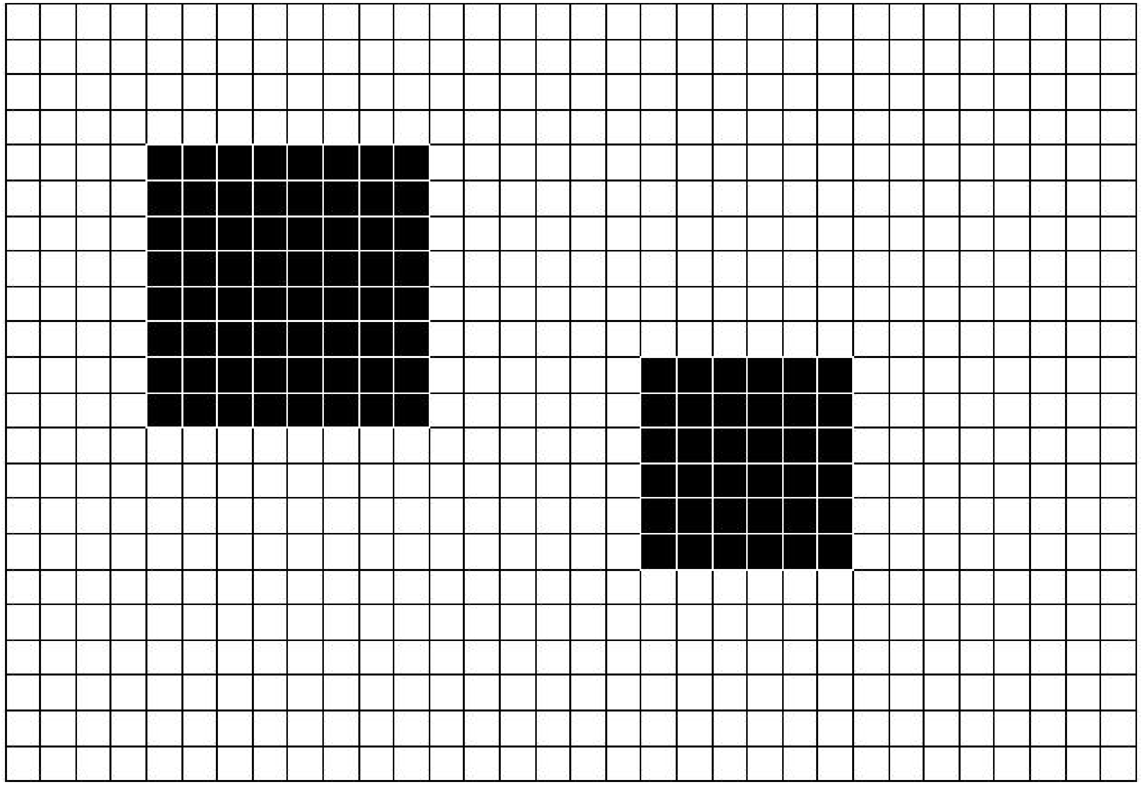
Redundancy



What is present in this world and where is it?

Attneave, Some informational aspects of visual perception

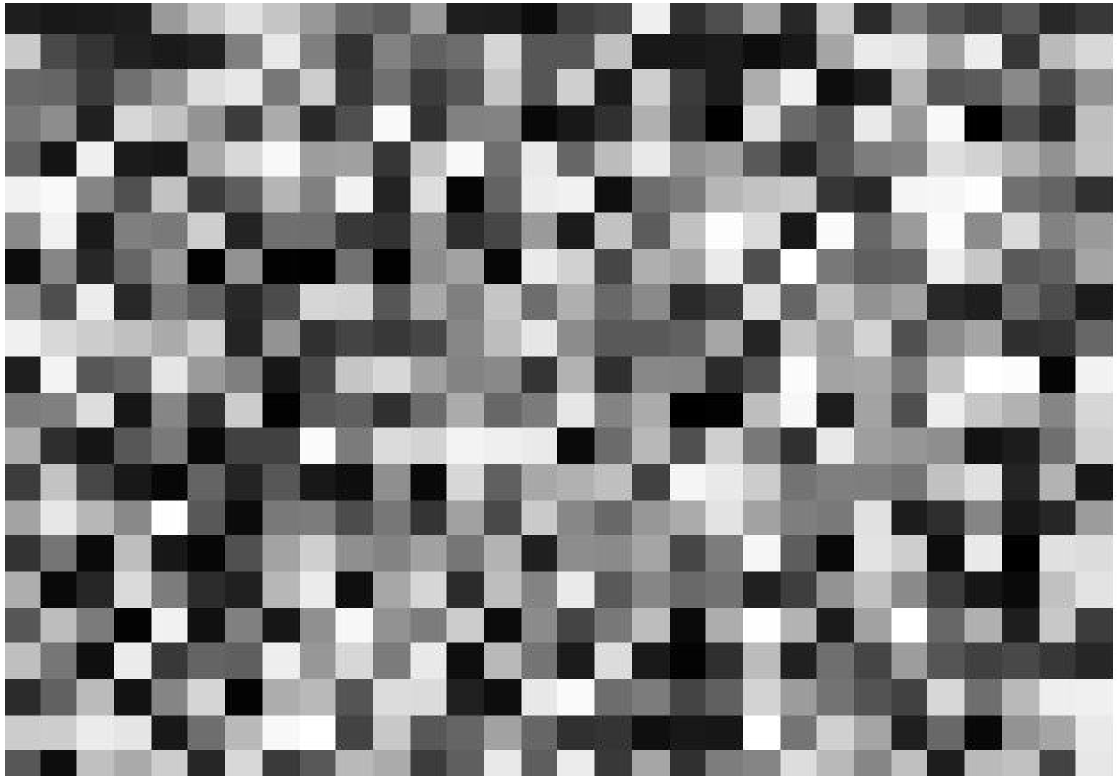
Redundancy



What is present in this world and where is it?

Black square, 8, (4,4)
Black square, 6, (18,14)

Redundancy



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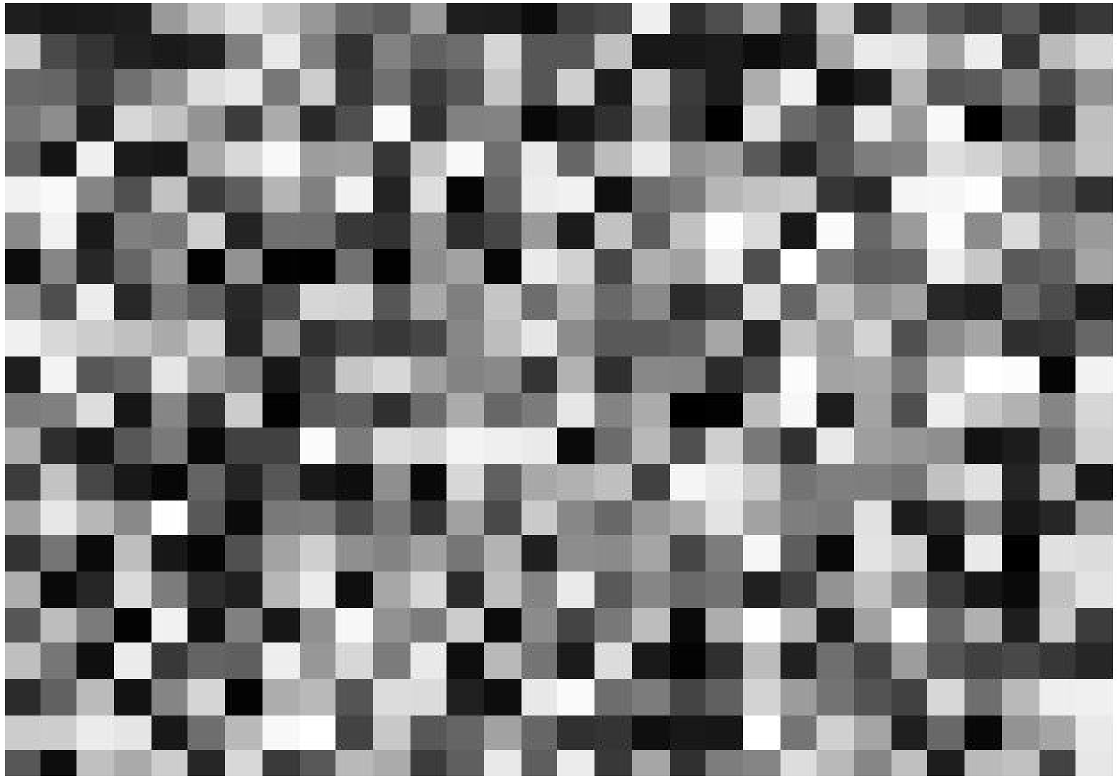

What is present in this world and where is it?

