PSE Quant Sampling Algorithm

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We try to formulate a way to compute path probabilities using `symbolic execution` and `testing` based technique.

```
int main(void)
{
        int a; // unintialized
        int d = std::uniform_distribution<rd_seed>(0, 650);

        // forall variable : (INT_MIN to INT_MAX)
        klee_make_symbolic(&a, sizeof(a), "a_sym");

        // PSE variable : Uniformly distributed [0 to 650]
        make_pse_symbolic<int>(&d, sizeof(d), "d_prob_sym", 0, 650);

        int c = a + 100;

        // case 1 : Pure Forall Predicate
        if (a > 50) {
          c = a + 75;
        } else {
          c = a - 75;
        }

        // case 2 : Pure PSE Predicate
        if (d > 60) d = 250;

        // case 3 : Dependence Case
        if (c > d) c = d;

        // Probabilistic query : assert(P(c != d) < 0.5)
        // Optimize here :
        //        Optimal value of forall (a) such that P(c != d) is close to 0.5
        return 0;
}
```

Algorithm 1 Candidates : (Testing Based Estimation)

```
for each p ∈ Paths do
    c := ConstraintSet(p)                               ▷ Path Constraints for p
    m := Optimize(query, c)                   ▷ solution for the path constraints
    concreteSet = {}
    for each v ∈ ForallVars(p) do                           ▷ ForallVars p → forall
        concreteSet.append({key : v, val : m[v]})             ▷ Candidate Values
    end for each
    executeCV(program, concreteSet)
end for each
```

Algorithm 2 executeCV : PSE Sampled Normal Execution

```
function EXECUTECV(P : program, C : concreteSet)
    for each v ∈ ForallVars(p) do
        value(v) := concreteSet(v)               ▷ Use values from ConcreteSet
    end for each
    ...                                         ▷ proceed with normal execution
end function
```

For the sample program given above, we first resort to using `symbolic execution` to generate `path constraints` for all the feasible paths that this program can take and then convert the `path constraints` into an `formal logic` optimization problem that gives an `assignment` to `forall` variables such that it leads to optimum violation of the *query*.

```python
def generateCandidates(k: int): # Candidates Algorithm
        opt = z3.Optimize()
        a = z3.Int("a_sym")
        d = z3.Int("d_prob_sym")

        opt.add(d >= 0)
        opt.add(d <= 650)
        opt.add(a > 50)
        opt.add(z3.Not(d > 60))
        opt.add(a + 75 > d)

        opt.maximize(a - d - 75)          # Query to optimize
        n = 0
        while opt.check() == z3.sat and n != k:
                m = opt.model()
                n += 1
                print("%s = %s" % (a, m[a]))
                print("%s = %s" % (d, m[d]))
                opt.add(a != m[a])
```

We now explore a slightly different example which is more involved in terms of the constraints and query that the user can pose at the end of the `symbolic execution`. In the below example we bound the values for `foralls` for example sake.

```
// forall variable
klee_make_symbolic(&a, sizeof(a), "a_sym");         // [0, 1]
klee_make_symbolic(&c, sizeof(b), "c_sym");         // [1, 10]
klee_make_symbolic(&d, sizeof(c), "d_sym");         // [0, 5]
klee_make_symbolic(&win, sizeof(win), "win_sym"); // win == 1

// PSE variable
make_pse_symbolic<int>(&b, sizeof(b), "b_prob_sym", 0, 1);
make_pse_symbolic<int>(&e, sizeof(e), "e_prob_sym", 1, 6);

klee_assume(a >= 0 && a <= 1);
klee_assume(c >= 1 && c <= 10);
klee_assume(d >= 0 && d <= 5);

if (a > b)                                       // maximize a
{
        if (c + e < 15)                          // minimize c
        {
                win = 1;
                win_ones++;
        }
        else
        {
                win = 0;
                win_zeros++;
        }
}
else
{
        if (d + e > 1)                          // maximize d
        {
                win = 1;
                win_ones++;
        }
        else
        {
                win = 0;
                win_zeros++;
        }
}
```

Below we show a sample of the type of the `queries` that a user can make in the context of the example shown above.

```
assert(P(win == 1) > 0.8);
```

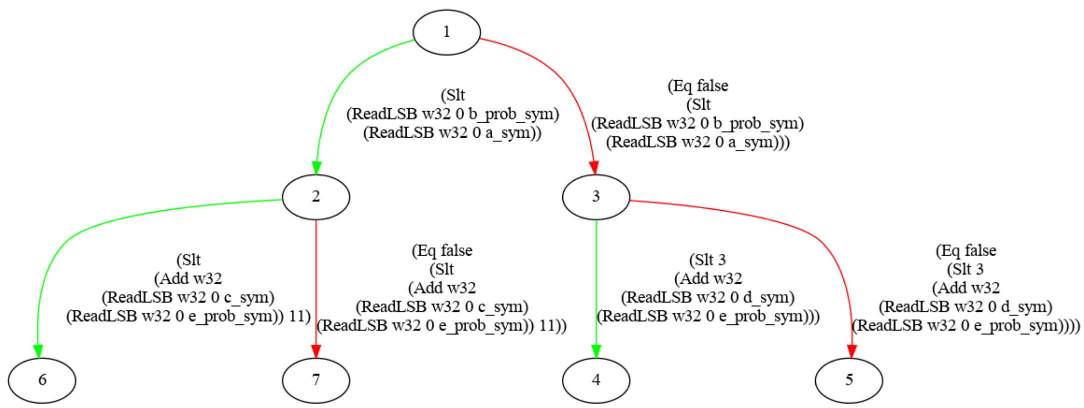
Based on the query posed, we get `candidate` models by converting the code into a `optimization` query and solve this optimization problem using any off-the-shelf `SMT Solver` to get model values for the forall variables that contribute to maximum violation of the `query` constraint.

