

Natural images statistics

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U2IS - ENSTA - IPParis

ecampus moodle: MI210 - Modèles neuro-
computationnels de la vision (P4 - 2020-21)

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Contents

1. What is vision?
2. Information theory a la Shannon
3. The redundancy reduction hypothesis
4. Statistics and the Fourier Transform
5. Natural images statistics

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What is vision?

"Vision is the process of discovering from images what is present in the world, and where it is"

David Marr, 1982

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David Marr, 1982

... in order to solve tasks efficiently.

Daniela Pamplona, 2017

What are the requirements of a working visual system?

Constrains:

- To run real time
- To be robust to noise
- To adapt to lightness, contrast, etc
- To be energetically cheap
- To use limited memory
- To cope with imperfect imaging process

Functions:

- To select and extract relevant information from the environment to solve tasks
- To represent the environment for navigation, reasoning, memory,
- To learn new objects
- To predict location, motion, shape
- ...

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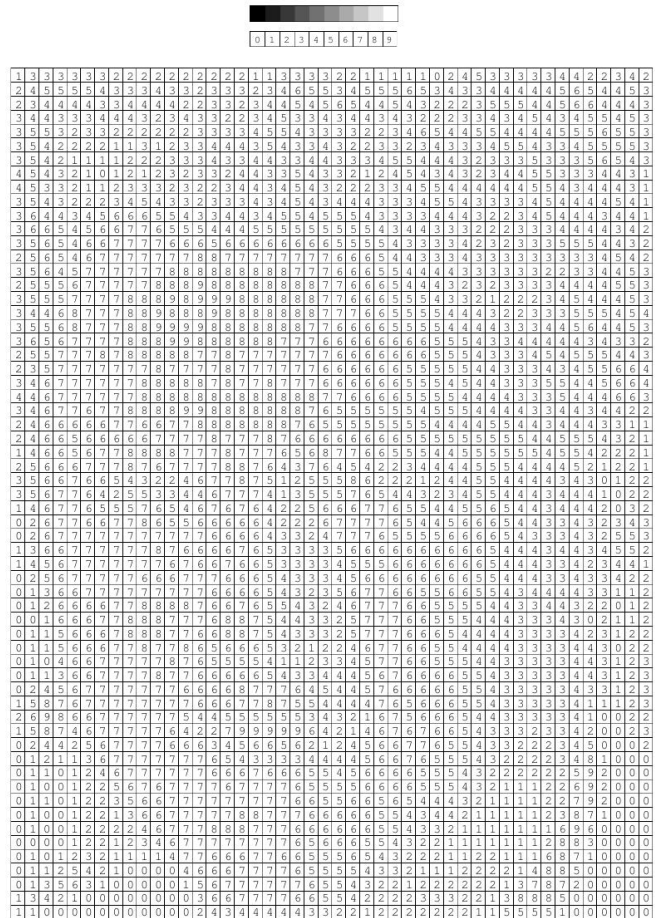
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What is in here?

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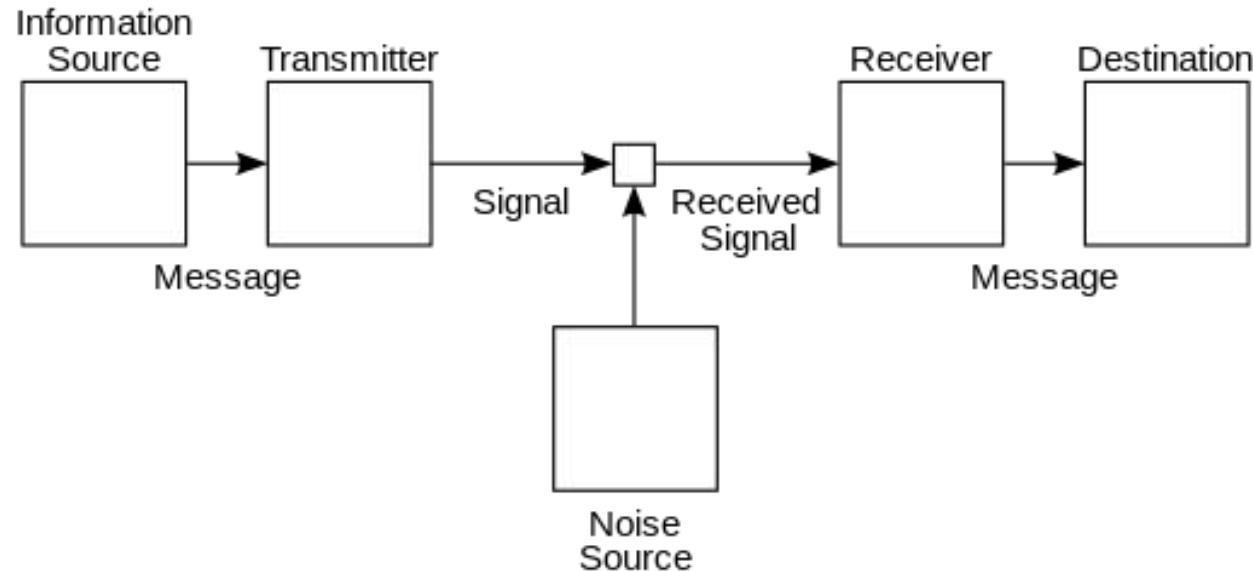
What is in here?



Contents

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General Communication System



Definitions

Information Source: Produces messages

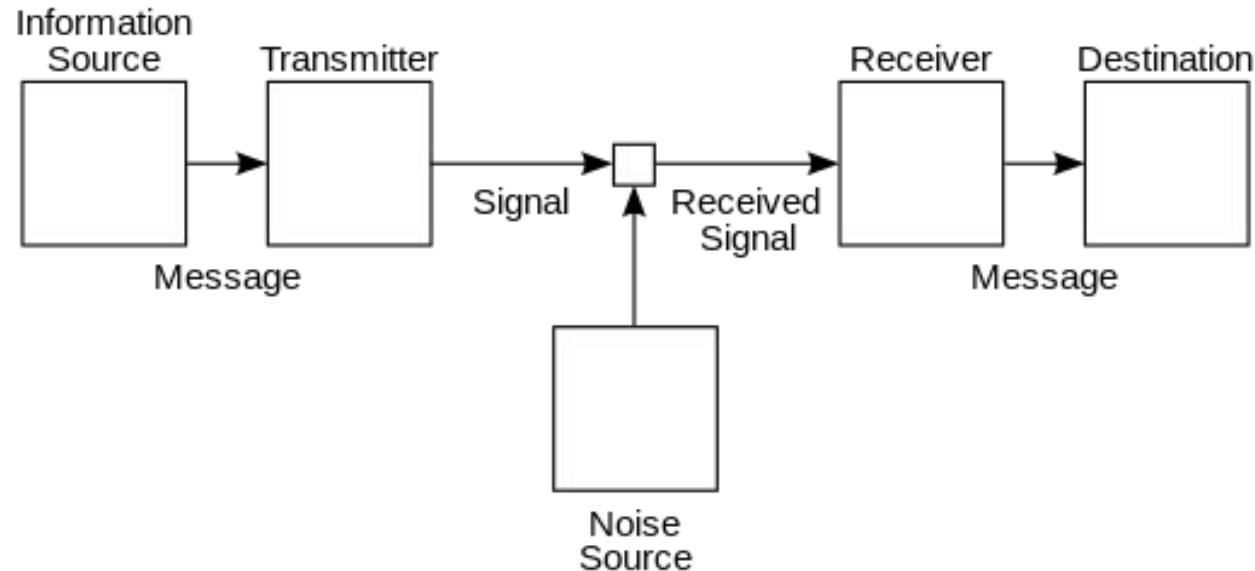
Transmitter: Transform/encode the message into a signal

Channel: Medium to carry the message

Receiver: Transforms/decode the signal into a message

Destination: Entity that the message is intended

General Communication System



Example: WhatsApp

Information Source: Alice

Transmitter: Mobile phone

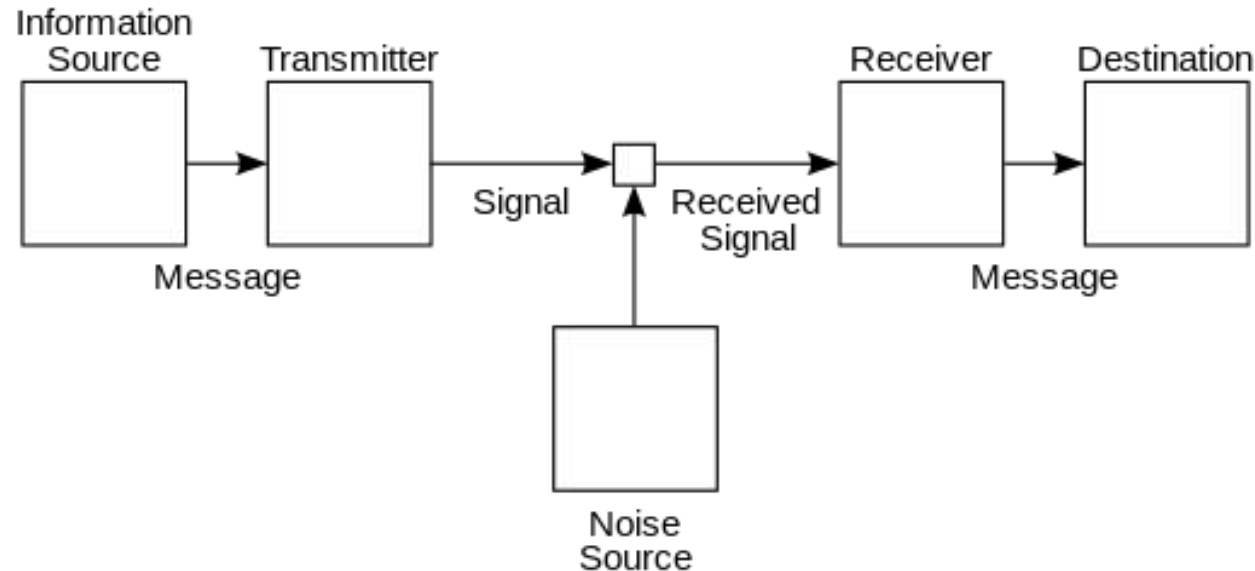
Channel: WiFi

Noise: SNR:30dB

Receiver: Mobile phone

Destination: Bob

General Communication System



Example: Visual System

Information Source: Environment

Transmitter: Eye

Channel: Early visual system

Noise: Unknown

Receiver: Higher areas (MT, TE, MIP,...)

Destination: Other brain areas (PMC,...)
(ultimately the environment)

Self-Information of a message

Information content of an event, $h(E)$, with probability $P\{E\}$ must:

1. be a decreasing function of $P\{E\}$: more an event is likely, the less information its occurrence brings to us.

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$$h(E) = \log \frac{1}{P\{E\}} = -\log P\{E\}$$

Entropy

- Entropy: measure of randomness of a random variable (r. v.)
- Entropy is the expected value of self information

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- Entropy is the expected value of self information

X : discrete r.v. taking values in $\{x_1, x_2, \dots, x_n\}$ with $p_i = P\{X = x_i\}$

$$H(X) = -\sum_{i=1}^n p_i \log p_i$$

Maximum entropy of a discrete r.v.

- X : Bernoulli r. v.,
 $H_{max}(X) = 1$ with $p = 0.5$
- X : discrete r.v. taking
values in $\{x_1, x_2, \dots, x_n\}$
with $p_i = 1/n$
 $H_{max}(X) = \text{Log}(N)$

Redundancy of a r.v.

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- Redundancy, or relative entropy, compares the entropy of a random variable with the maximal entropy
 $r = 1 - H(X)/H_{max}(X)$

Redundancy of a r.v.