Black pixel (0,0)
Black pixel (0,1)
Black pixel (0,2)
Black pixel (0,3)
Gray pixel (0,4)
Light gray pixel (0,4)
...

Redundancy

What is present in this world and where is it?

Redundancy

What is present in this world and where is it?

Branches and leaves on top
Mountain on left to center
Reflects on left to center bottom
Dog and person on bottom
Dense trees on right
...

What is the message?
What are natural images?

Non Natural Natural

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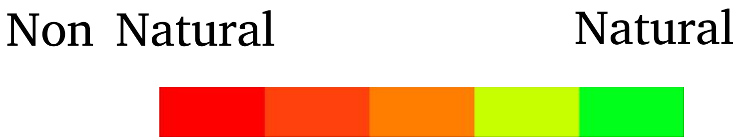
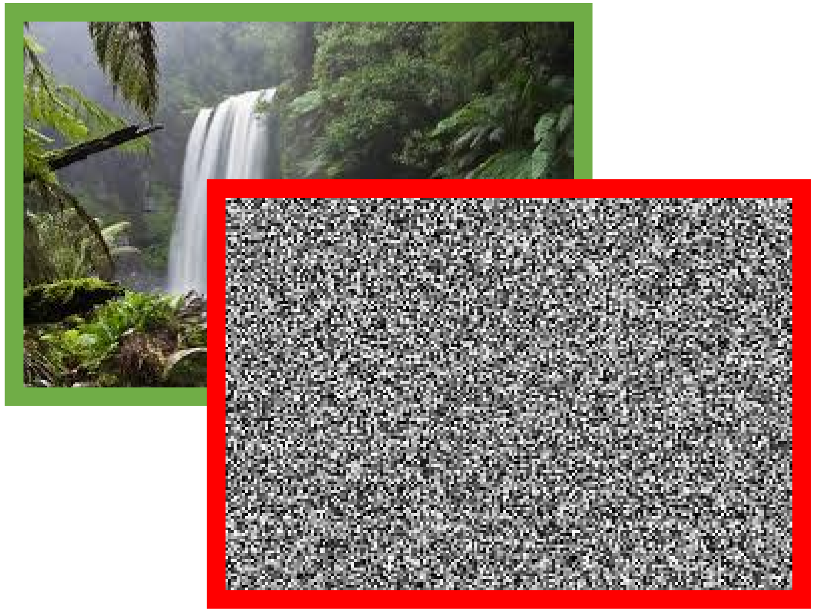
What are natural images?





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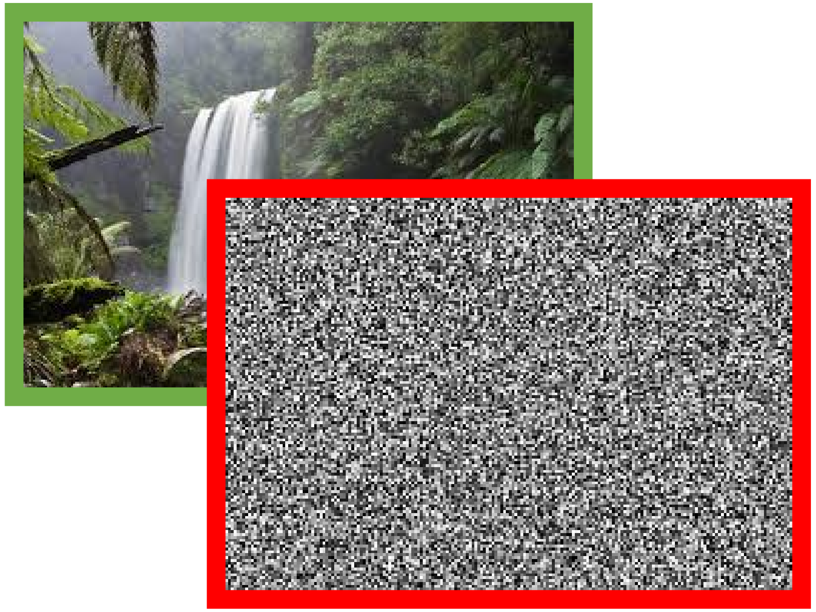

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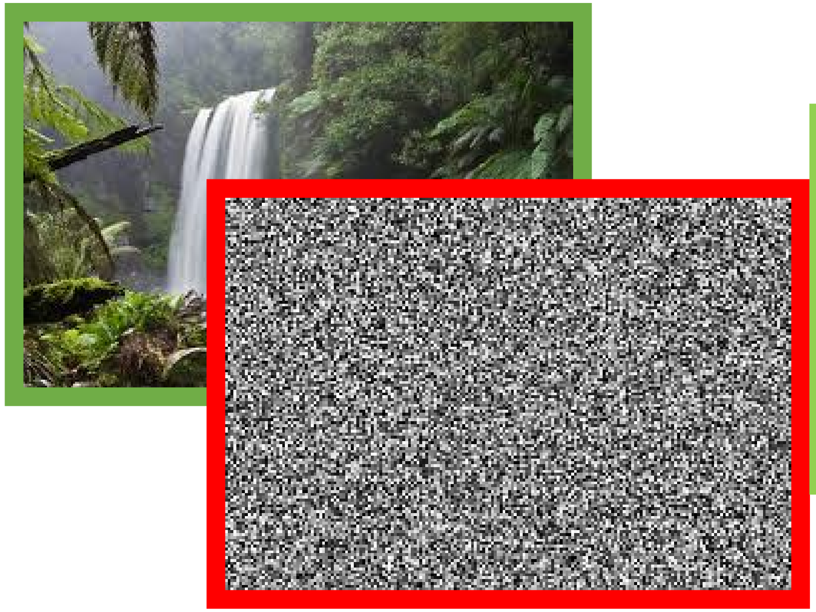

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Answer: No, because we do not know the probability distribution of natural images
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Question: How?

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Question: Can we, at least, approximate it?

Answer: Yes!

Question: How?

Answer: Looking at the statistics of the natural images

Natural images are redundant

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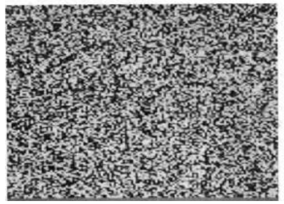

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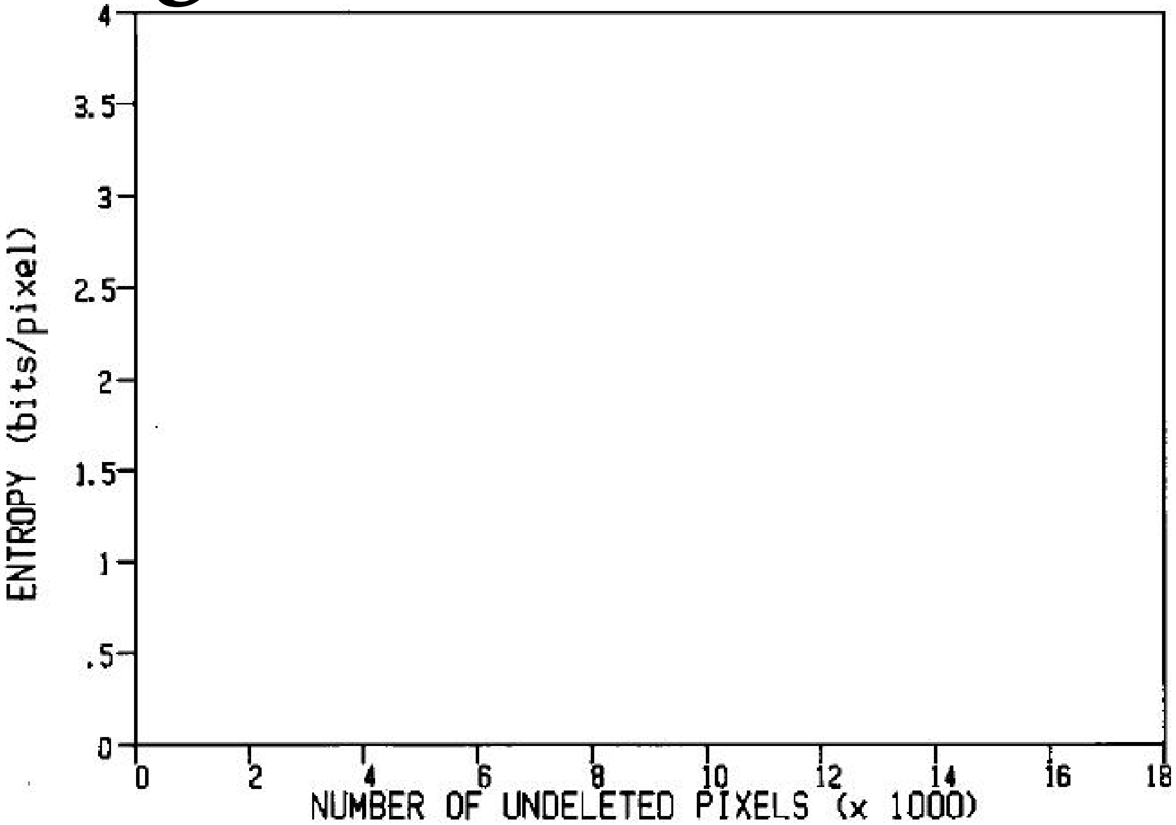
Kersten Predictability and redundancy of natural images,

