# INTERFERENCE

**Q1. Obtain the condition for maxima and minima of the light reflected from a thin, transparent film of uniform thickness. Why is the visibility of the fringe much higher in the reflected system than in the transmitted system.**

(M.U. May 2015, 2017, 2018; Dec 2013, 2014, 2016, 2017)[8 Marks]

Consider a plane parallel thin film of thickness 't' and refractive index '$\mu$'. Let a ray of light 'AB' be incident obliquely on the film making an incident angle 'i' with the normal 'N'. The incident ray 'AB' gets reflected as well as refracted at point 'B' into $R_1$ and a refracted ray 'BC' respectively. Angle of reflection is equal to that of incidence and hence in the diagram 'r' is denoting the angle of refraction. At point 'C' the light ray 'BC' gets reflected into ray 'CD' with an angle 'r' as the angle of reflection, and gets refracted into "transmitted ray" $T_1$ . Ray 'CD' is travelling through the film and at point 'D' it gets reflected and refracted into ray 'DE' and $R_2$ respectively. Ray 'DE' then gets refracted as "transmitted ray" $T_2$ as shown in *Figure 1*.

**Construction:** Draw a perpendicular from point 'C' which meets 'BD' at point 'G' and draw another perpendicular from point D which meets Ray 1 origin at 'H', to obtain optical path difference (OPD) between Ray 1 and Ray 2.

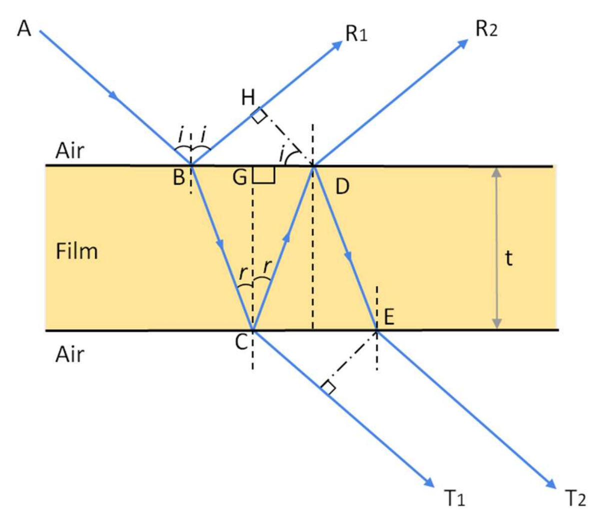


*Figure 1: Interference in Thin Films*

Let the path difference between the paths travelled by Ray 1 and Ray2 be represented by $\Delta physical$ which can be calculated from the diagram as follows:

$$\Delta physical = (BC + CD) - (BH)$$

This path is travelled by light, light has different velocities in different mediums depending on the density of the medium. This fact aspect gets incorporated in the physical path difference when we multiply the path by the refractive index of the medium in which it is travelled. Hence, BC and CD get multiplied by $\mu$ as they are inside the film while BH gets multiplied by 1 as it is outside and the medium is air with refractive index 1 to get resultant path difference for light also called as optical path difference (OPD).

$$\Delta_{OPD} = \Delta = (BC + CD) * \mu - (BH * 1) - - - - - (1)$$

From geometry we can see that $\Delta BGC$ is congruent to $\Delta DGC$ is by ASA test

Therefore, we can write BC=CD and hence BC+CD=2BC -----(2)

putting (2) in (1)

$$\Delta = 2\mu BC - BH \text{ -----(3)}$$

In $\Delta$ , BC in terms of cos r can be replaced as

$$\Delta = \frac{2\mu t}{cosr} - BH \text{ ----- (4)}$$

In $\Delta$ , angle D is complement of angle B (which is 90-i) and hence equal to 'i', hence BH in (4) in terms of sin i can be replaced as

$$\Delta = \frac{2\mu t}{cosr} - BD(sin\ i) \text{ ----- (5)}$$

$\Delta BGC$is congruent to $\Delta DGC$ also gives us BG=GD;

therefore

BD = 2BG ----- (6)

