```
def count_heads(prob : float, n : int):
    x = 0
    while(n > 0):
        d = bernoulli.rvs(size=1, p=prob)[0]
        x = x + d
        n = n - 1
    return x
```

Fig. 1. Program to *count* heads over *n* bernoulli trials.

```
if (prob <= 0.5) E(x) < 0.4 * n else E(x) >= 0.6 * n
```

Fig. 2. Assert we want to check for failure.

```
double prob, path_prob = 1, choice_prob = 1;
int sum = 0, n = 0;

klee_make_symbolic(&prob, f"prob_sym_{i}");
klee_make_symbolic(&n, f"n_symbolic");
for i in range(n):
    int d;
    klee_make_symbolic(&d, f"d_sym_{i}");

    d = bernoulli(prob);
    (d == 1) ? choice_prob = prob : choice_prob = (1 - prob);
    path_prob = path_prob * choice_prob;

    sum = sum + d;

klee_dump(path_prob)
klee_dump(sum) // E[heads in "n" runs]
```

Fig. 3. Program listing for *n* bernoulli trails experiment with transformation for KLEE.

We consider a independent *bernoulli* trials here of flipping a fair coin "n" times.

$$choice\_prob = \begin{cases} p & \text{if } d \text{ value is 1 corresponding to getting a "heads"} \\ 1-p & \text{if } d \text{ value is 0 corresponding to getting a "tails"} \end{cases}$$

concretely, on the  $i^{th}$  run  $\vec{d}$  can have a value as below, one-hot encoded w.r.t the outcome of heads or tails.

$$\vec{d}_i = encode(<0,0,0,1,1,0,1,1,1,0>)$$

Based on the value of the  $\vec{d}_i$ , we get  $w_i$  value using *choice\_prob*.

$$w_i = (p)^{x_i} * (1-p)^{n-x_i}$$
 (1)

where  $x_i$  denotes the number of *heads* in the  $i^{th}$  randomized run and for n runs. n = 10 for the case in the above example.

$$Objective_1 = maximize(\sum_{i=1}^k (p)^{x_i} * (1-p)^{n-x_i}) \quad | \forall (i,j) \ [x_i \neq x_j]$$
 (4)

Fig. 4. Optimization Expression for k randomized paths

$$w_i = (p)^{x_i} * (1-p)^{10-x_i}$$
 (2)

We consider top "k" randomized runs now for the optimization query. The expression for optimization thus becomes

$$maximize(\sum_{i=1}^{k} w_i)$$

On substituting the value of  $w_i$  from (1).

$$maximize(\sum_{i=1}^{k} (p)^{x_i} * (1-p)^{n-x_i})$$
 (3)

After performing the optimization above, we get different values of  $\vec{d}_i$ . For  $i^{th}$  randomized run. we get a single one hot encoded  $\vec{d}$  vector. We show below the encoding for a few i values.

$$\begin{split} \vec{d}_1 &= encode(<0,0,0,1,1,0,1,1,1,0>)\\ \vec{d}_3 &= encode(<0,1,0,1,1,0,0,0,1,0>)\\ \vec{d}_4 &= encode(<1,0,1,0,1,0,1,1,0,1>) \end{split}$$

....

We run the optimization by renaming the k pse variables set appropriately and then impose the distinct clause so that we don't run the optimization on the same randomized runs again.

$$\forall (i,j) \ [\vec{d}_i \neq \vec{d}_j] \tag{5}$$

We approximate the value of *expected* heads in the above program, with the following equations.

$$w_i = \prod_{i=1}^n choice\_prob_i(j), \quad sum_i = \sum_{i=1}^n components(\vec{d}_i), \tag{6}$$

$$EV(heads) = (\sum_{i=1}^{k} w_i * sum_i), \quad Error = n * prob - EV(heads)$$
 (7)

where both  $w_i$  and  $sum_i$  can both be computed from the corresponding  $\vec{d}_i$  expression we get from the *model* of the *optimization* query Eq 4