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 $H_{max}(X) = 1$ with $p = 0.5$
- X : discrete r.v. taking values in $\{x_1, x_2, \dots, x_n\}$ with $p_i = 1/n$
 $H_{max}(X) = \text{Log}(N)$
- Redundancy, or relative entropy, compares the entropy of a random variable with the maximal entropy
 $r = 1 - H(X)/H_{max}(X)$

REDUCE THE REDUNDANCY \Leftrightarrow INCREASE THE ENTROPY

Contents

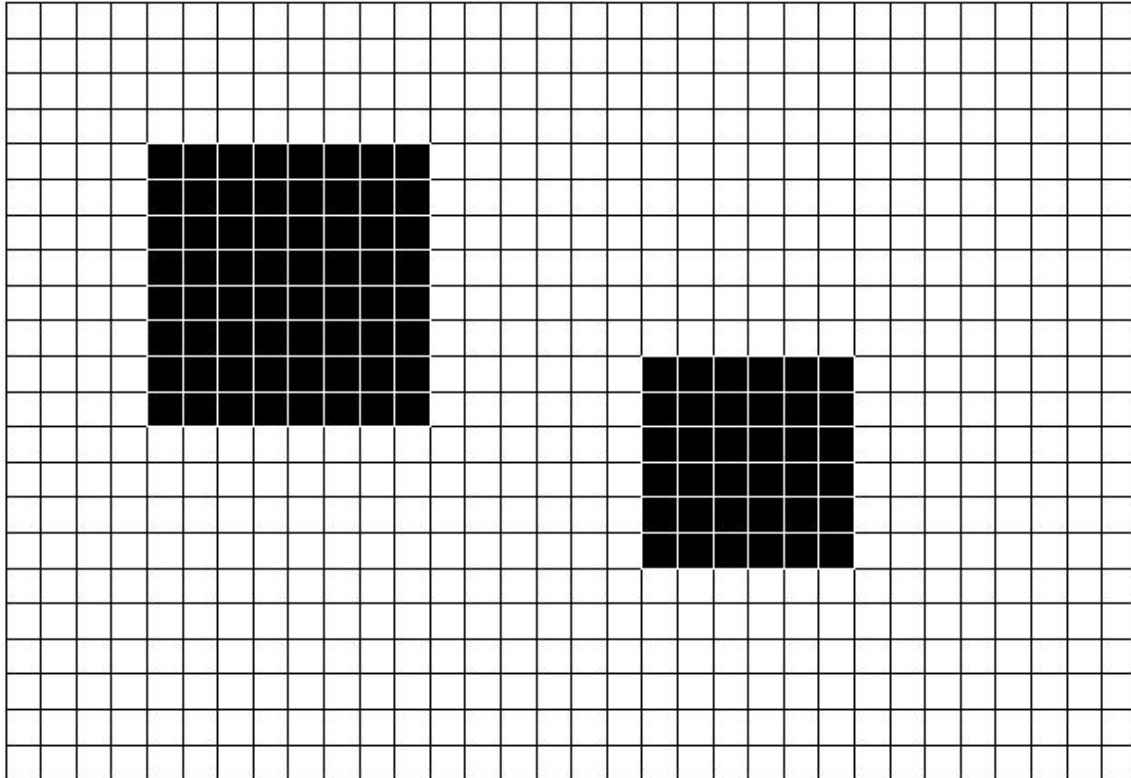
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What is the function of the early visual system?

They [The sensory relays] recode sensory messages, extracting signals of high relative entropy from the highly redundant sensory input.

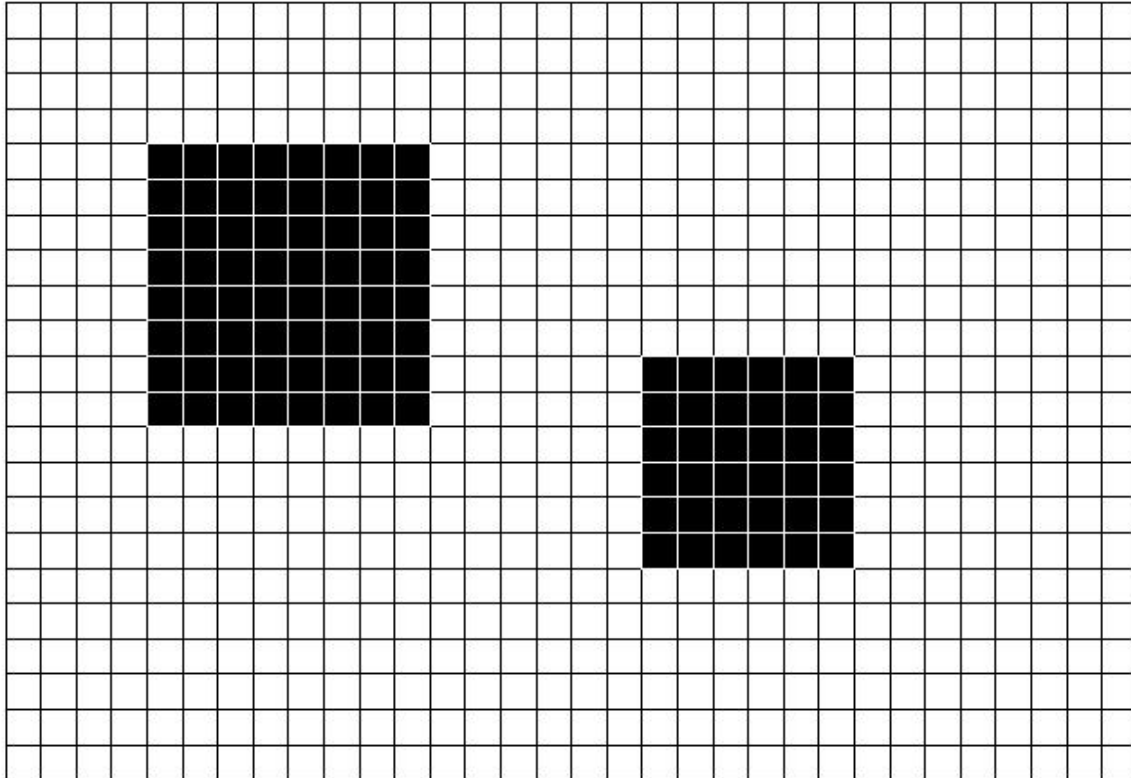
Barlow 1961

Redundancy



What is present in this world and where is it?

Redundancy

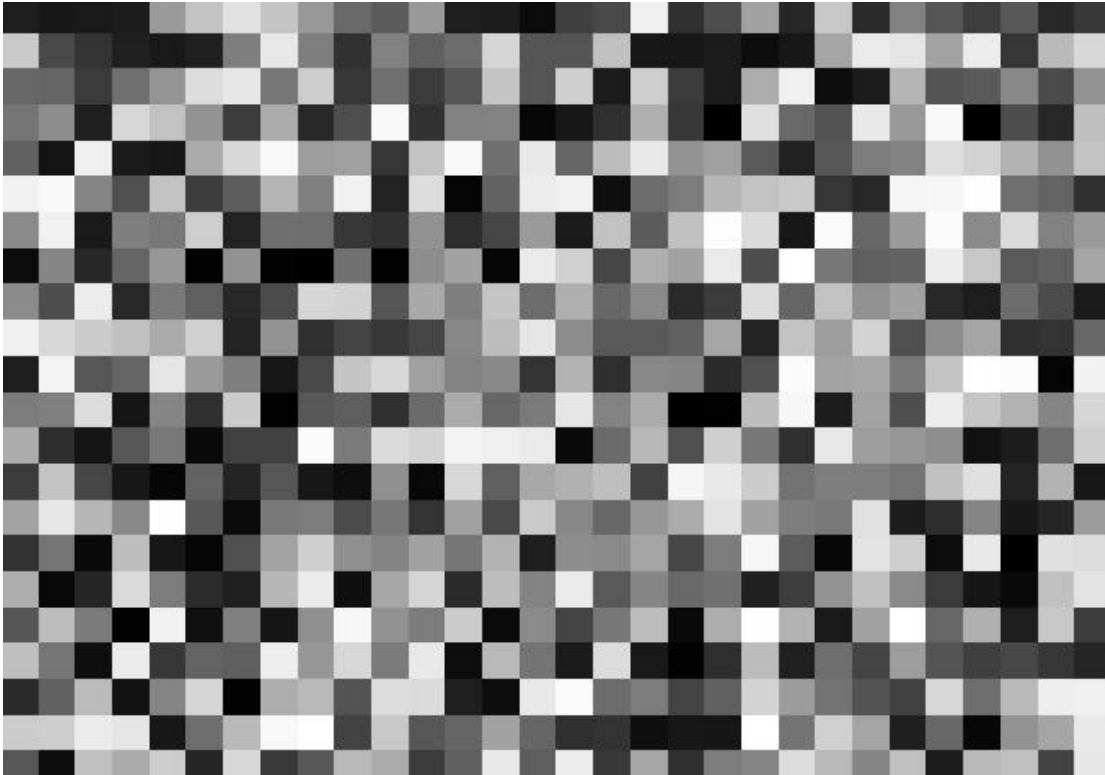


What is present in this world and where is it?

Black square, 8, (4,4)

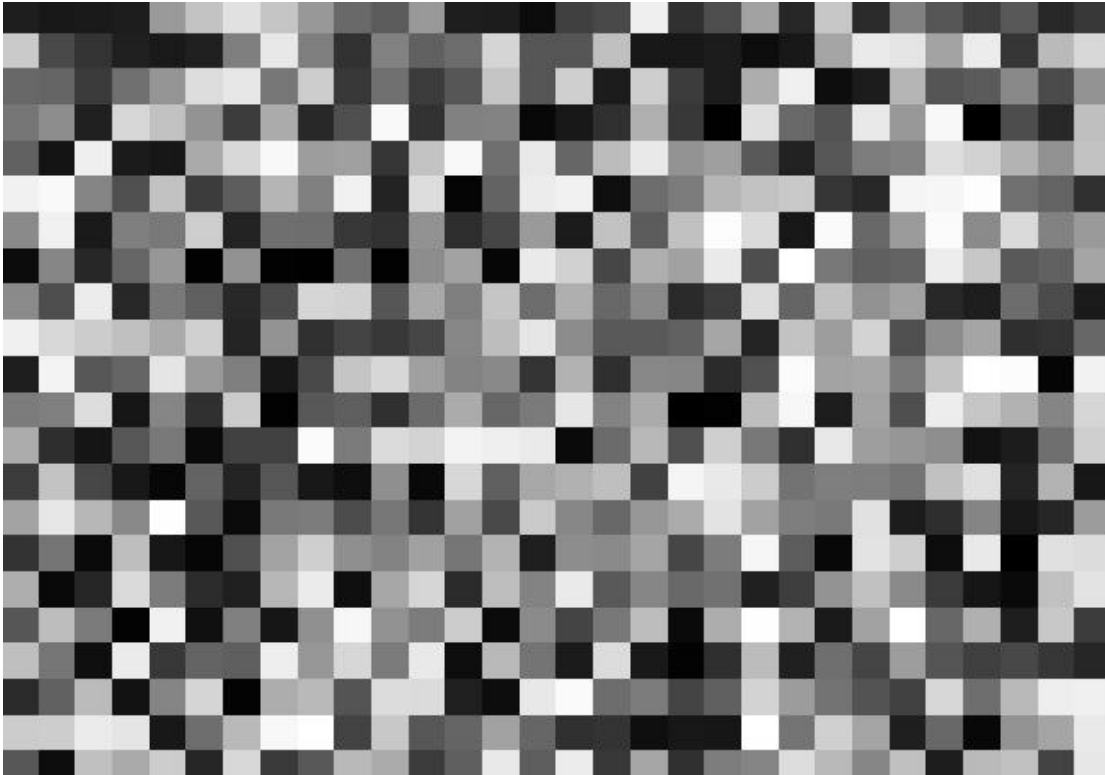
Black square, 6, (18,14)

Redundancy



What is present in this world and where is it?