Putting (6) in (5) we get;

$$\Delta = \frac{2\mu t}{cosr} - 2BG(sin\ i)$$

In Δ , BG in terms of tan r can be replaced as

$$\Delta = \frac{2\mu t}{cosr} - 2t(tanr)(sini) \text{ ----- } (7)$$

Snell's law for the air and film boundary is

$$\mu = \frac{sini}{sinr} \text{ ----- } (8)$$

Putting (8) in (7) for sin I we get;

$$\Delta = \frac{2\mu t}{cosr} - 2t(tanr)\mu(sinr)$$

$$\Delta = \frac{2\mu}{cosr}(1 - sin^2 r)$$

$$\Delta = 2\mu t(cos\ r)$$

Hence, the expression for optical path difference is as follows:

$$\Delta_{OPD} = 2\mu t(cos\ r) \text{ ----- } (9)$$

While travelling from a rarer medium whenever there is a reflection at the boundary of a denser medium, additional phase difference of $\pi$ is obtained which results in extra path difference of $\frac{\lambda}{2}$.

**Reflected system:**

At point B in the Figure 1 we get $\frac{\lambda}{2}$ for BH because reflection is happening at the boundary of the film which is the denser medium in this case hence the net path difference is ;

$$\Delta = 2\mu t(cosr) - \frac{\lambda}{2} \text{ ------ } (10)$$

1. **For constructive interference**: Path difference is integral multiple of wavelength; which is mathematically

   $\Delta$=n $\lambda$ -----(11)

   Putting (10) in (11)

$$2\mu t(cosr) - \frac{\lambda}{2} = n\lambda$$

$$2\mu t(cosr) = (2n - 1)\frac{\lambda}{2}$$

2. **For destructive interference**: Path difference is odd integral multiple of wavelength which is mathematically;

   $\Delta = (2n - 1)\frac{\lambda}{2}$-----(12)

   Putting (10) in (12)

   $2\mu t(cosr) - \frac{\lambda}{2} = 2n\frac{\lambda}{2} - \frac{\lambda}{2}$

   $2\mu t(cosr) = n\,\lambda$

**Transmitted system:**

The reverse of reflected system happens in a transmitted light system:

1. **For constructive interference**:

   $2\mu t(cosr) = n\,\lambda$

2. **For destructive interference**:

   $$2\mu t(cosr) = (2n - 1)\frac{\lambda}{2}$$

The visibility of fringes in a reflected light system is much higher because there is a higher contrast in the intensities of the maxima and minima, while the transmitted light system has poor contrast between these.

**Q2. Write the condition for maxima and minima due to interference in a wedge-shaped film observed in reflected light.**

When two surfaces of a film are not parallel and making a very small angle '$\theta$' between them such that thickness 't' goes on increasing as we move away from the point of contact of two surfaces of the film, then the film is said to be a wedge-shaped thin film. The conditions for interference in a wedge-shaped thin film are obtained just like for a plane-parallel thin film. The only difference is the angle 'r' is larger for this case. In *Figure 2* we can see that 'r' for the wedge-shaped thin film is the exterior angle for triangle BCE hence new r= r+ $\theta$. So by substituting 'r' in the conditions for the plane parallel thin film we obtain the following conditions for **minima and maxima in reflected system:**