For k = 5 & n = 10 the two constraint sets and optimization expressions are as follows:

$$w_1 = \prod_{j=1}^{10} choice\_prob_1(j), \quad sum_1 = \sum_{j=1}^{10} components(\vec{d}_1), \tag{8}$$

$$w_2 = \prod_{j=1}^{10} choice\_prob_2(j), \quad sum_2 = \sum_{j=1}^{10} components(\vec{d}_2), \tag{9}$$

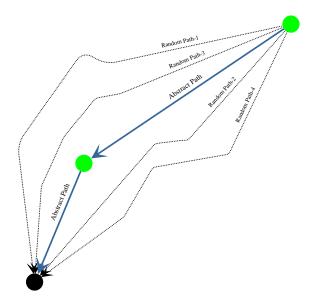


Fig. 5. Multiple random paths corresponding to different  $\vec{d}$ 's but a single abstract path.

prob = 
$$0.435$$
, n = 8 (8 coin flips), k = 240 (randomized paths)  
prob =  $0.551$ , n = 5 (5 coin flips), k = 27 (randomized paths)

Fig. 6. Values for Assert failure.

$$w_3 = \prod_{j=1}^{10} choice\_prob_3(j), \quad sum_3 = \sum_{j=1}^{10} components(\vec{d}_3), \tag{10}$$

$$w_4 = \prod_{j=1}^{10} choice\_prob_4(j), \quad sum_4 = \sum_{j=1}^{10} components(\vec{d}_4), \tag{11}$$

$$w_5 = \prod_{j=1}^{10} choice\_prob_5(j), \quad sum_5 = \sum_{j=1}^{10} components(\vec{d}_5), \tag{12}$$

$$\vec{d}_1 \neq \vec{d}_2 \neq \vec{d}_3 \neq \vec{d}_4 \neq \vec{d}_5$$
 (13)

$$EV(heads) = (\sum_{i=1}^{k} w_i * sum_i), \tag{14}$$

$$EV(heads) = (w_1 * sum_1 + w_2 * sum_2 + w_3 * sum_3 + w_4 * sum_4 + w_5 * sum_5)$$
 (15)

 $Objective_2 = maximize(w_1 * sum_1 + w_2 * sum_2 + w_3 * sum_3 + w_4 * sum_4 + w_5 * sum_5)$  (16)

We can make multiple optimization *objectives* be fulfilled in one query, but the best results are produced for the  $Objective_2$  since it minimizes the error most.

$$Objective_2 = maximize(\sum_{i=1}^k w_i * x_i)$$
 (17)

## 1 QUICKSORT EXAMPLE

Consider a pivot vector  $\vec{p_{abs}}$  that KLEE produces over a single abstract path. We consider a 10 element array as shown above. The *pivot* indices for  $\vec{p_{abs}}$  as choosen from a *random* distribution.

A possible assignment of the indexes choosen as *pivot* indices along this *abstract* path can be as shown below.

$$\vec{p_{abs}} = \boxed{\begin{array}{c|cccc} a_9 & a_8 & a_2 & a_3 & a_8 & a_8 \end{array}} \tag{19}$$

For same forall values in arr[10], we can have different random runs and on each run, the  $\vec{pivot}$  vector will have a different assignment of values as indices. We are showing 5 different  $\vec{pvot}_{r_j}$  vectors, each corresponding to a valid random run j.

The probability mass associated with the  $i_{th}$  random run as is given below considering each pivot selection being an independent selection. The variable  $count_i$  denotes the number of comparisions made in  $t^{th}$  random run.

$$w_i = (1/n) * \prod_{j=1}^{k} 1/(\#size_{partition_j}) \quad | \quad for \ k-partitons$$
 (27)

$$E[comparisions] = \sum_{i=1}^{r} (w_i * count_i) | for r - runs$$
 (28)