Redundancy



What is present in this world and where is it?

Black pixel (0,0)

Black pixel (0,1)

Black pixel (0,2)

Black pixel (0,3)

Gray pixel (0,4)

Light gray pixel (0,4)

Redundancy



What is present in this world and where is it?

Redundancy



What is present in this world and where is it?

Branches and leaves on top
Mountain on left to center
Reflects on left to center
bottom

Dog and person on bottom
Dense trees on right

What is the message?

What are natural images?

Non Natural

Natural

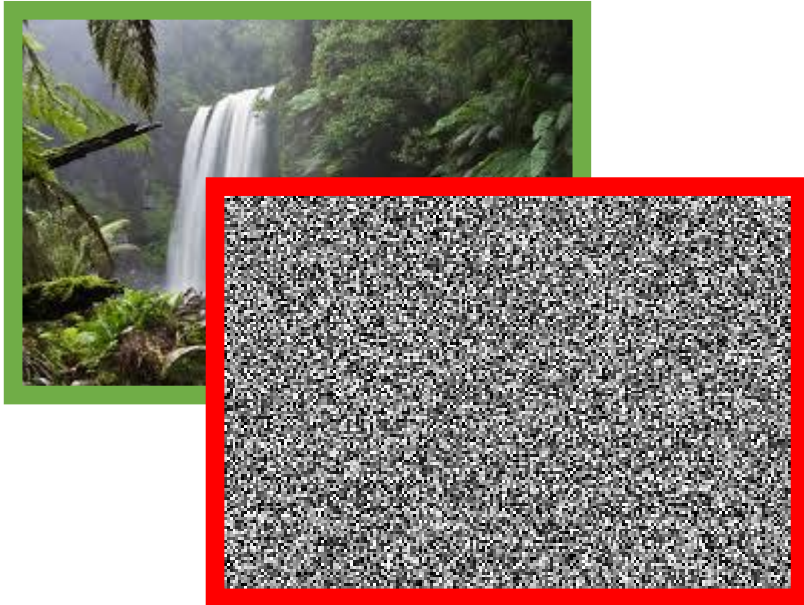


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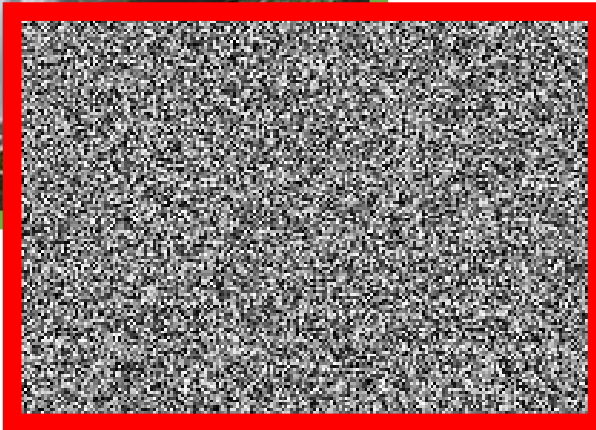


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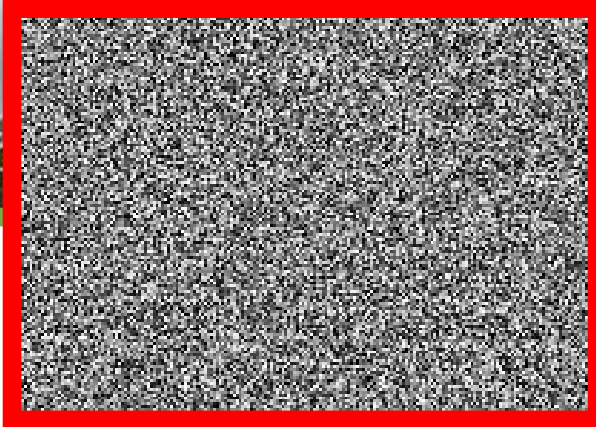


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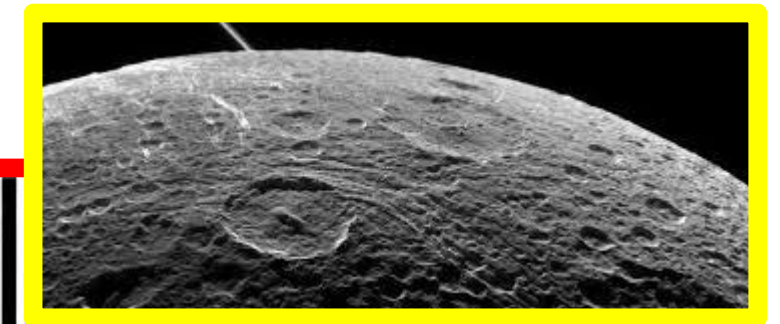
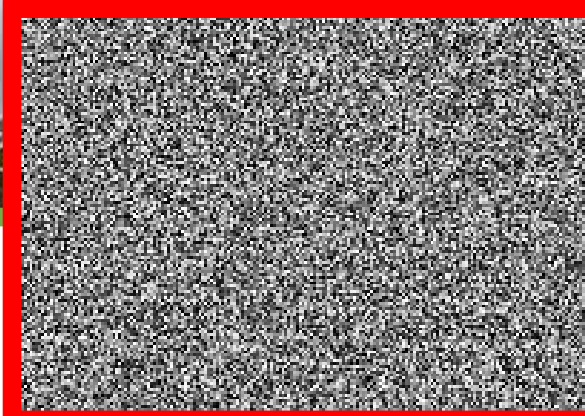


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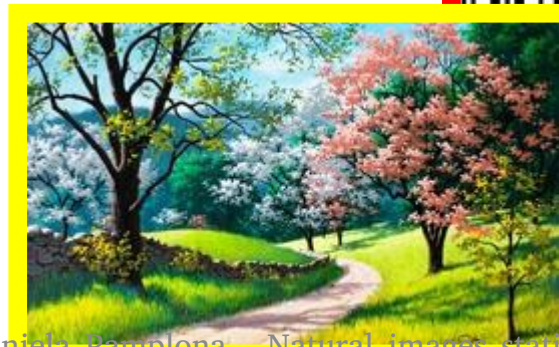
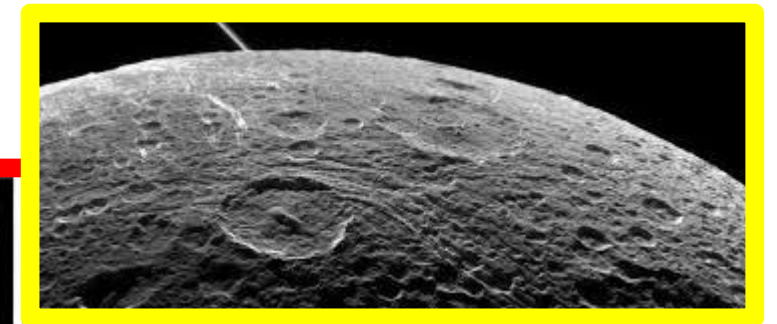
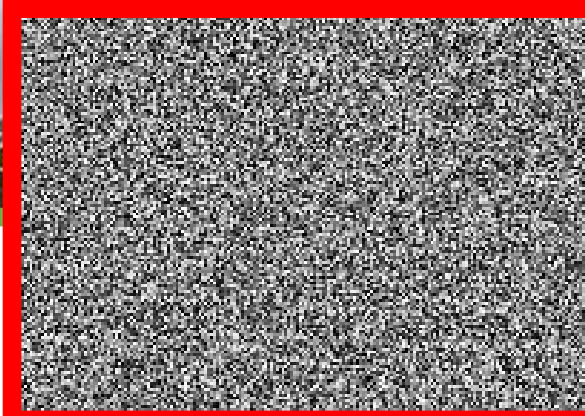


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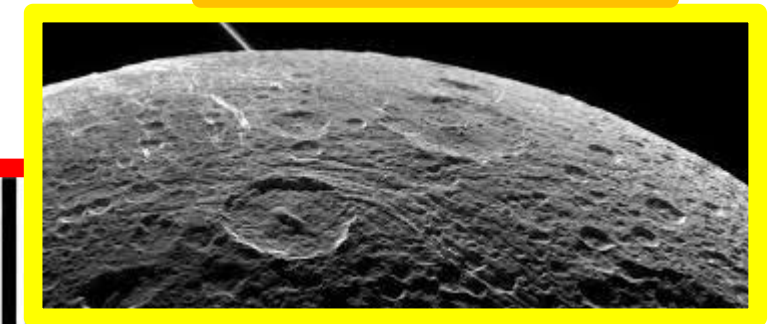
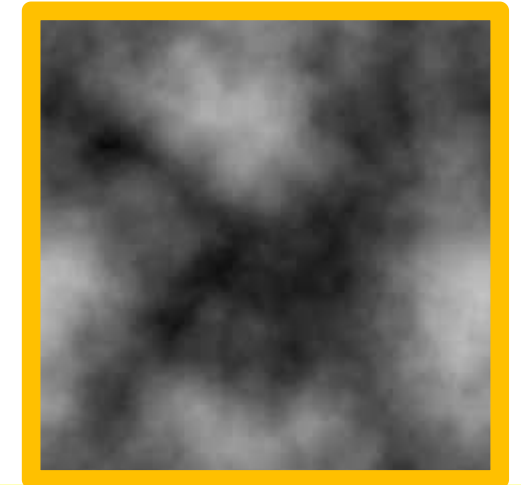
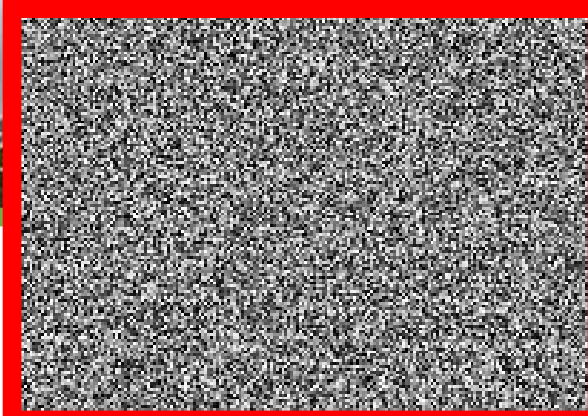


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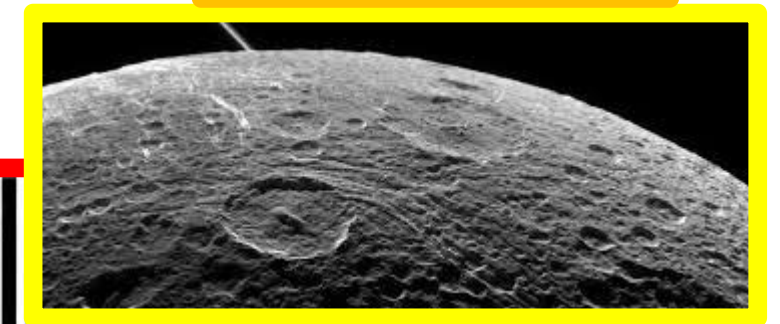
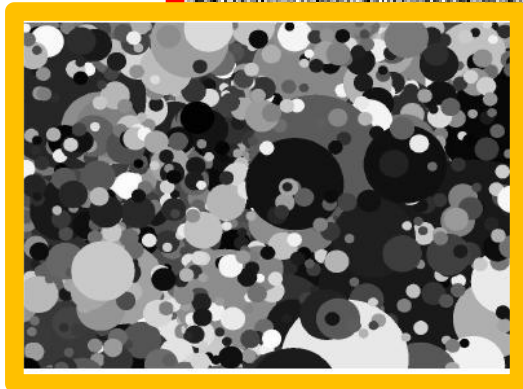
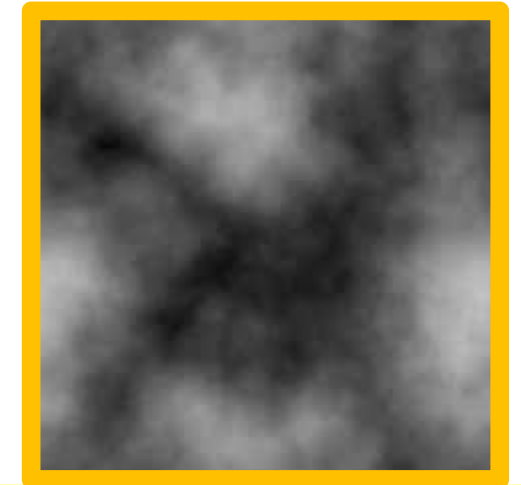
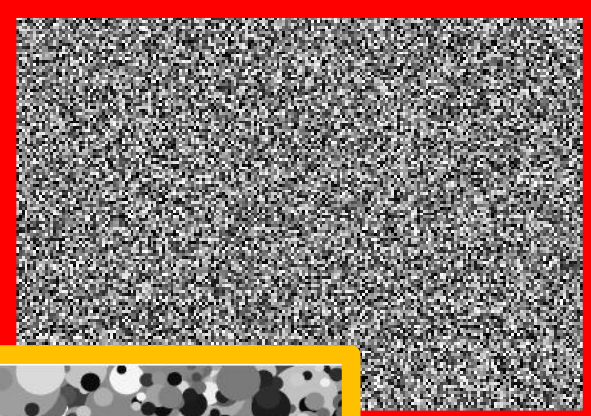


What is the message?