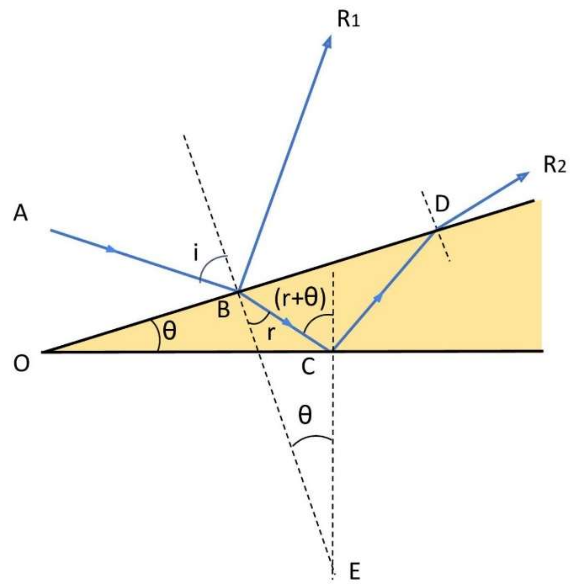


*Figure 2: Interference in Wedge-shaped Film*

1. **For constructive interference (MAXIMA):**

   $2\mu t cos(r+\theta) = (2n-1)\frac{\lambda}{2}$ ----- (1)

2. **For destructive interference (MINIMA):**

   $2\mu t cos(r+\theta) = n\lambda$ -----(2) Where, n=1,2,3,4……….

Q3. Derive the expression for fringe width.

**Fringe width:** In an interference pattern the distance between two consecutive dark or bright bands is called as Fringe width. **The symbol used to denote fringe width for the amplitude division type of interference is 'β'.** The easiest way to derive an expression for fringe width is in wedge-shaped thin film. So, consider a wedge-shaped thin film with '$\theta$' as the angle between its two surfaces and thickness 't' goes on increasing as we move away from the point of contact of two surfaces of the film.

**Conditions for interference in a wedge-shaped thin film:**

1. **For constructive interference (MAXIMA):**

   $$2\mu t cos(r+\theta) = (2n-1)\frac{\lambda}{2} \text{-----}(1)$$

2. **For destructive interference (MINIMA):**

   $$2\mu t cos(r+\theta) = n\lambda \text{-----}(2)$$ Where, n=1,2,3,4……….

Condition for minima is simpler so we shall find the distance between two consecutive dark bands in order to find the expression for fringe width.

**Destructive interference condition for the nth dark band:**

$$2\mu t_1 cos(r+\theta) = n\lambda \text{-----}(3)$$

**Destructive interference condition for the (n+1)th dark band:**

$$2\mu t_2 cos(r+\theta) = (n+1)\lambda \text{-----}(4)$$

where $t_1$ and $t_2$ are the thickness of the wedge-shaped film when n[th] and (n+1)[th] dark bands occur as shown in the *Figure 3*.

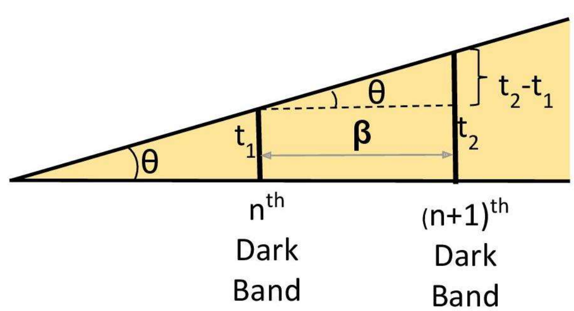

*Figure 3: Fringe Width*

In case of normal incidence, the angle of refraction 'r' tends to zero and as ‘ϴ’ is very small (r+ϴ) tends to zero and hence cos(r+ϴ)=1.

Substituting this in (3) and (4) we get;

$$2\mu t_1 = n\lambda \text{ -----(5)}$$

$$2\mu t_2 = (n+1)\lambda \text{ -----(6)}$$

Subtracting (6)-(5) we get;

$$2\mu(t_2 - t_1) = \lambda \text{ -----(7)}$$

From *Figure 3* for fringe width(β) we can write;

$$tan\theta = \frac{t_2 - t_1}{\beta} \text{ -----(8)}$$

Substituting $(t_2 - t_1)$ from (7) in (8) we get ;

$$tan\theta = \frac{\lambda}{2\mu}$$

Rearranging terms to find expression for fringe width ‘$\beta$’

$$\beta = \frac{\lambda}{2\mu tan}$$

Here, ϴ is very small and hence ϴ~tanϴ. So we can approximate ;

$$\beta = \frac{\lambda}{2\mu\theta}$$

This is the expression for fringe width.