Natural images are redundant



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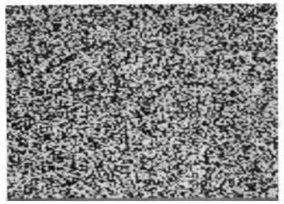

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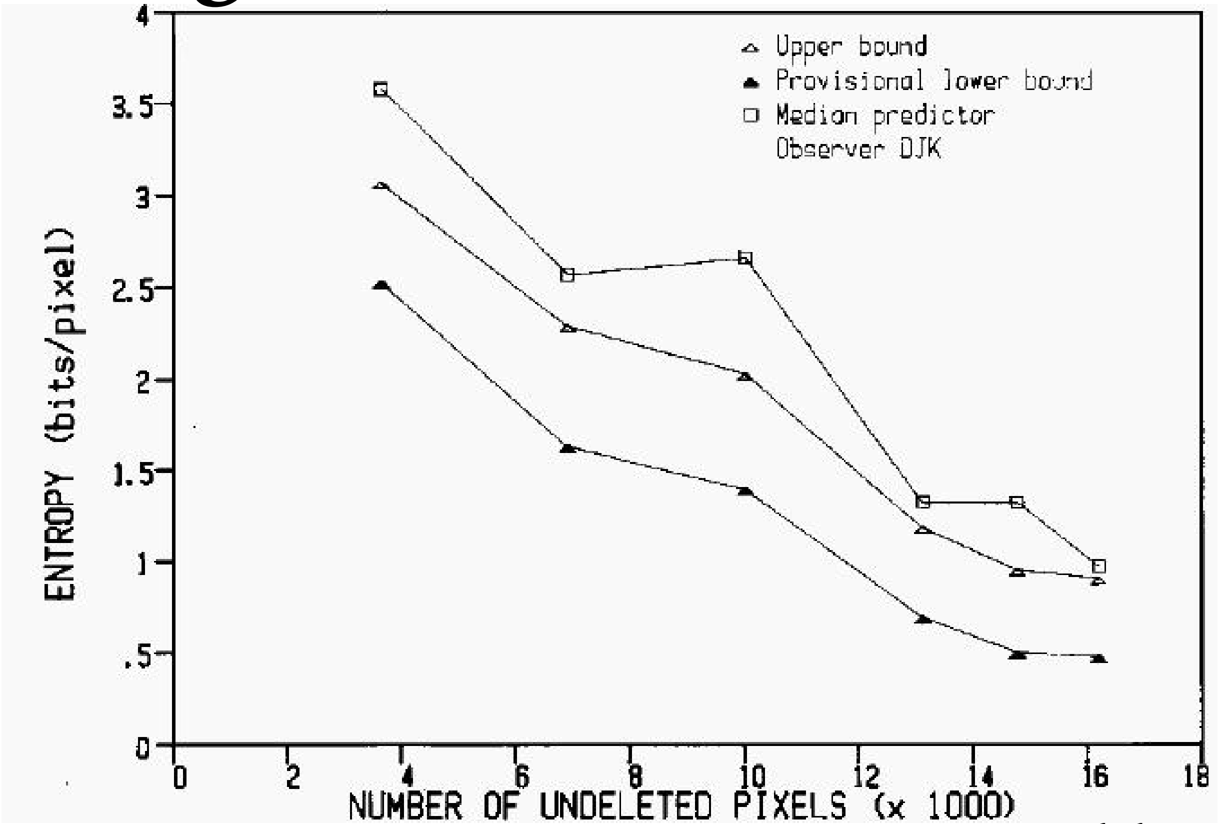

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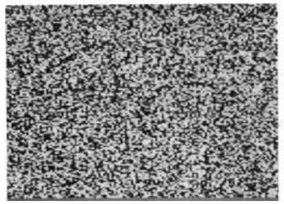

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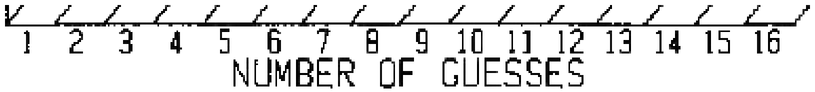

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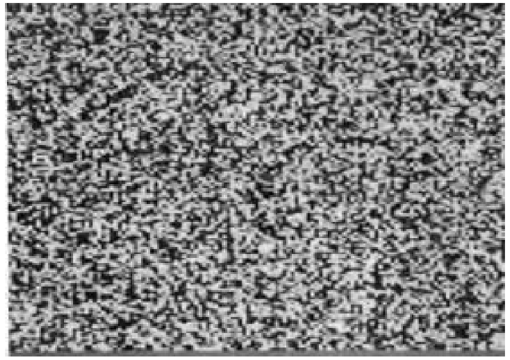

16 gray levels

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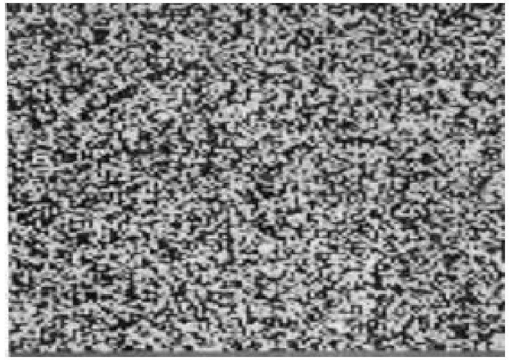

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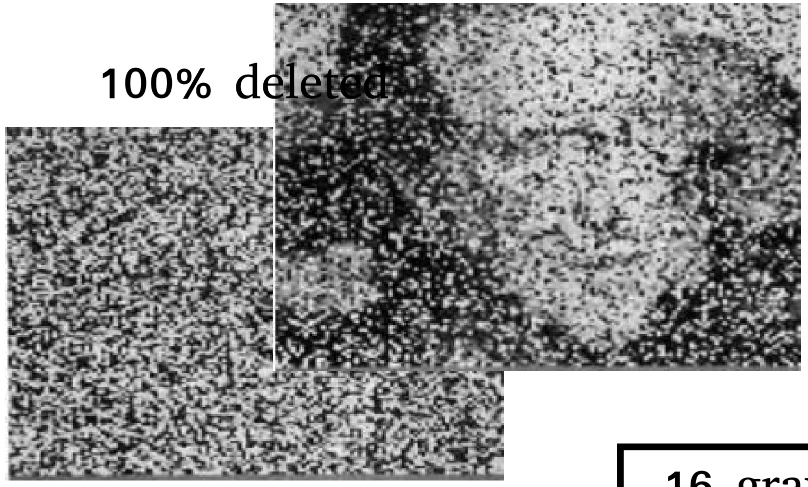

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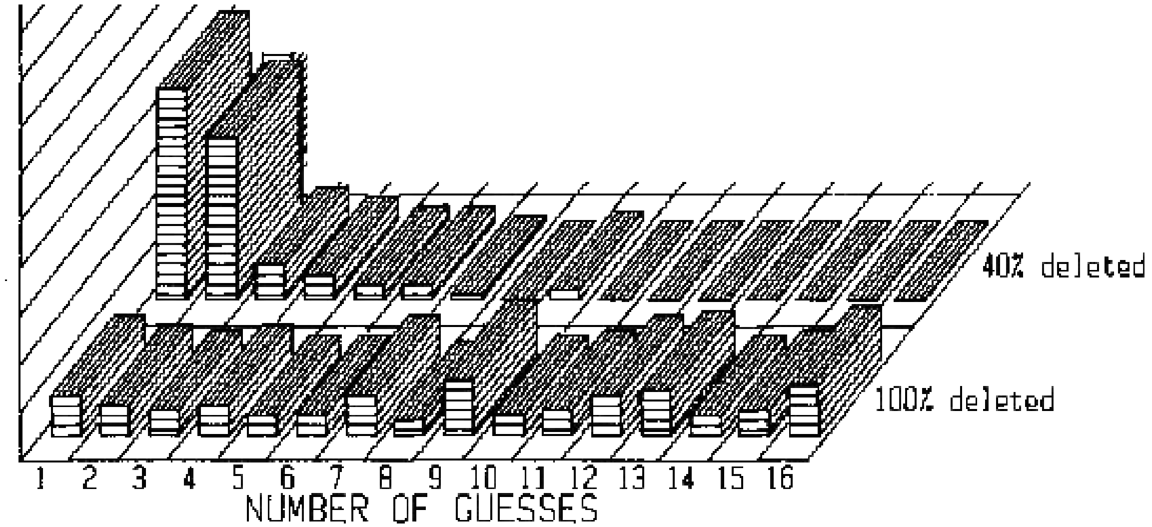