Based on the example we do the following `optimizations`. one on `Path 1` and the other on `Path 3` where `win == 1`

```
Path 1 : maximize((a - b) + 11 - (c + e))

Path 3 : maximize((b - a) + (d + e) - 1)
```

On the other two paths, we `minimize` the same. `win == 1` on the `green` edged paths.




```
Path 1 : [And(a_sym >= 0, a_sym <= 1), And(c_sym >= 1, c_sym <= 10),
 And(d_sym >= 0, d_sym <= 5), And(b_prob_sym >= 0, b_prob_sym <= 1),
 And(e_prob_sym >= 1, e_prob_sym <= 6), win_sym == 1, b_prob_sym <
 a_sym, c_sym + e_prob_sym < 15]
        Model : 1
                a_sym = 1
                c_sym = 1
                d_sym = 0
        Model : 2
                a_sym = 1
                c_sym = 2
                d_sym = 0
        Model : 3
                a_sym = 1
                c_sym = 3
```

```
                d_sym = 0
Path 2 : [And(a_sym >= 0, a_sym <= 1), And(c_sym >= 1, c_sym <= 10),
And(d_sym >= 0, d_sym <= 5), And(b_prob_sym >= 0, b_prob_sym <= 1),
And(e_prob_sym >= 1, e_prob_sym <= 6), b_prob_sym < a_sym, Not(c_sym
+ e_prob_sym < 15)]
        Model : 1
                a_sym = 1
                c_sym = 9
                d_sym = 0
        Model : 2
                a_sym = 1
                c_sym = 10
                d_sym = 0
Path 3 : [And(a_sym >= 0, a_sym <= 1), And(c_sym >= 1, c_sym <= 10),
And(d_sym >= 0, d_sym <= 5), And(b_prob_sym >= 0, b_prob_sym <= 1),
And(e_prob_sym >= 1, e_prob_sym <= 6), win_sym == 1, Not(b_prob_sym
< a_sym), d_sym + e_prob_sym > 1]
        Model : 1
                a_sym = 0
                c_sym = 1
                d_sym = 0
        Model : 2
                a_sym = 0
                c_sym = 2
                d_sym = 0
        Model : 3
                a_sym = 0
                c_sym = 3
                d_sym = 1

                ...
```

Here `P(win == 1)` is the query of interest to us so we optimize along those paths where this condition holds.

We now do a `transformation` pass over the program and instrument the count of the given `condition` failing. In the `transforamtion` pass, we make the `program` take values that we find as `candidate` models upon following `Algorithm 1`

```
// Take in candidate vector for [foralls]
scanf("%d", &a); // [0, 1]
scanf("%d", &c); // [1, 10]
scanf("%d", &d); // [0, 5]

// // PSE variable : Random Sampling
```

```
std::default_random_engine generator;
std::uniform_int_distribution<int> distribution1(0, 1); // b
std::uniform_int_distribution<int> distribution2(1, 6); // e

while (term_count--) // term_count > 500000 (large sample)
{
        b = distribution1(generator);
        e = distribution2(generator);

        if (a > b) { //
                if (c + e < 15) {
                        win = 1;
                        win_ones++;
                } else {
                        ...
                }
        } else {
                if (d + e > 1) {
                        win = 1;
                        win_ones++;
                } else {
                        ...
                }
        }
        run++;
} // P(win==1) is win_ones/run;
```

We tweaked the values in such a way that for a very small number of candidate vectors the (probabilistic) assert fails and our algorithm must now catch that. Indeed we find that for the following assignments to `forall` variables, the assert fails.

```
        Fail : P(win == 1)  :  0.754100
               Vals -> a : 1, c : 10, d : 0
```

For other `candidate` vectors, we find that the `probabilistic` asserts actually hold and will not cause a violation during execution under normal condiditons.

```
        Pass : P(win == 1)  :  1.000000
               Vals -> a : 0, c : 7, d : 4
        Pass : P(win == 1)  :  1.000000
               Vals -> a : 1, c : 4, d : 4
        Pass : P(win == 1)  :  1.000000
               Vals -> a : 1, c : 1, d : 3
        Pass : P(win == 1)  :  1.000000
               Vals -> a : 0, c : 6, d : 4
```

```
Pass : P(win == 1)  :  0.834900
        Vals -> a : 0, c : 7, d : 0
Pass : P(win == 1)  :  0.834900
        Vals -> a : 0, c : 2, d : 0
Pass : P(win == 1)  :  1.000000
        Vals -> a : 0, c : 10, d : 2
Pass : P(win == 1)  :  0.917800
        Vals -> a : 1, c : 3, d : 0
                ...
```