**Q4. Give any one application of a wedge-shaped thin film. Or Explain the method to determine the diameter of a very thin wire.**

Consider a wire of diameter ‘d’, where ‘d’ is to be determined. The wire is kept between two optically flat surfaces which are in contact with each other at another end as shown in *Figure 4*. Let ‘L’ be the distance of the wire from the

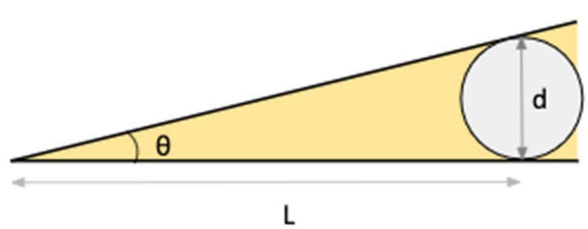

*Figure 4: Use of wedge-shaped thin film to find ‘d’*

point of contact of two optically flat surfaces and 'Θ' be the angle between the two flat surfaces.

From the *Figure 4* we can say;

$$tan\theta = \frac{d}{L} \text{-----}(1)$$

As 'Θ' is very small for a wedge-shaped thin film tanΘ~Θ, hence

$$\theta = \frac{d}{L} \text{-----}(2)$$

From the expression of Fringe width for wedge-shaped thin film we have;

$$\beta = \frac{\lambda}{2\mu\theta} \text{-----}(3)$$

Substituting (2) in (3) we have

$$\beta = \frac{\lambda L}{2\mu d}$$

Rearranging the above equation for 'd'

$$d = \frac{\lambda L}{2\mu\beta} \text{-----}(4)$$

Where, $\lambda$ is the wavelength of incident light, $\mu$ is the refractive index of film, $\beta$ is the fringe width.

Q5. Explain a method to test optical flatness of a surface.

(M.U. May 2018)[3 Marks]

Consider a wedge-shaped thin film made of two surfaces shown above (OM and ON) as shown in the *Figure 5*. Let one of them be optically flat and other one which is to tested for flatness. The test is based on the phenomenon of interference. If the surface put to test is also optically flat the formed wedge-

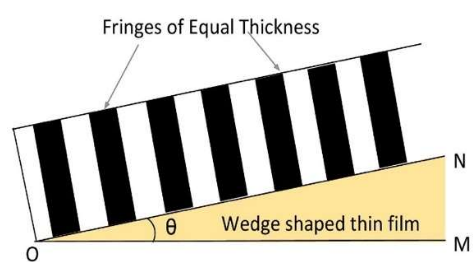

*Figure 5: Testing of optical flatness*

shaped film will give interference pattern which varies consistently in thickness from O to M as shown in *Figure 5*. Hence, if the fringes have equal thickness, then it can be deduced that the new surface is indeed optically flat.

**Q6. Describe the experimental set up for obtaining Newton's Rings.**

**Experimental Setup of Newton's Rings:**

Newton's Rings (NR) setup consist of the plano-convex lens placed on a glass plate. Once the setup is illuminated alternate bright and dark rings are formed due to amplitude division type of interference in the thin film that is created between the plano-convex lens and glass plate. The alternate bright and dark rings can be viewed by a microscope. In order to avoid the disturbance between the light source (sodium lamp) and microscope they are kept perpendicular to each other and a glass plate 'P' is placed at $45^0$ such that light reaches the set up normally as shown in the *Figure 6a*. Once the set-up is arranged as shown in the *Figure 6a*, alternate bright and dark rings as shown in *Figure 6b* can be seen in the microscope.