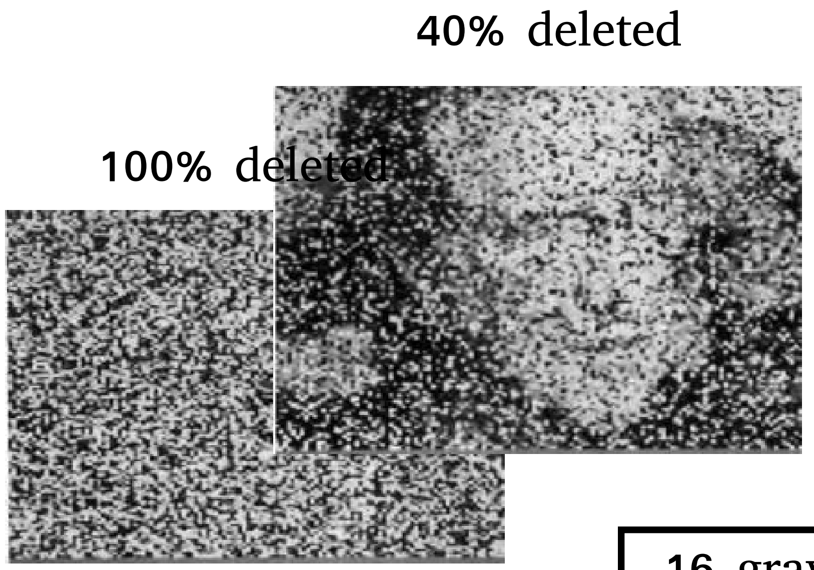

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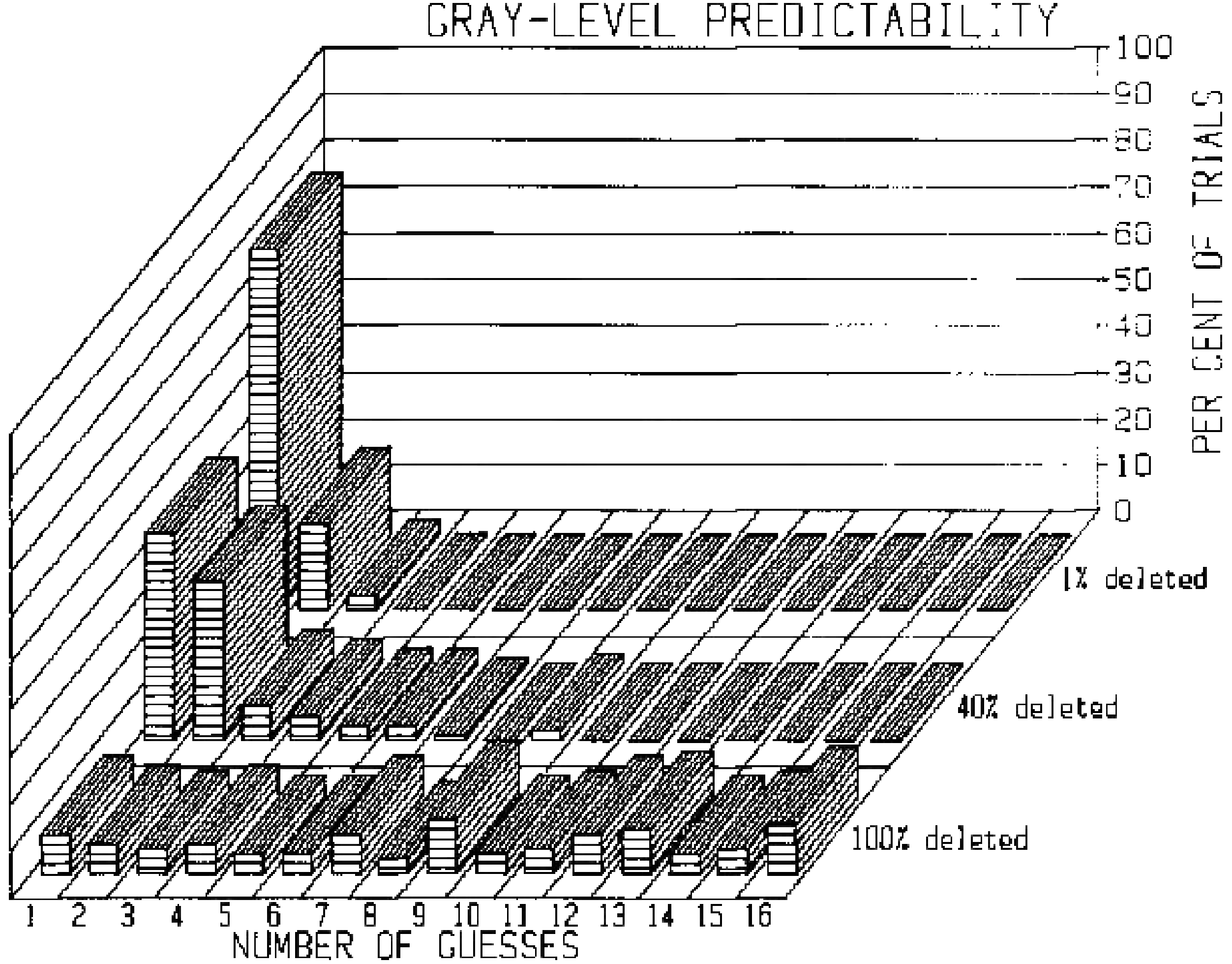

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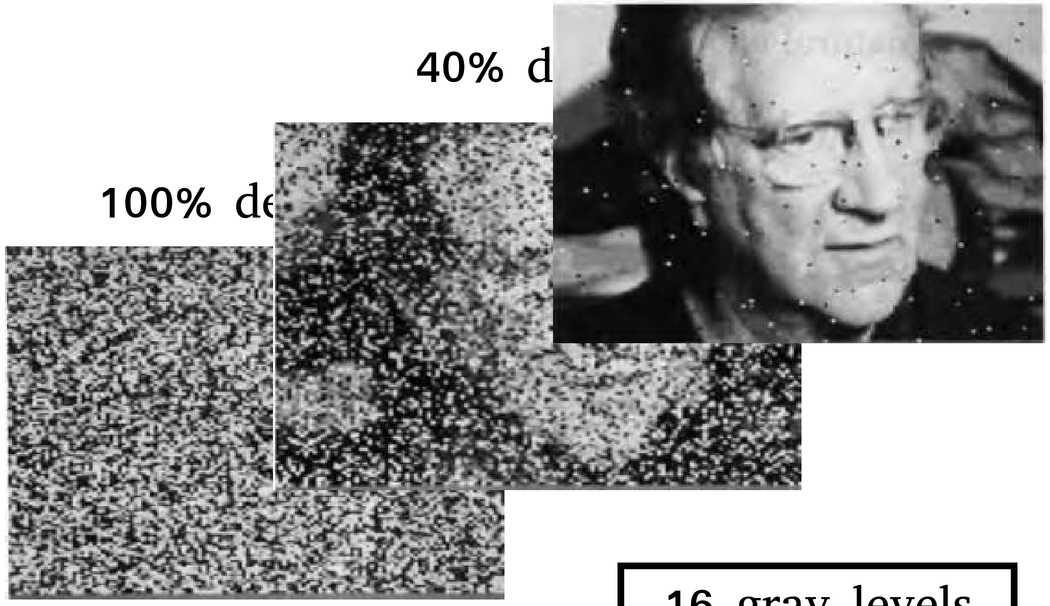

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Contents

General idea (method of the moments):

- μ^n moment of order n: $\mu^n[X] = E[(X-\mu)^n]$
- X r. v. with n - first moments well defined
- $X_1, X_2, \ldots, X_k$ sequence of r. v.

If $\lim_{k\to\infty} \mu^n[X^k] = \mu^n[X]$ then $X^k \xrightarrow{d} X$

Moments

- $\mu^1[X] = E[X-\mu]=0$
- $\mu^2[X] = E[(X-\mu)^2]$ (auto correlation)
- $\mu^3[X] = E[(X-\mu)^3]$ (skewness)
- $\mu^4[X] = E[(X-\mu)^4]$ (kurtosis)
- ...

Fourier Transform

- The Fourier transform decomposes a function of time (a signal) into the frequencies that make it up.

