

We consider a independent *bernoulli* trials here of flipping a fair coin " n " times.

$$choice_prob = \begin{cases} p & \text{if } d \text{ value is 1 corresponding to getting a "heads"} \\ 1 - p & \text{if } d \text{ value is 0 corresponding to getting a "tails"} \end{cases}$$

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probability_0 >= 0.0000001,
probability_0 <= 0.999999999999999,

d_0_0 >= 0,                d_0_0 <= 1,
choice_0_0 ==
    If(d_0_0 == 1, probability_0, 1 - probability_0),

d_1_0 >= 0,                d_1_0 <= 1,
choice_1_0 ==
    If(d_1_0 == 1, probability_0, 1 - probability_0),

d_2_0 >= 0,                d_2_0 <= 1,
choice_2_0 ==
    If(d_2_0 == 1, probability_0, 1 - probability_0),

d_3_0 >= 0,                d_3_0 <= 1,
choice_3_0 ==
    If(d_3_0 == 1, probability_0, 1 - probability_0),

d_4_0 >= 0,                d_4_0 <= 1,
choice_4_0 ==
    If(d_4_0 == 1, probability_0, 1 - probability_0),

d_5_0 >= 0,                d_5_0 <= 1,
choice_5_0 ==
    If(d_5_0 == 1, probability_0, 1 - probability_0),

d_6_0 >= 0,                d_6_0 <= 1,
choice_6_0 ==
    If(d_6_0 == 1, probability_0, 1 - probability_0),

d_7_0 >= 0,                d_7_0 <= 1,
choice_7_0 ==
    If(d_7_0 == 1, probability_0, 1 - probability_0),

d_8_0 >= 0,                d_8_0 <= 1,
choice_8_0 ==
    If(d_8_0 == 1, probability_0, 1 - probability_0),

d_9_0 >= 0,                d_9_0 <= 1,
choice_9_0 ==
    If(d_9_0 == 1, probability_0, 1 - probability_0),

```

Fig. 1. Constraints over choice variables.

Path weight is the product of the choice probability values.

$$w_i = (p)^{x_i} * (1 - p)^{10-x_i} \quad (1)$$

```
path_prob_0 == 1 *choice_0_0 * choice_1_0*
choice_2_0 * choice_3_0 * choice_4_0 * choice_5_0 *
choice_6_0 * choice_7_0 * choice_8_0 * choice_9_0
```

Fig. 2. Expression for path_prob variable.

For approximation of the expected value of heads in n trials,

$$EV(heads) = \left(\sum_{i=1}^k w_i * sum_i \right) \quad (2)$$

```
ToReal(sum_d_heads_0) ==
ToReal(0 + d_0_0 + d_1_0 + d_2_0 + d_3_0 +
d_4_0 + d_5_0 + d_6_0 + d_7_0 + d_8_0 + d_9_0) * 1 *
(choice_0_0 * choice_1_0 * choice_2_0 * choice_3_0 *
choice_4_0 * choice_5_0 * choice_6_0 * choice_7_0 *
choice_8_0 * choice_9_0)
```

Fig. 3. Expression for expected heads value.

We want to maximize the value of path prob here.

$$maximize\left(\sum_{i=1}^k (p)^{x_i} * (1 - p)^{n-x_i}\right) \quad (3)$$

```
d_9_0 = 0,
d_3_0 = 0,
d_7_0 = 0,
d_5_0 = 0,
d_1_0 = 0,
d_8_0 = 0,
d_2_0 = 0,
sum_d_heads_0 = 0,
d_0_0 = 0,
d_6_0 = 0,
d_4_0 = 0,
```

Fig. 4. One possible model given by z3 after computing for 2 hours

```
path_prob_0 = 0.951110130465771,  
choice_9_0 = 0.995,  
choice_8_0 = 0.995,  
choice_7_0 = 0.995,  
choice_6_0 = 0.995,  
choice_5_0 = 0.995,  
choice_4_0 = 0.995,  
choice_3_0 = 0.995,  
choice_2_0 = 0.995,  
choice_1_0 = 0.995,  
choice_0_0 = 0.995,  
probability_0 = 0.005
```

Fig. 5. One possible model given by z3 after computing for 2 hours