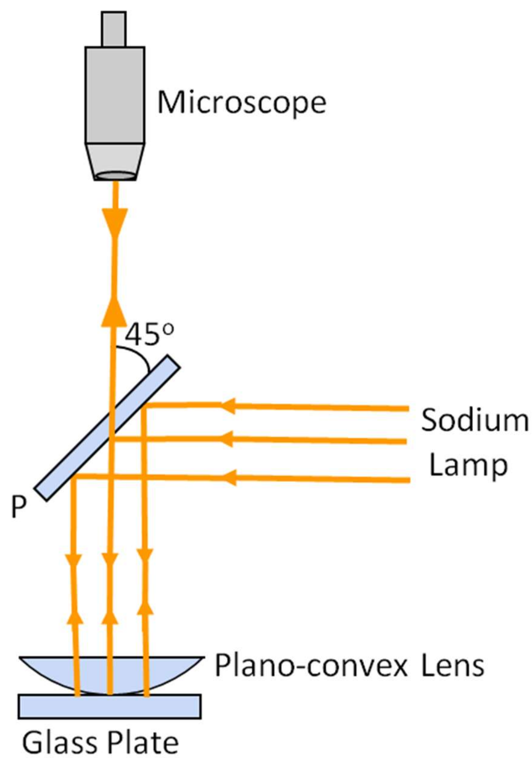


*Figure 6a: Experimental arrangement for NR*

*Figure 6b: Newton's rings*

**Q7. Show that in a Newton's Rings setup radius of nth dark ring is proportional to the square root of the ring number.**

(M.U. May 2015; Dec 2018)[5 Marks]

Consider a plano-convex lens placed on a glass plate as shown in the *Figure 7*.

Let arc ABC be a part of a circle which has its centre at point O and a radius 'R' called as the radius of curvature. In such a Newton's Ring (NR) set up let us consider $n^{th}$ dark ring with radius '$r_n$' formed at a place where the thickness of the yellow film between the plano-convex lens and glass plate is 't'.

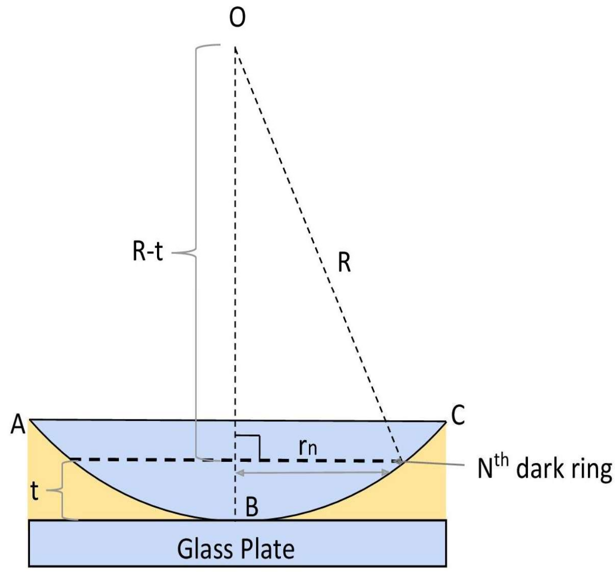

*Figure 7: Newton's Rings set-up*

In the Figure 7 by Pythagoras theorem in the right angled triangle we have,

$$R^2 = (R-t)^2 + r_n^2$$

Opening the square we get,

$$R^2 = R^2 + t^2 - 2Rt + r_n^2 \text{ -----(1)}$$

't' (of the order of $10^{-7}$) is very small, so, $t^2$ even smaller, hence neglected,

$$2Rt = r_n^2$$

$$2t = \frac{r_n^2}{R} \text{ -----(2)}$$

**The interference condition for dark band in reflected system is ,**

$$\mathbf{2\mu t(\cos r) = n\,\lambda}$$

For normal incidence r ~0 and hence cosr~1

$$2\mu t = n\lambda \text{-----(3)}$$

Comparing (2) and (3) we get,

$$\frac{r_n^2}{R} = \frac{n\,\lambda}{\mu}$$

Rearranging terms gives;