1D

$$F(x) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi(x\frac{n}{N})}$$

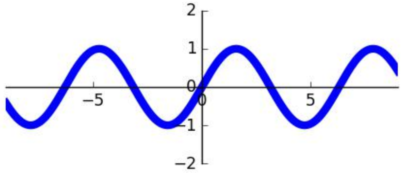
$$f(n) = \frac{1}{N} \sum_{n=0}^{N-1} F(x) e^{j2\pi(x\frac{n}{N})}$$

$$PS(x) = |F(x)|^2$$

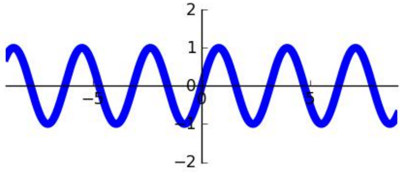
1D Fourier Transform

time domain

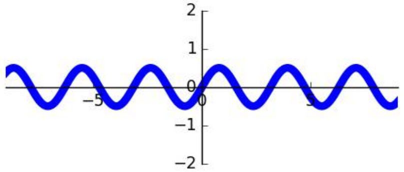
sin(x)



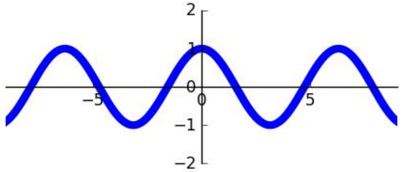
sin(2x)



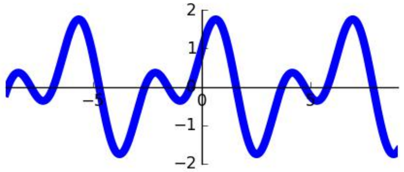
0.5sin(2x)



sin(x+pi/2)



sin(2x)+sin(x+pi/2)

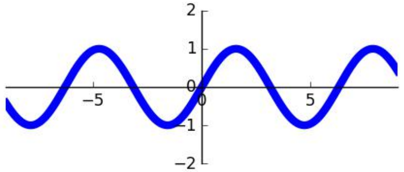


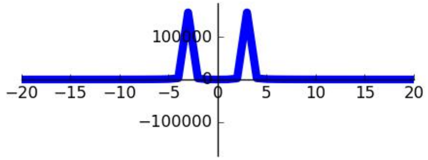
1D Fourier Transform

time domain

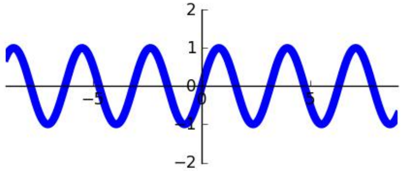
ps (fft)

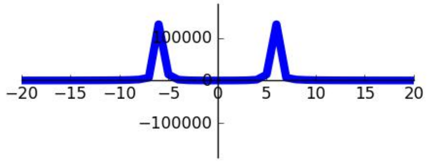
sin(x)





sin(2x)





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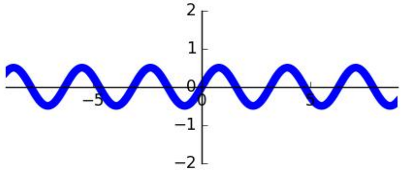
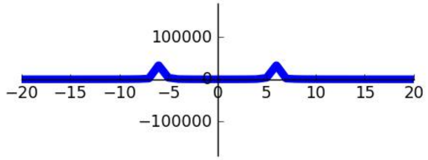
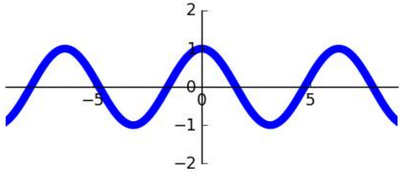
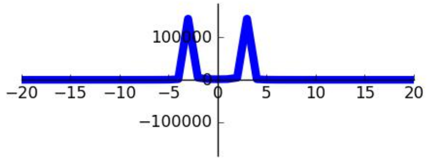




sin(x+pi/2)





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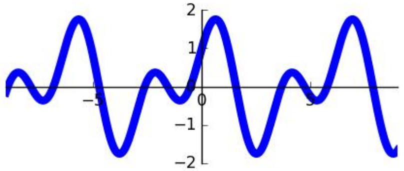

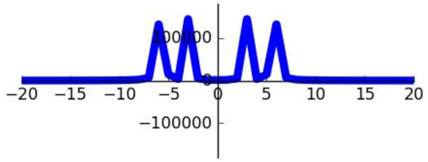

Fourier Transform

- The Fourier transform decomposes a function of time (a signal) into the frequencies that make it up.

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2D

$$F(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n) e^{-j2\pi(x\frac{m}{M}+y\frac{n}{N})}$$

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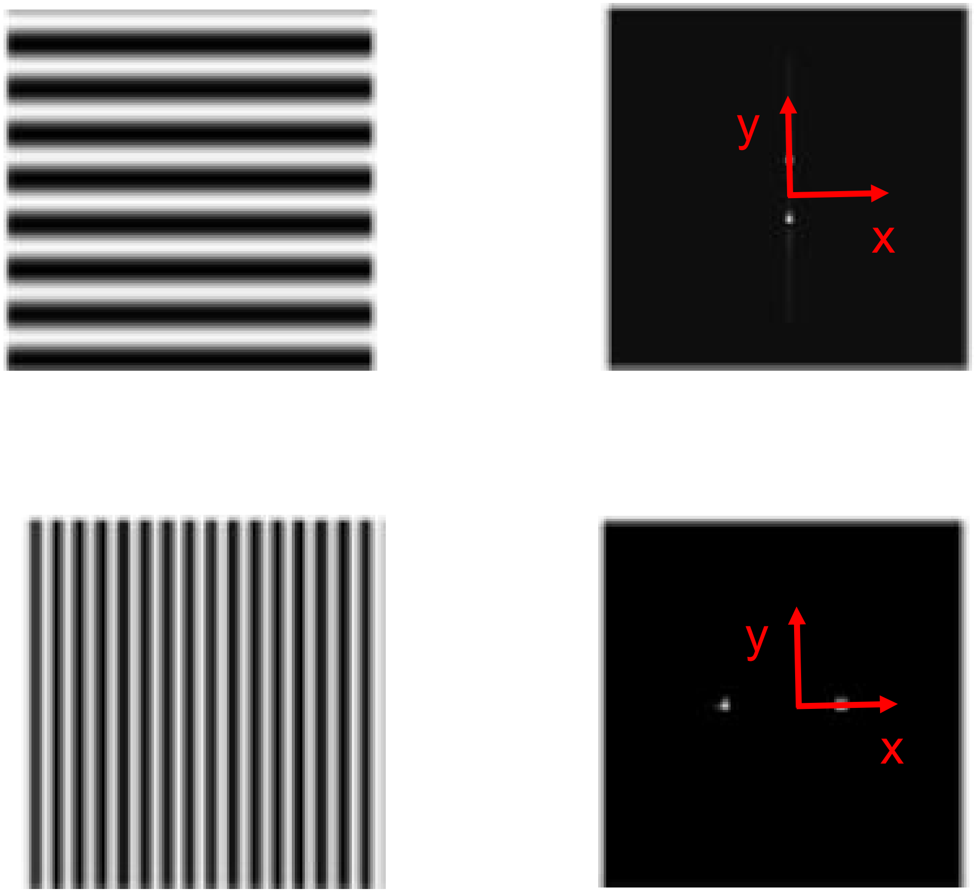
2 D Fourier Transform of waves



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$$f(m,n) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} F(x,y) e^{j2\pi(x\frac{m}{M}+y\frac{n}{N})}$$

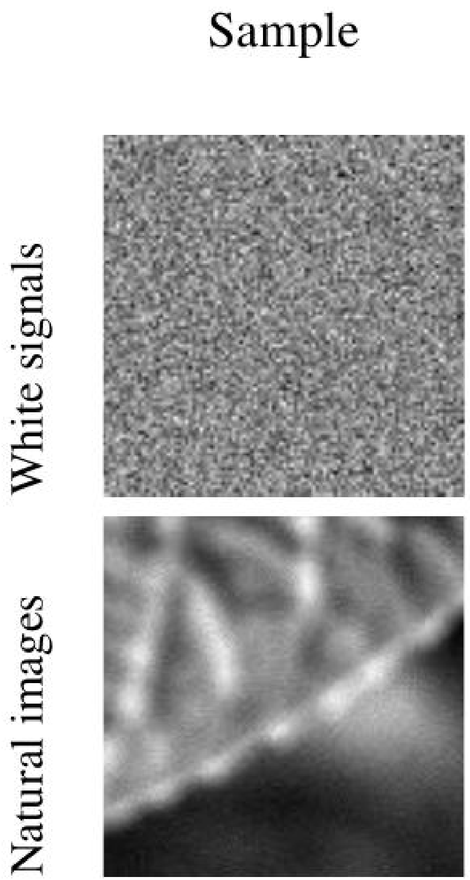
$$PS(x,y) = |F(x,y)|^2$$

Wiener-Khinchin theorem: If $X = \{ x\_1, \ldots, x\_n \}$ is a stationary process and $\mu^2[X]$ exists and is finite, then:

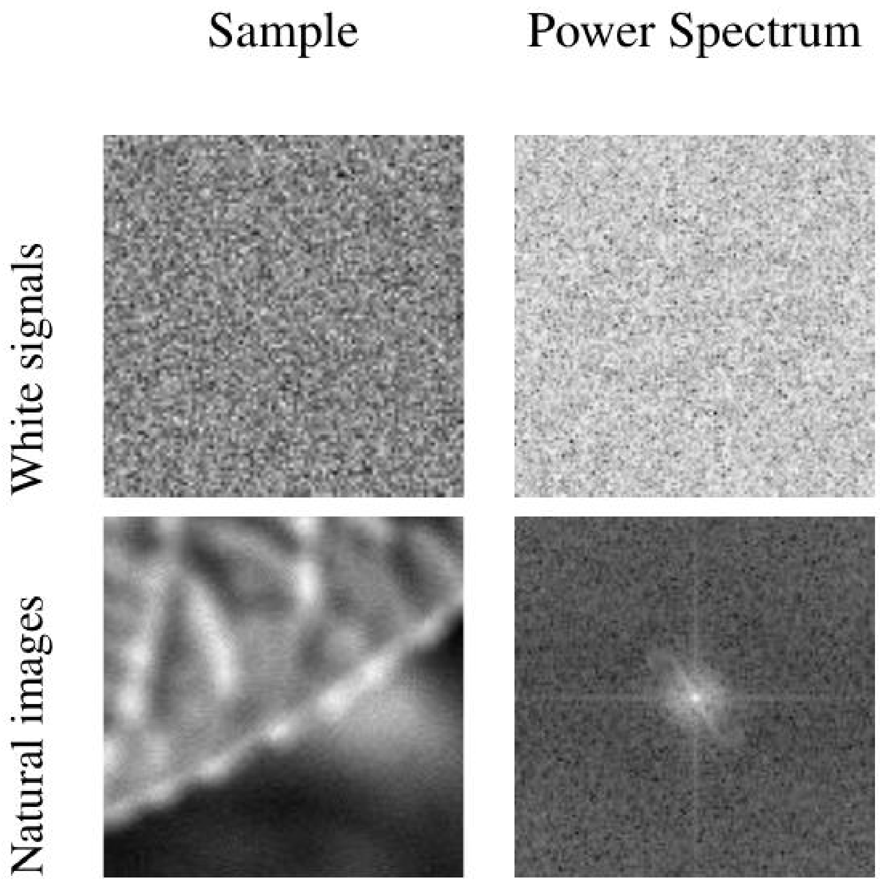
$$E[PS(X)) = F(\mu^2[X])$$

Contents

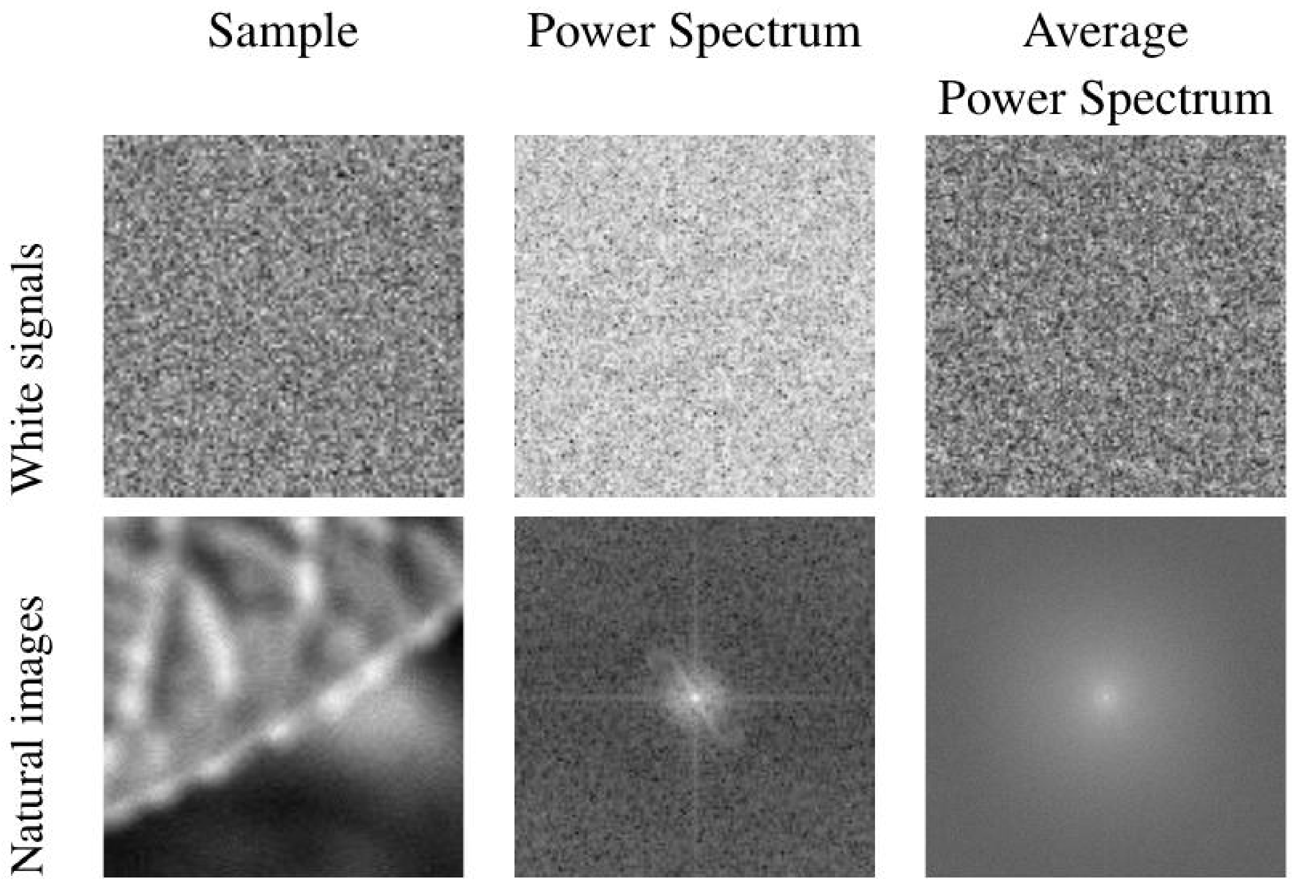
Power Spectrum of Natural Images



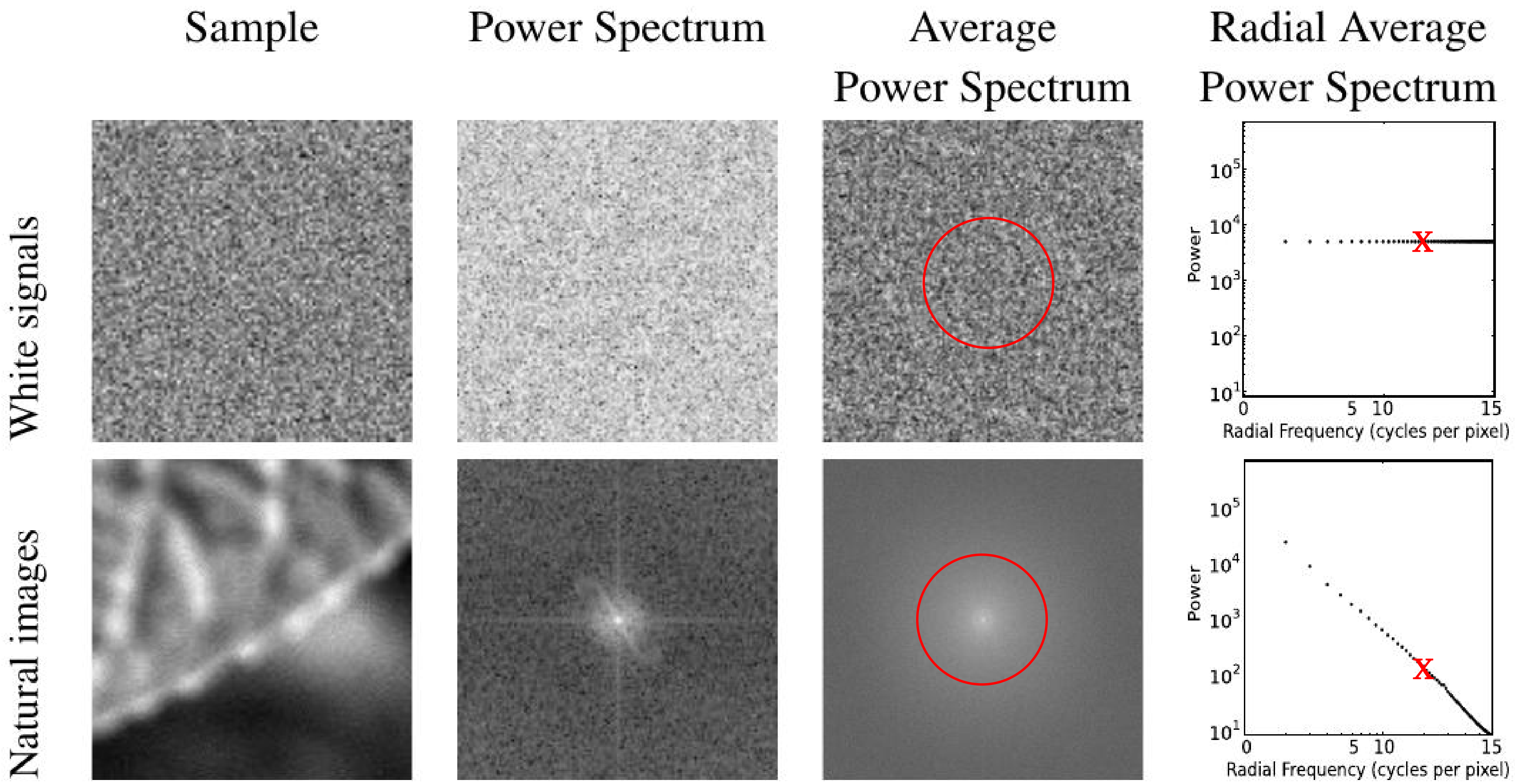
Power Spectrum of Natural Images



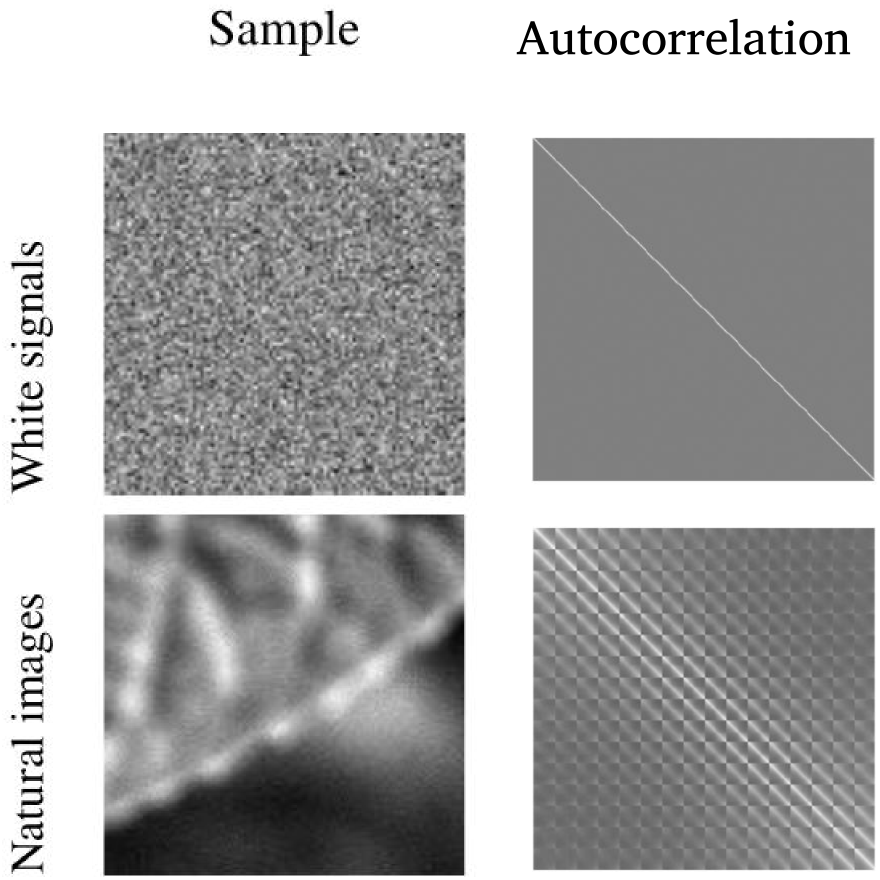
Power Spectrum of Natural Images



Power Spectrum of Natural Images



Auto correlation of natural images

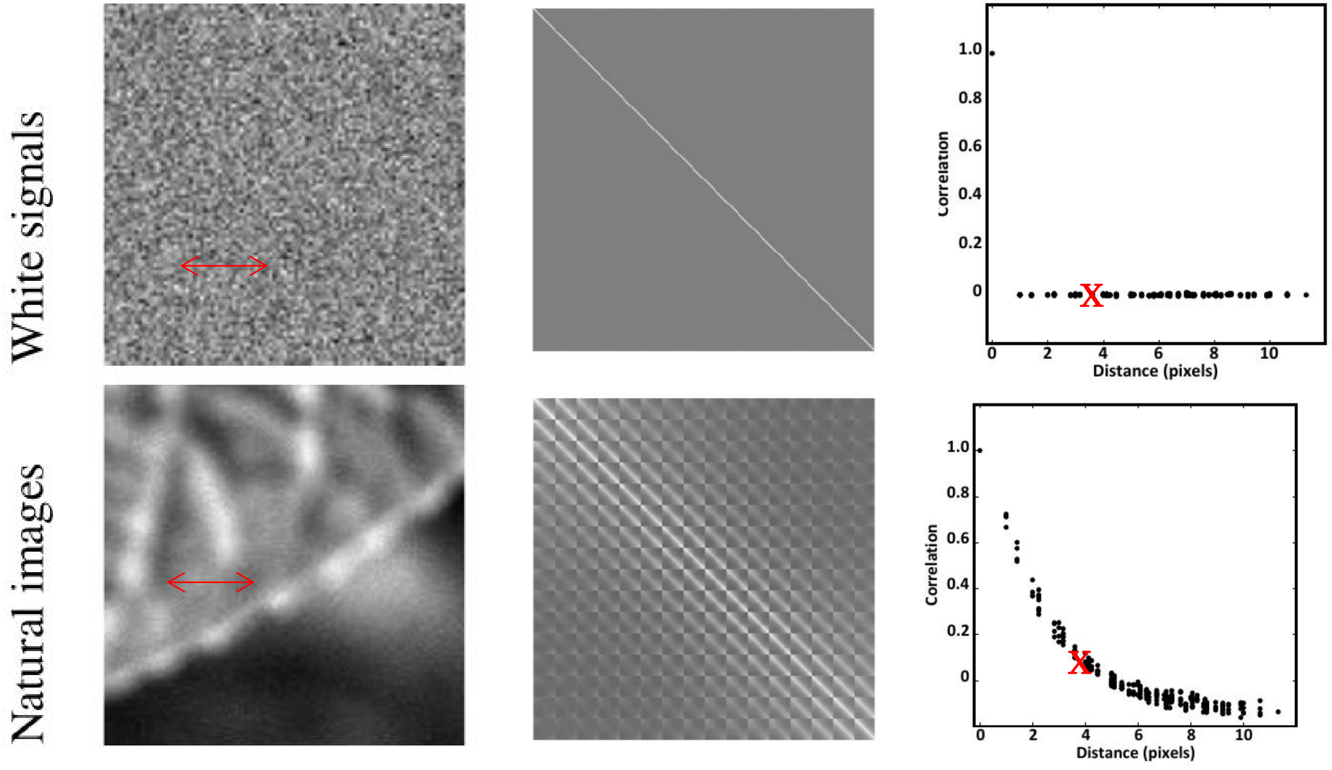


Auto correlation of natural images

Sample

Autocorrelation

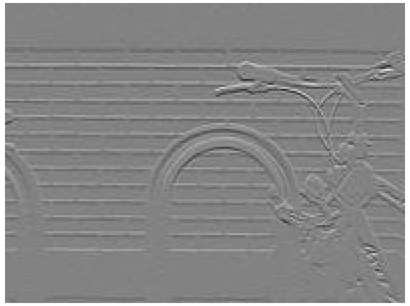
Autocorrelation in function of pixel distance



Sobel Filters: Introduction

- It performs a 2-D spatial gradient measurement on images in order to emphasizes edges.
- Pairs of convolution kernels (K_x and K_y) designed to respond maximally to edges running vertically and horizontally relative to the pixel grid