What are natural images?

Non Natural

Natural



Question: Can we measure the redundancy of natural images?

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Answer: No, because we do not know the probability distribution of natural images
(we do know the distribution of white noise images, e.g. $N(0,1)$)

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Question: Can we, at least, approximate it?

Answer: Yes!

Question: Can we measure the redundancy of natural images?

Answer: No, because we do not know the probability distribution of natural images
(we do know the distribution of white noise images, e.g. $N(0,1)$)

Question: Can we, at least, approximate it?

Answer: Yes!

Question: How?

Question: Can we measure the redundancy of natural images?

Answer: No, because we do not know the probability distribution of natural images
(we do know the distribution of white noise images, e.g. $N(0,1)$)

Question: Can we, at least, approximate it?

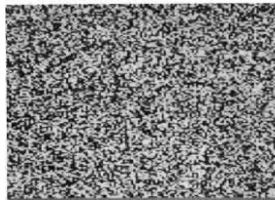
Answer: Yes!

Question: How?

Answer: Looking at the statistics of the natural images

Natural images are redundant

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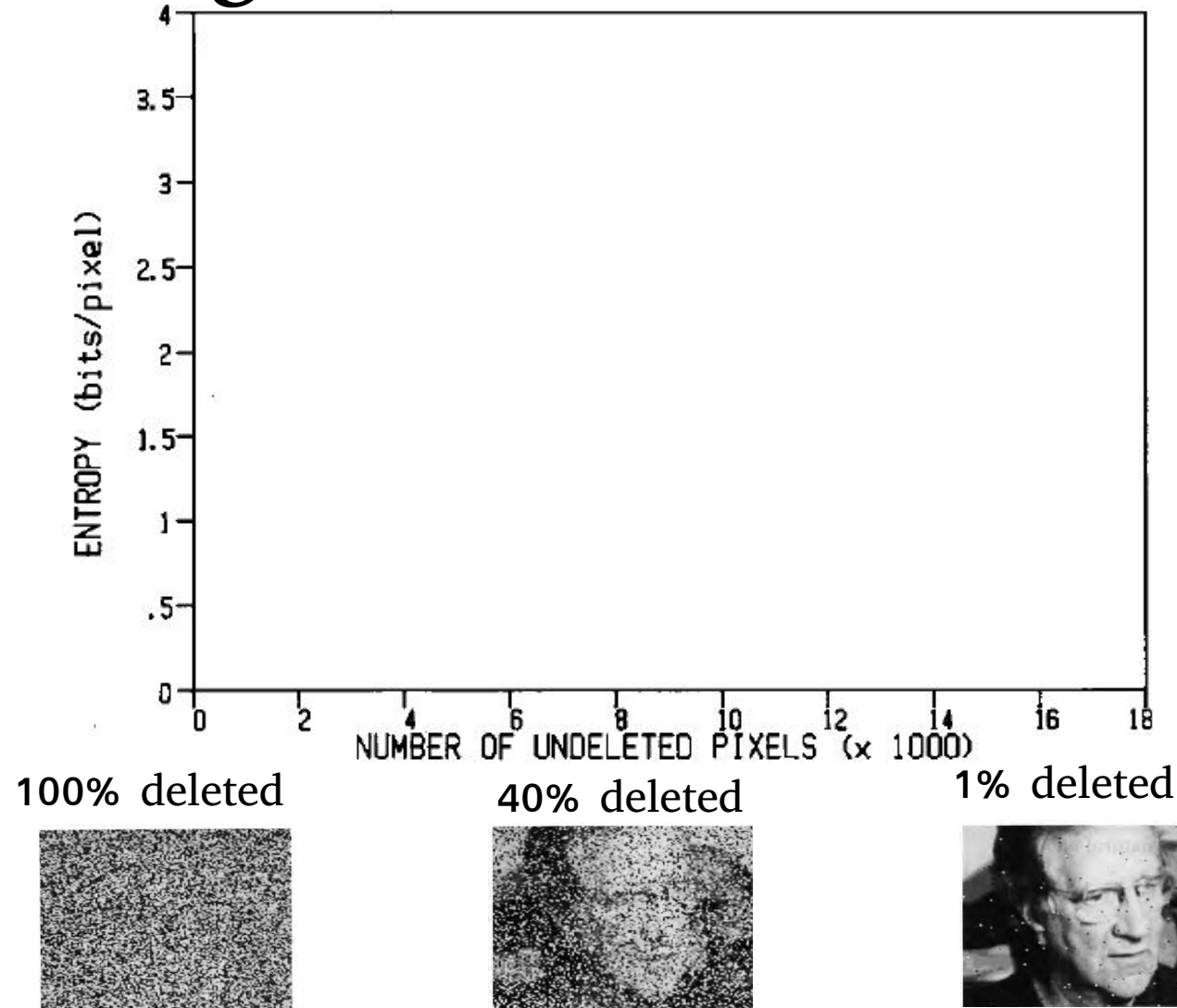
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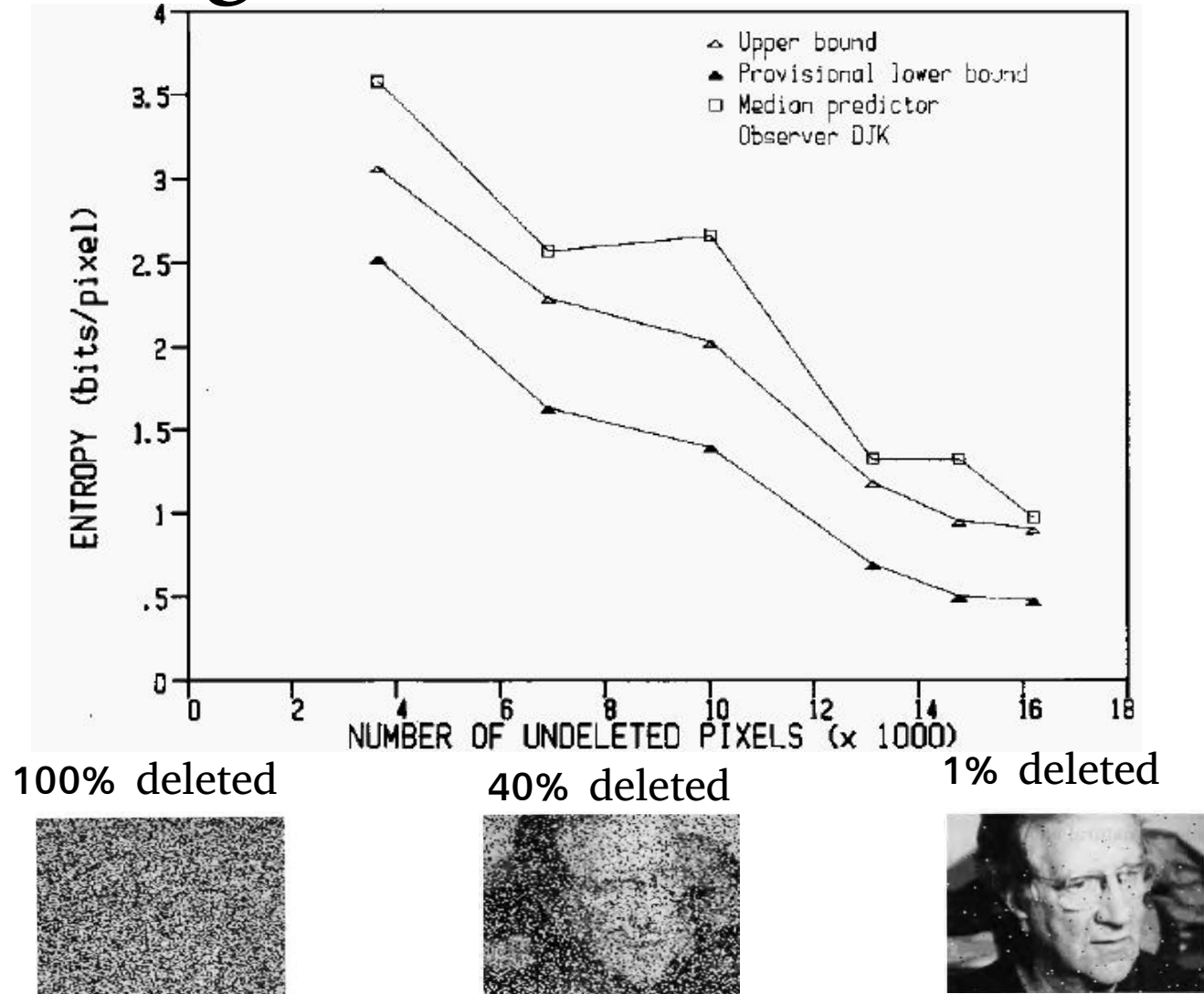
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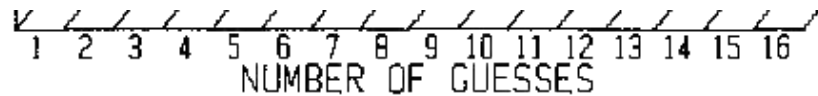
Natural images are redundant



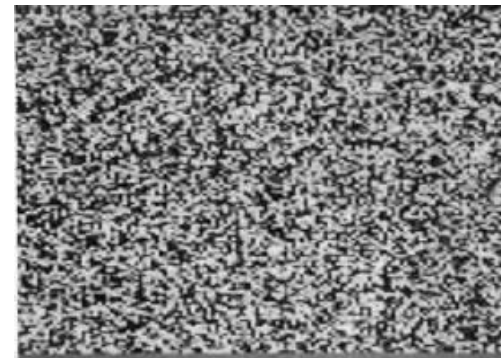
Natural images are redundant



Natural images are redundant. So what?