$$r_n^2 = \frac{n\,R\,\lambda}{\mu}$$

If the lens, light source and film medium remain same, 'R', 'λ' and '$\mu$' remain constant and in that case we can say ;

$$\boldsymbol{r_n\ \alpha\ \sqrt{n}}\quad \text{where, n =0,1,2,3....}$$

Extra :<u>For a Bright ring:</u>

**The interference condition for bright band in reflected system is ,**

$$\boldsymbol{2\mu t(cosr) = (2n-1)\frac{\lambda}{2}} \text{-----(4)}$$

Comparing (4) and (2) for normal incidence we have

$$\frac{r_n^2}{R} = \frac{(2n-1)\,\lambda}{2\mu}$$

Rearranging terms gives;

$$r_n^2 = \frac{(2n-1)\,R\,\lambda}{2\mu}$$

If the lens, light source and film medium remain same, 'R', 'λ' and '$\mu$' remain constant and in that case we can say ;

$$\boldsymbol{r_n\ \alpha\ \sqrt{(2n-1)}}\quad \text{where, n =1,2,3....}$$

Q8. Explain how Newton's Ring setup is used determine wavelength of light.

(M.U. May 2013, 2017; Dec 2014, 2018)[5-8 Marks]

The light for which wavelength is to be determined is used as a source in the experimental set-up shown in *Figure 8*. Once the rings are in the view from microscope one has to measure the diameter of every ring. Then plot a graph of diameter square Vs the ring number as shown in *Figure 8*. The points obtained on the graph connect into a line of which we need to find the slope.

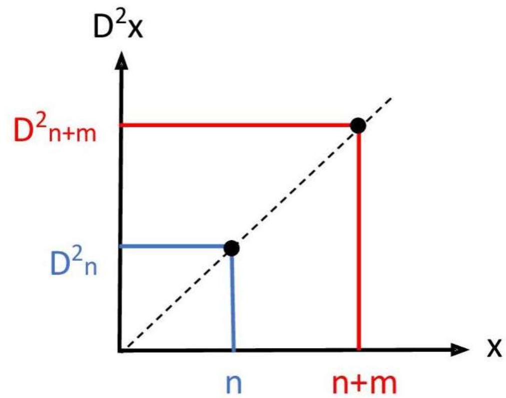


*Figure 8: Diameter square Vs Ring number*

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{D_{n+m}^2 - D_n^2}{n+m-n} = \frac{D_{n+m}^2 - D_n^2}{m} \text{ ----- (1)}$$

We know that radius of the dark ring is related to square root of the ring number as

$$r_n^2 = \frac{n\,R\,\lambda}{\mu}$$

For diameter (D=2r) of the ring this changes to ;

$$D_n^2 = \frac{4\,n\,R\,\lambda}{\mu} \text{ ----- (2)}$$

Similarly

$$D_{n+m}^2 = \frac{4\,(n+m)\,R\,\lambda}{\mu} \text{ ----- (3)}$$

Using (2) and (3) to obtain expression for slope

$$slope = \frac{D_{n+m}^2 - D_n^2}{m} = \frac{4\;R\,\lambda}{\mu} \text{ ----- (4)}$$

For air film $\mu = 1$ and rearranging the above equation for wavelength

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4\,m\,R} = \frac{slope}{4\,R}$$

So wavelength can be determined by dividing slope of the graph by 4 times radius of curvature of the lens used for obtaining Newton's Rings.

## Q9. Explain a method to determine the refractive index of a liquid

**(M.U. May 2014, 2016, 2017)[4 Marks]**

The Newton's Ring experiment is performed with an air film (yellow) once and then is performed again with a liquid film (red) for which refractive index is to be determined. Set up is as shown in the *Figure 9*.