$G_x = I * K_x$ $\qquad$ $G_y = I * K_y$



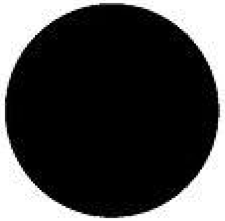
Sobel Filters: Introduction

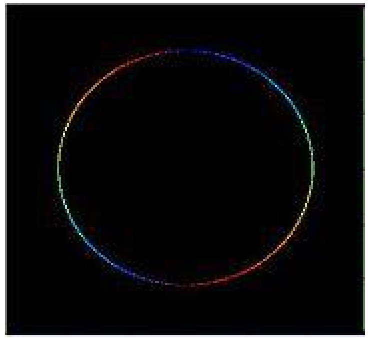
- Magnitude of the gradient:

$$|G| = \sqrt{(G_x^2 + G_y^2)}$$

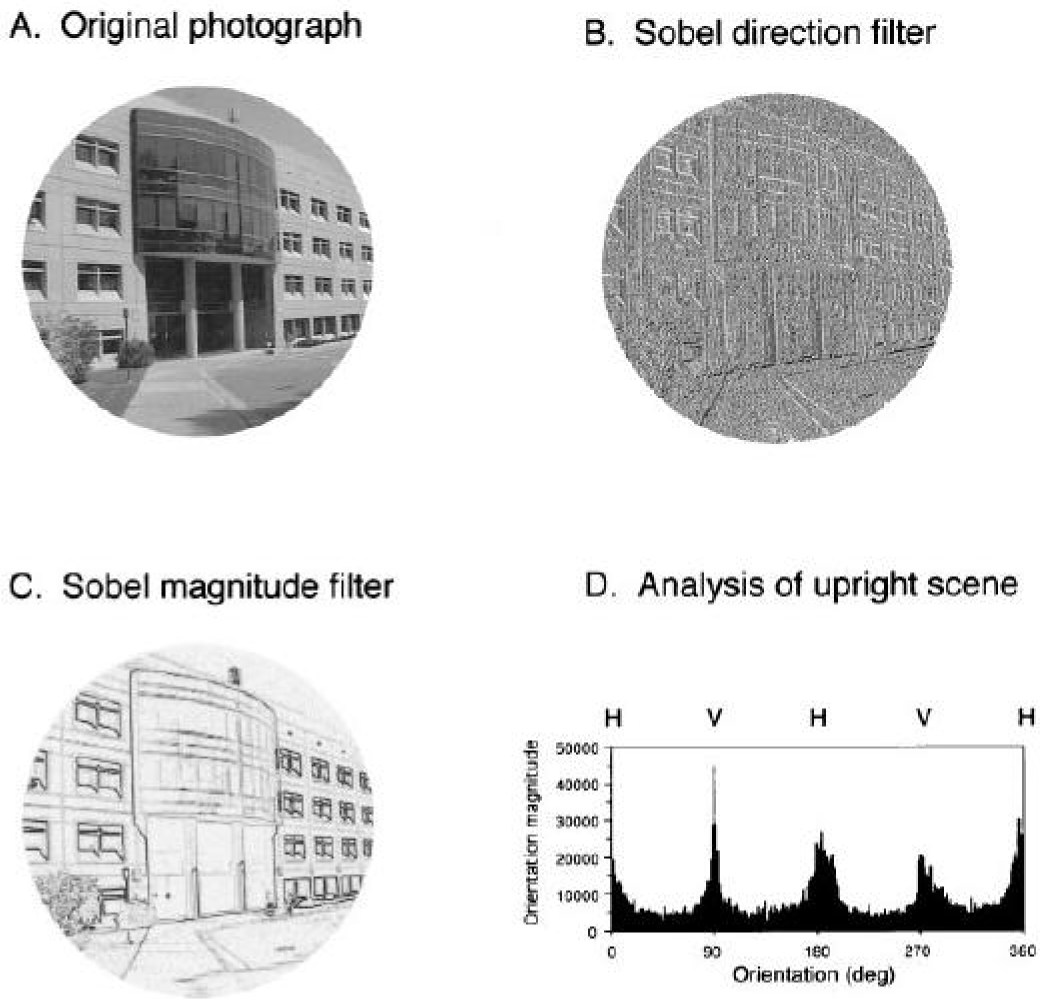
- Angle of the gradient:

$$\theta = \arctan(G_y/G_x)$$



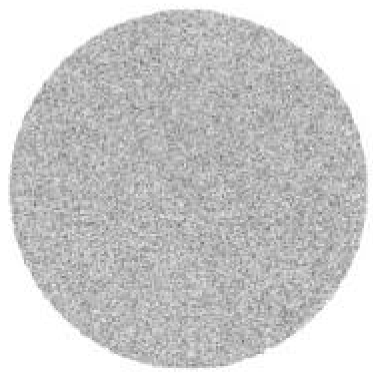


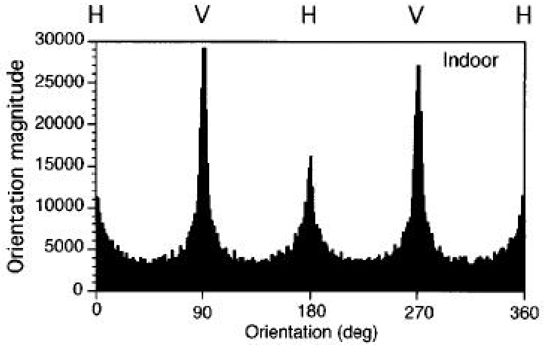
Analysis of edges orientations

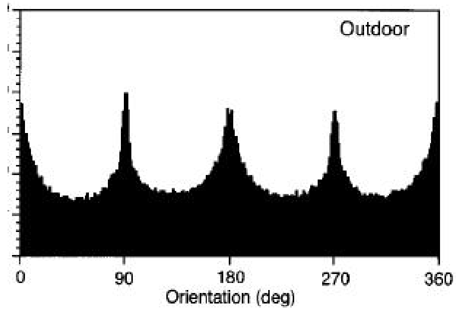


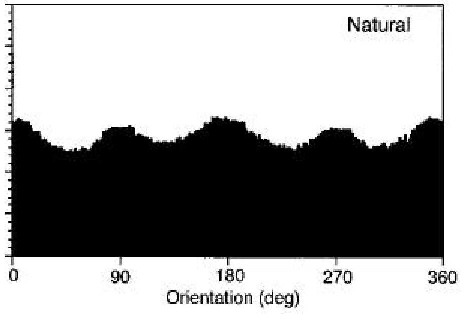
The distribution of oriented contoursin the real world, Coppola et al

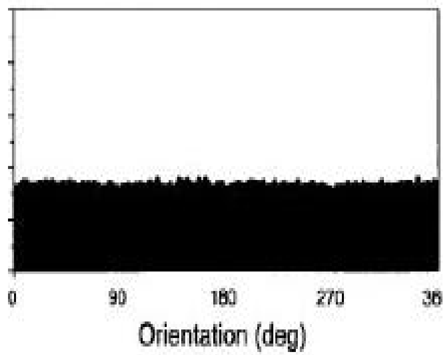
Analysis of edges orientations

Indoor	Outdoor	Natural	White Noise
			





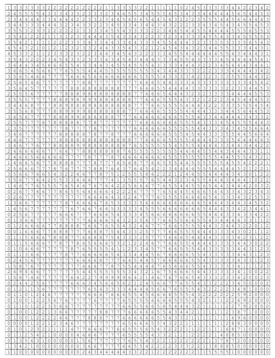




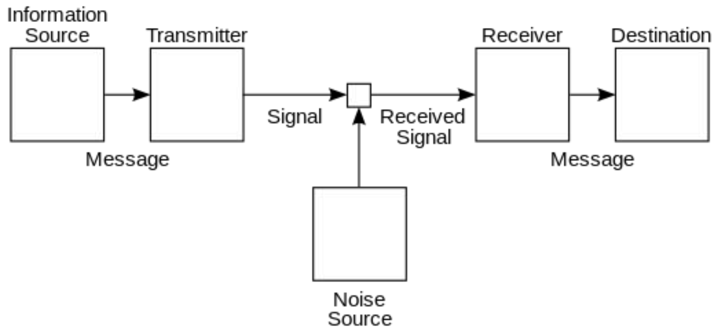
The distribution of oriented contoursin the real world, Coppola et al

Summary

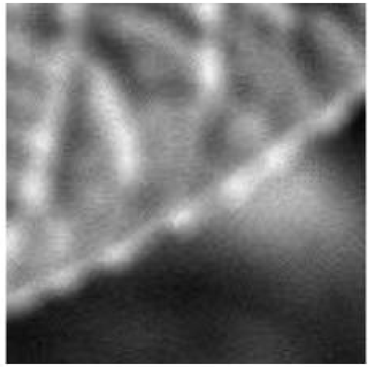
1. What is vision?

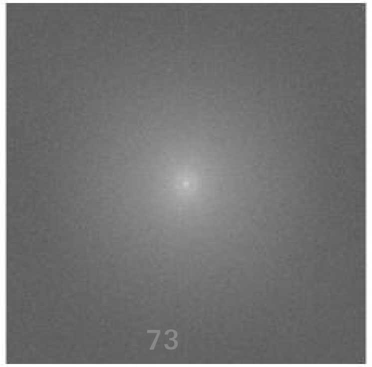


2. The redundancy reduction hypothesis



3. Natural images statistics





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Extra Slides