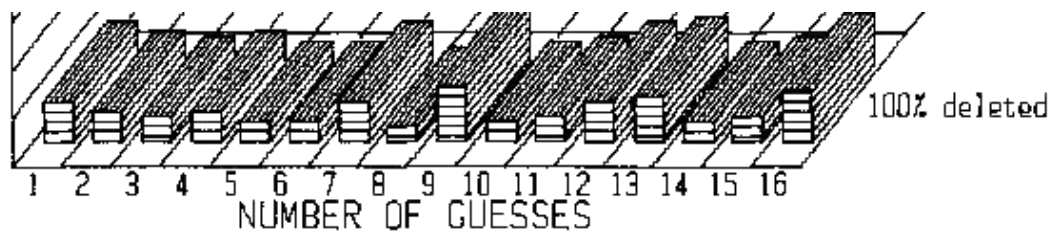


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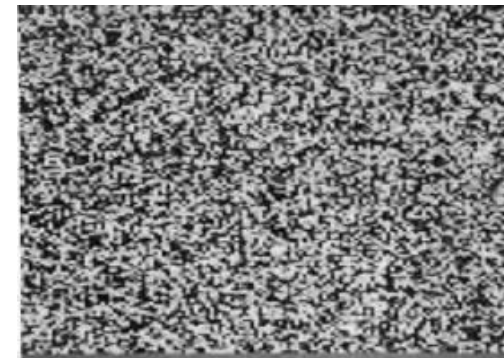


16 gray levels

Natural images are redundant. So what?

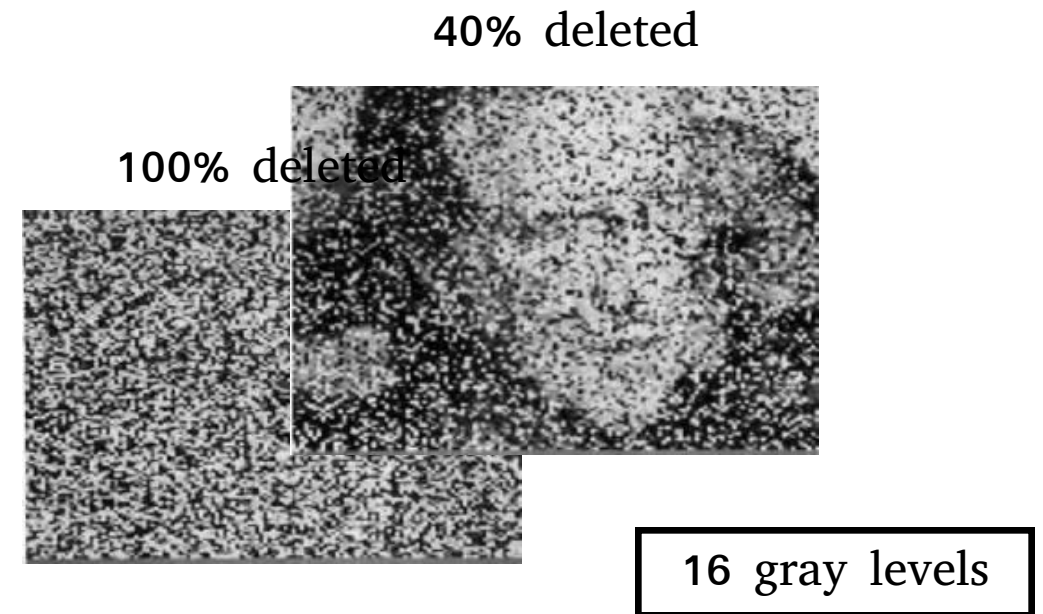
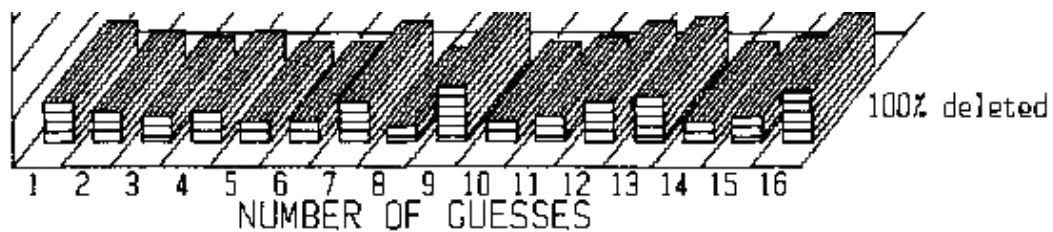


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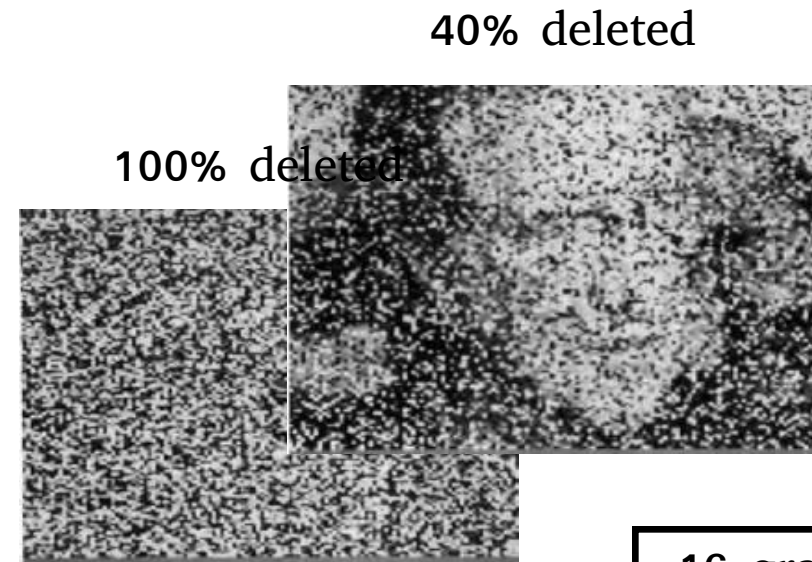
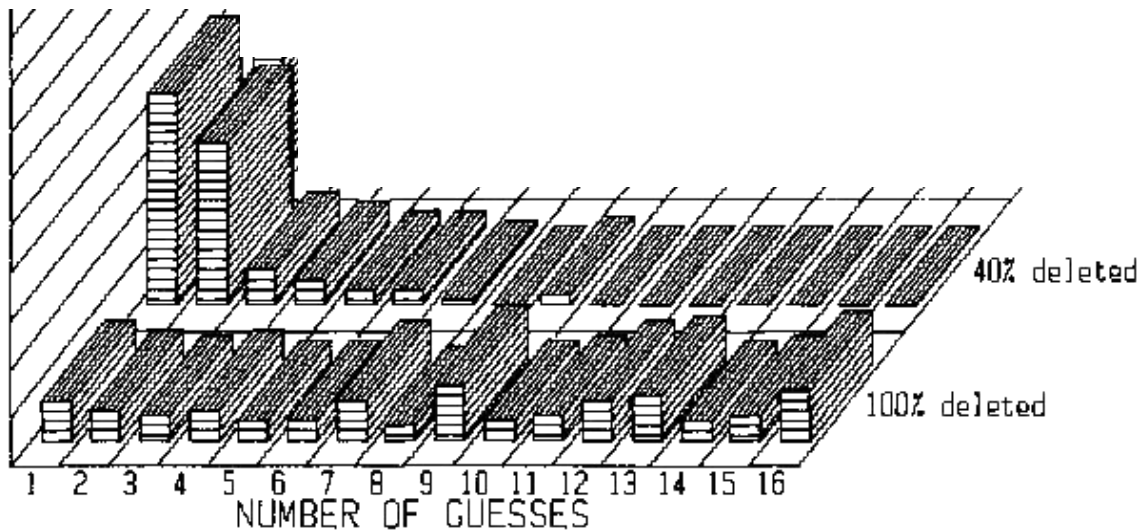


16 gray levels

Natural images are redundant. So what?

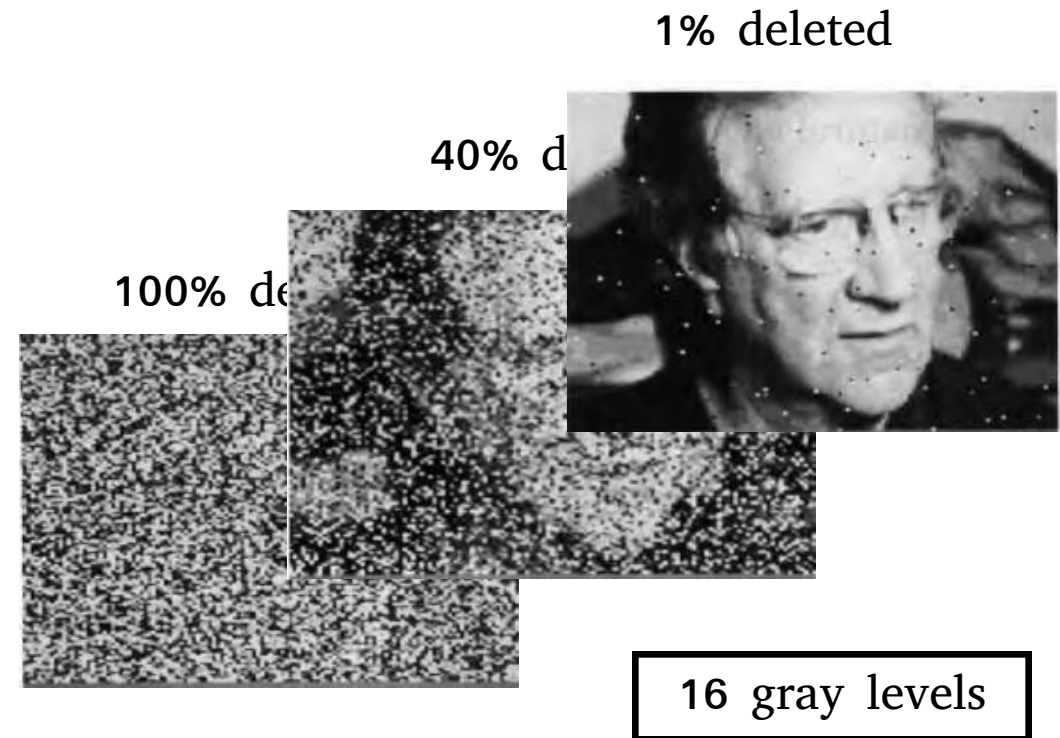
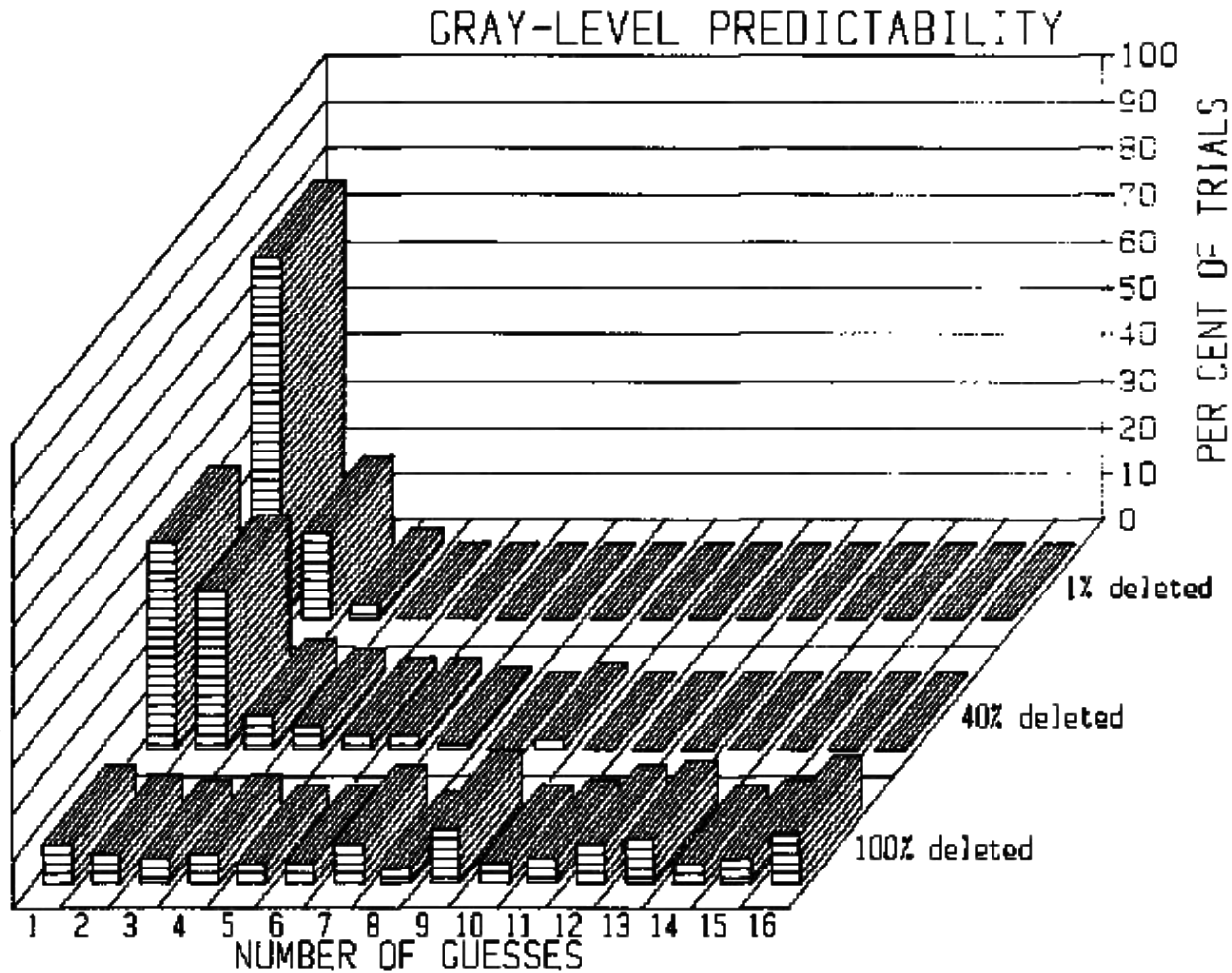


Natural images are redundant. So what?