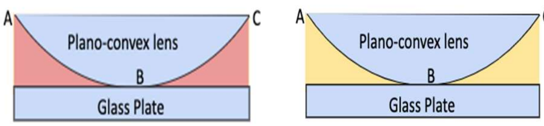


*Figure 9: Newton's Ring set-up with air(yellow) and liquid (red) film*

In both the cases rings are viewed from microscope and a graph of diameter square Vs the ring number is plotted as shown in *Figure 9*. The points obtained on the graph are connected to form a line. Slope of this line is from (4) in the question above is written as:

$$slope_{air} = \frac{4\ R\ \lambda}{\mu_{air}} = [\frac{D_{n+m}^2 - D_n^2}{m}]_{air} \text{-----}(1)$$

$$slope_{liq} = \frac{4\ R\ \lambda}{\mu_{liq}} = [\frac{D_{n+m}^2 - D_n^2}{m}]_{liq} \text{-----}(2)$$

Dividing (1) and (2) we get

$$\frac{\mu_{liq}}{\mu_{air}} = \frac{slope_{air}}{slope_{liq}} = \frac{D_{n+m}^2 - D_n^2{}_{air}}{D_{n+m}^2 - D_n^2{}_{liq}}$$

As the refractive index of air is one we can write expression for refractive index as follows:

$$\mu_{liq} = \frac{D_{n+m}^2 - D_n^2{}_{air}}{D_{n+m}^2 - D_n^2{}_{liq}}$$

From the equation we can say that a Newton's ring shrinks with increase in refractive index of liquid. This is how Newton's rings can be used to find out the unknown refractive index of a given liquid.

Q10. Why are Newton's rings circular and the fringes in wedge shaped films straight?

(M.U. May 2014, 2017; Dec 2016, 2017)[3 Marks]

The "fringes" in both the Newton's rings experiment and wedge-shaped film represent the locus of points of equal thickness of the film.

In case of Newton's rings, the points of equal thickness are found on a circular path, in concentric circles, whose centre is the point of contact between the glass plate and the Plano convex lens. Hence, these fringes are circular and concentric.

In case of wedge-shaped films, points of equal thickness are found as straight, lines parallel to the contact edge of the wedge.

Q11. Why is the centre of interference pattern of Newton's rings dark?

(M.U. May 2014, 2017; Dec 2016, 2017)[3 Marks]

Conditions for interference in Newton's rings:

Bright rings: $2\mu t cos(r + \theta) = (2n + 1)\frac{\lambda}{2}$

Dark rings: $2\mu t cos(r + \theta) = n\lambda$

*Figure 11a: Newton's rings*

The thickness of air film at the point of contact of glass plate and convex lens (at point B in *Figure 11b*) in Newton's rings is '0'. Also, for a lens of large radius of curvature the wedge angle (θ) between the plano-convex lens and glass plate is very small (θ~0), and for normal incidence i=0 that implies r = 0. Thus $cos(r + \theta) = cos0 = 1$. For an air film $\mu = 1$.

Conditions for interference in Newton's rings for all possible values of 'n' reduce to:

Bright rings: $2t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ .... so on

Dark rings: $2t = 0, \lambda, 2\lambda, 3\lambda$,.....so on

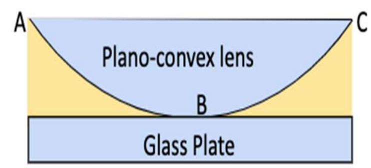


*Figure 11b: Newton's rings*

At the centre (point of contact B), t = 0. From the above equations, it is evident that there is no value of 'n' for bright rings which can give t=0, while for the dark rings we can get t=0 at n=0. Hence the centre of Newton's rings is dark as shown in the *Figure 11a*.