16 gray levels

Natural images are redundant. So what?



Contents

1. What is vision?
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General idea (method of the moments):

- μ^n moment of order n : $\mu^n[X] = E[(X-\mu)^n]$
- X r. v. with n - first moments well defined
- X_1, X_2, \dots, X_k sequence of r. v.

If $\lim_{k \rightarrow \infty} \mu^n[X^k] = \mu^n[X]$ then $X^k \xrightarrow{d} X$

Moments

- $\mu^1[X] = E[X - \mu] = 0$
- $\mu^2[X] = E[(X - \mu)^2]$ (auto correlation)
- $\mu^3[X] = E[(X - \mu)^3]$ (skewness)
- $\mu^4[X] = E[(X - \mu)^4]$ (kurtosis)
- ...

Fourier Transform

- The Fourier transform decomposes a function of time (a signal) into the frequencies that make it up.

1D

$$F(x) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi(x\frac{n}{N})}$$

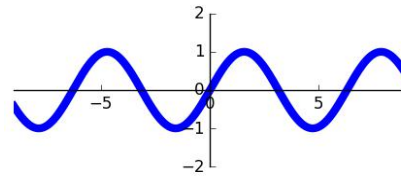
$$f(n) = \frac{1}{N} \sum_{x=0}^{N-1} F(x) e^{j2\pi(x\frac{n}{N})}$$

$$PS(x) = |F(x)|^2$$

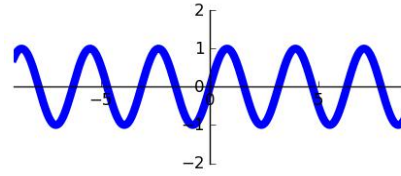
1D Fourier Transform

time domain

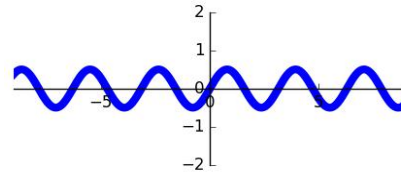
$$\sin(x)$$



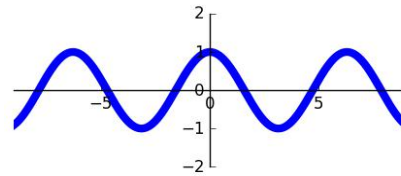
$$\sin(2x)$$



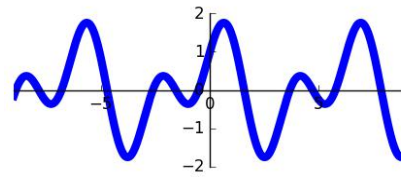
$$0.5\sin(2x)$$



$$\sin(x+\pi/2)$$



$$\sin(2x)+\sin(x+\pi/2)$$

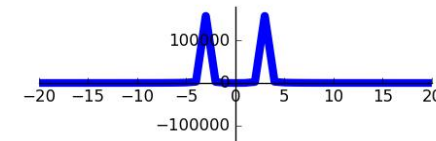
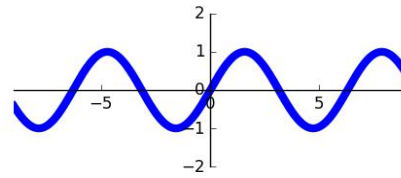


1D Fourier Transform

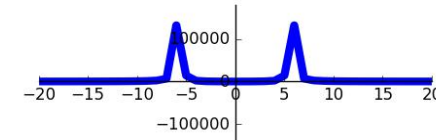
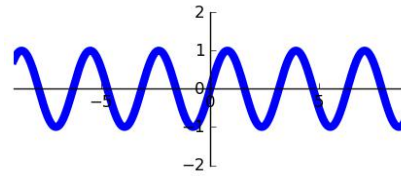
time domain

ps (fft)

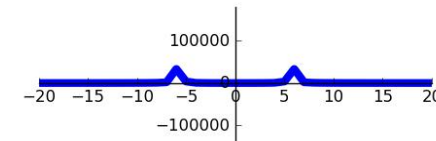
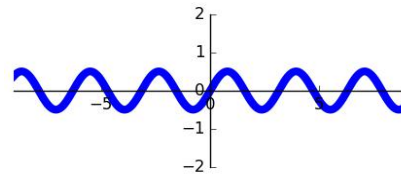
$\sin(x)$



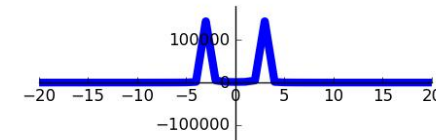
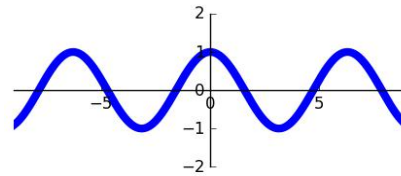
$\sin(2x)$



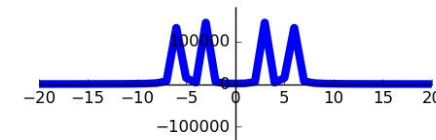
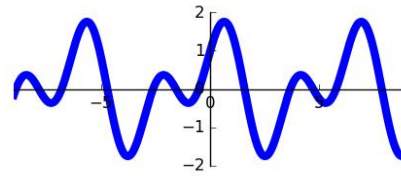
$0.5\sin(2x)$



$\sin(x+\pi/2)$



$\sin(2x)+\sin(x+\pi/2)$



Fourier Transform

- The Fourier transform decomposes a function of time (a signal) into the frequencies that make it up.

$$\text{1D}$$
$$F(x) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi(x\frac{n}{N})}$$

$$f(n) = \frac{1}{N} \sum_{x=0}^{N-1} F(x) e^{j2\pi(x\frac{n}{N})}$$

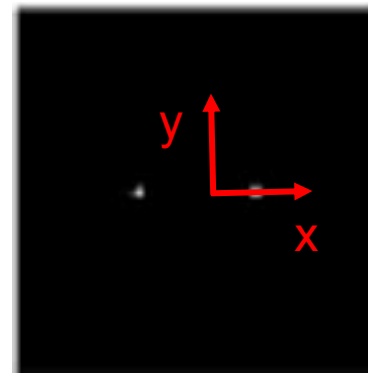
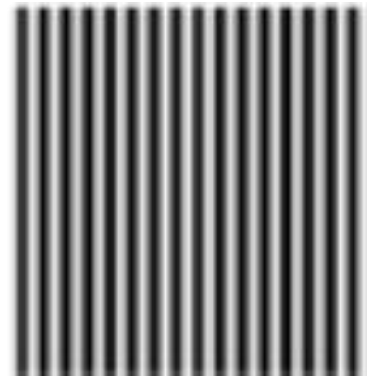
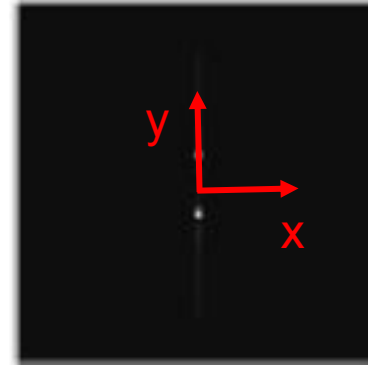
$$PS(x) = |F(x)|^2$$

$$\text{2D}$$
$$F(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{-j2\pi(x\frac{m}{M}+y\frac{n}{N})}$$

$$f(m,n) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(x,y) e^{j2\pi(x\frac{m}{M}+y\frac{n}{N})}$$

$$PS(x,y) = |F(x,y)|^2$$

2 D Fourier Transform of waves



Fourier Transform

- The Fourier transform decomposes a function of time (a signal) into the frequencies that make it up.

$$\begin{aligned} &\text{1D} \\ F(x) &= \sum_{n=0}^{N-1} f(n) e^{-j2\pi(x\frac{n}{N})} \\ f(n) &= \frac{1}{N} \sum_{x=0}^{N-1} F(x) e^{j2\pi(x\frac{n}{N})} \end{aligned}$$

$$PS(x) = |F(x)|^2$$

$$\begin{aligned} &\text{2D} \\ F(x,y) &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{-j2\pi(x\frac{m}{M}+y\frac{n}{N})} \\ f(m,n) &= \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(x,y) e^{j2\pi(x\frac{m}{M}+y\frac{n}{N})} \end{aligned}$$

$$PS(x,y) = |F(x,y)|^2$$

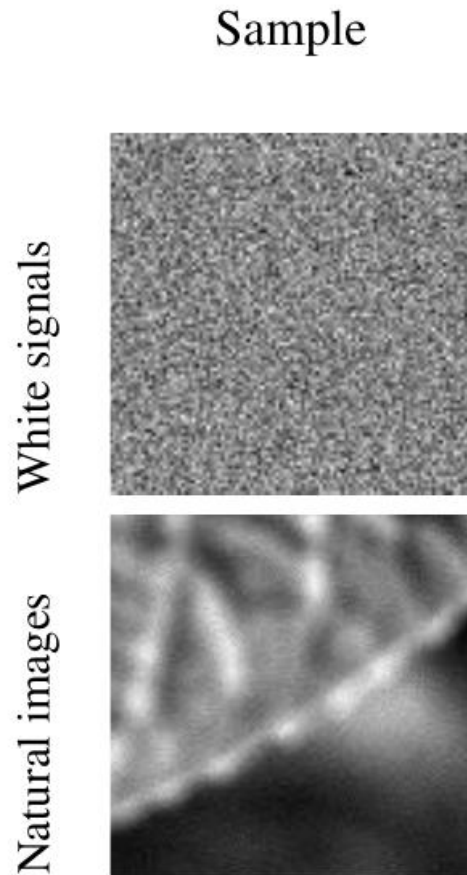
Wiener-Khinchin theorem: If $X = \{x_{-1}, \dots, x_{-n}\}$ is a stationary process and $\mu^2[X]$ exists and is finite, then:

$$E[PS(X)] = F(\mu^2[X])$$

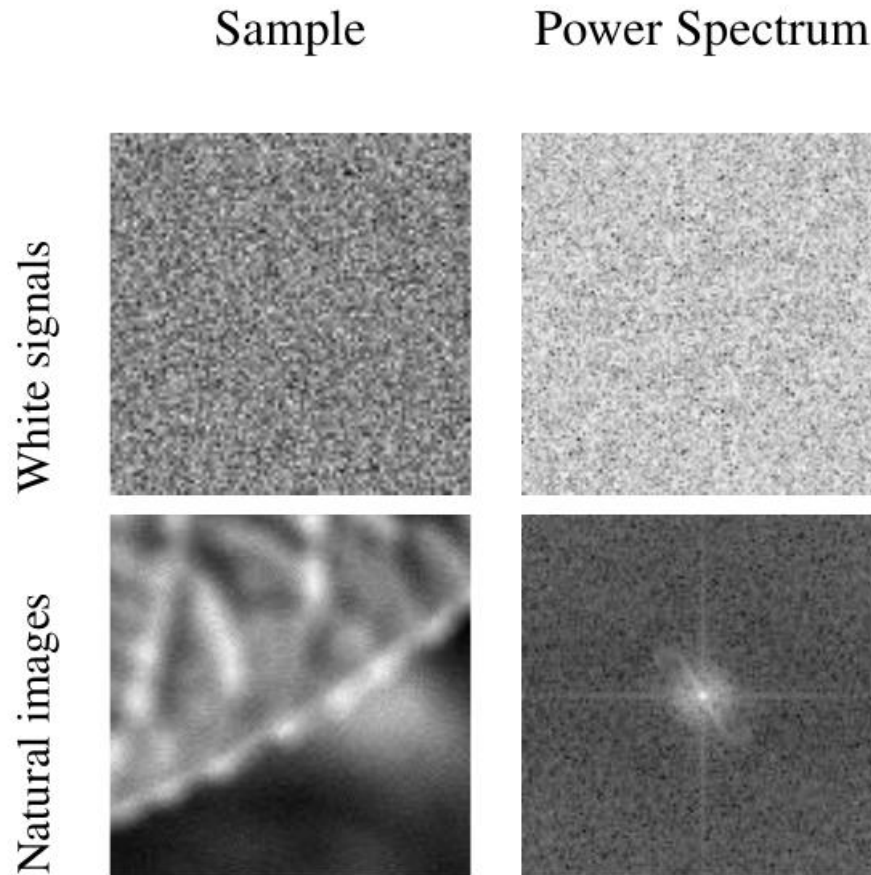
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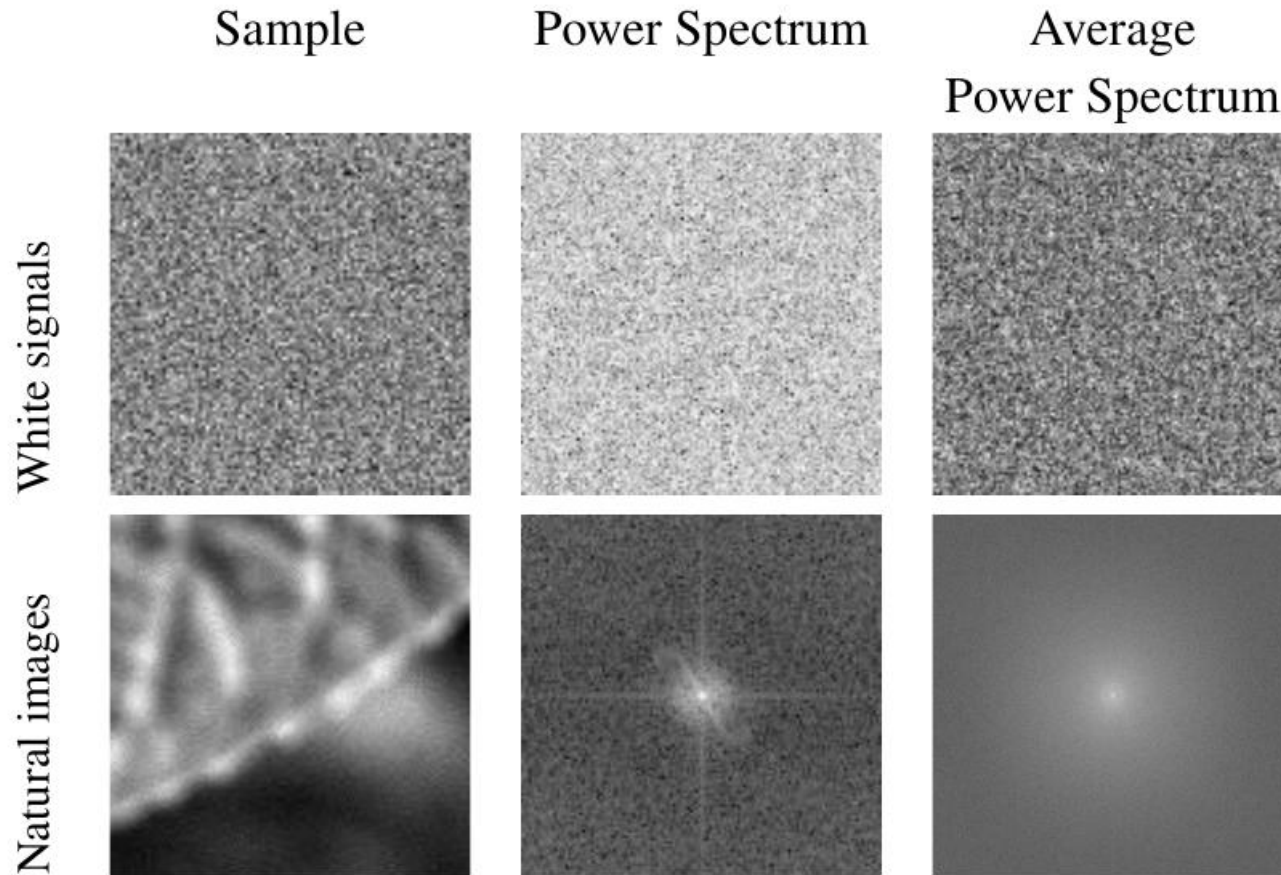
Power Spectrum of Natural Images