**Q12. Write a detailed description of an anti-reflecting film. Mention the conditions for refractive index and thickness of the film in order to act as anti-reflection coating.**

(M.U. May 2012; Dec 2013, 2017, 2018)[3 Marks]

Antireflection film is a transparent coating that has the capability of curtailing reflection from a surface by covering it. For a film to work as an antireflection coating it should satisfy following three conditions

1. $\mu_a < \mu_f < \mu_g$
2. $\mu_f = \sqrt{\mu_g}$
3. $t_{min} = \frac{\lambda}{4\mu_f}$

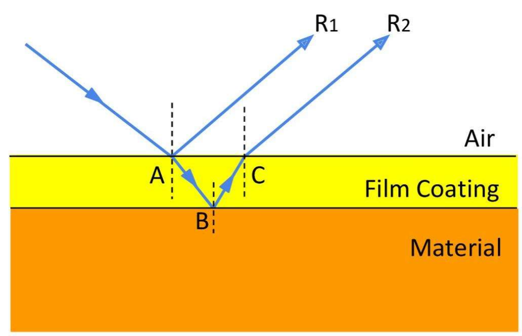


*Figure 12: Antireflection film coating*

**Explanation:**

1. The refractive index of the film ($\mu_f = yellow$) should be greater than that of air ($\mu_a = White$) but less than that of the material ($\mu_g = orange$) on which it is supposed to be coated (usually glass) as shown in *Figure 12*.

2. The refractive index of the film ($\mu_f = yellow$) should equal to the square root of the refractive index of the material ($\mu_g = orange$) on which it is supposed to be coated.
3. The minimum thickness of the coating should be $t_{min} = \frac{\lambda}{4\mu_f}$ where '$\lambda$' is the mean wavelength of the incident light of which we wish to curtail reflection.

**Derivation for refractive index of the film:**

For a (yellow in Figure 12)film to act an antireflection coating the Ray 1 (R1) and Ray 2 (R2) should undergo destructive interference such that none of them is able to leave the film ensuring **NO REFLECTION**. If we want the destructive interference to be perfect we need that the amplitudes of both the rays should be equal.

$$Amplitude\ of\ Ray\ 1 = Amplitude\ of\ Ray\ 2$$

Amplitude of a ray is given by following relations between refractive index of mediums from which it is reflected.

$$\left(\frac{\mu_f + \mu_a}{\mu_f - \mu_a}\right)^2 = \left(\frac{\mu_g + \mu_f}{\mu_g - \mu_f}\right)^2$$

Taking square root on both side,

$$\frac{\mu_f + \mu_a}{\mu_f - \mu_a} = \frac{\mu_g + \mu_f}{\mu_g - \mu_f}$$

$$(\mu_f + \mu_a)(\mu_g - \mu_f) = (\mu_g + \mu_f)(\mu_f - \mu_a)$$

Taking air as the outer medium, $\mu_a = 1$,

$$(\mu_f + 1)(\mu_g - \mu_f) = (\mu_g + \mu_f)(\mu_f - 1)$$

$$\mu_f(\mu_g - \mu_f) + \mu_g - \mu_f = \mu_f(\mu_g + \mu_f) - \mu_g - \mu_f$$

$$\mu_f\mu_g - \mu_f{}^2 + \mu_g - \mu_f = \mu_f\mu_g + \mu_f{}^2 - \mu_g - \mu_f$$

$$2\mu_g = 2\mu_f{}^2$$

$$\mu_f = \sqrt{\mu_g}$$

Hence, film material is to be chosen such that its refractive index is equal to the square root of the refractive index of the material on which it is supposed to be coated.

Derivation for **Thickness of film:**

For a set of mediums such that $\mu_a < \mu_f < \mu_g$ the condition for destructive interference is given as

$2\mu_f\, t\, \cos r = (2n+1)\frac{\lambda}{2}$ where n=0,1,2,3....

To find minimum thickness of the film we use n=0 and normal incidence such that i=0 hence r=0 so cos r=1. Above equation will reduce to

$t_{min} = \frac{\lambda}{4\mu_f}$

It means that optical thickness of anti-reflecting coating should be one fourth of wavelength.