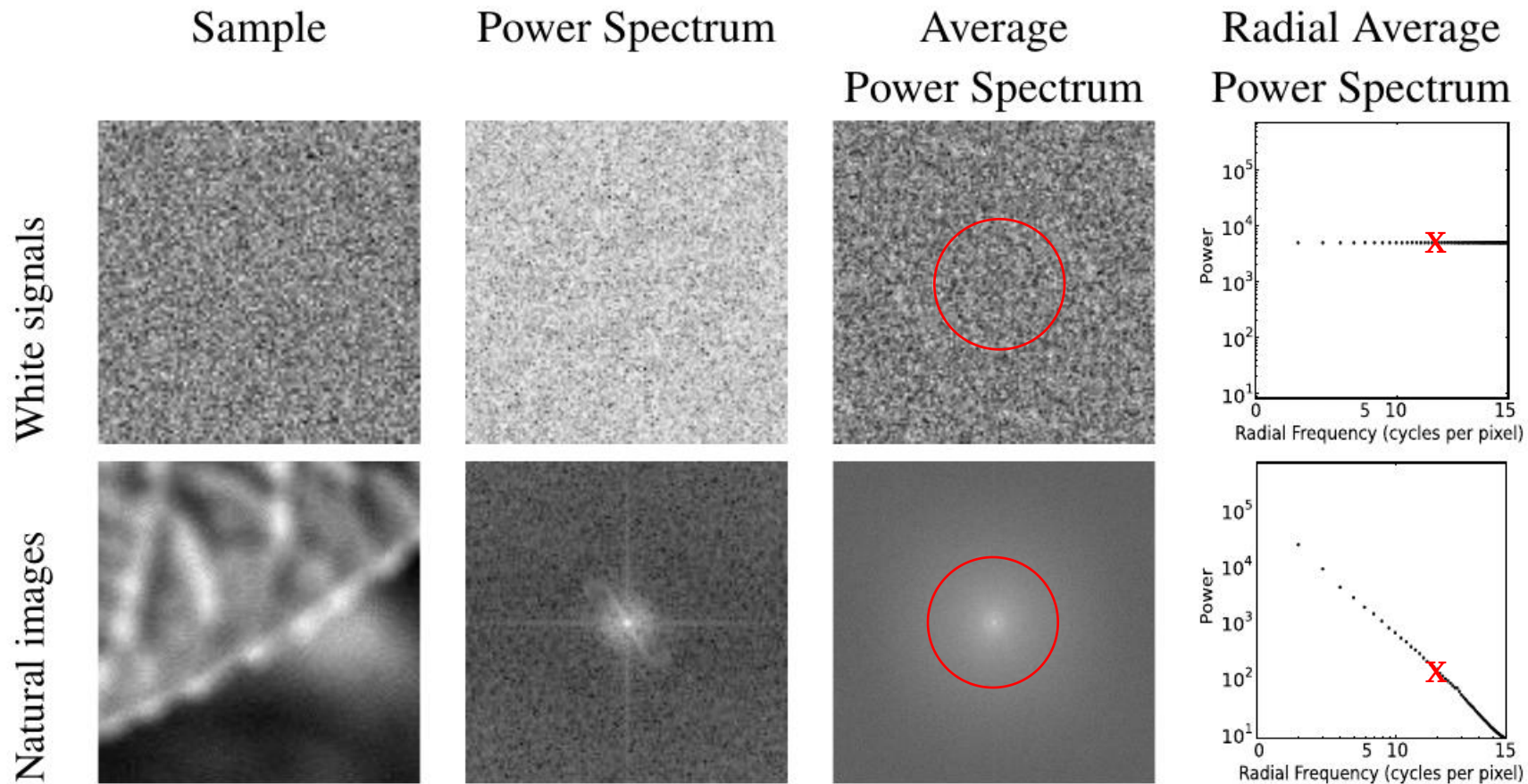
Power Spectrum of Natural Images



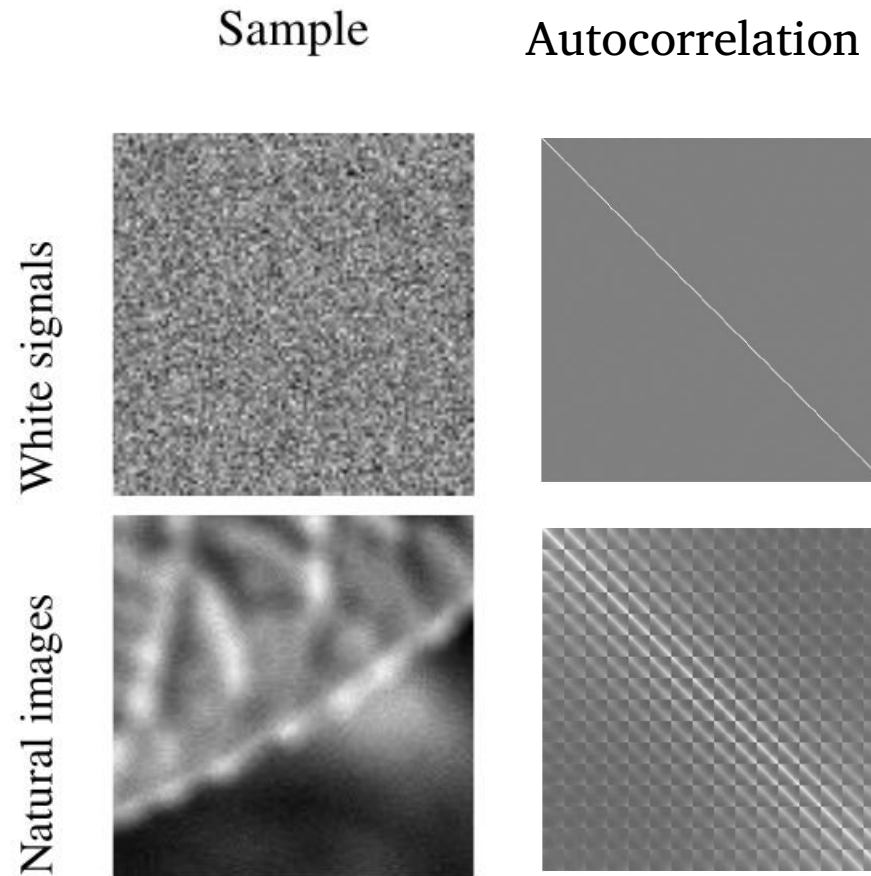
Power Spectrum of Natural Images



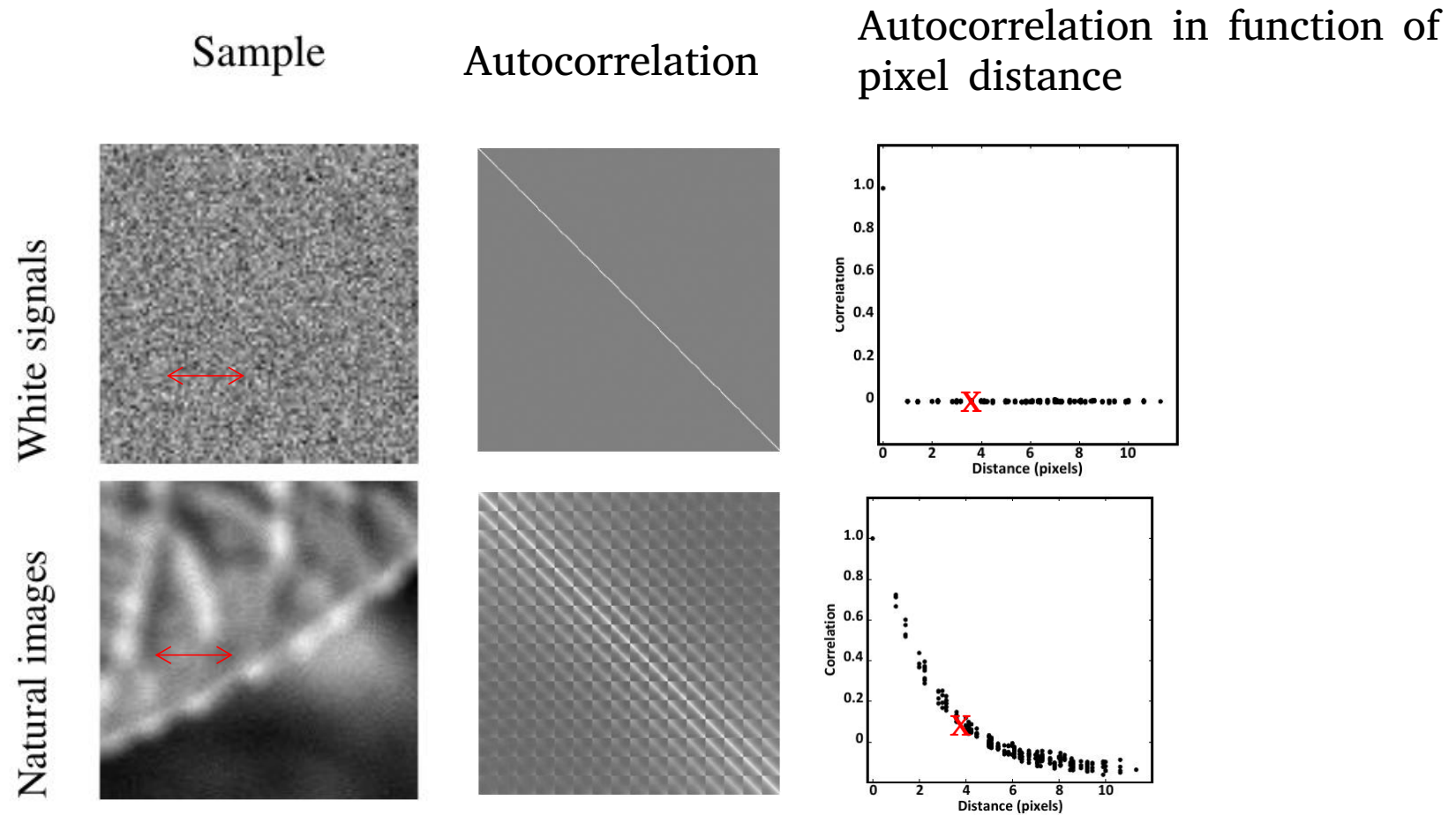
Power Spectrum of Natural Images



Auto correlation of natural images



Auto correlation of natural images



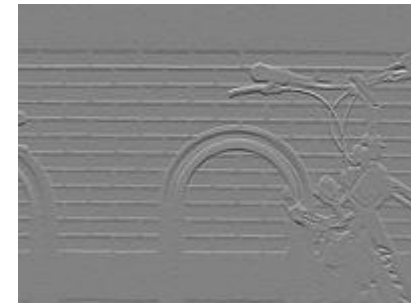
Sobel Filters: Introduction

- It performs a 2-D spatial gradient measurement on images in order to emphasize edges.
- Pairs of convolution kernels (K_x and K_y) designed to respond maximally to edges running vertically and horizontally relative to the pixel grid

$$G_x = I * K_x$$



$$G_y = I * K_y$$



Sobel Filters: Introduction

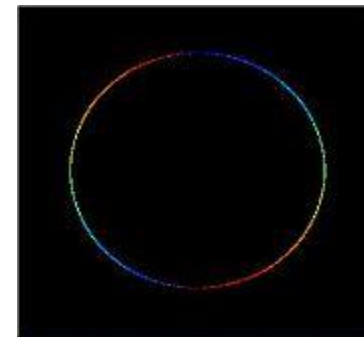
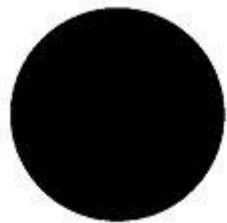
- Magnitude of the gradient:

$$|G| = \sqrt{G_x^2 + G_y^2}$$



- Angle of the gradient:

$$\theta = \arctan(G_y/G_x)$$

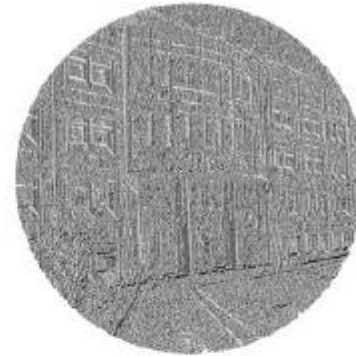


Analysis of edges orientations

A. Original photograph



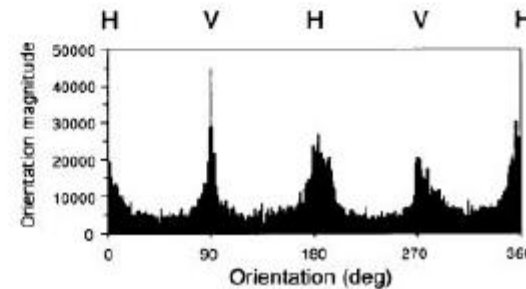
B. Sobel direction filter



C. Sobel magnitude filter



D. Analysis of upright scene



Analysis of edges orientations

Indoor



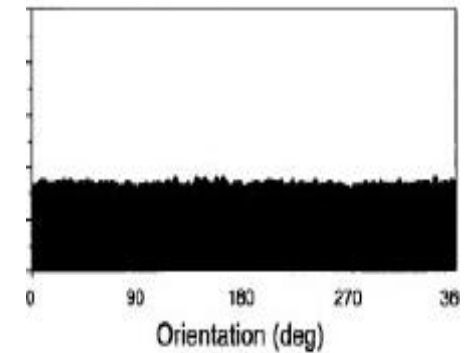
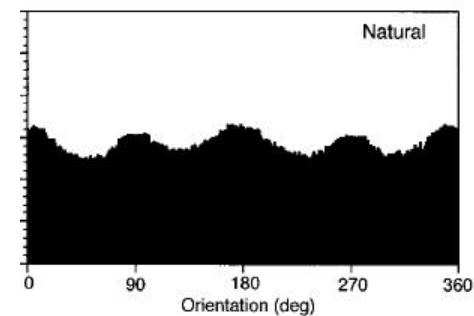
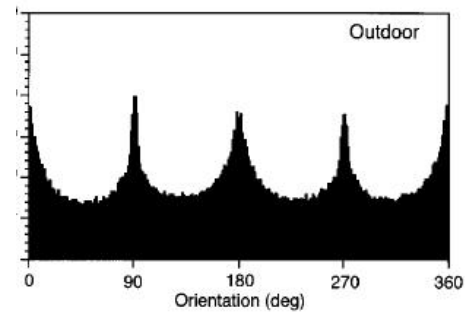
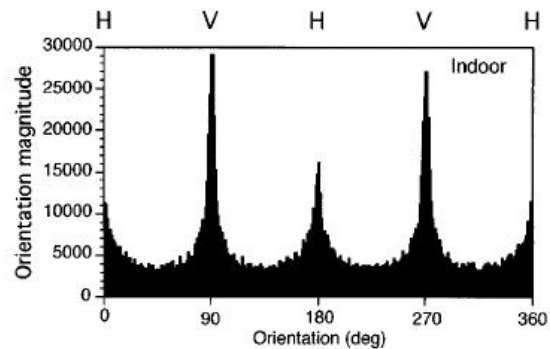
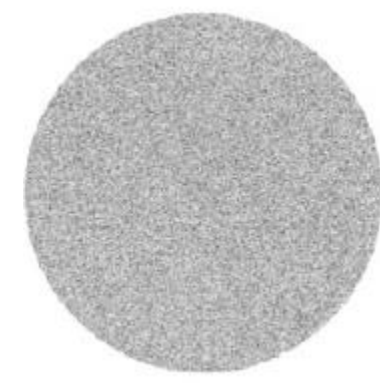
Outdoor



Natural

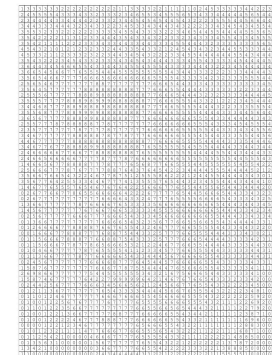


White Noise

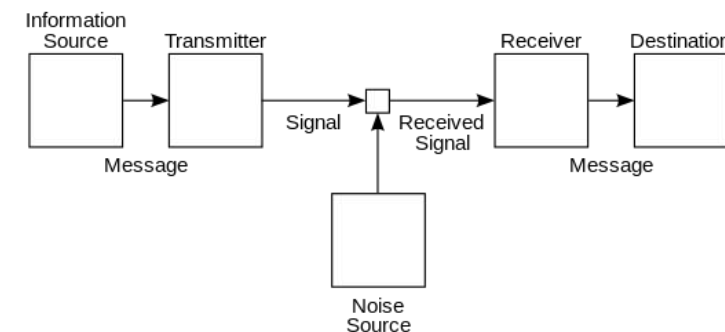


Summary

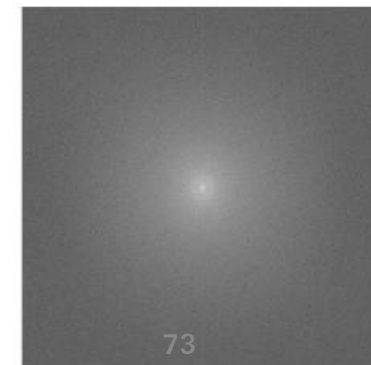
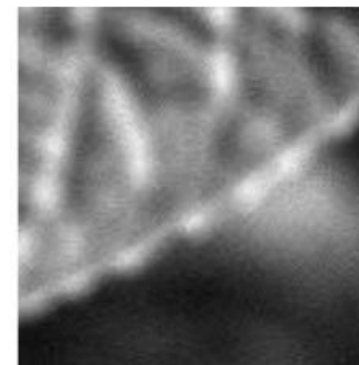
1. What is vision?



2. The redundancy reduction hypothesis



3. Natural images statistics



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